دانشگاه سمنان
Semnan University
Faculty of Mechanical Engineering
دانشکده مهندسی مکانیک

دانشکده مهندسی مکانیک

درس مکاترونیک ۱

MECHATRONICS 1

Section 3:

Dynamics Review

Reference:

Engineering Mechanics
Dynamics
9th Edition
Meriam, Kraige & Bolton

Chapters 1 - 6

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CONTENTS:

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1/1 History and Modern Applications

History of Dynamics

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Galileo Galilei

Portrait of Galileo Galilei (1564–1642) (oil on canvas), Sustermans, Justus (1597–1681) (school of)/ Galleria Palatina, Florence, Italy/Bridgeman Art Library.

Applications of Dynamics

WENN Ltd/Alamy Stock Photo

Artificial hand

1/2 Basic Concepts

- **Space**
- ***Time***
- ***Mass***
- ***Force***
- A ***Particle***
- A ***Rigid Body***
- ***Vector*** and ***Scalar*** quantities

1/3 Newton's Laws

- ***Law I.*** A particle remains at rest or continues to move with uniform velocity (in a straight line with a constant speed) if there is no unbalanced force acting on it.

- ***Law II.*** The acceleration of a particle is proportional to the resultant force acting on it and is in the direction of this force.

$$\mathbf{F} = m\mathbf{a}$$

- ***Law III.*** The forces of action and reaction between interacting bodies are equal in magnitude, opposite in direction, and collinear.

1/4 Units

- International System of metric units (SI)
- U.S. Customary system of units

Quantity	Dimensional Symbol	SI Units: Unit	SI Units: Symbol	U.S. Customary Units: Unit	U.S. Customary Units: Symbol
Mass	M	kilogram (Base units)	kg	slug	—
Length	L	meter* (Base units)	m	foot (Base units)	ft
Time	T	second (Base units)	s	second (Base units)	sec
Force	F	newton	N	pound (Base units)	lb

*Also spelled *metre*.

Omikron/Photo Researchers, Inc.

The U.S. standard kilogram at the National Bureau of Standards.

SI Units	U.S. Customary Units
$(1\ N) = (1\ kg)(1\ m/s^2)$	$(1\ lb) = (1\ slug)(1\ ft/sec^2)$
$N = kg \cdot m/s^2$	$slug = lb \cdot sec^2/ft$

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1/5 Gravitation

- Newton's law of gravitation, which governs the mutual attraction between bodies

$$F = G\frac{m_1 m_2}{r^2}$$

$F =$ the mutual force of attraction between two particles

$G =$ a universal constant called the *constant of gravitation*

$m_1, m_2 =$ the masses of the two particles

$r =$ the distance between the centers of the particles

$G = 6.673(10^{-11})\ \text{m}^3/(\text{kg}\cdot\text{s}^2)$

$$\mathbf{W} = m\mathbf{g}$$

$9.81\ \text{m/s}^2$ in SI units

$32.2\ \text{ft/sec}^2$ in U.S. customary units

1/6 Dimensions

- A given dimension such as length can be expressed in a number of different units such as meters, millimeters, or kilometers.

- Thus, a *dimension* is different from a *unit*.

- The *principle of dimensional homogeneity* states that all physical relations must be dimensionally homogeneous

$$F = ML/T^2$$

CONTENTS:

دانشگاه سمنان

2/1 Introduction

- Kinematics is the branch of dynamics which describes the motion of bodies without reference to the forces which either cause the motion or are generated as a result of the motion.

- Kinematics is often described as the "geometry of motion."

- A thorough working knowledge of kinematics is a prerequisite to kinetics, which is the study of the relationships between motion and the corresponding forces which cause or accompany the motion.

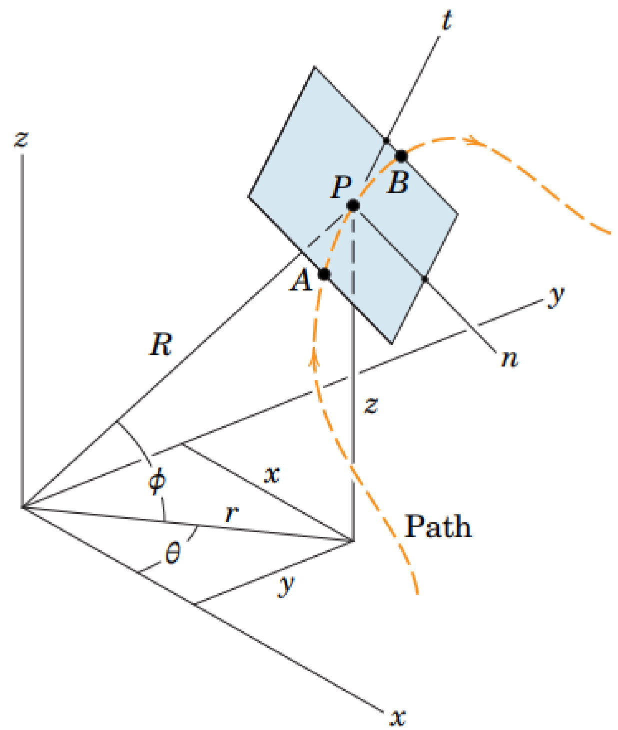
2/1 Introduction

Particle Motion

- A particle is a body whose physical dimensions are so small compared with the radius of curvature of its path that we may treat the motion of the particle as that of a point.

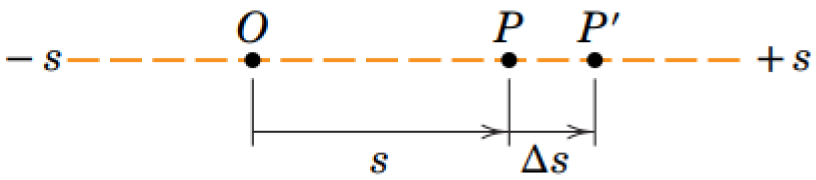
Choice of Coordinates

- Rectangular coordinates x, y, z (Cartesian)
- Cylindrical coordinates r, θ, z
- Spherical coordinates R, θ, ϕ
- Tangent t and normal n to the curve (path variables)



2/2 Rectilinear Motion

- Particle P moving along a straight line



Velocity and Acceleration

- Average velocity of the particle during the interval Δt is the displacement divided by the time interval or $v_{av} = \Delta s / \Delta t$

2/2 Rectilinear Motion

Velocity and Acceleration

- *instantaneous velocity*

$$v = \frac{ds}{dt} = \dot{s}$$

- The average acceleration of the particle during the interval Δt is the change in its velocity divided by the time interval or $a_{av} = \Delta v / \Delta t$.

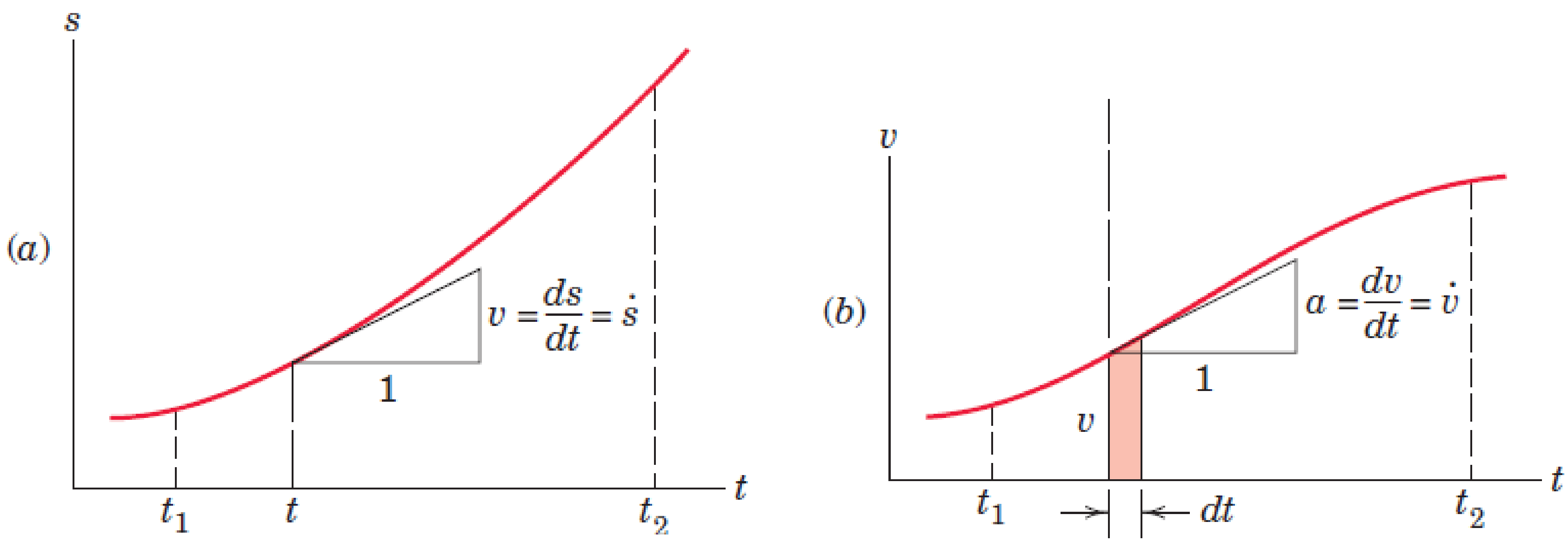
- *instantaneous acceleration*

$$a = \frac{dv}{dt} = \dot{v} \quad \text{or} \quad a = \frac{d^2s}{dt^2} = \ddot{s}$$

→ $$v\,dv = a\,ds \quad \text{or} \quad \dot{s}\,d\dot{s} = \ddot{s}\,ds$$

2/2 Rectilinear Motion

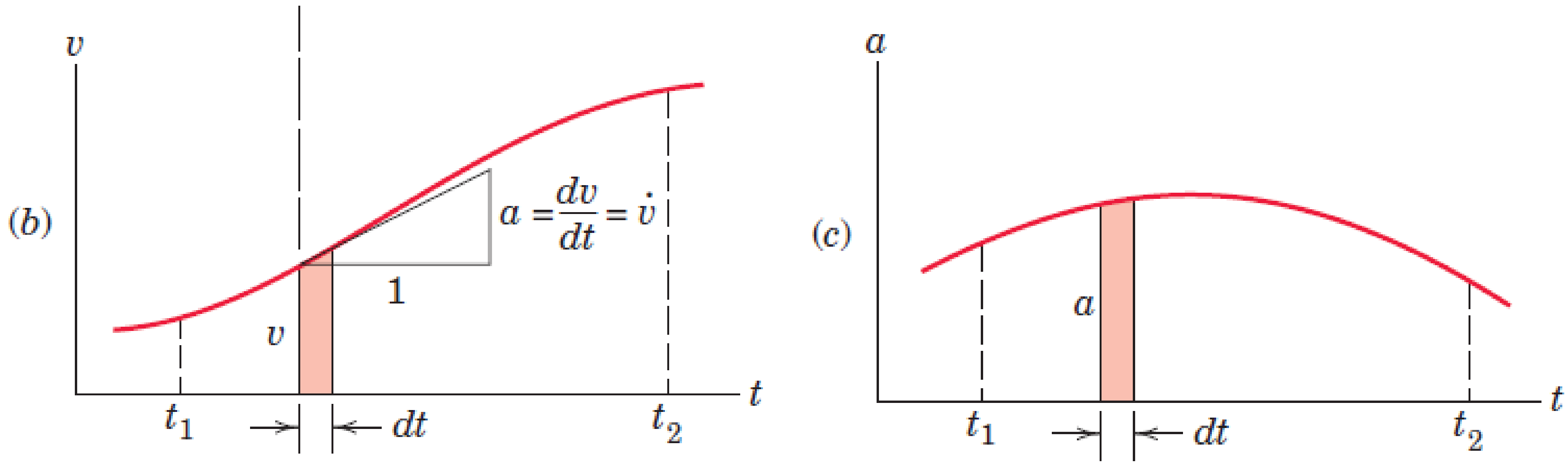
Graphical Interpretations



$$\int_{s_1}^{s_2} ds = \int_{t_1}^{t_2} v\,dt \qquad \text{or} \qquad s_2 - s_1 = (\text{area under } v\text{-}t \text{ curve})$$

2/2 Rectilinear Motion

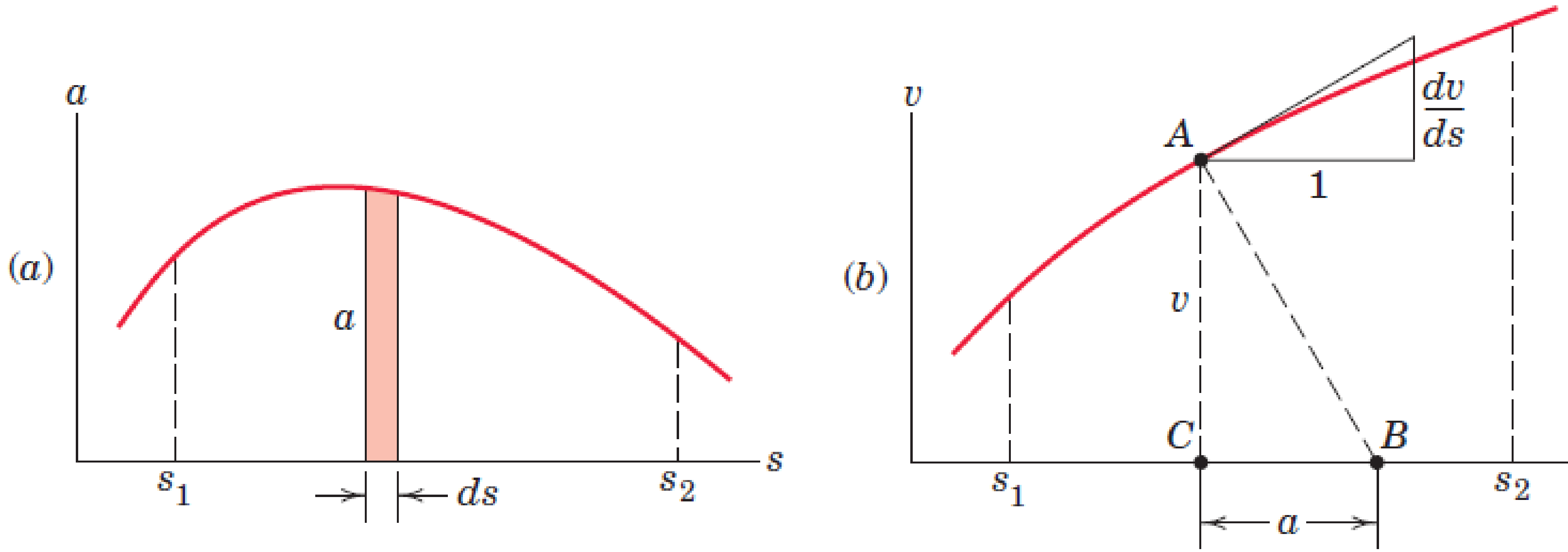
Graphical Interpretations



$$\int_{v_1}^{v_2} dv = \int_{t_1}^{t_2} a\, dt \qquad \text{or} \qquad v_2 - v_1 = (\text{area under } a\text{-}t \text{ curve})$$

2/2 Rectilinear Motion

Graphical Interpretations



$$\int_{v_1}^{v_2} v\,dv = \int_{s_1}^{s_2} a\,ds \qquad \text{or} \qquad \frac{1}{2}(v_2{}^2 - v_1{}^2) = (\text{area under } a\text{-}s \text{ curve})$$

2/2 Rectilinear Motion

Analytical Integration

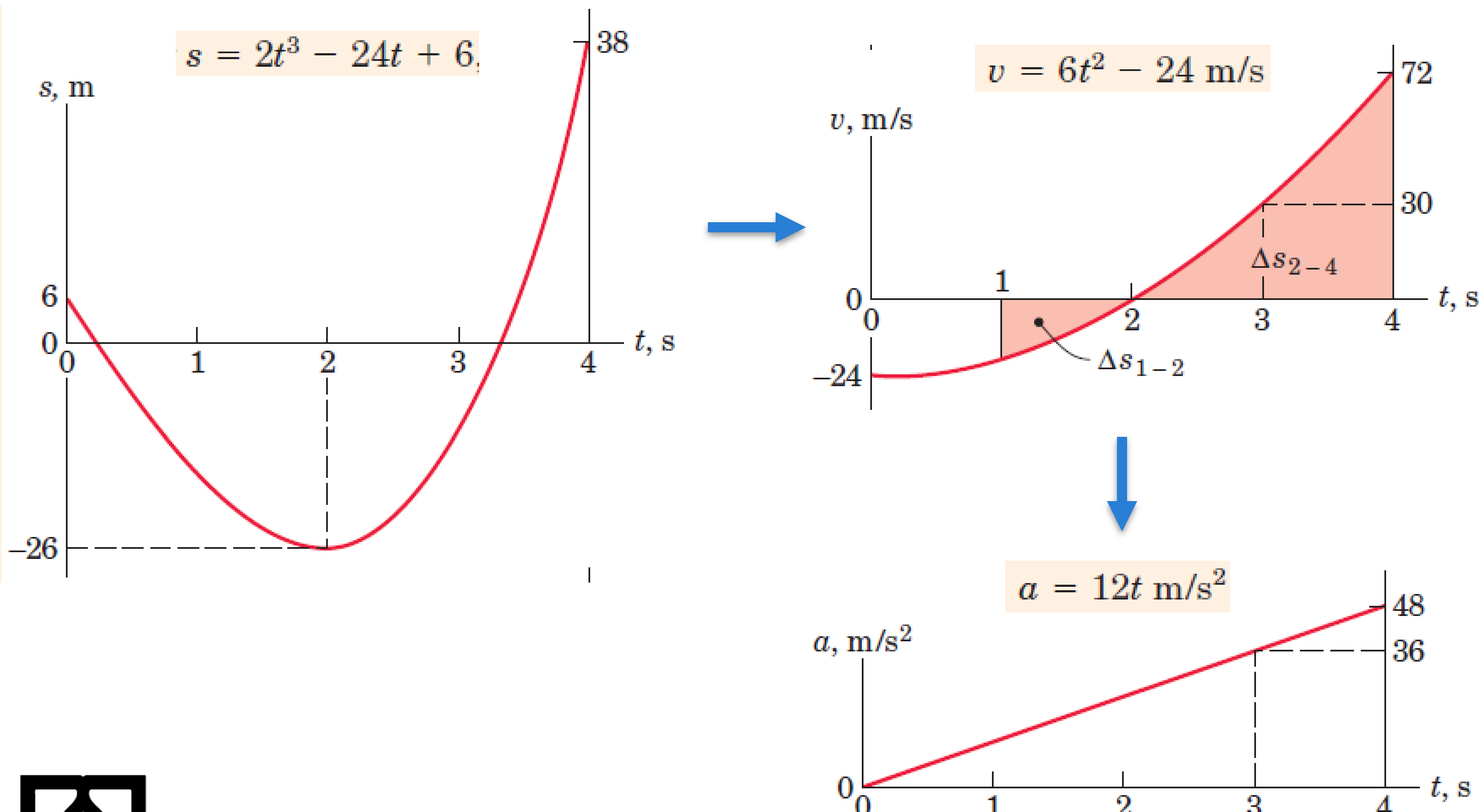
- *(a) Constant Acceleration*

$$\int_{v_0}^{v} dv = a\int_0^t dt \qquad \text{or} \qquad v = v_0 + at$$

$$\int_{v_0}^{v} v\,dv = a\int_{s_0}^{s} ds \qquad \text{or} \qquad v^2 = {v_0}^2 + 2a(s - s_0)$$

$$\int_{s_0}^{s} ds = \int_0^t (v_0 + at)\,dt \qquad \text{or} \qquad s = s_0 + v_0 t + \frac{1}{2}at^2$$

SAMPLE PROBLEM 2/1



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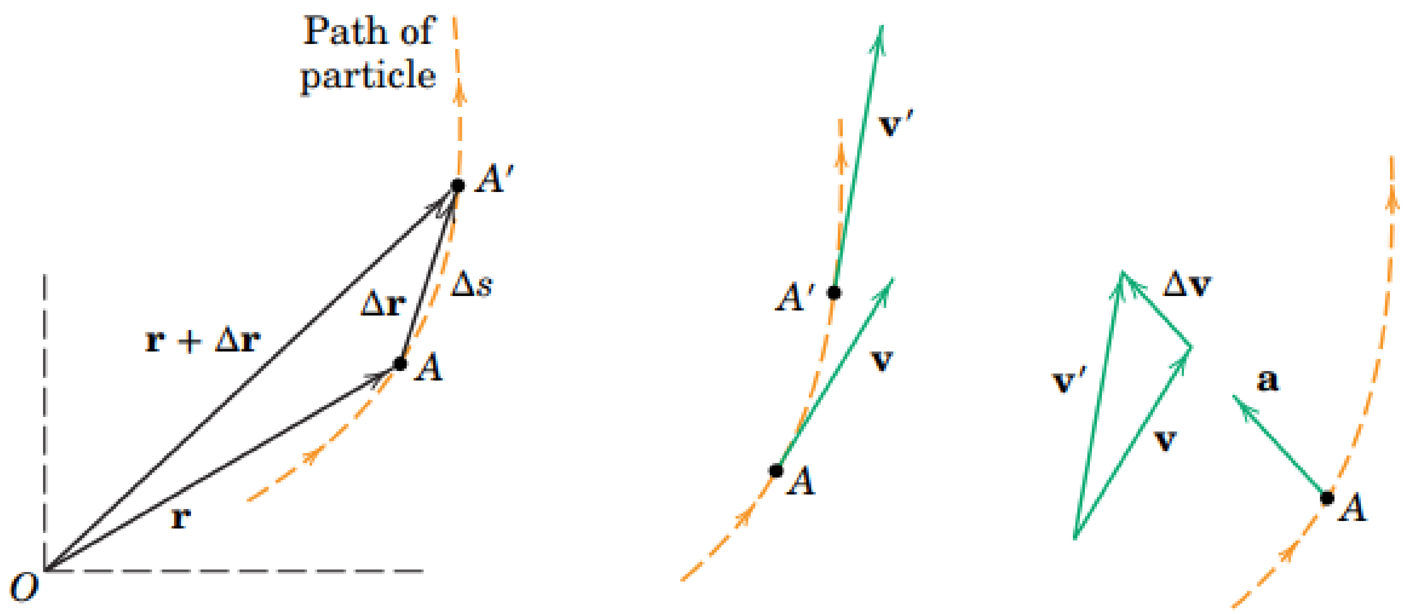
2/3 Plane Curvilinear Motion

- *Position Vector* **r**



2/3 Plane Curvilinear Motion

Velocity

- *Average velocity* of the particle between *A* and *A′* is defined as $\mathbf{v}_{av} = \Delta\mathbf{r} / \Delta t$

- *instantaneous velocity (*approaches tangent to the path*)*

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = \dot{\mathbf{r}}$$

- The magnitude of **v** is called the *speed* and is the scalar

$$v = |\mathbf{v}| = \frac{ds}{dt} = \dot{s}$$

2/3 Plane Curvilinear Motion

Acceleration

- The *average acceleration* of the particle between A and A' is defined as $\Delta \mathbf{v} / \Delta t$

- *instantaneous acceleration*

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = \dot{\mathbf{v}}$$

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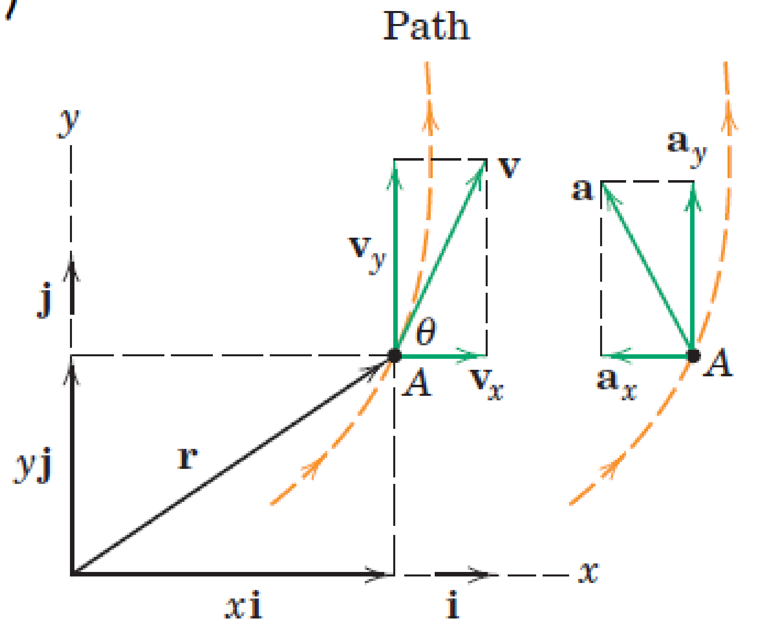
2/4 Rectangular Coordinates (*x*-*y*)

Vector Representation

$$\mathbf{r} = x\mathbf{i} + y\mathbf{j}$$

$$\mathbf{v} = \dot{\mathbf{r}} = \dot{x}\mathbf{i} + \dot{y}\mathbf{j}$$

$$\mathbf{a} = \dot{\mathbf{v}} = \ddot{\mathbf{r}} = \ddot{x}\mathbf{i} + \ddot{y}\mathbf{j}$$



$$v^2 = {v_x}^2 + {v_y}^2 \qquad v = \sqrt{{v_x}^2 + {v_y}^2} \qquad \tan\theta = \frac{v_y}{v_x}$$

$$a^2 = {a_x}^2 + {a_y}^2 \qquad a = \sqrt{{a_x}^2 + {a_y}^2}$$

2/4 Rectangular Coordinates (*x*-*y*)

Projectile Motion

- An important application of two-dimensional kinematic theory is the problem of projectile motion.

- For a first treatment of the subject, we neglect aerodynamic drag and the curvature and rotation of the earth, and we assume that the altitude change is small enough so that the acceleration due to gravity can be considered constant.

$$a_x = 0 \qquad a_y = -g$$

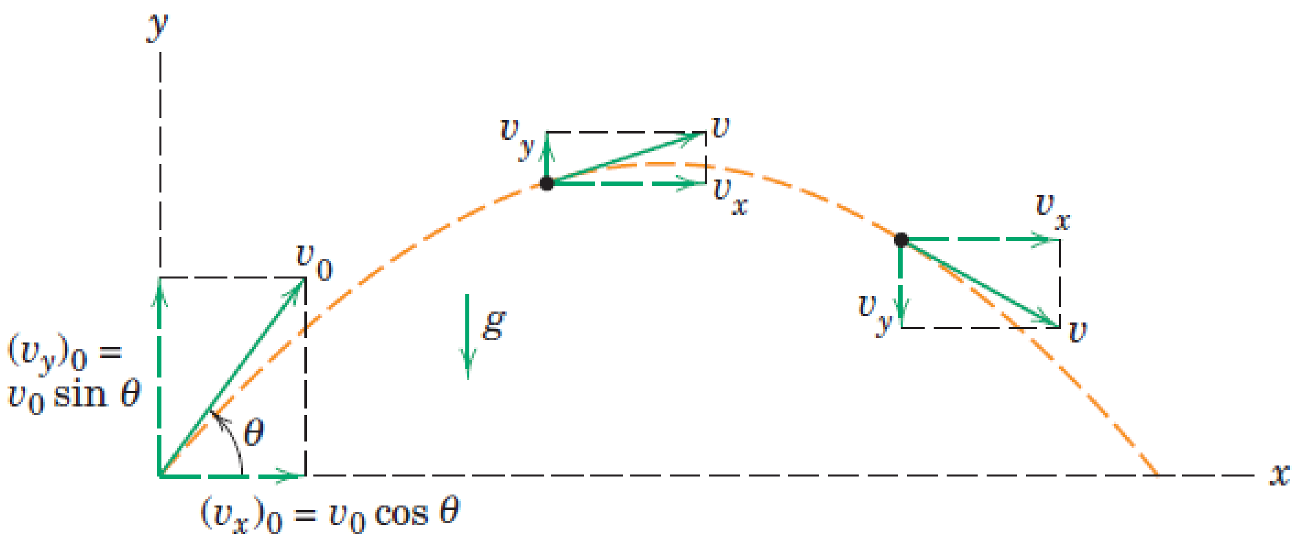
2/4 Rectangular Coordinates (*x*-*y*)

Projectile Motion

$$v_x = (v_x)_0 \qquad v_y = (v_y)_0 - gt$$

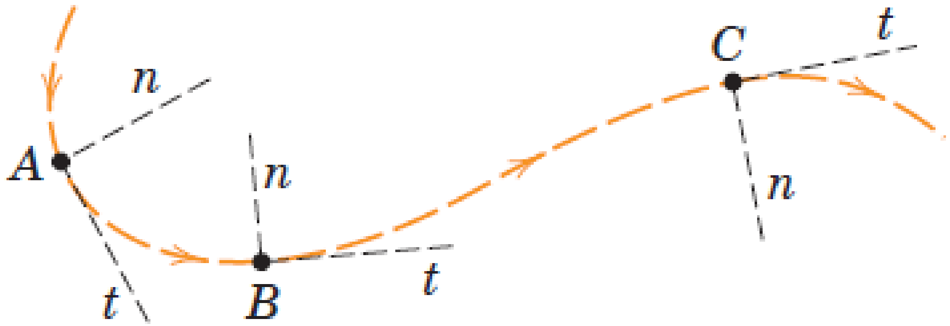
$$x = x_0 + (v_x)_0 t \qquad y = y_0 + (v_y)_0 t - \frac{1}{2}gt^2$$

$$v_y^2 = (v_y)_0^2 - 2g(y - y_0)$$



2/5 Normal and Tangential Coordinates (*n*-*t*)

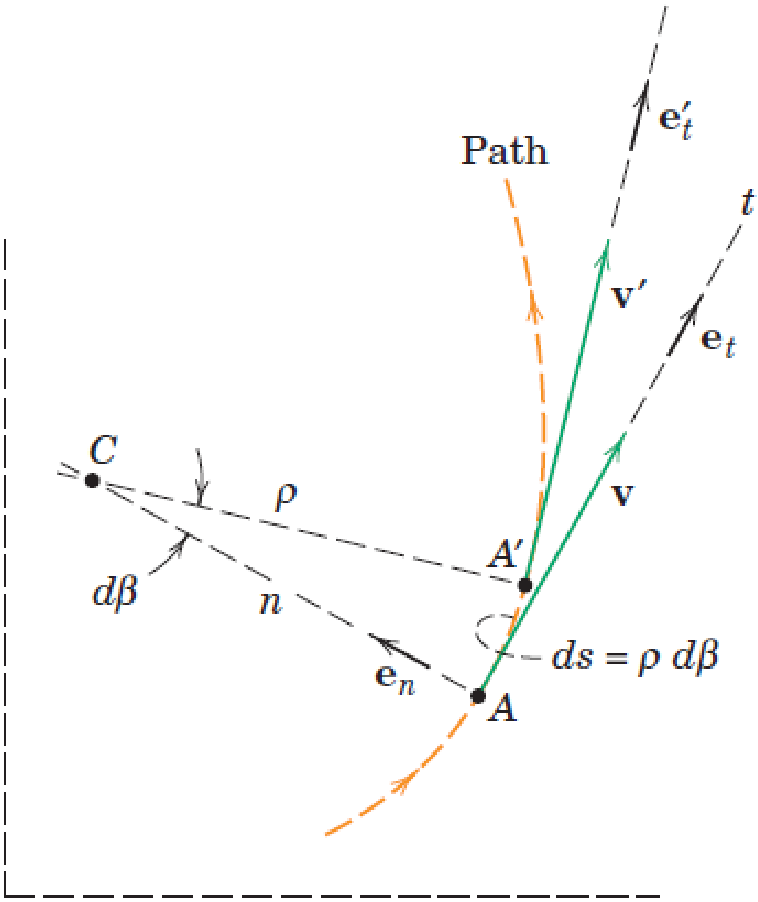
- Measurements made along the tangent *t* and normal *n* to the path of the particle



2/5 Normal and Tangential Coordinates (*n*-*t*)

Velocity and Acceleration

$$\mathbf{v} = v\mathbf{e}_t = \rho\dot{\beta}\mathbf{e}_t$$



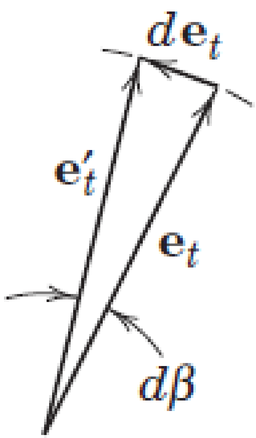
2/5 Normal and Tangential Coordinates (*n*-*t*)

Velocity and Acceleration

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = \frac{d(v\mathbf{e}_t)}{dt} = v\dot{\mathbf{e}}_t + \dot{v}\mathbf{e}_t$$

$$d\mathbf{e}_t = \mathbf{e}_n \, d\beta$$

$$\dot{\mathbf{e}}_t = \dot{\beta}\mathbf{e}_n$$



2/5 Normal and Tangential Coordinates (*n*-*t*)

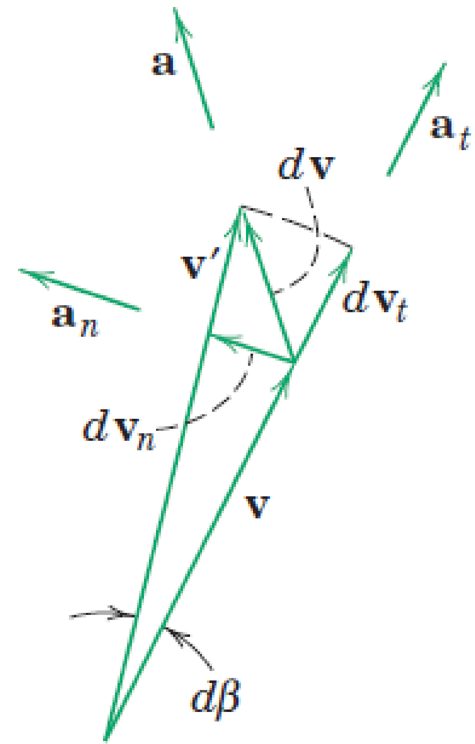
Velocity and Acceleration

$$\mathbf{a} = \frac{v^2}{\rho}\mathbf{e}_n + \dot{v}\mathbf{e}_t$$

$$a_n = \frac{v^2}{\rho} = \rho\dot{\beta}^2 = v\dot{\beta}$$

$$a_t = \dot{v} = \ddot{s} \quad \rightarrow \quad a_t = \dot{v} = d(\rho\dot{\beta})/dt = \rho\ddot{\beta} + \dot{\rho}\dot{\beta}$$

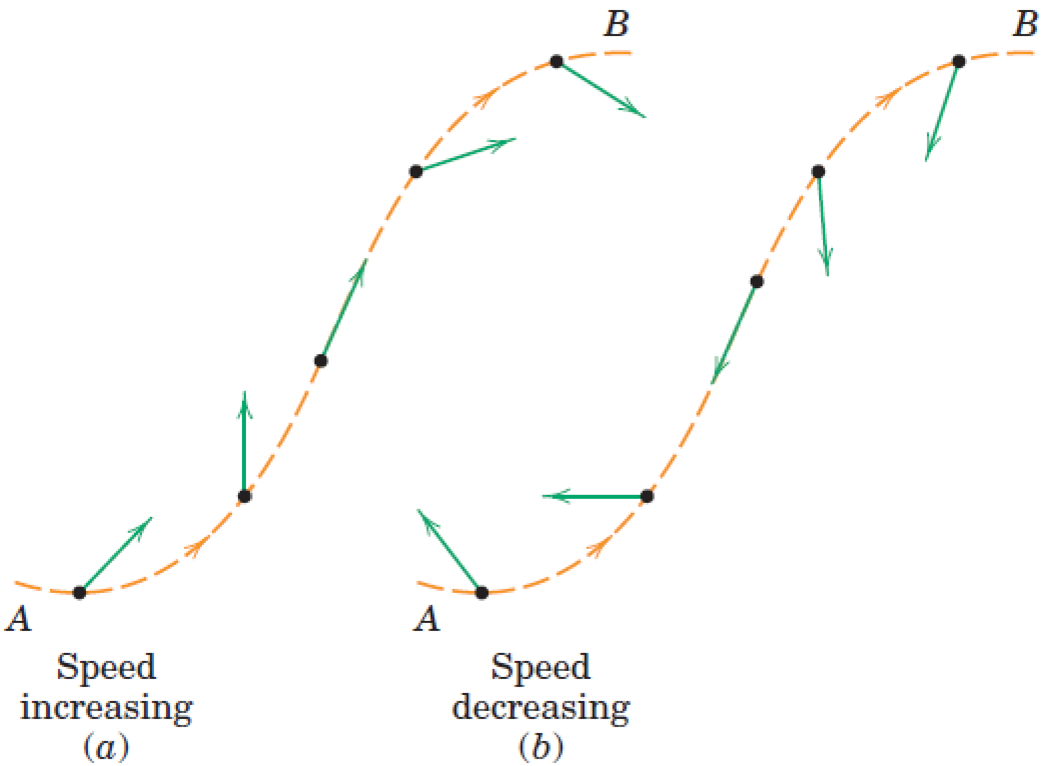
$$a = \sqrt{{a_n}^2 + {a_t}^2}$$



2/5 Normal and Tangential Coordinates (*n*-*t*)

Geometric Interpretation

- a_n is *always directed toward the center of curvature C*



2/5 Normal and Tangential Coordinates (*n*-*t*)

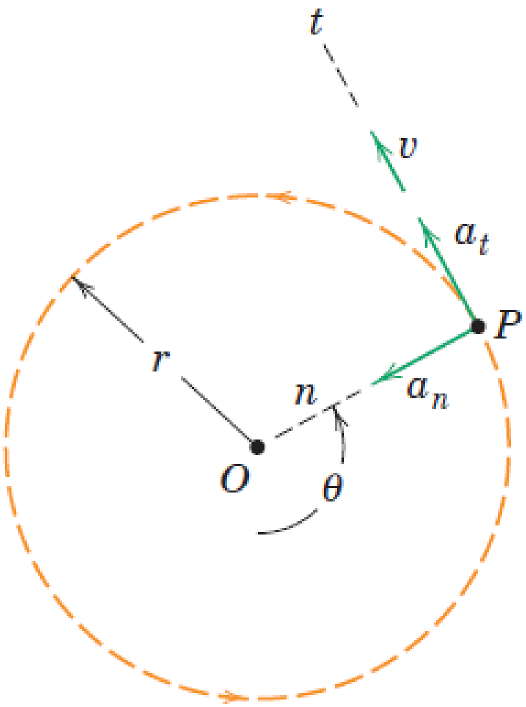
Circular Motion

- Circular motion is an important special case of plane curvilinear motion where the radius of curvature ρ becomes the constant radius r of the circle.

$$v = r\dot{\theta}$$

$$a_n = v^2/r = r\dot{\theta}^2 = v\dot{\theta}$$

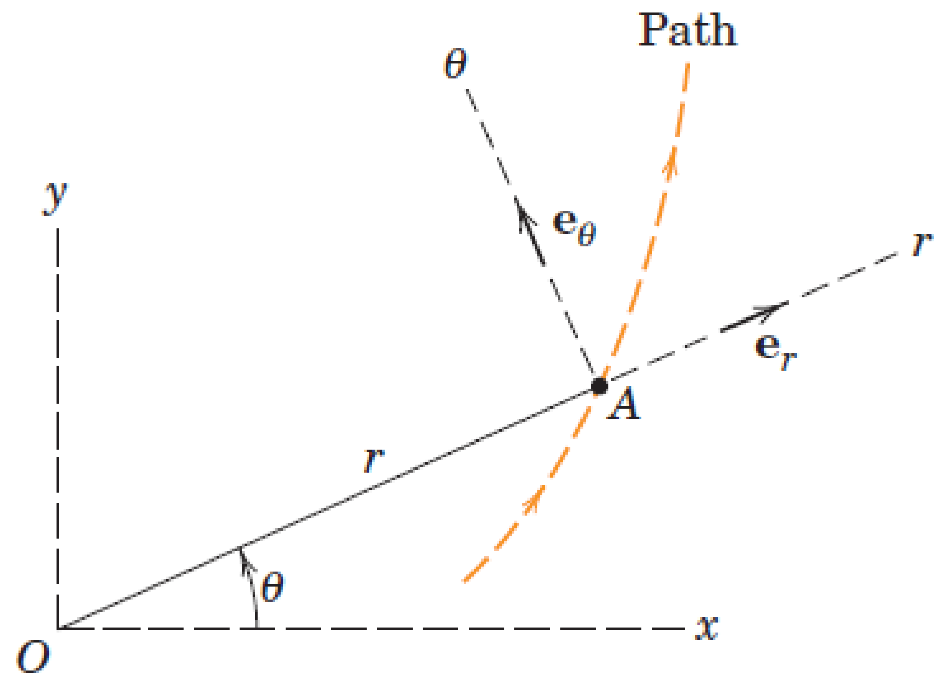
$$a_t = \dot{v} = r\ddot{\theta}$$



2/6 Polar Coordinates (r-θ)

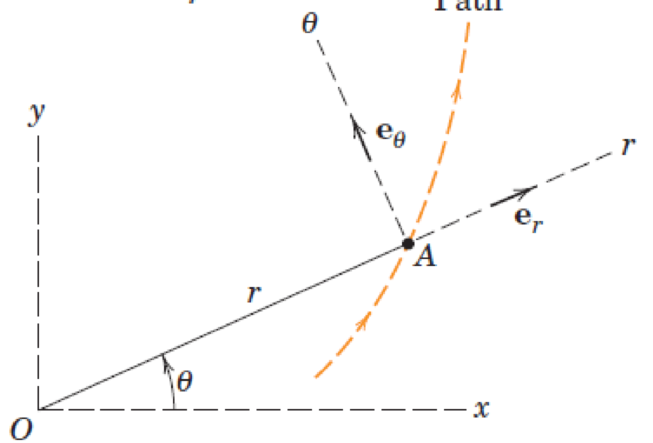
- Particle is located by the radial distance r from a fixed point and by an angular measurement θ to the radial line

$$\mathbf{r} = r\mathbf{e}_r$$



2/6 Polar Coordinates (r-θ)

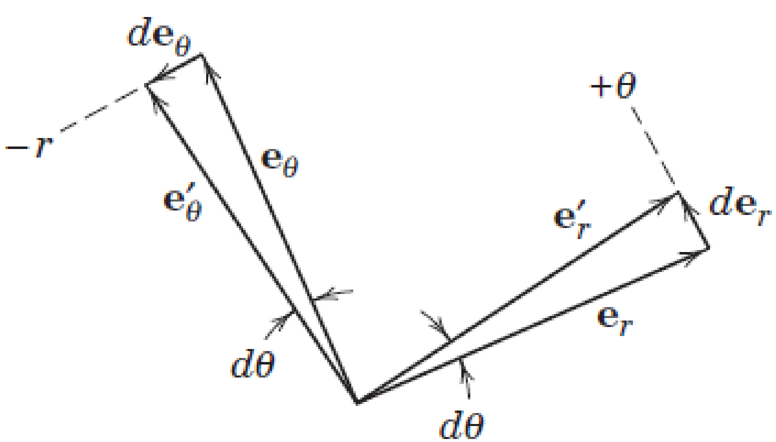
$$\mathbf{r} = r\mathbf{e}_r$$



Time Derivatives of the Unit Vectors

$$\frac{d\mathbf{e}_r}{d\theta} = \mathbf{e}_\theta \quad \text{and} \quad \frac{d\mathbf{e}_\theta}{d\theta} = -\mathbf{e}_r$$

$$\dot{\mathbf{e}}_r = \dot{\theta}\mathbf{e}_\theta \quad \text{and} \quad \dot{\mathbf{e}}_\theta = -\dot{\theta}\mathbf{e}_r$$



2/6 Polar Coordinates (r-θ)

Velocity

$$\mathbf{v} = \dot{\mathbf{r}} = \dot{r}\mathbf{e}_r + r\dot{\mathbf{e}}_r$$

$$\mathbf{v} = \dot{r}\mathbf{e}_r + r\dot{\theta}\mathbf{e}_\theta$$

$$v_r = \dot{r}$$

$$v_\theta = r\dot{\theta}$$

$$v = \sqrt{{v_r}^2 + {v_\theta}^2}$$

Acceleration

$$\mathbf{a} = \dot{\mathbf{v}} = (\ddot{r}\mathbf{e}_r + \dot{r}\dot{\mathbf{e}}_r) + (\dot{r}\dot{\theta}\mathbf{e}_\theta + r\ddot{\theta}\mathbf{e}_\theta + r\dot{\theta}\dot{\mathbf{e}}_\theta)$$

$$\mathbf{a} = (\ddot{r} - r\dot{\theta}^2)\mathbf{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\mathbf{e}_\theta$$

$$a_r = \ddot{r} - r\dot{\theta}^2$$

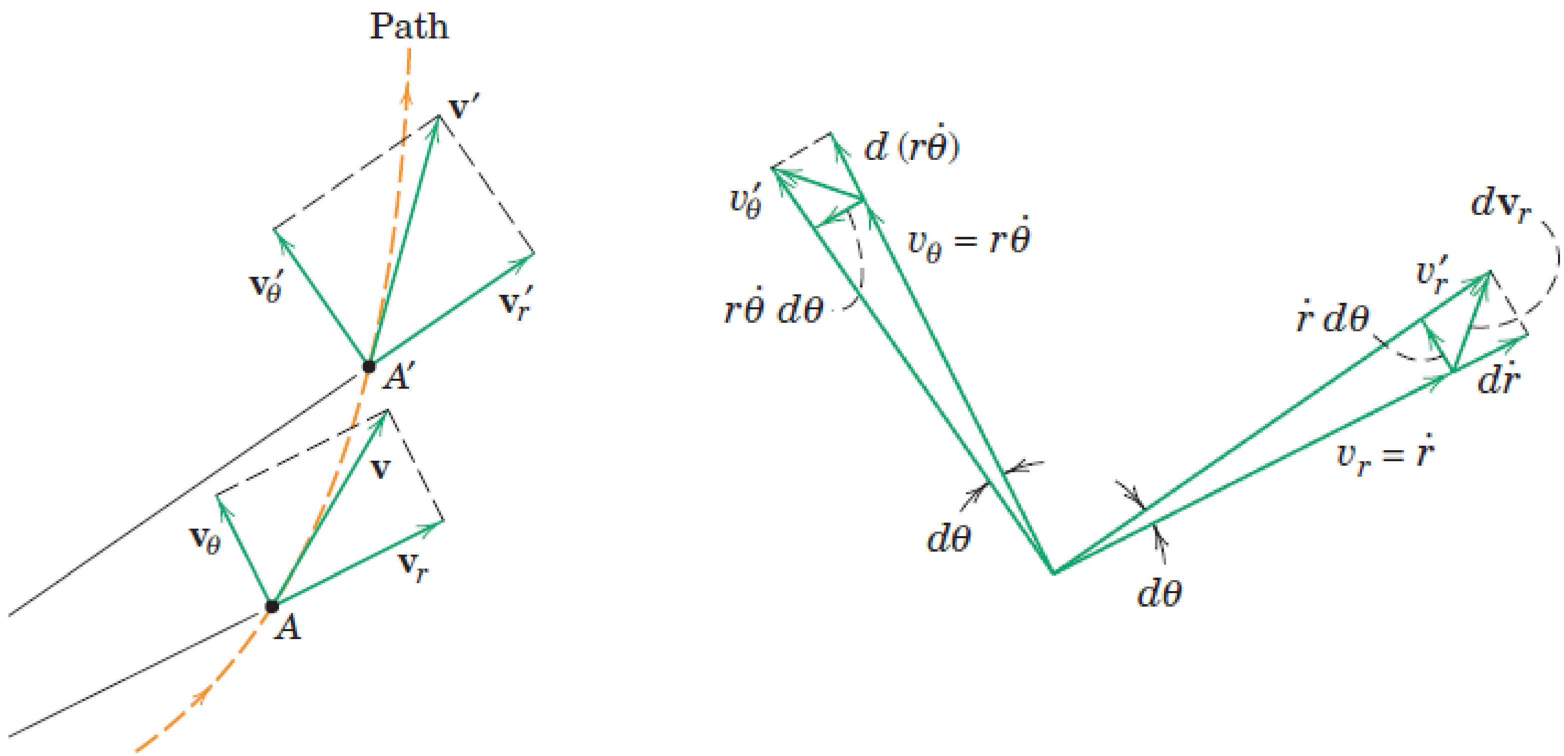
$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$$

$$a = \sqrt{{a_r}^2 + {a_\theta}^2}$$

$$a_\theta = \frac{1}{r}\frac{d}{dt}(r^2\dot{\theta})$$

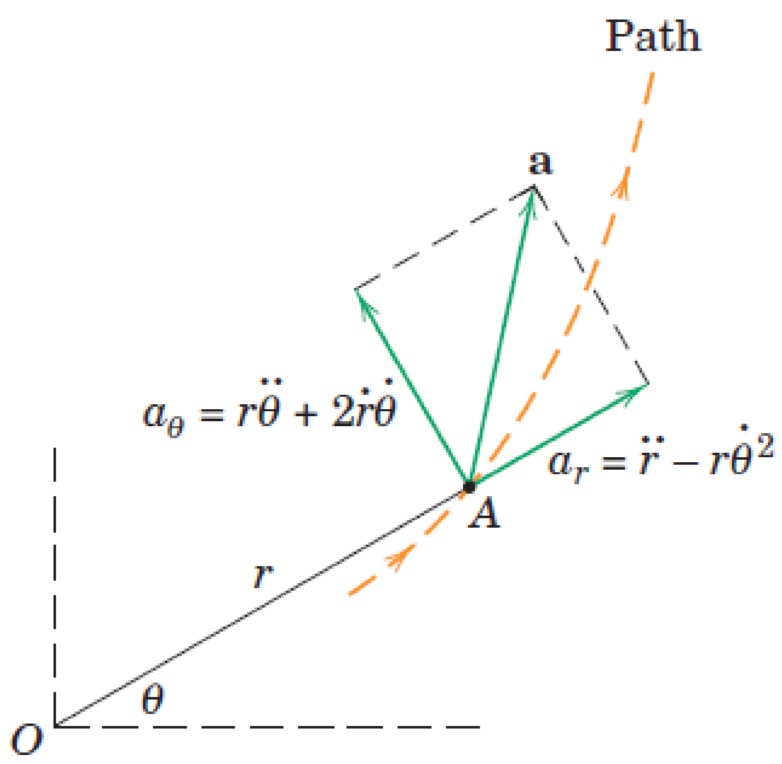
2/6 Polar Coordinates (r-θ)

Geometric Interpretation



2/6 Polar Coordinates (r-θ)

Geometric Interpretation



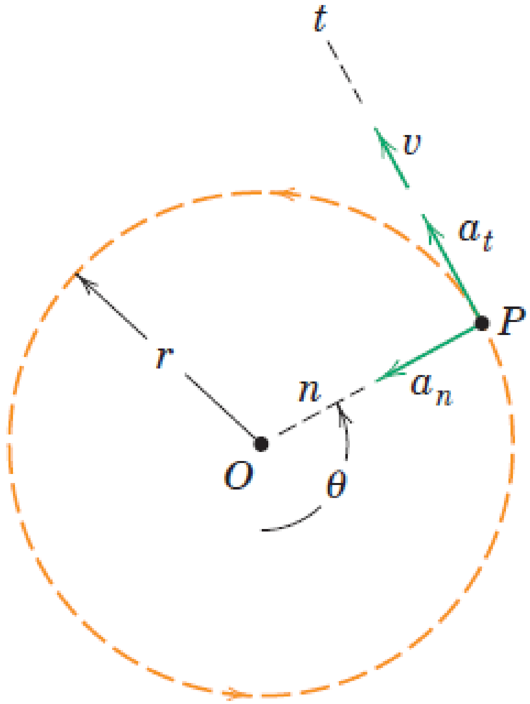
2/6 Polar Coordinates (r-θ)

Circular Motion

- Same as that obtained with *n*- and *t*-components, where the n- and *t*-directions coincide but the positive *r*-direction is in the negative *n*-direction.

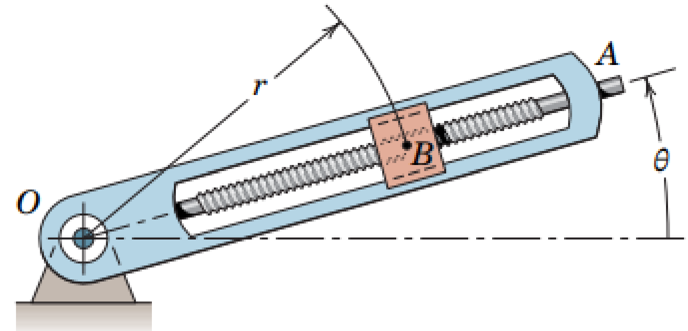
$$v = r\dot{\theta}$$
$$a_n = v^2/r = r\dot{\theta}^2 = v\dot{\theta}$$
$$a_t = \dot{v} = r\ddot{\theta}$$

$$v_r = 0 \qquad v_\theta = r\dot{\theta}$$
$$a_r = -r\dot{\theta}^2 \qquad a_\theta = r\ddot{\theta}$$



SAMPLE PROBLEM 2/9

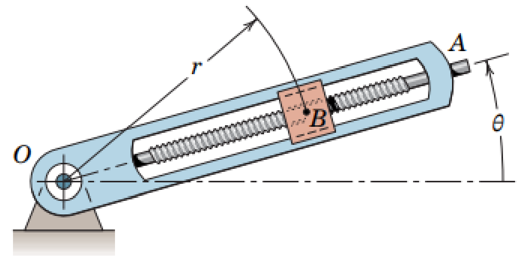
Rotation of the radially slotted arm is governed by $\theta = 0.2t + 0.02t^3$, where θ is in radians and t is in seconds. Simultaneously, the power screw in the arm engages the slider B and controls its distance from O according to $r = 0.2 + 0.04t^2$, where r is in meters and t is in seconds. Calculate the magnitudes of the velocity and acceleration of the slider for the instant when $t = 3$ s.



$r = 0.2 + 0.04t^2$	$r_3 = 0.2 + 0.04(3^2) = 0.56$ m
$\dot{r} = 0.08t$	$\dot{r}_3 = 0.08(3) = 0.24$ m/s
$\ddot{r} = 0.08$	$\ddot{r}_3 = 0.08$ m/s^2

$[v_\theta = r\dot{\theta}]$	$v_r = 0.24$ m/s
$[v_\theta = r\dot{\theta}]$	$v_\theta = 0.56(0.74) = 0.414$ m/s
$[v = \sqrt{v_r{}^2 + v_\theta{}^2}]$	$v = \sqrt{(0.24)^2 + (0.414)^2} = 0.479$ m/s

SAMPLE PROBLEM 2/9

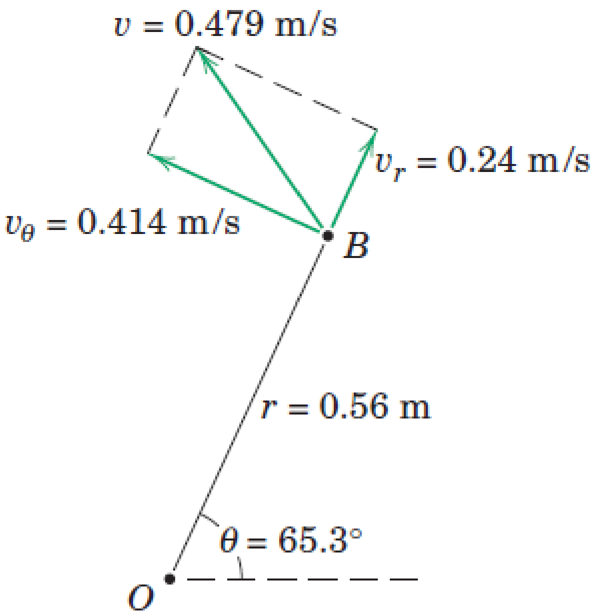


$$\theta = 0.2t + 0.02t^3 \qquad \theta_3 = 0.2(3) + 0.02(3^3) = 1.14 \text{ rad}$$

$$\text{or } \theta_3 = 1.14(180/\pi) = 65.3°$$

$$\dot{\theta} = 0.2 + 0.06t^2 \qquad \dot{\theta}_3 = 0.2 + 0.06(3^2) = 0.74 \text{ rad/s}$$

$$\ddot{\theta} = 0.12t \qquad \ddot{\theta}_3 = 0.12(3) = 0.36 \text{ rad/s}^2$$

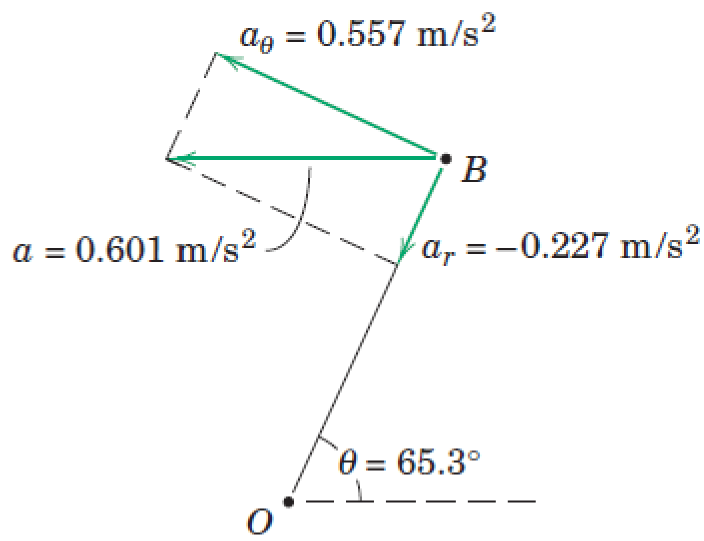


SAMPLE PROBLEM 2/9

$$[a_r = \ddot{r} - r\dot{\theta}^2] \qquad a_r = 0.08 - 0.56(0.74)^2 = -0.227 \text{ m/s}^2$$

$$[a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}] \qquad a_\theta = 0.56(0.36) + 2(0.24)(0.74) = 0.557 \text{ m/s}^2$$

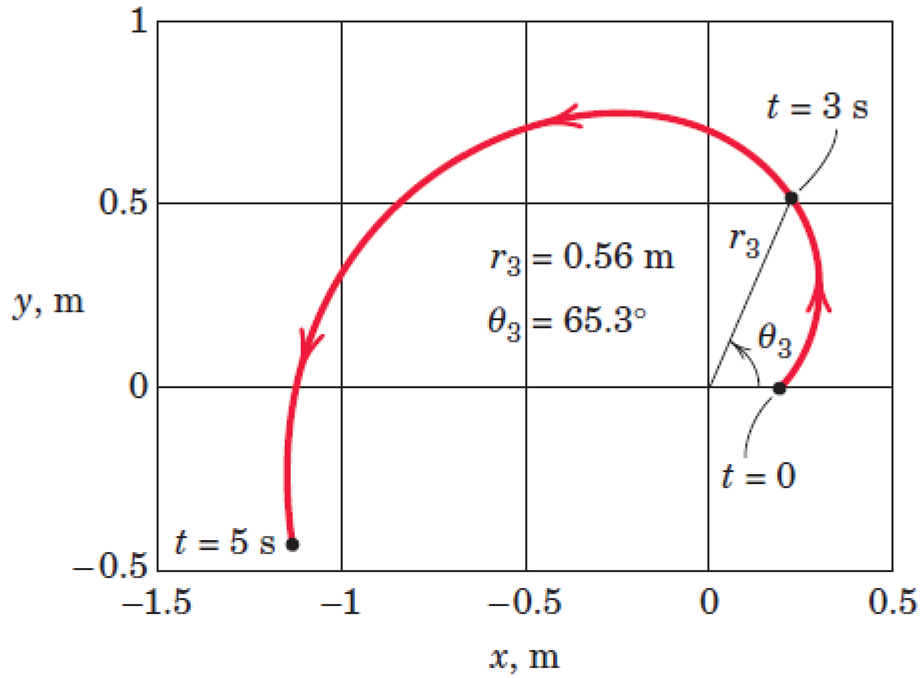
$$[a = \sqrt{a_r{}^2 + a_\theta{}^2}] \qquad a = \sqrt{(-0.227)^2 + (0.557)^2} = 0.601 \text{ m/s}^2$$



SAMPLE PROBLEM 2/9

- Conversion from polar to rectangular coordinates

$$x = r\cos\theta \qquad y = r\sin\theta$$



2/8 Relative Motion (Translating Axes)

- It is not always possible or convenient, however, to use a fi xed set of axes to describe or to measure motion.
- In addition, there are many engineering problems for which the analysis of motion is simplified by using measurements made with respect to a moving reference system.
- These measurements, when combined with the absolute motion of the moving coordinate system, enable us to determine the absolute motion in question.
- This approach is called a relative-motion analysis.

2/8 Relative Motion (Translating Axes)

Choice of Coordinate System

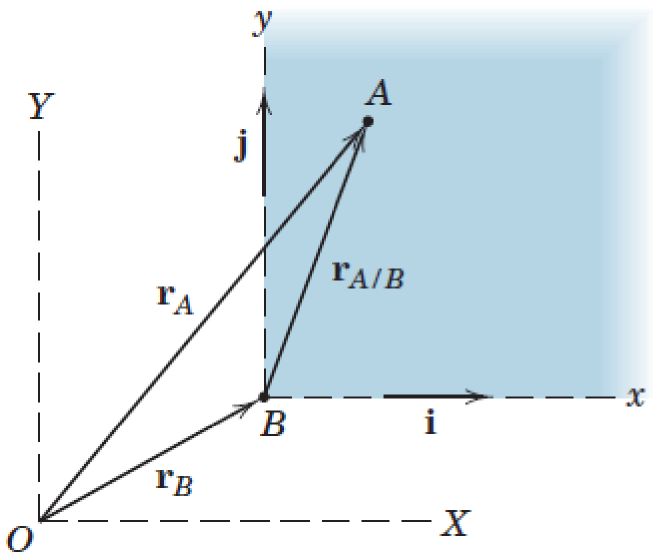
- The motion of the moving coordinate system is specified with respect to a fixed coordinate system.

Vector Representation

$$\mathbf{r}_A = \mathbf{r}_B + \mathbf{r}_{A/B}$$

$$\dot{\mathbf{r}}_A = \dot{\mathbf{r}}_B + \dot{\mathbf{r}}_{A/B} \quad \text{or} \quad \mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$$

$$\ddot{\mathbf{r}}_A = \ddot{\mathbf{r}}_B + \ddot{\mathbf{r}}_{A/B} \quad \text{or} \quad \mathbf{a}_A = \mathbf{a}_B + \mathbf{a}_{A/B}$$



2/9 Constrained Motion of Connected Particles

- Sometimes the motions of particles are interrelated because of the constraints imposed by interconnecting members.

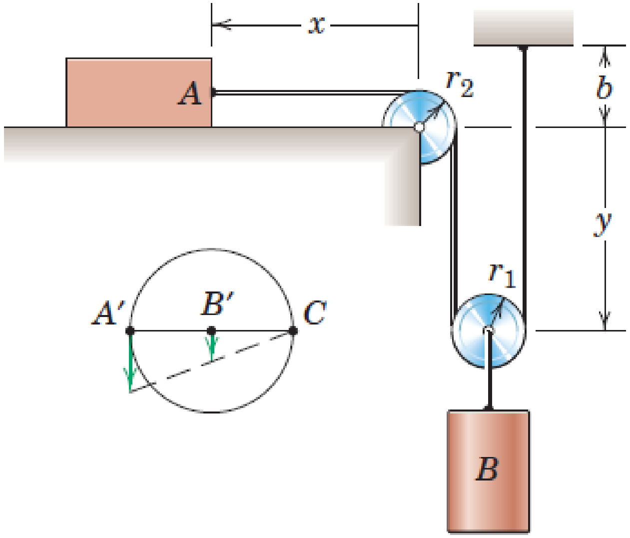
One Degree of Freedom

- One degree of freedom: only one variable, either x or y, is needed to specify the positions of all parts of the system.

$$L = x + \frac{\pi r_2}{2} + 2y + \pi r_1 + b$$

$$0 = \dot{x} + 2\dot{y} \qquad \text{or} \qquad 0 = v_A + 2v_B$$

$$0 = \ddot{x} + 2\ddot{y} \qquad \text{or} \qquad 0 = a_A + 2a_B$$



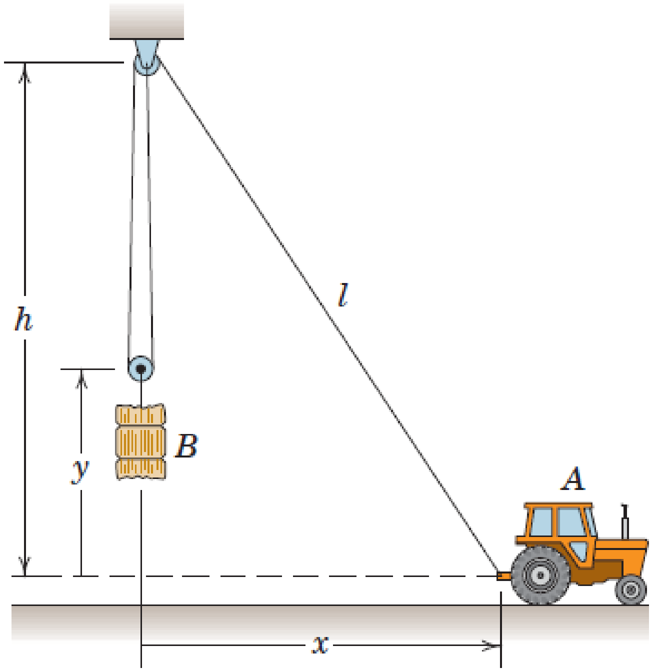
SAMPLE PROBLEM 2/16

The tractor A is used to hoist the bale B with the pulley arrangement shown. If A has a forward velocity v_A, determine an expression for the upward velocity v_B of the bale in terms of x.

$$L = 2(h - y) + l = 2(h - y) + \sqrt{h^2 + x^2}$$

$$0 = -2\dot{y} + \frac{x\dot{x}}{\sqrt{h^2 + x^2}}$$

$$v_B = \frac{1}{2}\frac{xv_A}{\sqrt{h^2 + x^2}}$$



Contents:

3/1 Introduction

- According to Newton's second law, a particle will accelerate when it is subjected to unbalanced forces.
- Kinetics is the study of the relations between unbalanced forces and the resulting changes in motion.
- We combine our knowledge of the properties of forces, which we developed in statics, and the kinematics of particle motion, and solve engineering problems involving force, mass, and motion.

3/1 Introduction

- General approaches to the solution of kinetics problems:
  - (A) Direct application of Newton’s second law

    (called the force-mass-acceleration method)
  - (B) Use of work and energy principles
  - (C) Solution by impulse and momentum methods.

SECTION A Force, Mass, and Acceleration

3/2 Newton's Second Law

- The ratios of applied force to corresponding acceleration all equal the same number, provided the units used for measurement are not changed in the experiments.

$$\frac{F_1}{a_1} = \frac{F_2}{a_2} = \cdots = \frac{F}{a} = C, \qquad \text{a constant}$$

$$\boxed{\Sigma \mathbf{F} = m\mathbf{a}}$$

- We conclude that the constant C is a measure of some invariable property of the particle. This property is the inertia of the particle, which is its resistance to rate of change of velocity.

3/3 Equation of Motion and Solution of Problems

Free-Body Diagram

- The only reliable way to account accurately and consistently for every force is to isolate the particle under consideration from all contacting and influencing bodies and replace the bodies removed by the forces they exert on the particle isolated.

- The resulting free body diagram is the means by which every force, known and unknown, which acts on the particle is represented and thus accounted for.

- Only after this vital step has been completed should you write the appropriate equation or equations of motion.

3/4 Rectilinear Motion

- If we choose the x-direction, for example, as the direction of the rectilinear motion of a particle:

$$\Sigma F_x = ma_x$$

$$\Sigma F_y = 0$$

$$\Sigma F_z = 0$$

- For cases where we are not free to choose a coordinate direction along the motion:

$$\Sigma F_x = ma_x$$

$$\Sigma F_y = ma_y$$

$$\Sigma F_z = ma_z$$

3/4 Rectilinear Motion

- Acceleration and resultant force:

$$\mathbf{a} = a_x\mathbf{i} + a_y\mathbf{j} + a_z\mathbf{k}$$

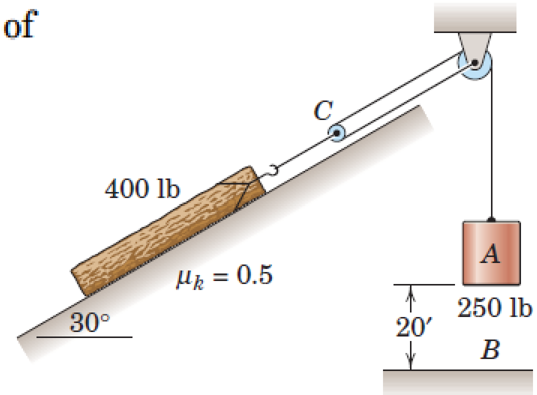
$$a = \sqrt{a_x{}^2 + a_y{}^2 + a_z{}^2}$$

$$\Sigma\mathbf{F} = \Sigma F_x\mathbf{i} + \Sigma F_y\mathbf{j} + \Sigma F_z\mathbf{k}$$

$$|\Sigma\mathbf{F}| = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2 + (\Sigma F_z)^2}$$

SAMPLE PROBLEM 3/3

The 250-lb concrete block A is released from rest in the position shown and pulls the 400-lb log up the 30° ramp. If the coefficient of kinetic friction between the log and the ramp is 0.5, determine the velocity of the block as it hits the ground at B.



$$L = 2s_C + s_A + \text{constant} \qquad 0 = 2a_C + a_A$$

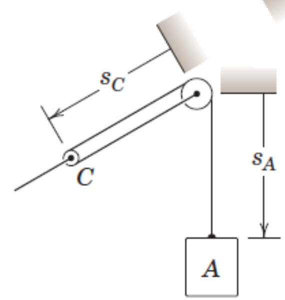
$$[\Sigma F_y = 0] \qquad N - 400 \cos 30° = 0 \qquad N = 346 \text{ lb}$$

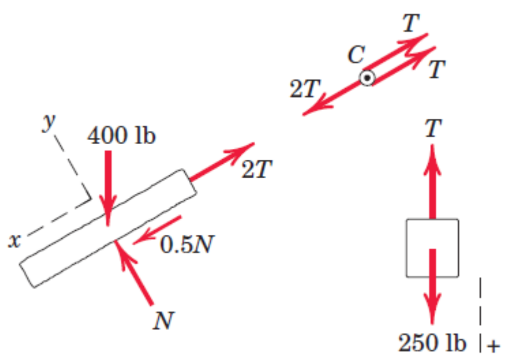
$$[\Sigma F_x = ma_x] \qquad 0.5(346) - 2T + 400 \sin 30° = \frac{400}{32.2} a_C$$

$$[+\downarrow \Sigma F = ma] \qquad 250 - T = \frac{250}{32.2} a_A$$

$$a_A = 5.83 \text{ ft/sec}^2 \qquad a_C = -2.92 \text{ ft/sec}^2 \qquad T = 205 \text{ lb}$$

$$[v^2 = 2ax] \qquad v_A = \sqrt{2(5.83)(20)} = 15.27 \text{ ft/sec}$$





3/5 Curvilinear Motion

- Rectangular coordinates:

$$\Sigma F_x = ma_x \qquad \Sigma F_y = ma_y$$

$$a_x = \ddot{x} \quad \text{and} \quad a_y = \ddot{y}$$

- Normal and tangential coordinates:

$$\Sigma F_n = ma_n \qquad \Sigma F_t = ma_t$$

$$a_n = \rho\dot{\beta}^2 = v^2/\rho = v\dot{\beta}, \quad a_t = \dot{v}, \quad \text{and} \quad v = \rho\dot{\beta}$$

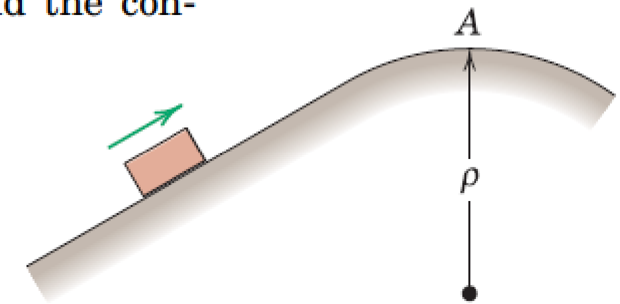
- Polar coordinates:

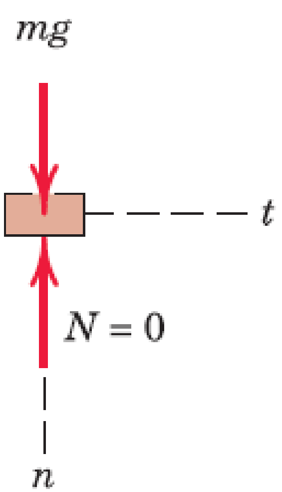
$$\Sigma F_r = ma_r \qquad \Sigma F_\theta = ma_\theta$$

$$a_r = \ddot{r} - r\dot{\theta}^2 \quad \text{and} \quad a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$$

SAMPLE PROBLEM 3/6

Determine the maximum speed v which the sliding block may have as it passes the topmost point A without losing contact with the lower surface. Assume a slightly loose fit between the slider and the constraint surfaces.





$$[\Sigma F_n = ma_n] \qquad mg = m\frac{v^2}{\rho} \qquad v = \sqrt{g\rho}$$

SECTION B Work and Energy

3/6 Work and Kinetic Energy

- There are two general classes of problems:
  - (1) Integration of the forces with respect to the displacement of the particle
  - (2) Integration of the forces with respect to the time they are applied.

- Integration with respect to displacement leads to the equations of work and energy.

3/6 Work and Kinetic Energy

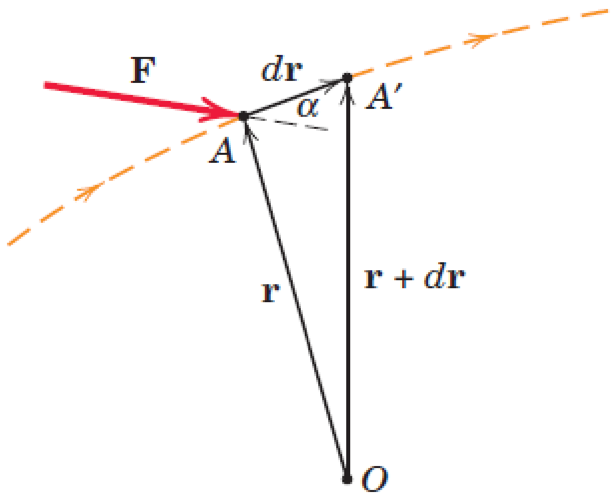
Definition of Work

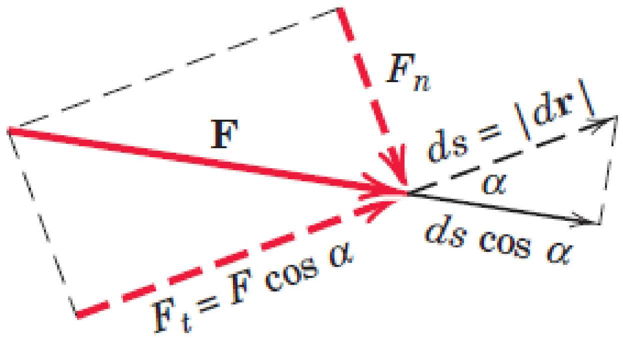
- The work done by the force ***F*** during the displacement ***dr***:

$$dU = \mathbf{F} \cdot d\mathbf{r}$$

→ $dU = F\, ds \cos\alpha$

$F_t = F \cos\alpha$ → $dU = F_t\, ds$



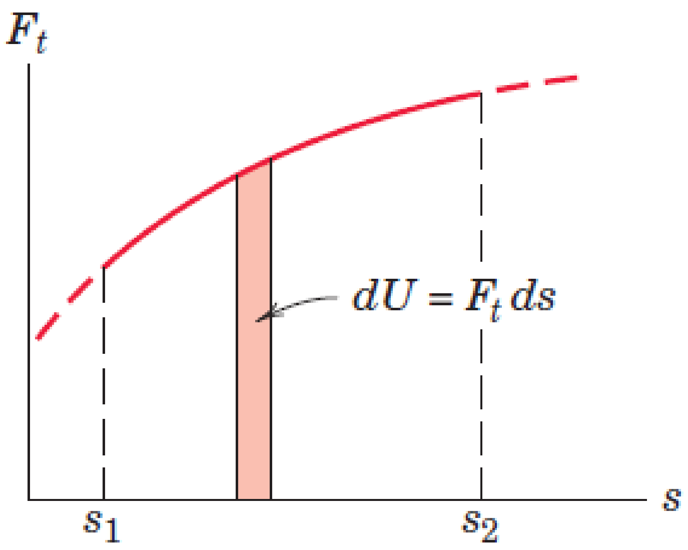


3/6 Work and Kinetic Energy

Calculation of Work

$$U = \int_1^2 \mathbf{F} \cdot d\mathbf{r} = \int_1^2 (F_x\,dx + F_y\,dy + F_z\,dz)$$

$$U = \int_{s_1}^{s_2} F_t\,ds$$



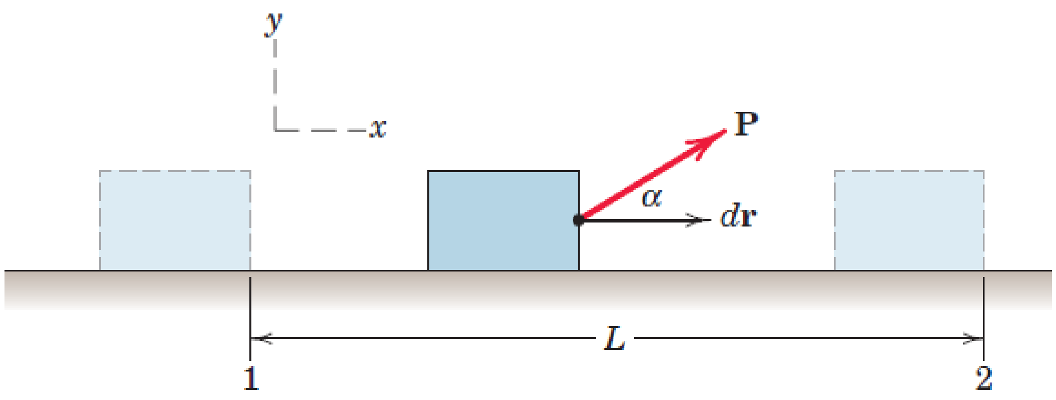
3/6 Work and Kinetic Energy

Examples of Work

- Work Associated with a Constant External Force

$$U_{1\text{-}2} = \int_1^2 \mathbf{F} \cdot d\mathbf{r} = \int_1^2 [(P\cos\alpha)\mathbf{i} + (P\sin\alpha)\mathbf{j}] \cdot dx\,\mathbf{i}$$

$$= \int_{x_1}^{x_2} P\cos\alpha\, dx = P\cos\alpha(x_2 - x_1) = PL\cos\alpha$$

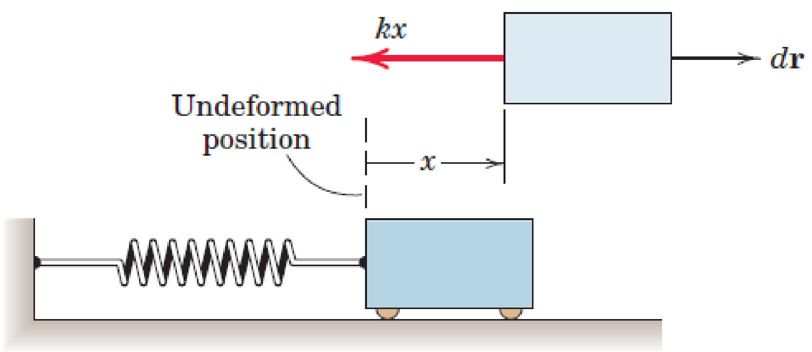


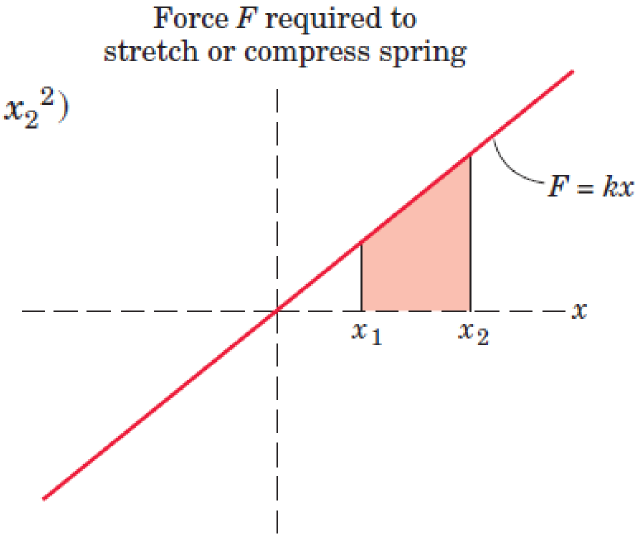
3/6 Work and Kinetic Energy

Examples of Work

- Work Associated with a Spring Force

$$U_{1\text{-}2} = \int_1^2 \mathbf{F}\cdot d\mathbf{r} = \int_1^2 (-kx\mathbf{i})\cdot dx\,\mathbf{i} = -\int_{x_1}^{x_2} kx\,dx = \frac{1}{2}k({x_1}^2 - {x_2}^2)$$





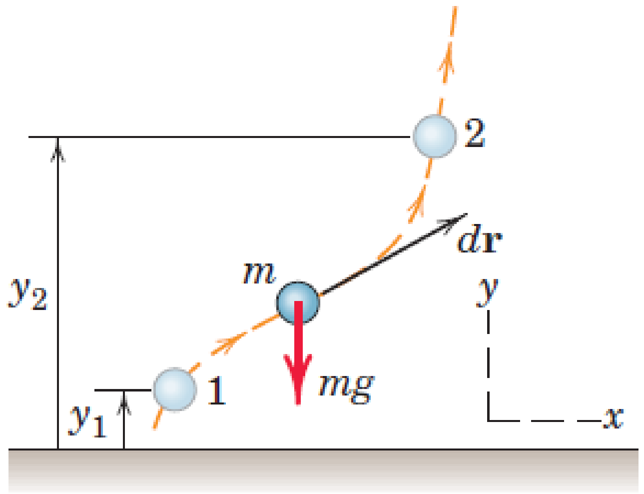
3/6 Work and Kinetic Energy

Examples of Work

- Work Associated with Weight

$$U_{1\text{-}2} = \int_1^2 \mathbf{F}\cdot d\mathbf{r} = \int_1^2 (-mg\mathbf{j})\cdot(dx\mathbf{i} + dy\mathbf{j})$$

$$= -mg\int_{y_1}^{y_2} dy = -mg(y_2 - y_1)$$



3/6 Work and Kinetic Energy

Principle of Work and Kinetic Energy

- The *kinetic energy T* of the particle:

$$T = \frac{1}{2}mv^2$$

- The *work-energy equation* for a particle:

$$T_1 + U_{1\text{-}2} = T_2$$

  - The equation states that the total work done by all forces acting on a particle as it moves from point 1 to point 2 equals the corresponding change in kinetic energy of the particle.

3/6 Work and Kinetic Energy

Power

- The capacity of a machine is rated by its power, which is defined as the time rate of doing work

$$P = dU/dt = \mathbf{F} \cdot d\mathbf{r}/dt \quad \rightarrow \quad \boxed{P = \mathbf{F} \cdot \mathbf{v}}$$

$$1 \text{ W} = 1 \text{ J/s}$$

$$1 \text{ hp} = 550 \text{ ft-lb/sec} = 33{,}000 \text{ ft-lb/min}$$

$$1 \text{ hp} = 746 \text{ W} = 0.746 \text{ kW}$$

3/6 Work and Kinetic Energy

Efficiency

- The ratio of the work done by a machine to the work done on the machine during the same time interval is called the mechanical efficiency e_m of the machine.

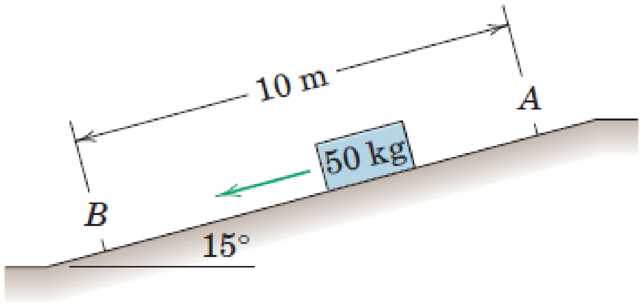
$$e_m = \frac{P_{\text{output}}}{P_{\text{input}}}$$

- In addition to energy loss by mechanical friction, there may also be electrical and thermal energy loss, in which case, the electrical efficiency e_e and thermal efficiency e_t are also involved. The overall efficiency e in such instances is:

$$e = e_m e_e e_t$$

SAMPLE PROBLEM 3/11

Calculate the velocity v of the 50-kg crate when it reaches the bottom of the chute at B if it is given an initial velocity of 4 m/s down the chute at A. The coefficient of kinetic friction is 0.30.



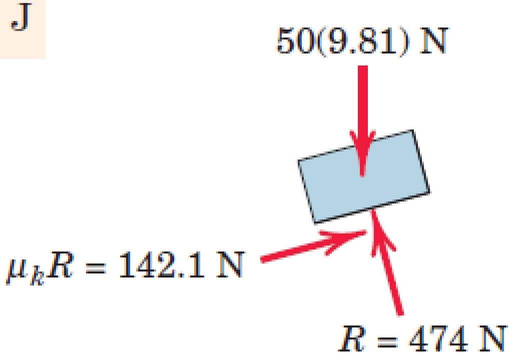
$[U = Fs]$ $\qquad U_{1\text{-}2} = 50(9.81)(10 \sin 15°) - 142.1(10) = -151.9 \text{ J}$

$[T_1 + U_{1\text{-}2} = T_2]$

$$\frac{1}{2}mv_1{}^2 + U_{1\text{-}2} = \frac{1}{2}mv_2{}^2$$

$$\frac{1}{2}(50)(4)^2 - 151.9 = \frac{1}{2}(50)v_2{}^2$$

$$v_2 = 3.15 \text{ m/s}$$



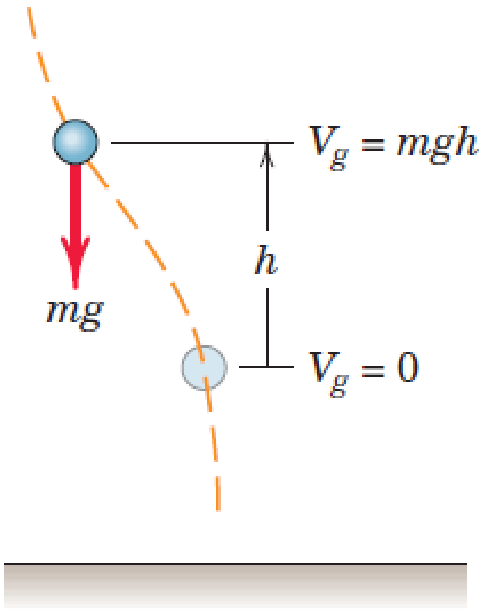
3/7 Potential Energy

Gravitational Potential Energy

- The *gravitational potential energy* V_g of the particle is defined as the work *mgh* done against the gravitational field to elevate the particle a distance h above some arbitrary reference plane.

$$V_g = mgh$$

$$\Delta V_g = mg(h_2 - h_1) = mg\Delta h$$



3/7 Potential Energy

Elastic Potential Energy

- The work which is done on the spring to deform it is stored in the spring and is called its *elastic potential energy* V_e.

$$V_e = \int_0^x kx\,dx = \frac{1}{2}kx^2$$

$$\Delta V_e = \frac{1}{2}k({x_2}^2 - {x_1}^2)$$

3/7 Potential Energy

Work-Energy Equation

- Work-energy equation modification to account for the potential-energy terms

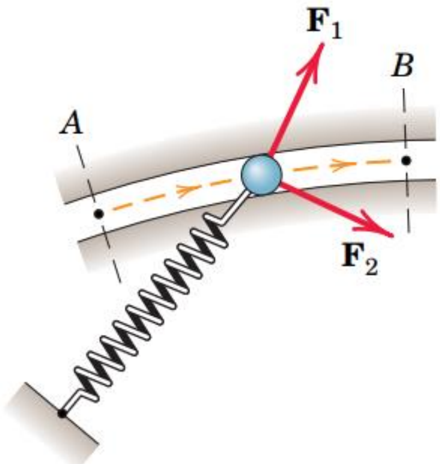
$$U'_{1\text{-}2} + (-\Delta V_g) + (-\Delta V_e) = \Delta T$$

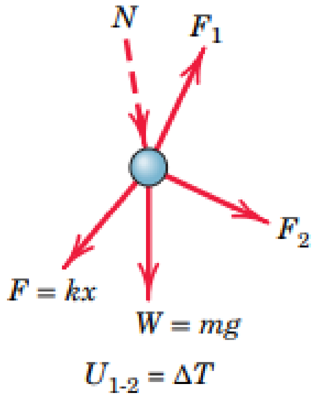
$$U'_{1\text{-}2} = \Delta T + \Delta V$$

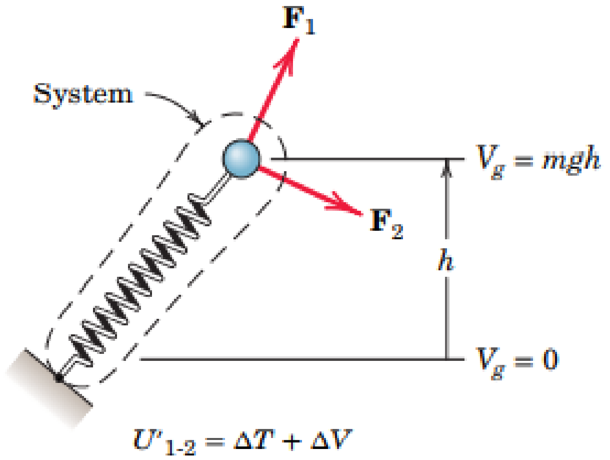
$$\boxed{T_1 + V_1 + U'_{1\text{-}2} = T_2 + V_2}$$

3/7 Potential Energy

Work-Energy Equation







3/7 Potential Energy

Work-Energy Equation

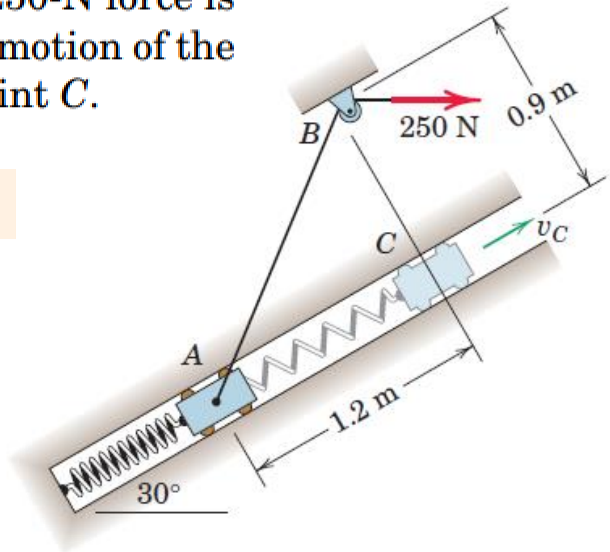
- For problems where the only forces are gravitational, elastic, and nonworking constraint forces:

$$T_1 + V_1 = T_2 + V_2 \qquad \text{or} \qquad E_1 = E_2$$

- E=T+V is the total mechanical energy of the particle and its attached spring.

SAMPLE PROBLEM 3/17

The 10-kg slider moves with negligible friction up the inclined guide. The attached spring has a stiffness of 60 N/m and is stretched 0.6 m in position A, where the slider is released from rest. The 250-N force is constant and the pulley offers negligible resistance to the motion of the cord. Calculate the velocity v_C of the slider as it passes point C.



$\overline{AB} - \overline{BC}$ or $1.5 - 0.9 = 0.6$ m.

$U'_{A\text{-}C} = 250(0.6) = 150$ J

$V_A = 0 \qquad V_C = mgh = 10(9.81)(1.2 \sin 30°) = 58.9$ J

$$V_A = \frac{1}{2}kx_A{}^2 = \frac{1}{2}(60)(0.6)^2 = 10.8 \text{ J}$$

$$V_C = \frac{1}{2}kx_B{}^2 = \frac{1}{2}60(0.6 + 1.2)^2 = 97.2 \text{ J}$$

$$[T_A + V_A + U'_{A\text{-}C} = T_C + V_C] \qquad 0 + 0 + 10.8 + 150 = \frac{1}{2}(10)v_C{}^2 + 58.9 + 97.2$$

$$v_C = 0.974 \text{ m/s}$$

دانشگاه سمنان

SECTION C Impulse and Momentum

3/8 Introduction

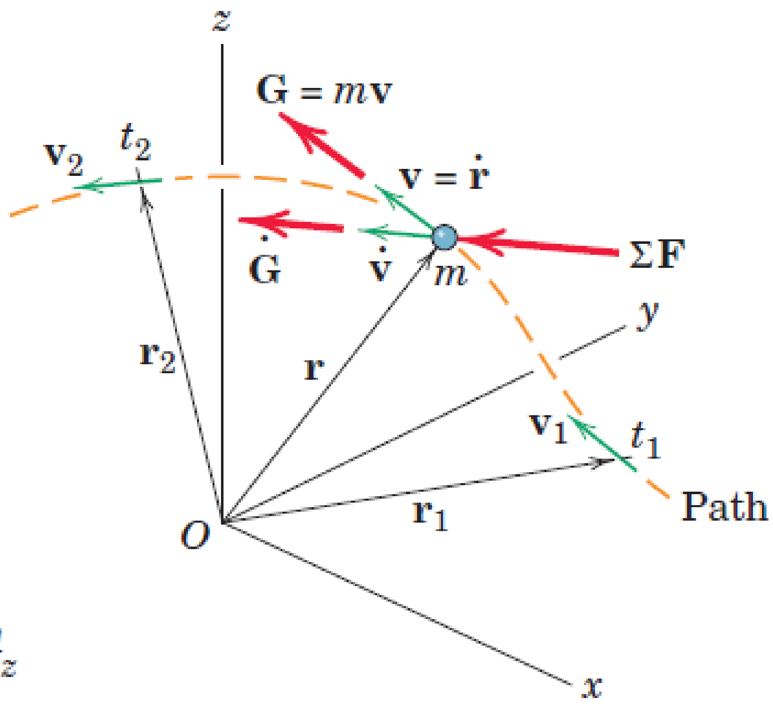
- We can integrate the equation of motion with respect to time rather than displacement.
- This approach leads to the equations of impulse and momentum.
- These equations greatly facilitate the solution of many problems in which the applied forces act during extremely short periods of time (as in impact problems) or over specified intervals of time.

3/9 Linear Impulse and Linear Momentum

- Linear momentum of the particle:
  - **G** = m**v**

$$\Sigma \mathbf{F} = m\dot{\mathbf{v}} = \frac{d}{dt}(m\mathbf{v})$$

$$\Sigma \mathbf{F} = \dot{\mathbf{G}}$$

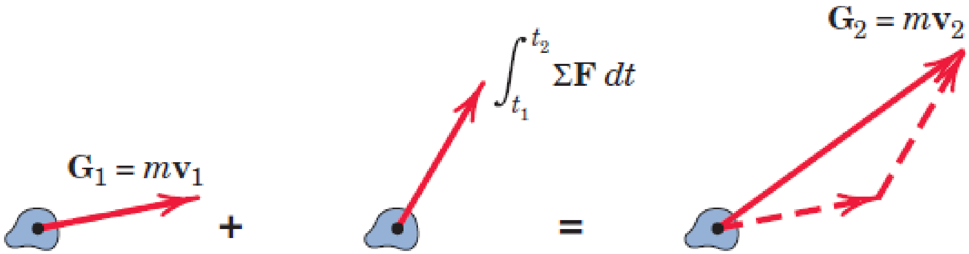


$$\Sigma F_x = \dot{G}_x \qquad \Sigma F_y = \dot{G}_y \qquad \Sigma F_z = \dot{G}_z$$

3/9 Linear Impulse and Linear Momentum

The Linear Impulse-Momentum Principle

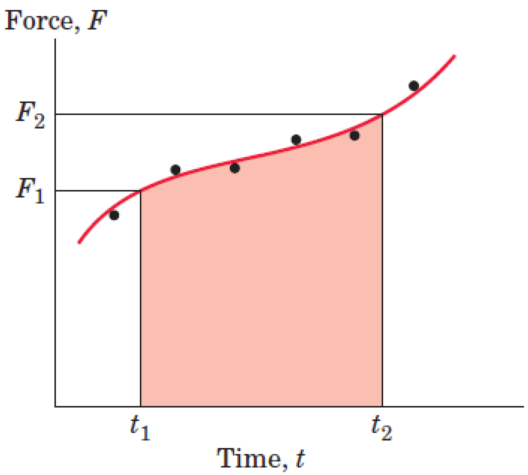
$$\mathbf{G}_1 + \int_{t_1}^{t_2} \Sigma\mathbf{F}\, dt = \mathbf{G}_2$$



$$m(v_1)_x + \int_{t_1}^{t_2} \Sigma F_x\, dt = m(v_2)_x$$

$$m(v_1)_y + \int_{t_1}^{t_2} \Sigma F_y\, dt = m(v_2)_y$$

$$m(v_1)_z + \int_{t_1}^{t_2} \Sigma F_z\, dt = m(v_2)_z$$



3/9 Linear Impulse and Linear Momentum

Conservation of Linear Momentum

- If the resultant force on a particle is zero during an interval of time, its linear momentum **G** remain constant.

- In this case, the linear momentum of the particle is said to be *conserved*.

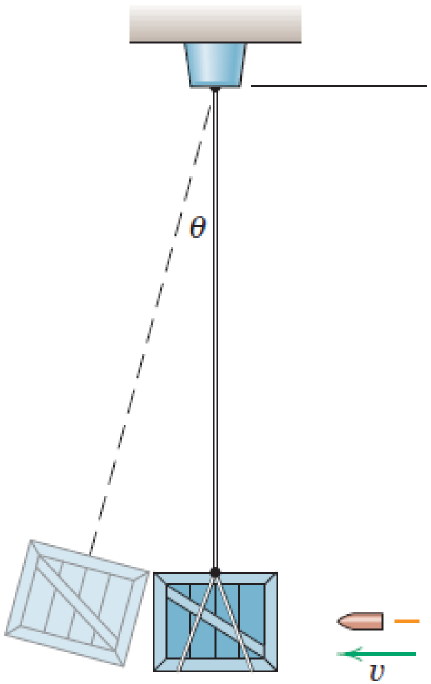
$$\Delta \mathbf{G} = \mathbf{0} \qquad \text{or} \qquad \mathbf{G}_1 = \mathbf{G}_2$$

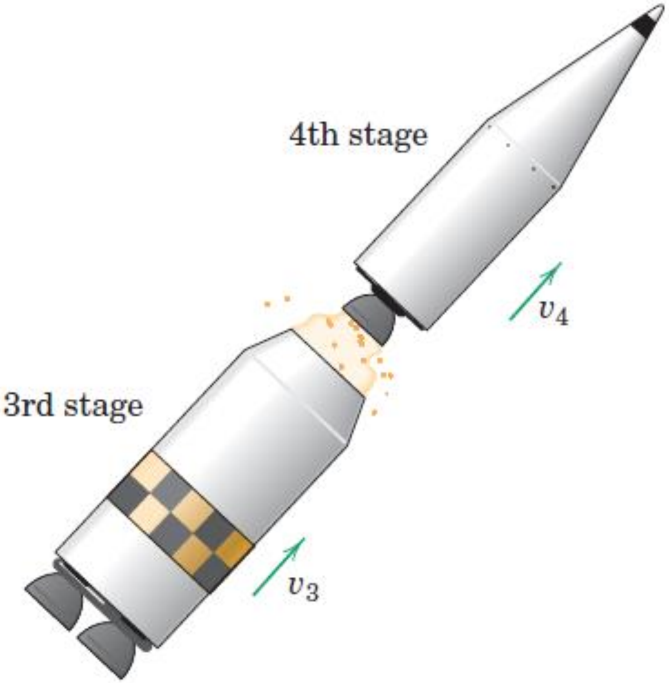
Principle of conservation of linear momentum

دانشگاه سمنان

3/9 Linear Impulse and Linear Momentum

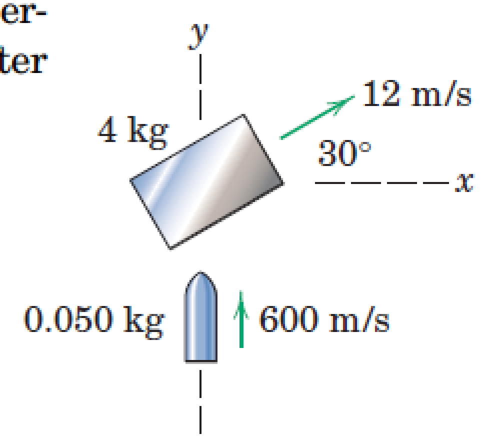
Conservation of Linear Momentum





SAMPLE PROBLEM 3/23

The 50-g bullet traveling at 600 m/s strikes the 4-kg block centrally and is embedded within it. If the block slides on a smooth horizontal plane with a velocity of 12 m/s in the direction shown prior to impact, determine the velocity $\mathbf{v}_2$ of the block and embedded bullet immediately after impact.

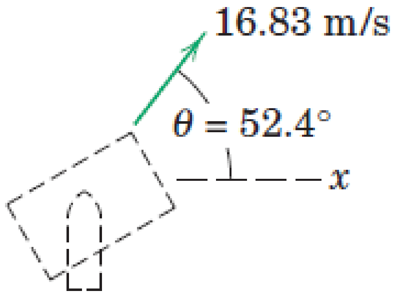


$$[\mathbf{G}_1 = \mathbf{G}_2] \quad 0.050(600\mathbf{j}) + 4(12)(\cos 30°\mathbf{i} + \sin 30°\mathbf{j}) = (4 + 0.050)\mathbf{v}_2$$

$$\mathbf{v}_2 = 10.26\mathbf{i} + 13.33\mathbf{j} \text{ m/s}$$

$$\left[v = \sqrt{v_x^2 + v_y^2}\right] \qquad v_2 = \sqrt{(10.26)^2 + (13.33)^2} = 16.83 \text{ m/s}$$

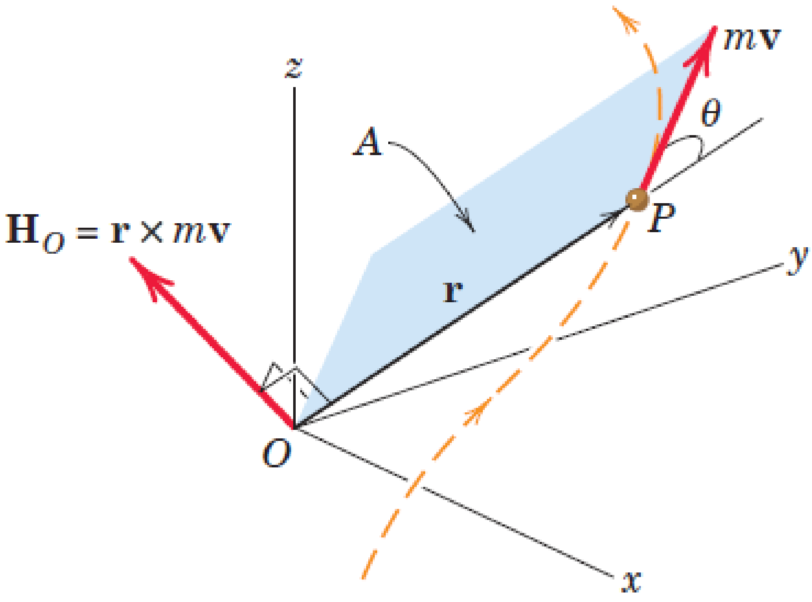
$$[\tan\theta = v_y/v_x] \qquad \tan\theta = \frac{13.33}{10.26} = 1.299 \qquad \theta = 52.4°$$



3/10 Angular Impulse and Angular Momentum

- A particle P of mass m moving along a curve in space.
- The velocity of the particle is v, and its linear momentum is G = mv.
- The moment of the linear momentum vector mv about the origin O is defined as the angular momentum H_O of P about O.

$$\mathbf{H}_O = \mathbf{r} \times m\mathbf{v}$$

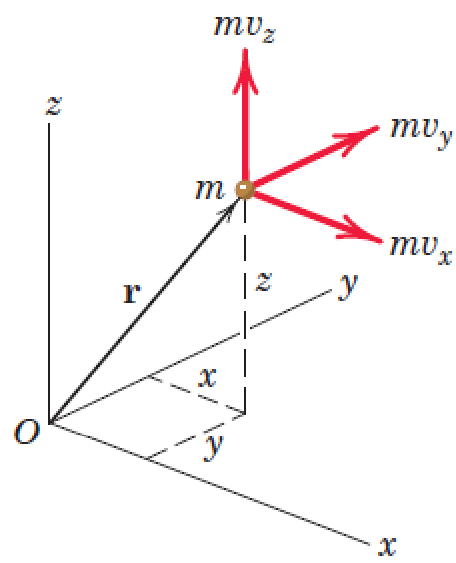


3/10 Angular Impulse and Angular Momentum

- The scalar components of angular momentum:

$$\mathbf{H}_O = \mathbf{r} \times m\mathbf{v} = m(v_z y - v_y z)\mathbf{i} + m(v_x z - v_z x)\mathbf{j} + m(v_y x - v_x y)\mathbf{k}$$

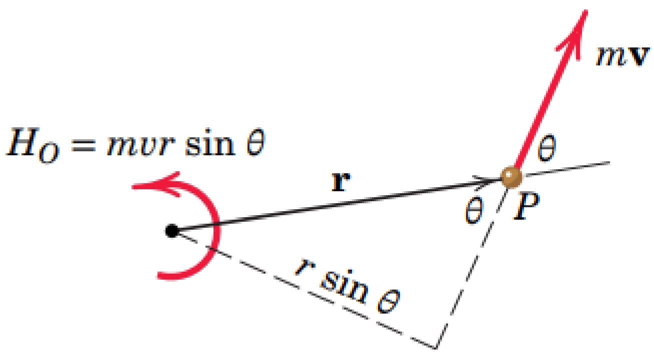
$$\mathbf{H}_O = m \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x & y & z \\ v_x & v_y & v_z \end{vmatrix}$$



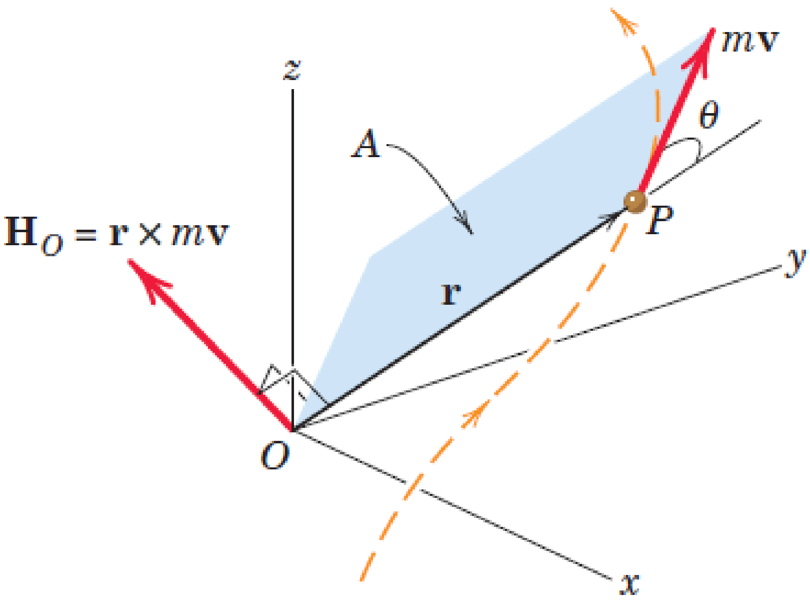
$$H_x = m(v_z y - v_y z) \qquad H_y = m(v_x z - v_z x) \qquad H_z = m(v_y x - v_x y)$$

3/10 Angular Impulse and Angular Momentum

- Two- dimensional representation:



View in plane A



3/10 Angular Impulse and Angular Momentum

Rate of Change of Angular Momentum

- The moment of the forces acting on the particle P to its angular momentum relation:

$$\Sigma \mathbf{M}_O = \mathbf{r} \times \Sigma \mathbf{F} = \mathbf{r} \times m\dot{\mathbf{v}}$$

- H_O Differentiation leads to:

$$\dot{\mathbf{H}}_O = \dot{\mathbf{r}} \times m\mathbf{v} + \mathbf{r} \times m\dot{\mathbf{v}} = \mathbf{v} \times m\mathbf{v} + \mathbf{r} \times m\dot{\mathbf{v}}$$

- So:

$$\Sigma \mathbf{M}_O = \dot{\mathbf{H}}_O$$

$$\Sigma M_{O_x} = \dot{H}_{O_x} \qquad \Sigma M_{O_y} = \dot{H}_{O_y} \qquad \Sigma M_{O_z} = \dot{H}_{O_z}$$

  - The moment about the fixed point O of all forces acting on m equals the time rate of change of angular momentum of m about O.

3/10 Angular Impulse and Angular Momentum

The Angular Impulse-Momentum Principle

- The total angular impulse on m about the fixed point O equals the corresponding change in angular momentum of m about O.

$$\int_{t_1}^{t_2} \Sigma \mathbf{M}_O \, dt = (\mathbf{H}_O)_2 - (\mathbf{H}_O)_1 = \Delta \mathbf{H}_O$$

$$(\mathbf{H}_O)_1 + \int_{t_1}^{t_2} \Sigma \mathbf{M}_O \, dt = (\mathbf{H}_O)_2$$

$$(H_{O_x})_1 + \int_{t_1}^{t_2} \Sigma M_{O_x} \, dt = (H_{O_x})_2$$

$$m(v_z y - v_y z)_1 + \int_{t_1}^{t_2} \Sigma M_{O_x} \, dt = m(v_z y - v_y z)_2$$

3/10 Angular Impulse and Angular Momentum

Conservation of Angular Momentum

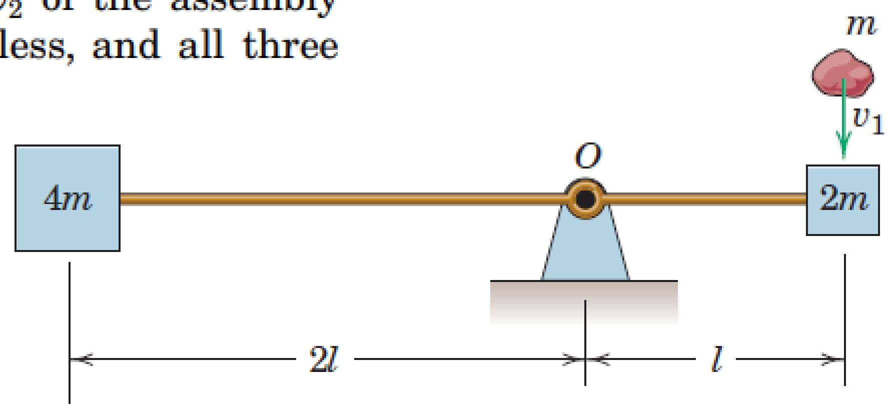
- If the resultant moment about a fixed point O of all forces acting on a particle is zero during an interval of time, its angular momentum H_O about that point remain constant.
- In this case, the angular momentum of the particle is said to be conserved.

$$\Delta \mathbf{H}_O = \mathbf{0} \quad \text{or} \quad (\mathbf{H}_O)_1 = (\mathbf{H}_O)_2$$

Principle of conservation of angular momentum

SAMPLE PROBLEM 3/26

The assembly of the light rod and two end masses is at rest when it is struck by the falling wad of putty traveling with speed v_1 as shown. The putty adheres to and travels with the right-hand end mass. Determine the angular velocity $\dot{\theta}_2$ of the assembly just after impact. The pivot at O is frictionless, and all three masses may be assumed to be particles.



$$(H_O)_1 = (H_O)_2$$

$$mv_1 l = (m + 2m)(l\dot{\theta}_2)l + 4m(2l\dot{\theta}_2)2l$$

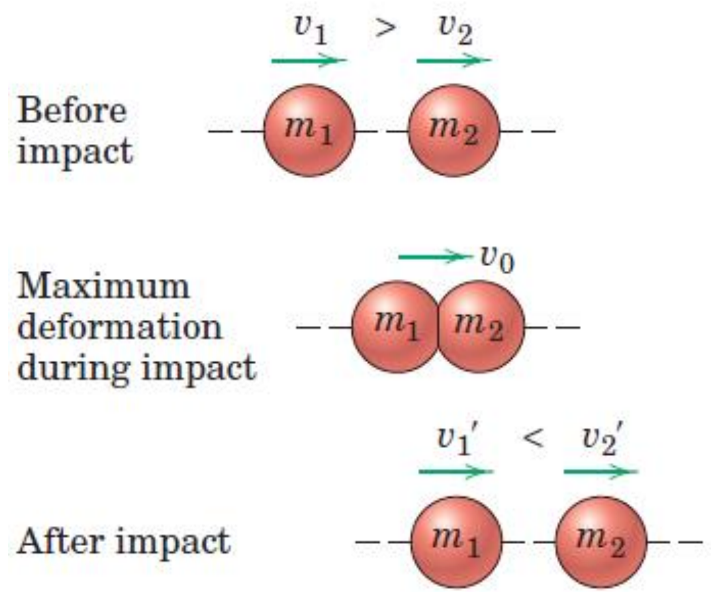
$$\dot{\theta}_2 = \frac{v_1}{19l} \text{ CW}$$

3/12 Impact

Direct Central Impact

- Collision of two spheres with collinear motion
  - Conservation of linear momentum:

$$m_1 v_1 + m_2 v_2 = m_1 {v_1}' + m_2 {v_2}'$$



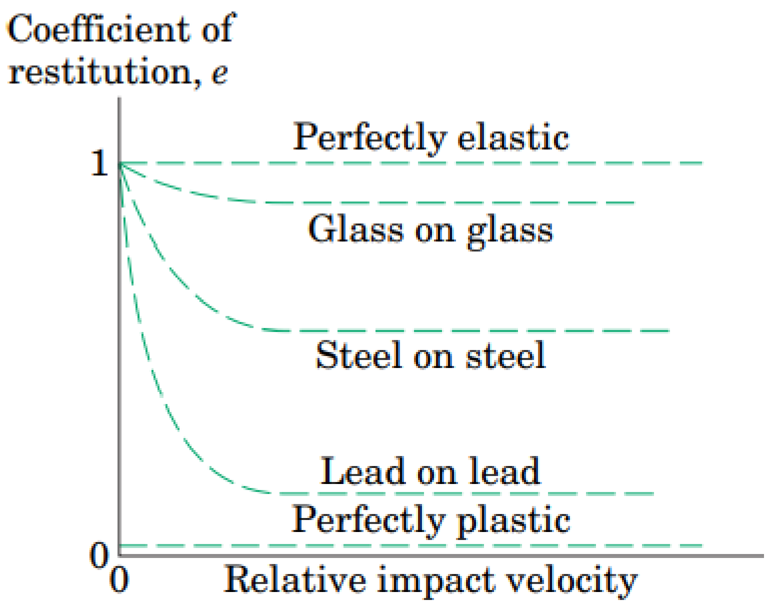
Coefficient of Restitution

$$e = \frac{{v_2}' - {v_1}'}{v_1 - v_2}$$

3/12 Impact

Energy Loss During Impact

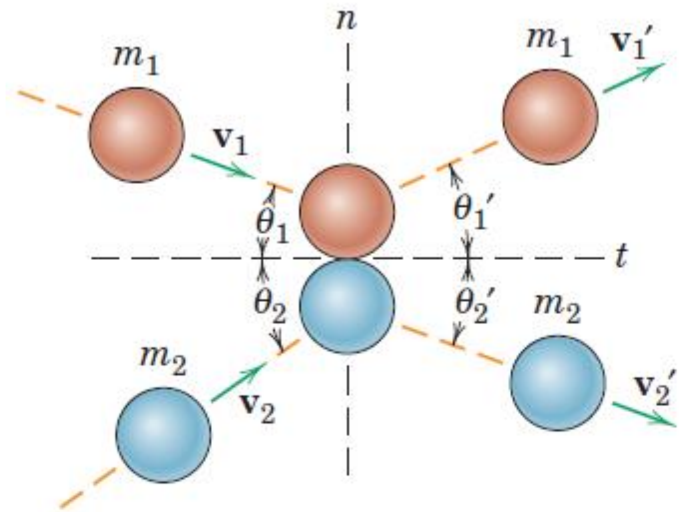
- Impact phenomena are almost always accompanied by energy loss, which may be calculated by subtracting the kinetic energy of the system just after impact from that just before impact.



3/12 Impact

Oblique Central Impact

- Tangent and normal directions



$$m_1(v_1)_t = m_1(v_1{}')_t$$

$$m_2(v_2)_t = m_2(v_2{}')_t$$

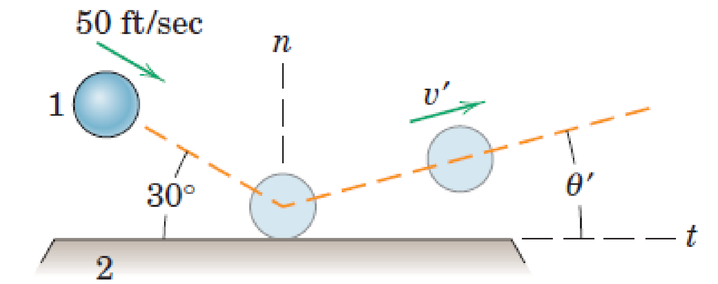
$$m_1(v_1)_n + m_2(v_2)_n = m_1(v_1{}')_n + m_2(v_2{}')_n$$

$$e = \frac{(v_2{}')_n - (v_1{}')_n}{(v_1)_n - (v_2)_n}$$

دانشگاه سمنان

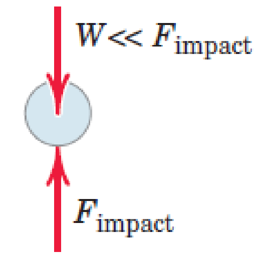
SAMPLE PROBLEM 3/29

A ball is projected onto the heavy plate with a velocity of 50 ft/sec at the 30° angle shown. If the effective coefficient of restitution is 0.5, compute the rebound velocity v' and its angle θ'.



$$e = \frac{(v_2')_n - (v_1')_n}{(v_1)_n - (v_2)_n} \qquad 0.5 = \frac{0 - (v_1')_n}{-50 \sin 30° - 0}$$

$$(v_1')_n = 12.5 \text{ ft/sec}$$



$$m(v_1)_t = m(v_1')_t \qquad (v_1')_t = (v_1)_t = 50 \cos 30° = 43.3 \text{ ft/sec}$$

$$v' = \sqrt{(v_1')_n{}^2 + (v_1')_t{}^2} = \sqrt{12.5^2 + 43.3^2} = 45.1 \text{ ft/sec}$$

$$\theta' = \tan^{-1}\left(\frac{(v_1')_n}{(v_1')_t}\right) = \tan^{-1}\left(\frac{12.5}{43.3}\right) = 16.10°$$

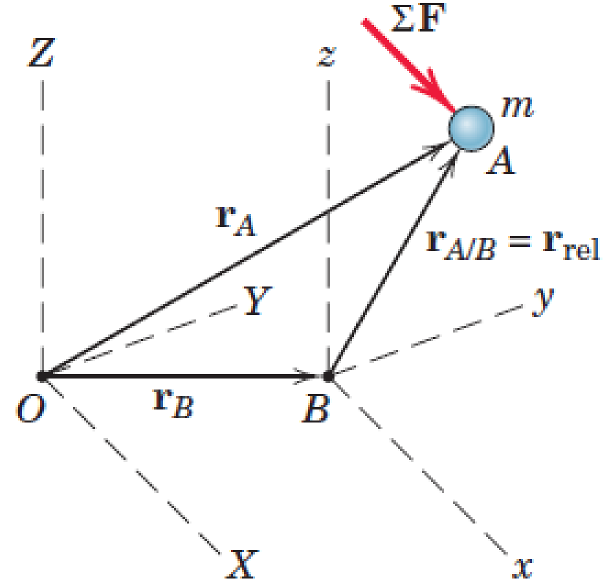
3/14 Relative Motion

Relative-Motion Equation

- A particle **A** of mass m whose motion is observed from a set of axes x-y-z which translate with respect to a fixed reference frame X-Y-Z.

$$\mathbf{a}_A = \mathbf{a}_B + \mathbf{a}_{\text{rel}}$$

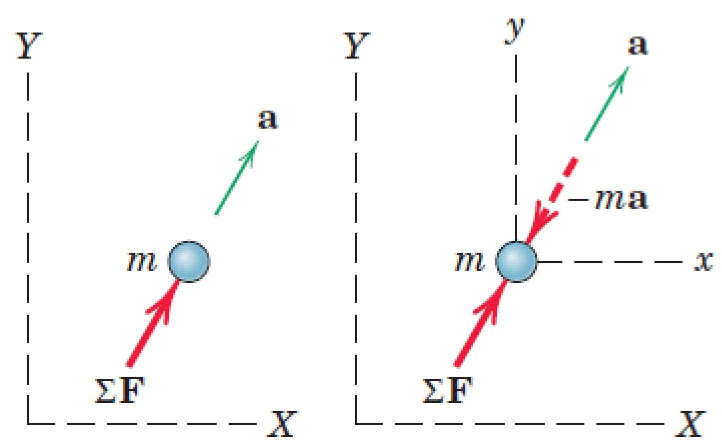
$$\Sigma \mathbf{F} = m(\mathbf{a}_B + \mathbf{a}_{\text{rel}})$$



3/14 Relative Motion

D'Alembert's Principle

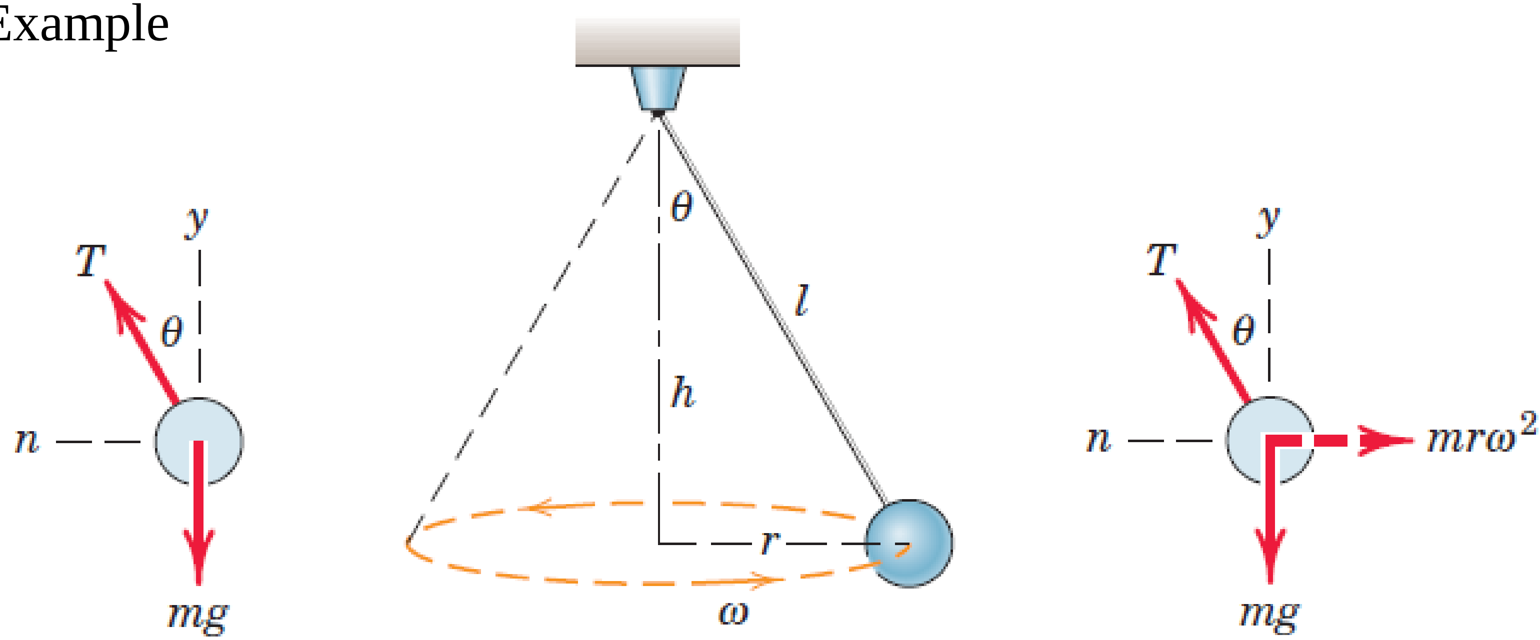
- The particle acceleration we measure from a fixed set of axes X-Y-Z, is its absolute acceleration a. In this case the familiar relation ΣF = ma applies.
- When we observe the particle from a moving system x-y-z attached to the particle, the particle necessarily appears to be at rest or in equilibrium in x-y-z.
- Thus, the observer who is accelerating with x-y-z concludes that a force −ma acts on the particle to balance ΣF.



3/14 Relative Motion

D'Alembert's Principle

- Example



- ***Contents:***

4/1 Introduction

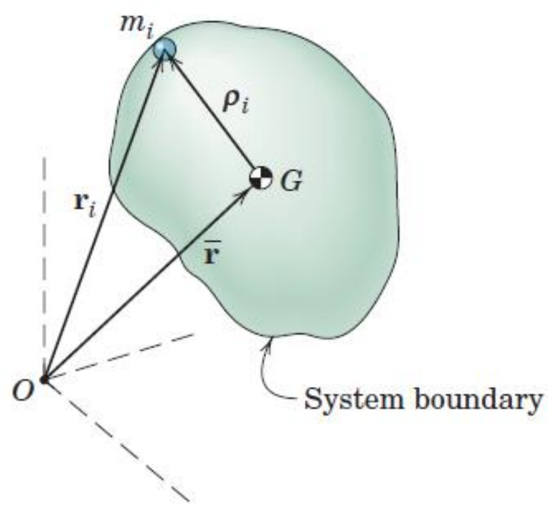
- In the previous two chapters, we have applied the principles of dynamics to the motion of a particle.

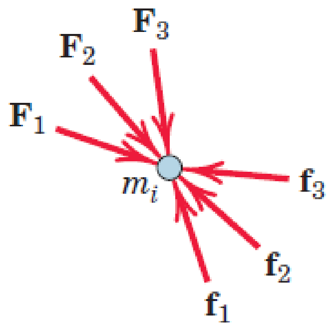
- Our next major step in the development of dynamics is to extend these principles, which we applied to a single particle, to describe the motion of a general system of particles.

- Recall that a rigid body is a solid system of particles wherein the distances between particles remain essentially unchanged.

4/2 Generalized Newton's Second Law

- Considering n mass particles bounded by a closed surface in space





- Forces F1, F2, F3, . . . acting on m_i from sources external to the envelop
- Forces f1, f2, f3, . . . acting on m_i from sources internal to the system boundary

4/2 Generalized Newton's Second Law

- The center of mass G of the isolated system of particles

$$m\bar{\mathbf{r}} = \Sigma m_i \mathbf{r}_i \qquad m = \Sigma m_i$$

- Newton's second law when applied to m_i gives:

$$\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \cdots + \mathbf{f}_1 + \mathbf{f}_2 + \mathbf{f}_3 + \cdots = m_i \ddot{\mathbf{r}}_i$$

$$\rightarrow \quad \Sigma\mathbf{F} + \Sigma\mathbf{f} = \Sigma m_i \ddot{\mathbf{r}}_i$$

- Substitution into the summation of the equations of motion gives:

$$\boxed{\Sigma\mathbf{F} = m\ddot{\bar{\mathbf{r}}} \qquad \text{or} \qquad \mathbf{F} = m\bar{\mathbf{a}}}$$

$$\Sigma F_x = m\bar{a}_x \qquad \Sigma F_y = m\bar{a}_y \qquad \Sigma F_z = m\bar{a}_z$$

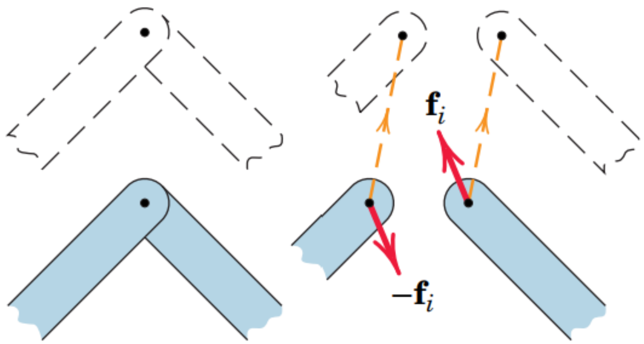
4/3 Work-Energy

Work-Energy Relation

$$(U_{1\text{-}2})_i = \Delta T_i \quad \rightarrow \quad U_{1\text{-}2} = \Delta T \quad \text{or} \quad T_1 + U_{1\text{-}2} = T_2$$

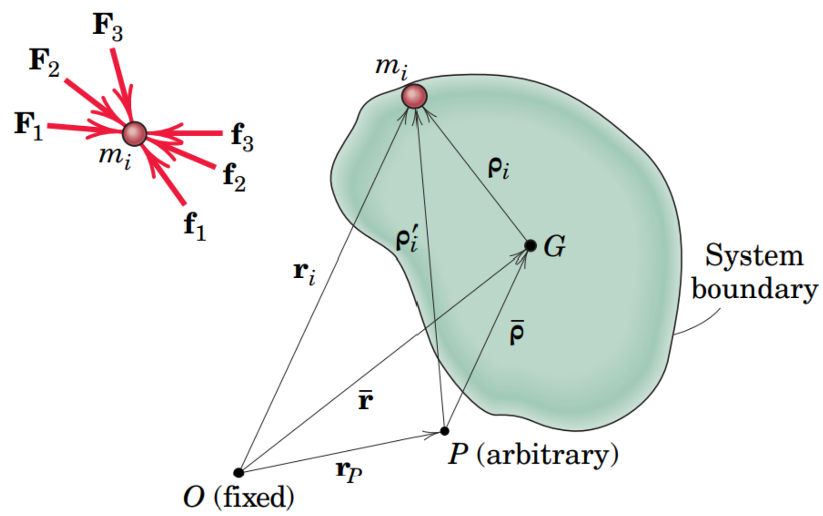
$$U'_{1\text{-}2} = \Delta T + \Delta V \qquad T_1 + V_1 + U'_{1\text{-}2} = T_2 + V_2$$

- For a rigid body or a system of rigid bodies joined by ideal frictionless connections, no net work is done by the internal interacting forces or moments in the connections.



4/3 Work-Energy

Kinetic Energy Expression



$$T = \Sigma \frac{1}{2} m_i v_i^{\,2}$$

$$\mathbf{v}_i = \bar{\mathbf{v}} + \dot{\boldsymbol{\rho}}_i$$

$$T = \Sigma \frac{1}{2} m_i \mathbf{v}_i \cdot \mathbf{v}_i = \Sigma \frac{1}{2} m_i (\bar{\mathbf{v}} + \dot{\boldsymbol{\rho}}_i) \cdot (\bar{\mathbf{v}} + \dot{\boldsymbol{\rho}}_i)$$
$$= \Sigma \frac{1}{2} m_i \bar{v}^2 + \Sigma \frac{1}{2} m_i |\dot{\boldsymbol{\rho}}_i|^2 + \Sigma m_i \bar{\mathbf{v}} \cdot \dot{\boldsymbol{\rho}}_i$$

Because $\boldsymbol{\rho}_i$ is measured from the mass center, $\Sigma m_i \boldsymbol{\rho}_i = \mathbf{0}$

$$T = \frac{1}{2} m \bar{v}^2 + \Sigma \frac{1}{2} m_i |\dot{\boldsymbol{\rho}}_i|^2$$

4/4 Impulse-Momentum

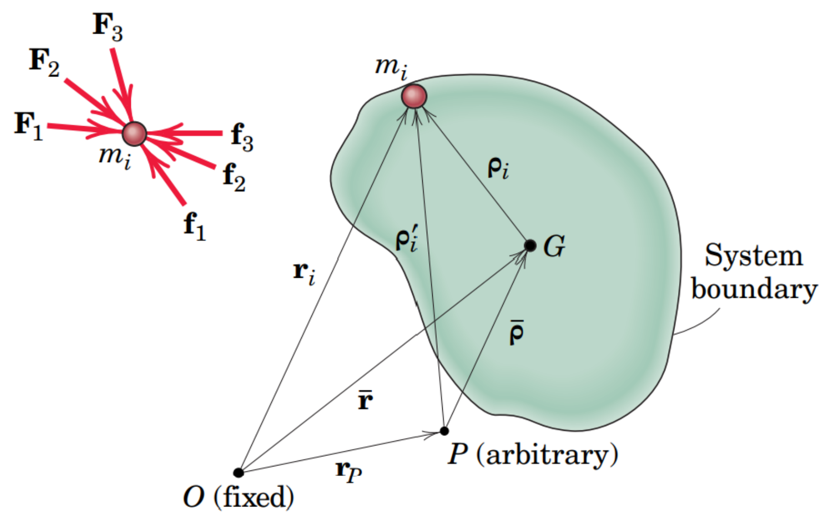
Linear Momentum

$$\mathbf{G} = \Sigma m_i(\bar{\mathbf{v}} + \dot{\boldsymbol{\rho}}_i) = \Sigma m_i \bar{\mathbf{v}} + \frac{d}{dt}\Sigma m_i \boldsymbol{\rho}_i$$

$$= \bar{\mathbf{v}}\,\Sigma m_i + \frac{d}{dt}(\mathbf{0})$$

$$\mathbf{G} = m\bar{\mathbf{v}}$$

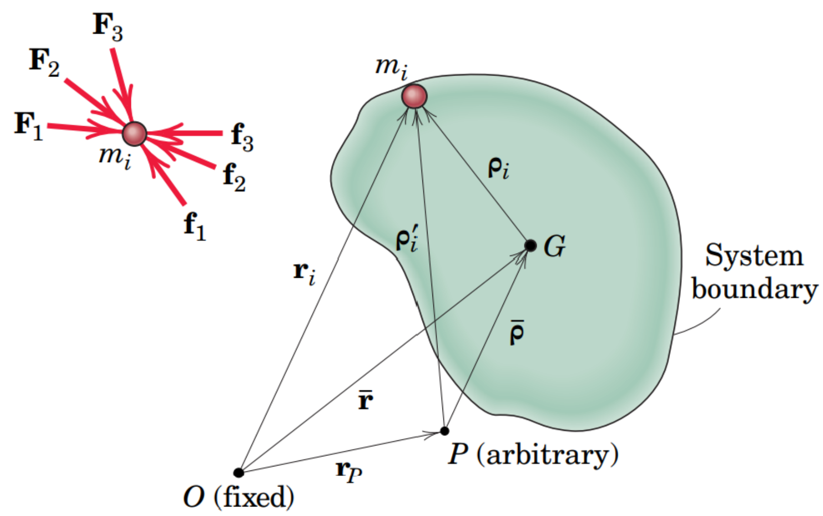
$$\Sigma\mathbf{F} = \dot{\mathbf{G}}$$



4/4 Impulse-Momentum

Angular Momentum

About a Fixed Point O.



$$\mathbf{H}_O = \Sigma(\mathbf{r}_i \times m_i\mathbf{v}_i)$$

$$\dot{\mathbf{H}}_O = \Sigma(\dot{\mathbf{r}}_i \times m_i\mathbf{v}_i) + \Sigma(\mathbf{r}_i \times m_i\dot{\mathbf{v}}_i)$$

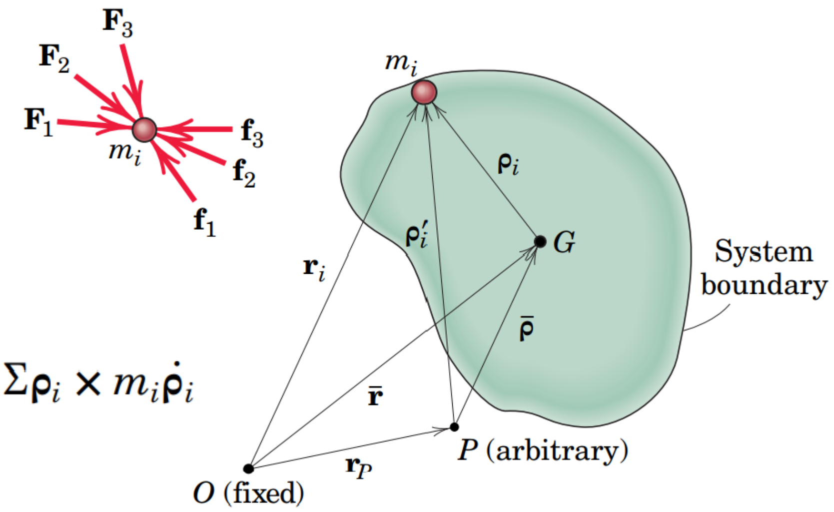
$$\Sigma(\mathbf{r}_i \times m_i\mathbf{a}_i) = \Sigma(\mathbf{r}_i \times \mathbf{F}_i)$$

$$\Sigma\mathbf{M}_O = \dot{\mathbf{H}}_O$$

4/4 Impulse-Momentum

Angular Momentum

About the Mass Center G.



$$\mathbf{H}_G = \Sigma \boldsymbol{\rho}_i \times m_i \dot{\mathbf{r}}_i$$

$$\mathbf{H}_G = \Sigma \boldsymbol{\rho}_i \times m_i(\dot{\bar{\mathbf{r}}} + \dot{\boldsymbol{\rho}}_i) = \Sigma \boldsymbol{\rho}_i \times m_i \dot{\bar{\mathbf{r}}} + \Sigma \boldsymbol{\rho}_i \times m_i \dot{\boldsymbol{\rho}}_i$$

$$-\dot{\bar{\mathbf{r}}} \times \Sigma m_i \boldsymbol{\rho}_i$$

$$\mathbf{H}_G = \Sigma \boldsymbol{\rho}_i \times m_i \dot{\boldsymbol{\rho}}_i$$

$$\dot{\mathbf{H}}_G = \Sigma \dot{\boldsymbol{\rho}}_i \times m_i(\dot{\bar{\mathbf{r}}} + \dot{\boldsymbol{\rho}}_i) + \Sigma \boldsymbol{\rho}_i \times m_i \ddot{\mathbf{r}}_i$$

$$\Sigma \mathbf{M}_G = \dot{\mathbf{H}}_G$$

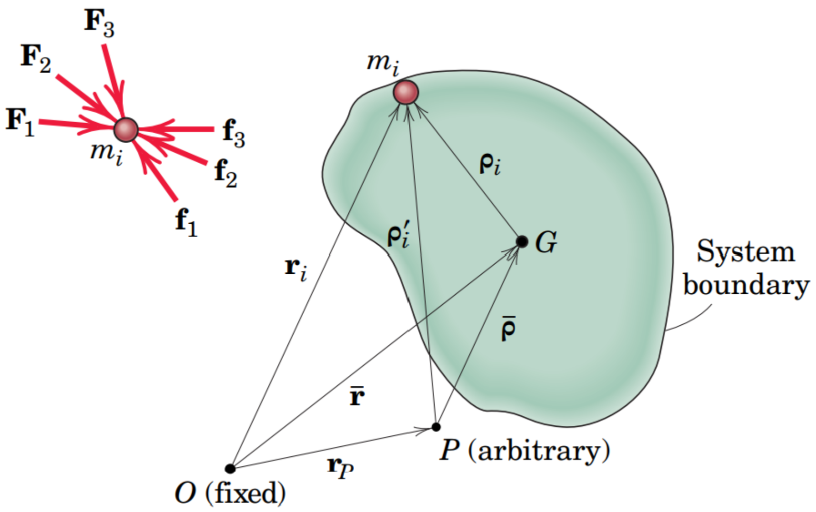
4/4 Impulse-Momentum

Angular Momentum

About an Arbitrary Point P.

$$\mathbf{H}_P = \Sigma \boldsymbol{\rho}_i' \times m_i \dot{\mathbf{r}}_i = \Sigma(\bar{\boldsymbol{\rho}} + \boldsymbol{\rho}_i) \times m_i \dot{\mathbf{r}}_i$$

$$\mathbf{H}_P = \mathbf{H}_G + \bar{\boldsymbol{\rho}} \times m\bar{\mathbf{v}}$$



$$\Sigma\mathbf{M}_P = \Sigma\mathbf{M}_G + \bar{\boldsymbol{\rho}} \times \Sigma\mathbf{F}$$

$$\Sigma\mathbf{M}_P = \dot{\mathbf{H}}_G + \bar{\boldsymbol{\rho}} \times m\bar{\mathbf{a}}$$

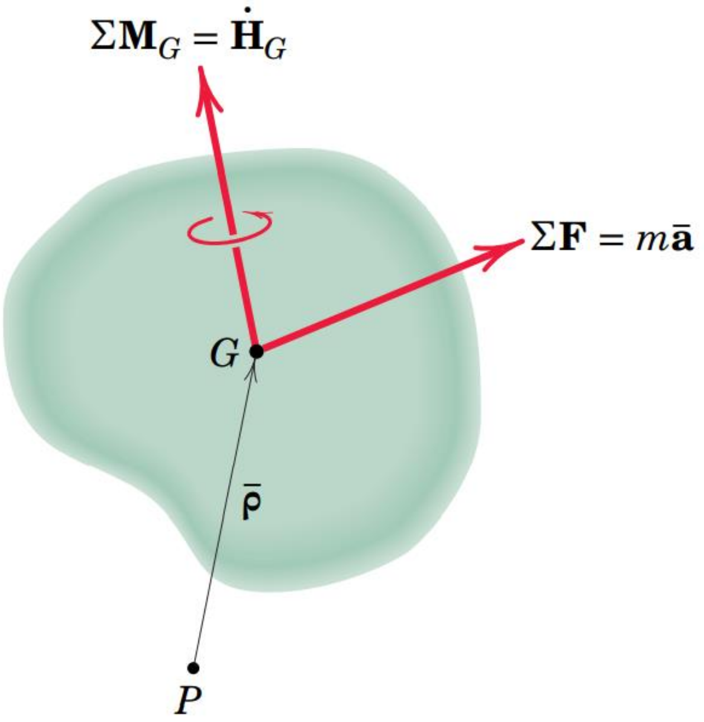
4/4 Impulse-Momentum

Angular Momentum

About an Arbitrary Point P.

$$\mathbf{H}_P = \mathbf{H}_G + \bar{\boldsymbol{\rho}} \times m\bar{\mathbf{v}}$$

$$\Sigma\mathbf{M}_P = \dot{\mathbf{H}}_G + \bar{\boldsymbol{\rho}} \times m\bar{\mathbf{a}}$$



4/5 Conservation of Energy and Momentum

Conservation of Energy

- A mass system is said to be conservative if it does not lose energy by virtue of internal friction forces which do negative work or by virtue of inelastic members which dissipate energy upon cycling.
- If no work is done on a conservative system during an interval of motion by external forces (other than gravity or other potential forces), then none of the energy of the system is lost.

$$\Delta T + \Delta V = 0$$

$$T_1 + V_1 = T_2 + V_2$$

4/5 Conservation of Energy and Momentum

Conservation of Momentum

- The principle of conservation of linear momentum

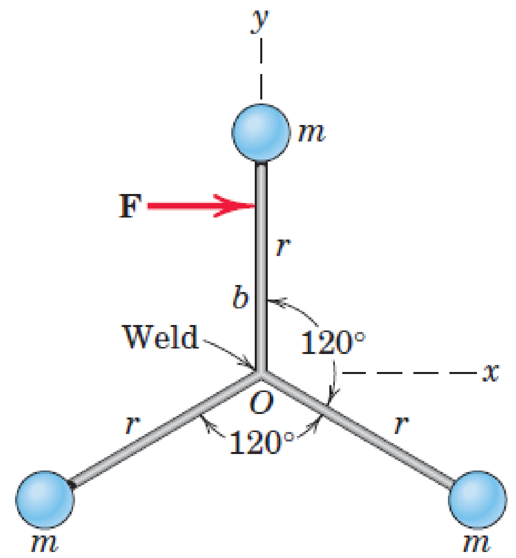
$$\mathbf{G}_1 = \mathbf{G}_2$$

- The principle of conservation of angular momentum

$$(\mathbf{H}_O)_1 = (\mathbf{H}_O)_2 \qquad \text{or} \qquad (\mathbf{H}_G)_1 = (\mathbf{H}_G)_2$$

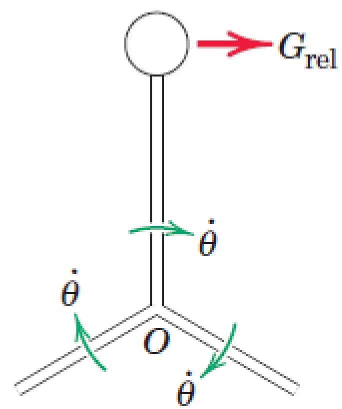
SAMPLE PROBLEM 4/2

Each of the three balls has a mass m and is welded to the rigid equiangular frame of negligible mass. The assembly rests on a smooth horizontal surface. If a force **F** is suddenly applied to one bar as shown, determine (a) the acceleration of point O and (b) the angular acceleration $\ddot{\theta}$ of the frame.



$$[\Sigma \mathbf{F} = m\bar{\mathbf{a}}] \qquad F\mathbf{i} = 3m\bar{\mathbf{a}} \qquad \bar{\mathbf{a}} = \mathbf{a}_O = \frac{F}{3m}\mathbf{i}$$

$$H_O = H_G = 3(mr\dot{\theta})r = 3mr^2\dot{\theta}$$



$$[\Sigma \mathbf{M}_G = \dot{\mathbf{H}}_G] \qquad Fb = \frac{d}{dt}(3mr^2\dot{\theta}) = 3mr^2\ddot{\theta} \qquad \text{so} \qquad \ddot{\theta} = \frac{Fb}{3mr^2}$$

❑ ***Contents:***

5/1 Introduction

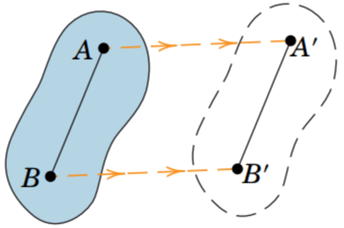
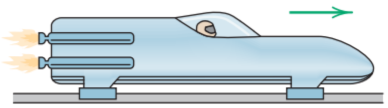
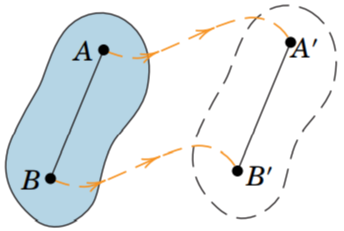
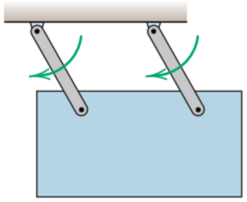
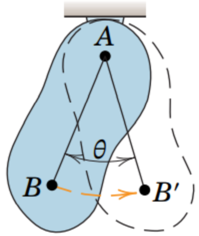
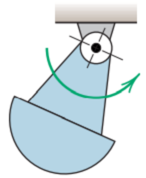
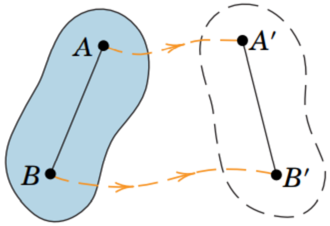
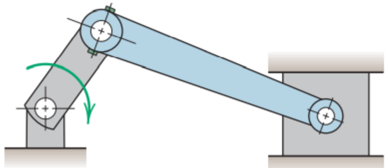
Rigid-Body Assumption

- A rigid body:
  - A system of particles for which the distances between the particles remain unchanged.
  - If the movements associated with the changes in shape are very small compared with the movements of the body as a whole, then the assumption of rigidity is usually acceptable.

5/1 Introduction

Plane Motion

- Translation
- Rotation
- General plane motion

	Type of Rigid-Body Plane Motion	Example
(*a*) Rectilinear translation		 Rocket test sled
(*b*) Curvilinear translation		 Parallel-link swinging plate
(*c*) Fixed-axis rotation		 Compound pendulum
(*d*) General plane motion		 Connecting rod in a reciprocating engine

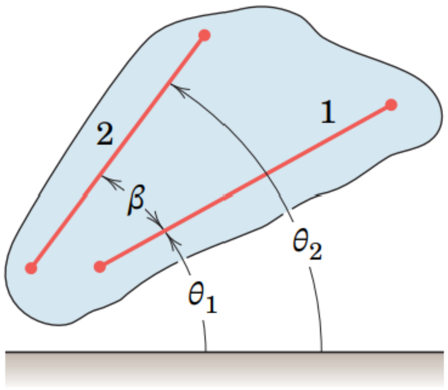
5/2 Rotation

- The rotation of a rigid body is described by its angular motion.

$$\theta_2 = \theta_1 + \beta$$

$$\dot{\theta}_2 = \dot{\theta}_1$$

$$\ddot{\theta}_2 = \ddot{\theta}_1$$



- All lines on a rigid body in its plane of motion have the same angular displacement, the same angular velocity, and the same angular acceleration

5/2 Rotation

Angular-Motion Relations

The angular velocity ω and angular acceleration α

$$\omega = \frac{d\theta}{dt} = \dot{\theta}$$

$$\alpha = \frac{d\omega}{dt} = \dot{\omega} \quad \text{or} \quad \alpha = \frac{d^2\theta}{dt^2} = \ddot{\theta}$$

$$\omega\, d\omega = \alpha\, d\theta \quad \text{or} \quad \dot{\theta}\, d\dot{\theta} = \ddot{\theta}\, d\theta$$

- For rotation with constant angular acceleration:

$$\omega = \omega_0 + \alpha t$$

$$\omega^2 = {\omega_0}^2 + 2\alpha(\theta - \theta_0)$$

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

5/2 Rotation

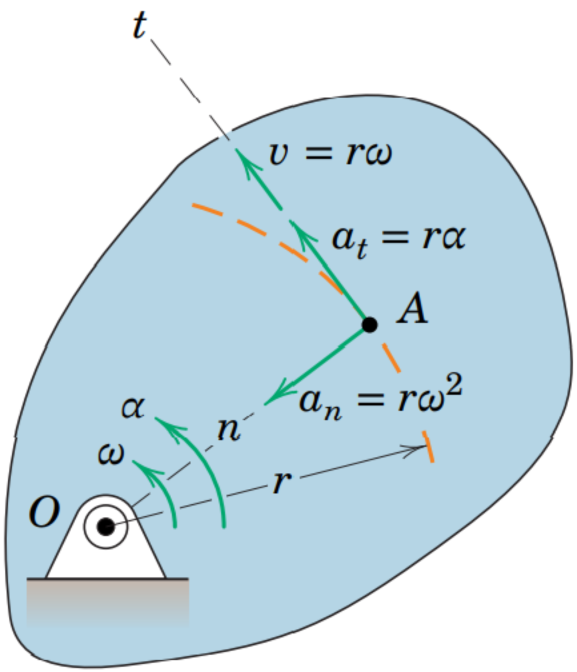
Angular-Motion Relations

Rotation about a Fixed Axis

$$v = r\omega$$

$$a_n = r\omega^2 = v^2/r = v\omega$$

$$a_t = r\alpha$$



5/2 Rotation

Angular-Motion Relations

- Using the cross-product relationship

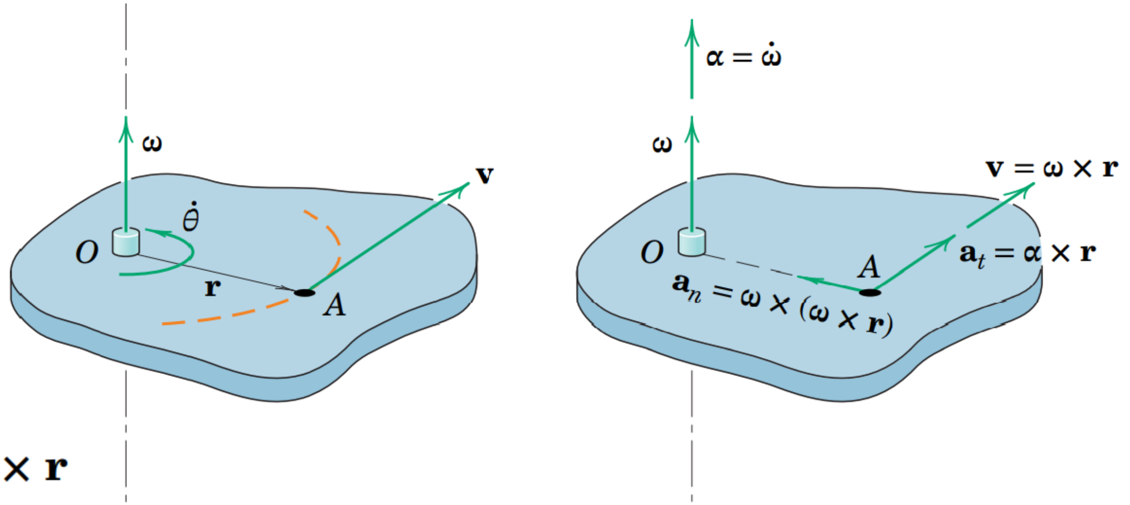
$$\mathbf{v} = \dot{\mathbf{r}} = \boldsymbol{\omega} \times \mathbf{r}$$

$$\mathbf{r} \times \boldsymbol{\omega} = -\mathbf{v}$$

$$\mathbf{a} = \dot{\mathbf{v}} = \boldsymbol{\omega} \times \dot{\mathbf{r}} + \dot{\boldsymbol{\omega}} \times \mathbf{r}$$

$$= \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) + \dot{\boldsymbol{\omega}} \times \mathbf{r}$$

$$= \boldsymbol{\omega} \times \mathbf{v} + \boldsymbol{\alpha} \times \mathbf{r}$$



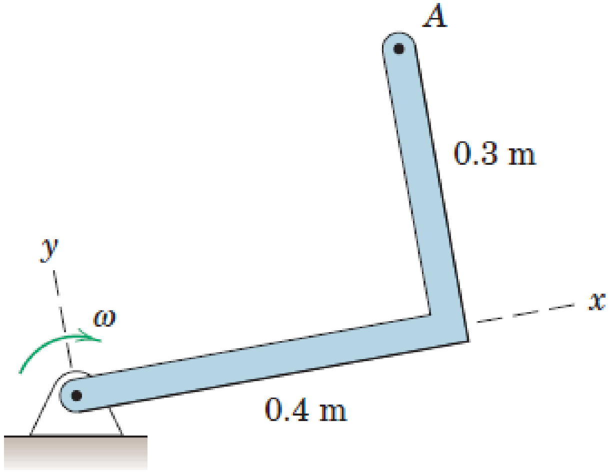
$$\mathbf{v} = \boldsymbol{\omega} \times \mathbf{r}$$

$$\mathbf{a}_n = \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r})$$

$$\mathbf{a}_t = \boldsymbol{\alpha} \times \mathbf{r}$$

SAMPLE PROBLEM 5/3

The right-angle bar rotates clockwise with an angular velocity which is decreasing at the rate of 4 rad/s^2. Write the vector expressions for the velocity and acceleration of point A when $\omega = 2$ rad/s.



$\boldsymbol{\omega} = -2\mathbf{k}$ rad/s and $\boldsymbol{\alpha} = +4\mathbf{k}$ rad/s^2

$[\mathbf{v} = \boldsymbol{\omega} \times \mathbf{r}]$	$\mathbf{v} = -2\mathbf{k} \times (0.4\mathbf{i} + 0.3\mathbf{j}) = 0.6\mathbf{i} - 0.8\mathbf{j}$ m/s
$[\mathbf{a}_n = \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r})]$	$\mathbf{a}_n = -2\mathbf{k} \times (0.6\mathbf{i} - 0.8\mathbf{j}) = -1.6\mathbf{i} - 1.2\mathbf{j}$ m/s^2
$[\mathbf{a}_t = \boldsymbol{\alpha} \times \mathbf{r}]$	$\mathbf{a}_t = 4\mathbf{k} \times (0.4\mathbf{i} + 0.3\mathbf{j}) = -1.2\mathbf{i} + 1.6\mathbf{j}$ m/s^2
$[\mathbf{a} = \mathbf{a}_n + \mathbf{a}_t]$	$\mathbf{a} = -2.8\mathbf{i} + 0.4\mathbf{j}$ m/s^2

$v = \sqrt{0.6^2 + 0.8^2} = 1$ m/s and $a = \sqrt{2.8^2 + 0.4^2} = 2.83$ m/s^2

5/3 Absolute Motion

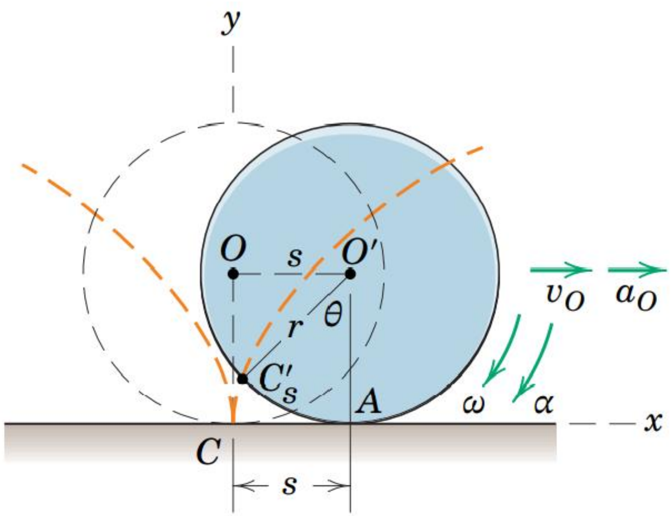
- We now develop the approach of absolute-motion analysis to describe the plane kinematics of rigid bodies.

- In this approach, we make use of the geometric relations which define the configuration of the body involved and then proceed to take the time derivatives of the defining geometric relations to obtain velocities and accelerations.

- The absolute-motion approach to rigid-body kinematics is quite straightforward, provided the configuration lends itself to a geometric description which is not overly complex. If the geometric configuration is awkward or complex, analysis by the principles of relative motion may be preferable.

SAMPLE PROBLEM 5/4

A wheel of radius r rolls on a flat surface without slipping. Determine the angular motion of the wheel in terms of the linear motion of its center O. Also determine the acceleration of a point on the rim of the wheel as the point comes into contact with the surface on which the wheel rolls.



$$s = r\theta$$

$$v_O = r\omega$$

$$a_O = r\alpha$$

$$x = s - r\sin\theta = r(\theta - \sin\theta) \qquad y = r - r\cos\theta = r(1 - \cos\theta)$$

$$\dot{x} = r\dot{\theta}(1 - \cos\theta) = v_O(1 - \cos\theta) \qquad \dot{y} = r\dot{\theta}\sin\theta = v_O\sin\theta$$

$$\ddot{x} = \dot{v}_O(1 - \cos\theta) + v_O\dot{\theta}\sin\theta \qquad \ddot{y} = \dot{v}_O\sin\theta + v_O\dot{\theta}\cos\theta$$

$$= a_O(1 - \cos\theta) + r\omega^2\sin\theta \qquad = a_O\sin\theta + r\omega^2\cos\theta$$

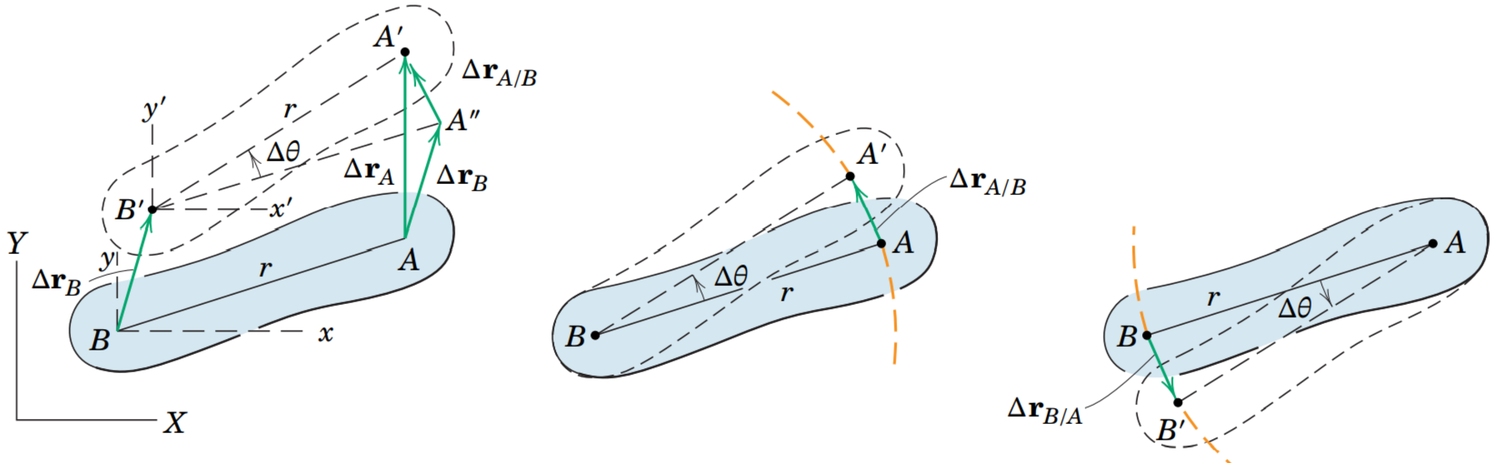
$\theta = 0$ → $\ddot{x} = 0$ and $\ddot{y} = r\omega^2$

$$\mathbf{v} = \dot{x}\mathbf{i} + \dot{y}\mathbf{j} \qquad \mathbf{a} = \ddot{x}\mathbf{i} + \ddot{y}\mathbf{j}$$

5/4 Relative Velocity

- The principles of relative motion: $\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$

Relative Velocity Due to Rotation



Motion relative to B

Motion relative to A

$$\Delta\mathbf{r}_A = \Delta\mathbf{r}_B + \Delta\mathbf{r}_{A/B}$$

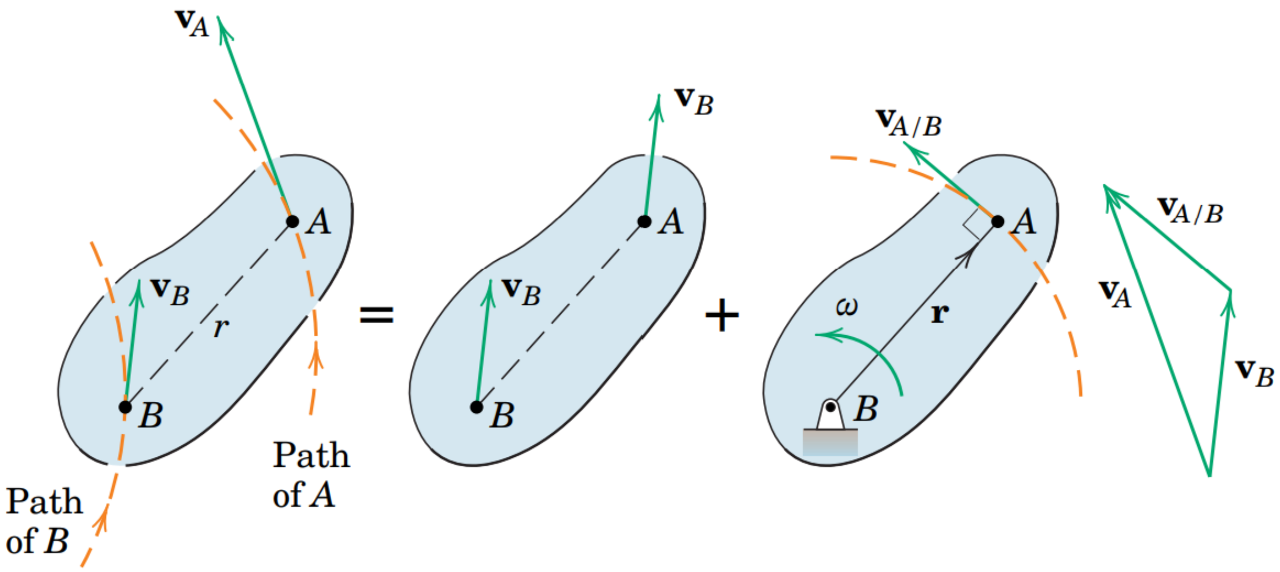
$$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$$

$$v_{A/B} = r\omega$$

$$\mathbf{v}_{A/B} = \boldsymbol{\omega} \times \mathbf{r}$$

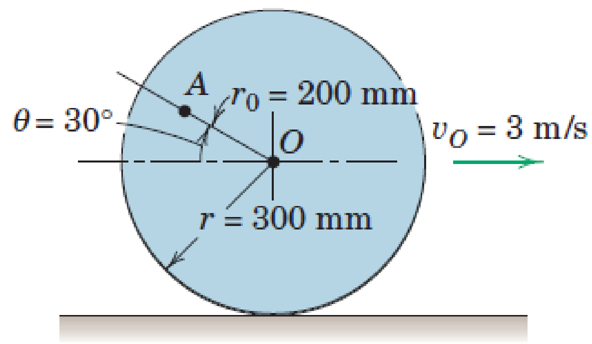
5/4 Relative Velocity

Interpretation of the Relative-Velocity Equation



SAMPLE PROBLEM 5/7

The wheel of radius $r = 300$ mm rolls to the right without slipping and has a velocity $v_O = 3$ m/s of its center O. Calculate the velocity of point A on the wheel for the instant represented.

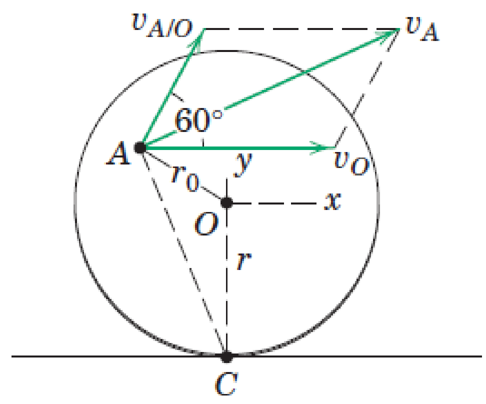


Solution I (Scalar-Geometric)

$$\mathbf{v}_A = \mathbf{v}_O + \mathbf{v}_{A/O}$$

$$[v_{A/O} = r_0\dot{\theta}] \qquad v_{A/O} = 0.2(10) = 2 \text{ m/s}$$

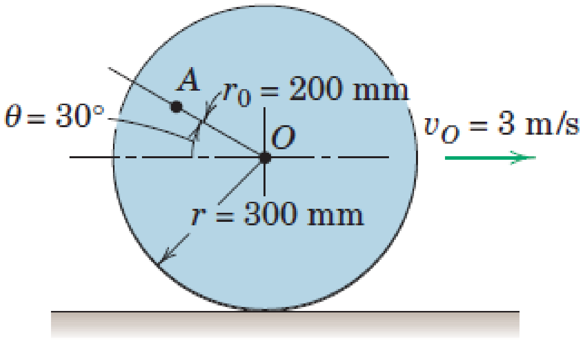
$$v_A{}^2 = 3^2 + 2^2 + 2(3)(2)\cos 60° = 19\ (\text{m/s})^2 \qquad v_A = 4.36 \text{ m/s}$$



$$v_{A/C} = \overline{AC}\omega = \frac{\overline{AC}}{\overline{OC}}v_O = \frac{0.436}{0.300}(3) = 4.36 \text{ m/s} \qquad v_A = v_{A/C} = 4.36 \text{ m/s}$$

SAMPLE PROBLEM 5/7

The wheel of radius $r = 300$ mm rolls to the right without slipping and has a velocity $v_O = 3$ m/s of its center O. Calculate the velocity of point A on the wheel for the instant represented.



Solution II (Vector)

$$\mathbf{v}_A = \mathbf{v}_O + \mathbf{v}_{A/O} = \mathbf{v}_O + \boldsymbol{\omega} \times \mathbf{r}_0$$

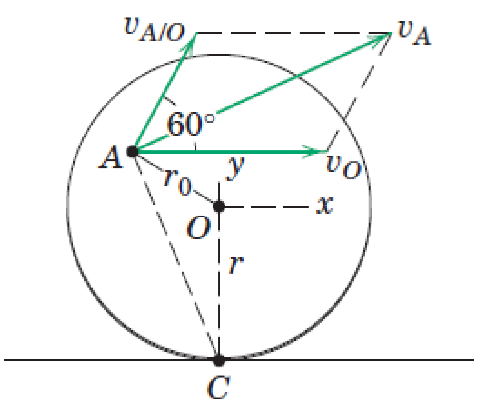
$$\boldsymbol{\omega} = -10\mathbf{k} \text{ rad/s}$$

$$\mathbf{r}_0 = 0.2(-\mathbf{i}\cos 30° + \mathbf{j}\sin 30°) = -0.1732\mathbf{i} + 0.1\mathbf{j} \text{ m}$$

$$\mathbf{v}_O = 3\mathbf{i} \text{ m/s}$$

$$\mathbf{v}_A = 3\mathbf{i} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & -10 \\ -0.1732 & 0.1 & 0 \end{vmatrix} = 3\mathbf{i} + 1.732\mathbf{j} + \mathbf{i}$$

$$= 4\mathbf{i} + 1.732\mathbf{j} \text{ m/s}$$



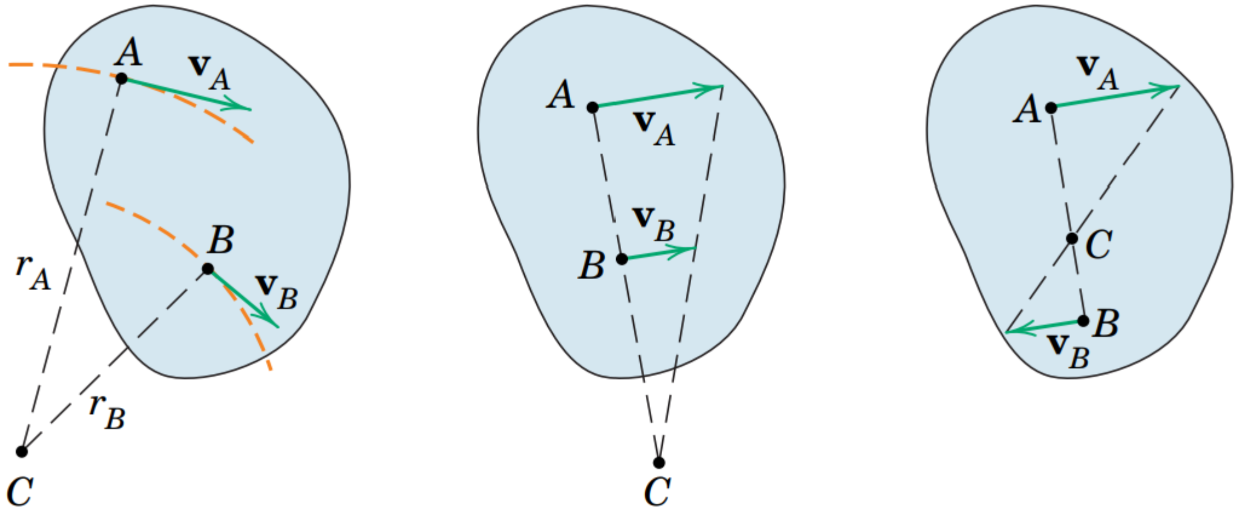
$$v_A = \sqrt{4^2 + (1.732)^2} = \sqrt{19} = 4.36 \text{ m/s}$$

5/5 Instantaneous Center of Zero Velocity

- We can solve the problem by choosing a unique reference point which momentarily has zero velocity.
- As far as velocities are concerned, the body may be considered to be in pure rotation about an axis, normal to the plane of motion, passing through this point.
- This axis is called the instantaneous axis of zero velocity, and the intersection of this axis with the plane of motion is known as the instantaneous center of zero velocity.

5/5 Instantaneous Center of Zero Velocity

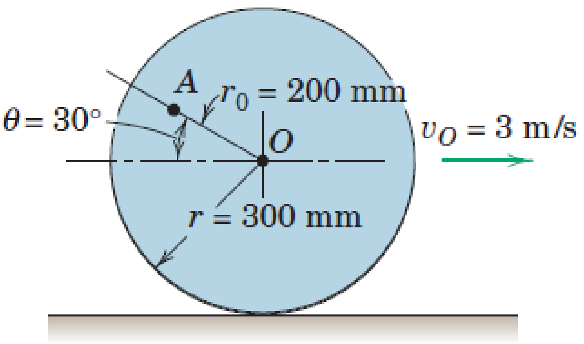
Locating the Instantaneous Center



$$\omega = \frac{v_A}{r_A}$$

SAMPLE PROBLEM 5/11

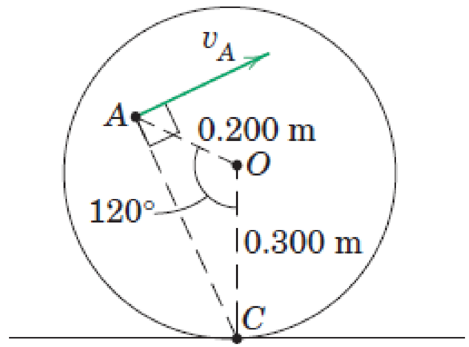
The wheel of Sample Problem 5/7, shown again here, rolls to the right without slipping, with its center O having a velocity $v_O = 3$ m/s. Locate the instantaneous center of zero velocity and use it to find the velocity of point A for the position indicated.



$[\omega = v/r]$ $\qquad \omega = v_O / \overline{OC} = 3/0.300 = 10$ rad/s

$$\overline{AC} = \sqrt{(0.300)^2 + (0.200)^2 - 2(0.300)(0.200)\cos 120^\circ} = 0.436 \text{ m}$$

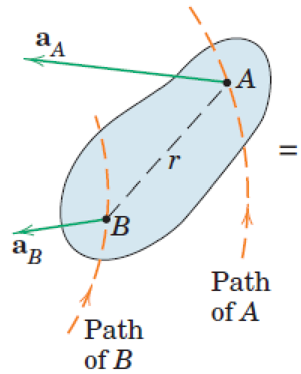
$[v = r\omega]$ $\qquad v_A = \overline{AC}\omega = 0.436(10) = 4.36$ m/s



5/6 Relative Acceleration

- The relative-acceleration equation:

$$\mathbf{a}_A = \mathbf{a}_B + \mathbf{a}_{A/B}$$

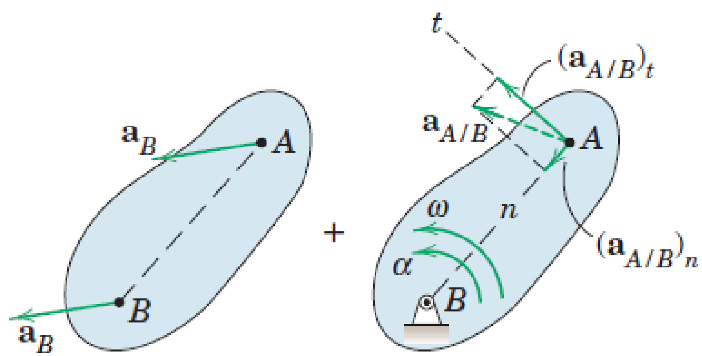


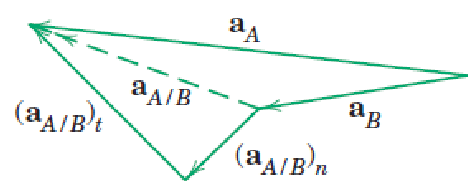
Relative Acceleration Due to Rotation

$$\mathbf{a}_A = \mathbf{a}_B + (\mathbf{a}_{A/B})_n + (\mathbf{a}_{A/B})_t$$

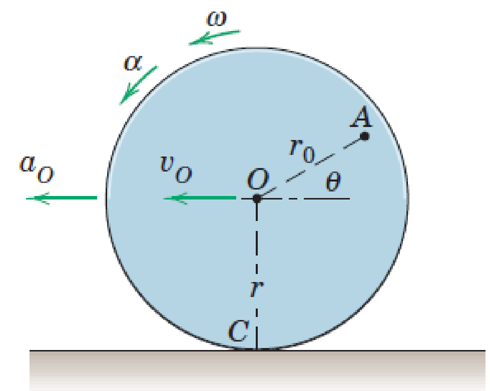
$$(a_{A/B})_n = {v_{A/B}}^2/r = r\omega^2$$
$$(a_{A/B})_t = \dot{v}_{A/B} = r\alpha$$

$$(\mathbf{a}_{A/B})_n = \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r})$$
$$(\mathbf{a}_{A/B})_t = \boldsymbol{\alpha} \times \mathbf{r}$$





SAMPLE PROBLEM 5/13



The wheel of radius r rolls to the left without slipping and, at the instant considered, the center O has a velocity $\mathbf{v}_O$ and an acceleration $\mathbf{a}_O$ to the left. Determine the acceleration of points A and C on the wheel for the instant considered.

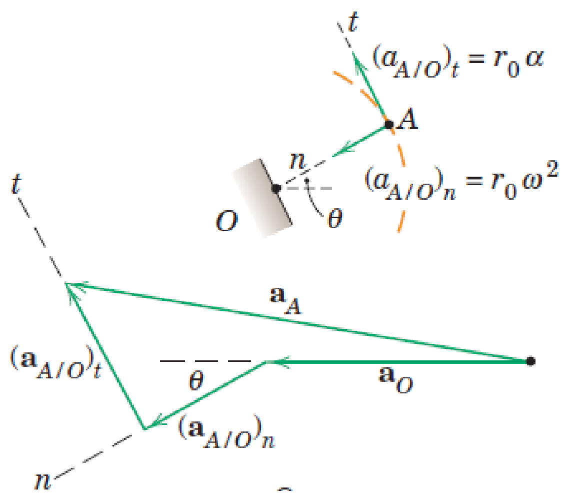
$$\omega = v_O/r \qquad \text{and} \qquad \alpha = a_O/r$$

$$\mathbf{a}_A = \mathbf{a}_O + \mathbf{a}_{A/O} = \mathbf{a}_O + (\mathbf{a}_{A/O})_n + (\mathbf{a}_{A/O})_t$$

$$(a_{A/O})_n = r_0\omega^2 = r_0\left(\frac{v_O}{r}\right)^2$$

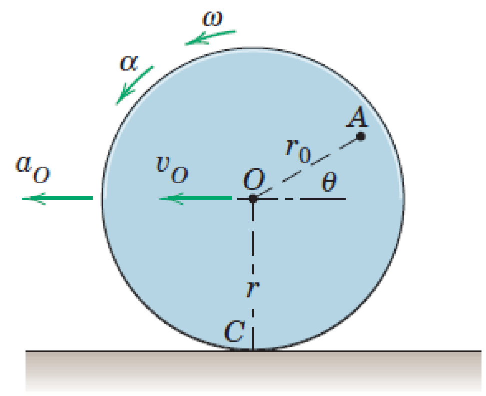
$$(a_{A/O})_t = r_0\alpha = r_0\left(\frac{a_O}{r}\right)$$

$$\begin{aligned} a_A &= \sqrt{(a_A)_n{}^2 + (a_A)_t{}^2} \\ &= \sqrt{[a_O \cos\theta + (a_{A/O})_n]^2 + [a_O \sin\theta + (a_{A/O})_t]^2} \\ &= \sqrt{(r\alpha \cos\theta + r_0\omega^2)^2 + (r\alpha \sin\theta + r_0\alpha)^2} \end{aligned}$$

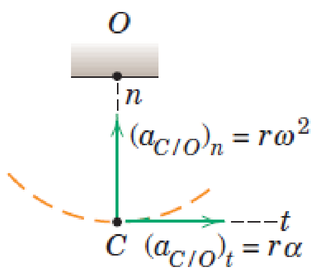


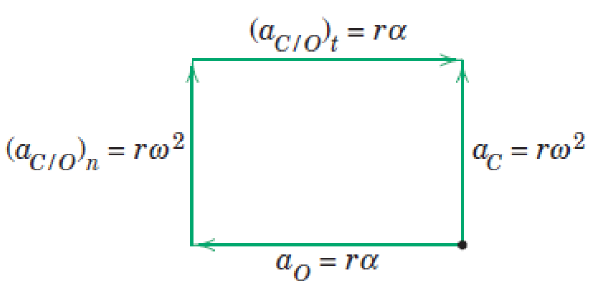
SAMPLE PROBLEM 5/13

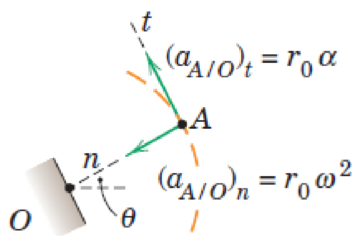
The wheel of radius r rolls to the left without slipping and, at the instant considered, the center O has a velocity $\mathbf{v}_O$ and an acceleration $\mathbf{a}_O$ to the left. Determine the acceleration of points A and C on the wheel for the instant considered.

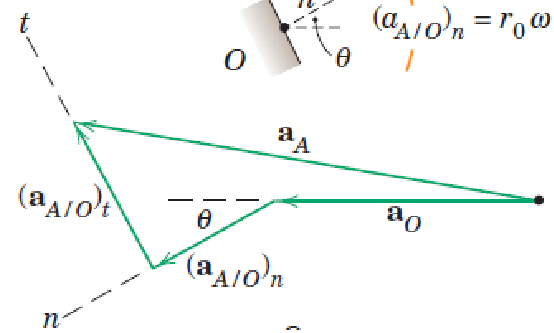


$$\mathbf{a}_C = \mathbf{a}_O + \mathbf{a}_{C/O} \quad \rightarrow \quad a_C = r\omega^2$$









5/7 Motion Relative to Rotating Axes

Rotating versus Nonrotating Systems

$$\mathbf{a}_A = \mathbf{a}_B + \underbrace{\dot{\boldsymbol{\omega}} \times \mathbf{r} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r})}_{\mathbf{a}_{P/B}} + \underbrace{2\boldsymbol{\omega} \times \mathbf{v}_{\text{rel}} + \mathbf{a}_{\text{rel}}}_{\mathbf{a}_{A/P}}$$

$$\mathbf{a}_A = \underbrace{\mathbf{a}_B + \mathbf{a}_{P/B}}_{\mathbf{a}_P} + \mathbf{a}_{A/P}$$

$$\mathbf{a}_A = \mathbf{a}_P + \mathbf{a}_{A/P}$$

$$\mathbf{a}_A = \mathbf{a}_B + \mathbf{a}_{A/B}$$

$$\boxed{\mathbf{a}_A = \mathbf{a}_P + 2\boldsymbol{\omega} \times \mathbf{v}_{\text{rel}} + \mathbf{a}_{\text{rel}}}$$

❑ ***CONTENTS:***

6/1 Introduction

- The kinetics of rigid bodies treats the relationships between the external forces acting on a body and the corresponding translational and rotational motions of the body

- For our purpose, a body which can be approximated as a thin slab with its motion confined to the plane of the slab will be considered to be in plane motion.

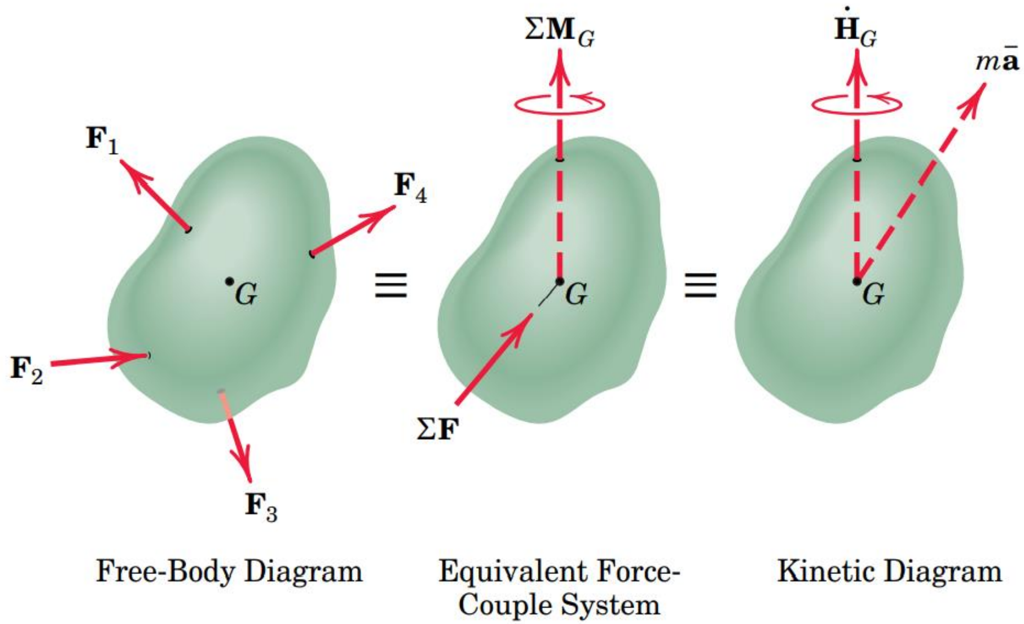
  - ✓Section A: forces and moments to instantaneous linear and angular accelerations relations
  - ✓Section B: method of work and energy
  - ✓Section C: methods of impulse and momentum

SECTION A Force, Mass, and Acceleration

6/2 General Equations of Motion

- The force equation: $\Sigma \mathbf{F} = m\bar{\mathbf{a}}$

- The moment equation taken about the mass center: $\Sigma \mathbf{M}_G = \dot{\mathbf{H}}_G$



6/2 General Equations of Motion

Plane-Motion Equations

- A rigid body moving with plane motion in the x-y plane
  - ✓ The mass moment of inertia:

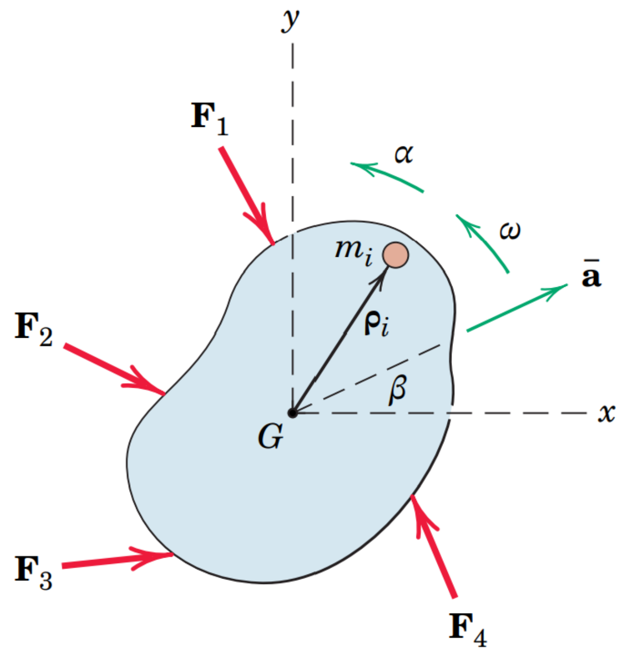
$$H_G = \Sigma \rho_i{}^2 m_i \omega = \omega \Sigma \rho_i{}^2 m_i \rightarrow \int \rho^2 \, dm \rightarrow \bar{I}$$

$$H_G = \bar{I}\omega \rightarrow \Sigma M_G = \dot{H}_G = \bar{I}\dot{\omega} = \bar{I}\alpha$$

$$\Sigma \mathbf{F} = m\bar{\mathbf{a}}$$

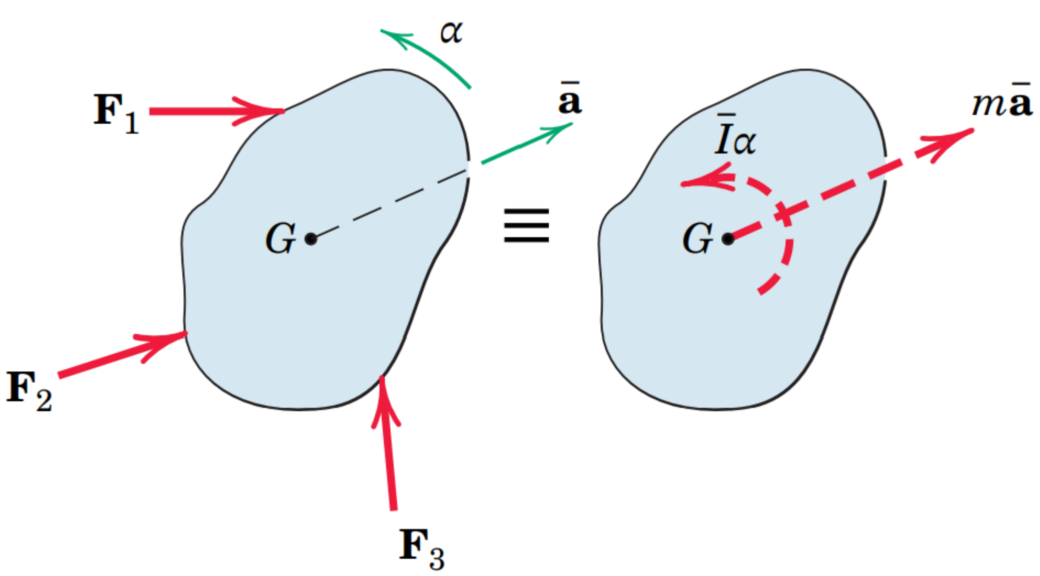
$$\Sigma M_G = \bar{I}\alpha$$

using x-y, n-t, or r-θ coordinates



6/2 General Equations of Motion

- ✓ The free-body diagram discloses the forces and moments appearing on the lefthand side of equations of motion.
- ✓ The kinetic diagram discloses the resulting dynamic response in terms of the translational term and the rotational term which appear on the right-hand side of equations of motion.



Free-Body Diagram Kinetic Diagram

6/2 General Equations of Motion

Alternative Moment Equations

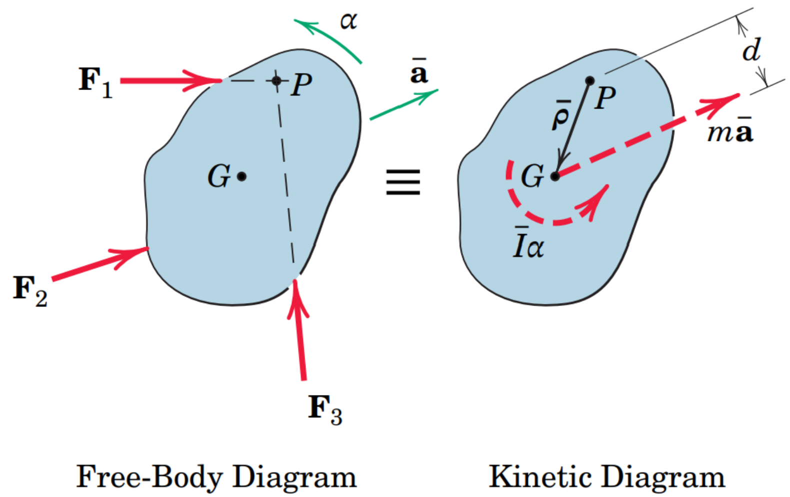
- General equation for moments about an arbitrary point P

$$\Sigma \mathbf{M}_P = \dot{\mathbf{H}}_G + \bar{\boldsymbol{\rho}} \times m\bar{\mathbf{a}}$$

$$\Sigma M_P = \bar{I}\alpha + m\bar{a}d$$

$$\Sigma \mathbf{M}_P = (\dot{\mathbf{H}}_P)_{\text{rel}} + \bar{\boldsymbol{\rho}} \times m\mathbf{a}_P$$

$$\Sigma \mathbf{M}_P = I_P\boldsymbol{\alpha} + \bar{\boldsymbol{\rho}} \times m\mathbf{a}_P$$



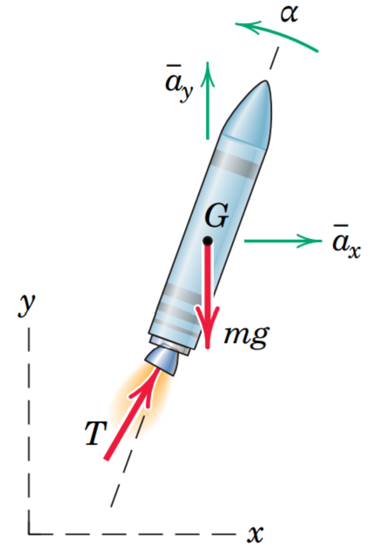
- ✓ When point P becomes a point O fixed in an inertial reference system

$$\Sigma M_O = I_O\alpha$$

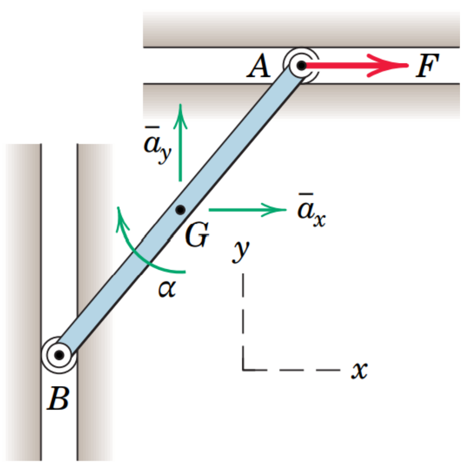
6/2 General Equations of Motion

Unconstrained and Constrained Motion

- The motion of a rigid body may be unconstrained or constrained.
- The two components a_x and a_y of the mass center acceleration and the angular acceleration α may be determined independently or not.
- In general, dynamics problems which involve physical constraints to motion require a kinematic analysis relating linear to angular acceleration before the force and moment equations of motion can be solved.



(a) Unconstrained Motion



(b) Constrained Motion

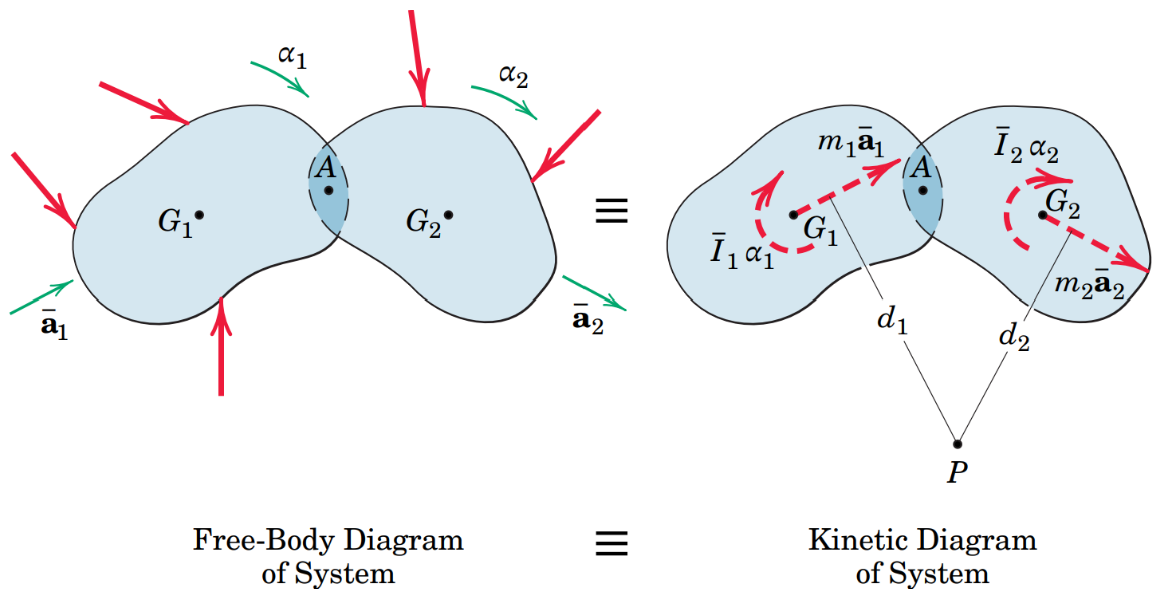
6/2 General Equations of Motion

Systems of Interconnected Bodies

✓ In problems dealing with two or more connected rigid bodies whose motions are related kinematically, it is convenient to analyze the bodies as an entire system.

$$\Sigma \mathbf{F} = \Sigma m\bar{\mathbf{a}}$$

$$\Sigma M_P = \Sigma \bar{I}\alpha + \Sigma m\bar{a}d$$



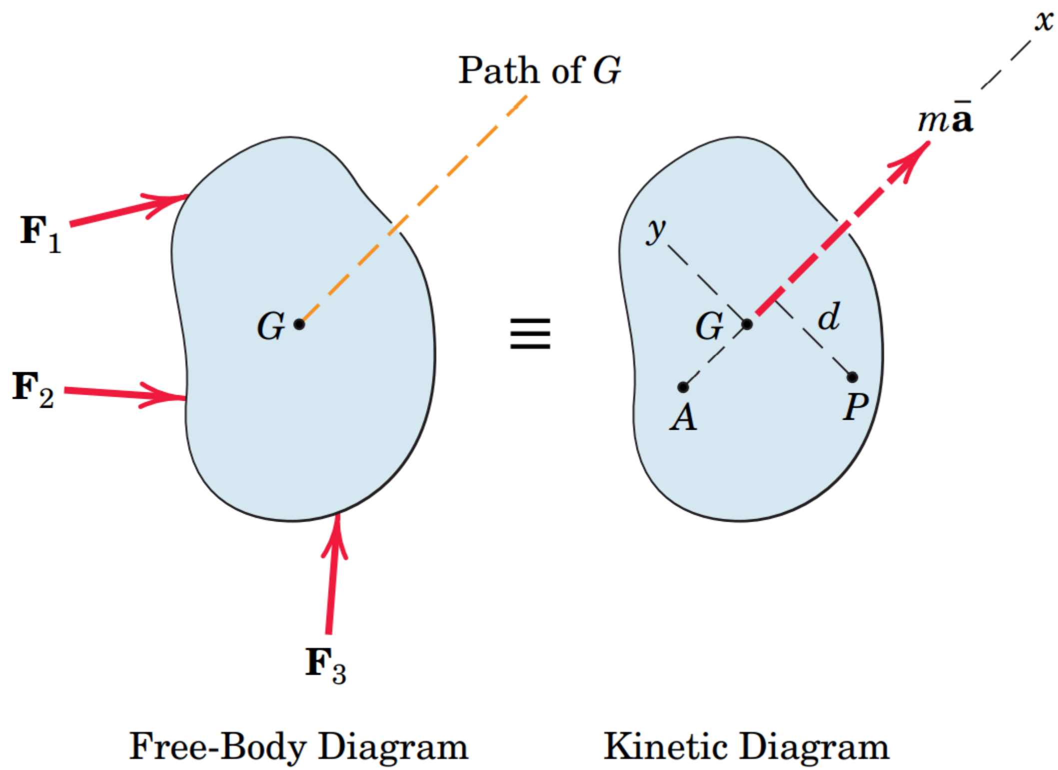
✓ If there are more than three remaining unknowns, the E.O.M. is not sufficient to solve the problem.

6/3 Translation

- There is no angular motion of the translating body, so that both ω and α are zero.
  - Rectilinear Translation:

$$\Sigma \mathbf{F} = m\bar{\mathbf{a}}$$

$$\Sigma M_G = \bar{I}\alpha = 0$$



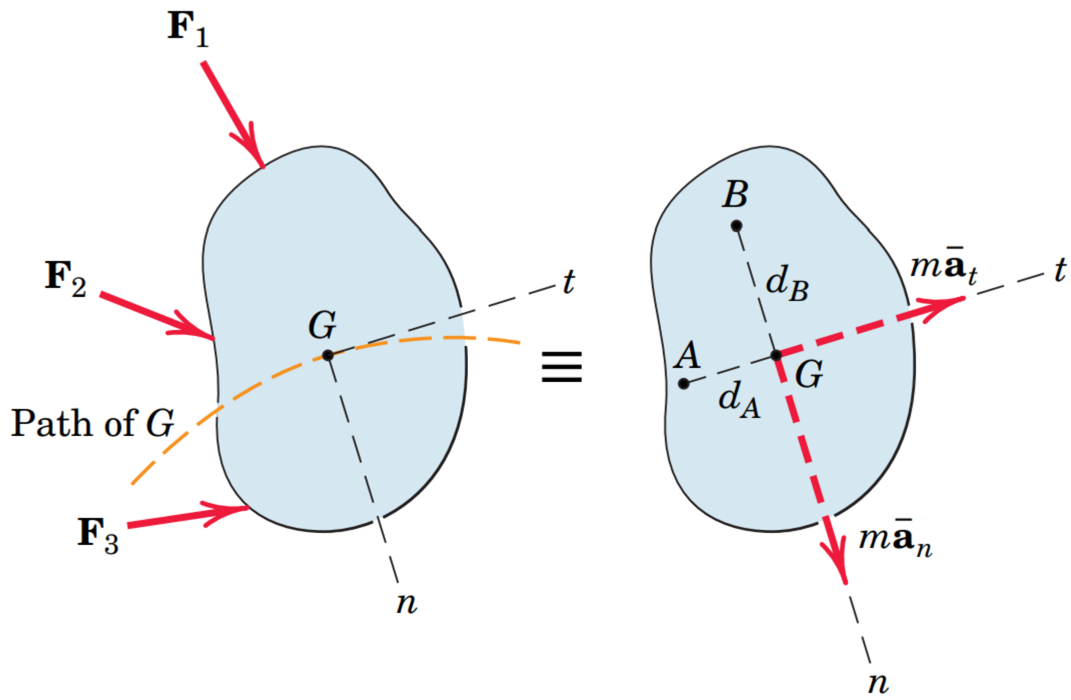
(a) Rectilinear Translation
($\alpha = 0$, $\omega = 0$)

6/3 Translation

- There is no angular motion of the translating body, so that both ω and α are zero.
  - Curvilinear Translation:

$$\Sigma \mathbf{F} = m\bar{\mathbf{a}}$$

$$\Sigma M_G = \bar{I}\alpha = 0$$

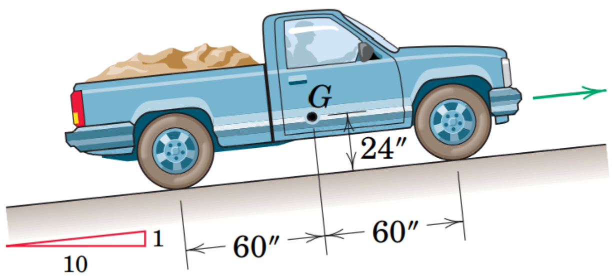


Free-Body Diagram   Kinetic Diagram

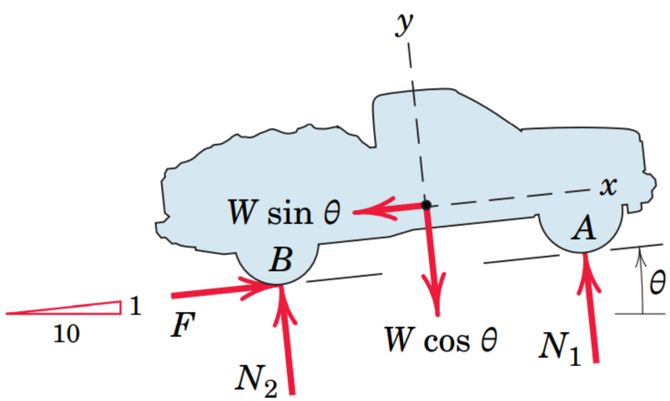
(b) Curvilinear Translation
($\alpha = 0$, $\omega = 0$)

SAMPLE PROBLEM 6/1

The pickup truck weighs 3220 lb and reaches a speed of 30 mi/hr from rest in a distance of 200 ft up the 10-percent incline with constant acceleration. Calculate the normal force under each pair of wheels and the friction force under the rear driving wheels. The effective coefficient of friction between the tires and the road is known to be at least 0.80.



$$m\bar{a} = \frac{3220}{32.2}(4.84) = 484 \text{ lb}$$

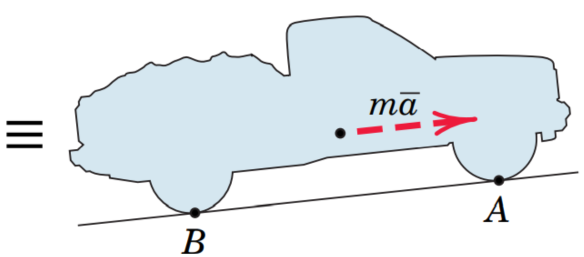


$[\Sigma F_x = m\bar{a}_x]$ $\quad F - 320 = 484 \qquad F = 804 \text{ lb}$

$[\Sigma F_y = m\bar{a}_y = 0]$ $\quad N_1 + N_2 - 3200 = 0$

$[\Sigma M_G = \bar{I}\alpha = 0]$ $\quad 60N_1 + 804(24) - N_2(60) = 0$

$$N_1 = 1441 \text{ lb} \qquad N_2 = 1763 \text{ lb}$$



SAMPLE PROBLEM 6/1

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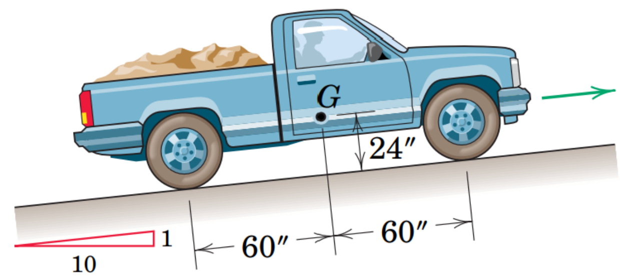


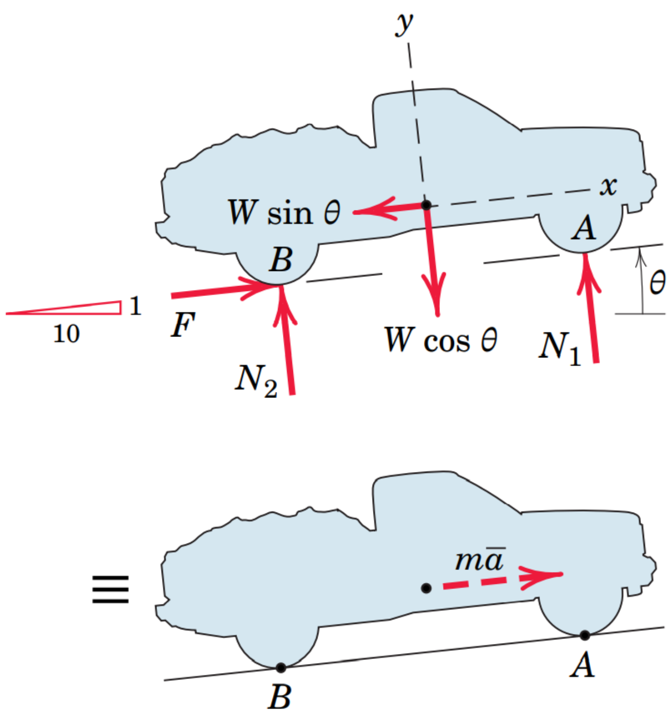
Alternative Solution

$[\Sigma M_A = m\bar{a}d]$ $$120N_2 - 60(3200) - 24(320) = 484(24)$$

$$N_2 = 1763 \text{ lb}$$

$[\Sigma M_B = m\bar{a}d]$ $$3200(60) - 320(24) - 120N_1 = 484(24)$$

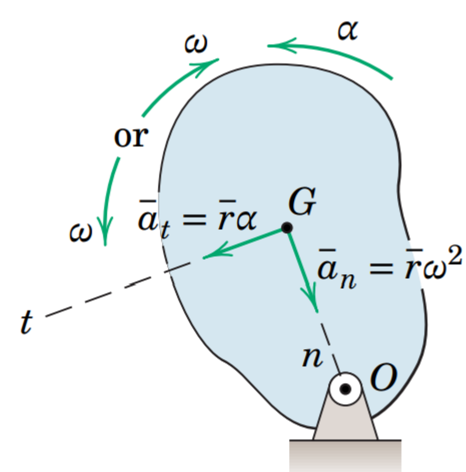
$$N_1 = 1441 \text{ lb}$$



6/4 Fixed-Axis Rotation

- All points in the body describe circles about the rotation axis, and all lines of the body in the plane of motion have the same angular velocity ω and angular acceleration α.

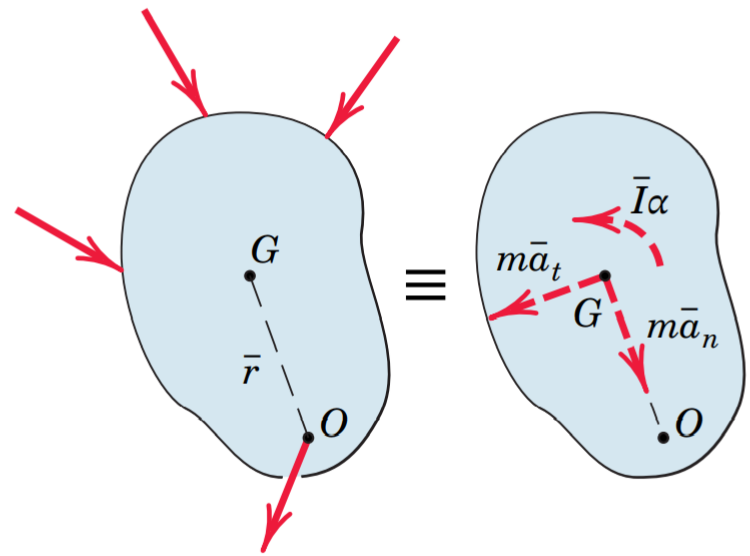
$$\Sigma M_O = \bar{I}\alpha + m\bar{a}_t\bar{r} \rightarrow I_O = \bar{I} + m\bar{r}^2 \rightarrow \Sigma M_O = (I_O - m\bar{r}^2)\alpha + m\bar{r}^2\alpha = I_O\alpha$$



Fixed-Axis Rotation

$$\Sigma \mathbf{F} = m\bar{\mathbf{a}}$$
$$\Sigma M_G = \bar{I}\alpha$$

$$\Sigma M_O = I_O\alpha$$



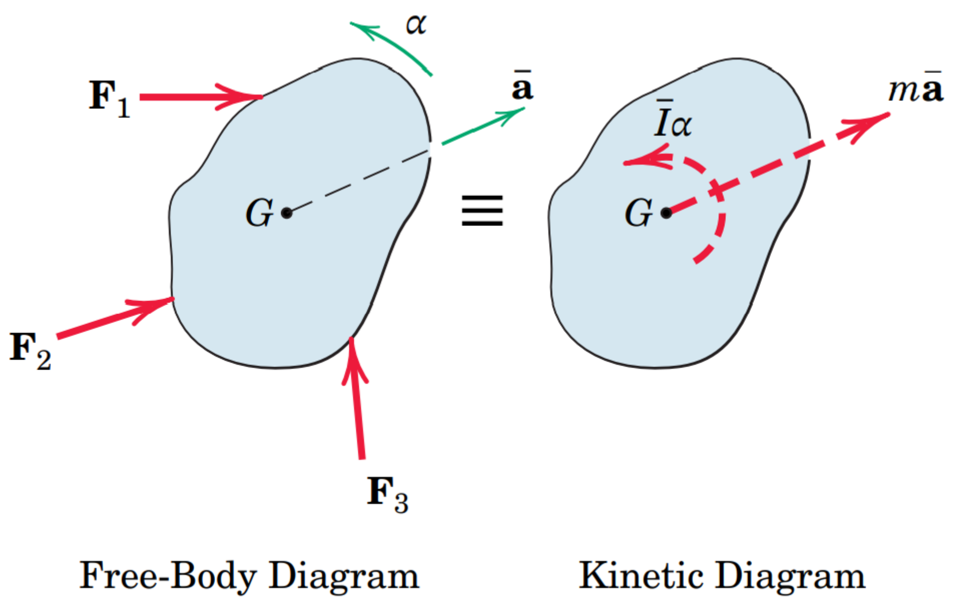
Free-Body Diagram Kinetic Diagram

6/5 General Plane Motion

- The dynamics of a rigid body in general plane motion combines translation and rotation.

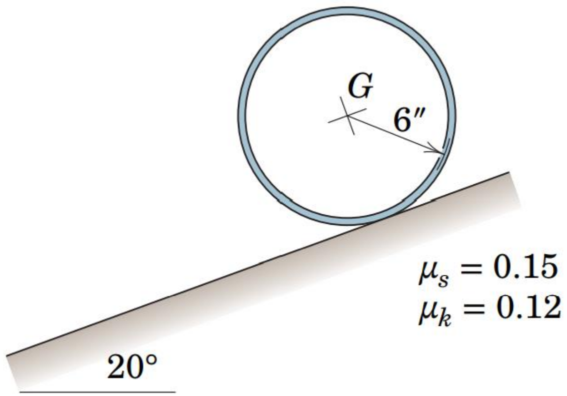
$$\Sigma \mathbf{F} = m\bar{\mathbf{a}}$$

$$\Sigma M_G = \bar{I}\alpha$$



SAMPLE PROBLEM 6/5

A metal hoop with a radius $r = 6$ in. is released from rest on the 20° incline. If the coefficients of static and kinetic friction are $\mu_s = 0.15$ and $\mu_k = 0.12$, determine the angular acceleration α of the hoop and the time t for the hoop to move a distance of 10 ft down the incline.



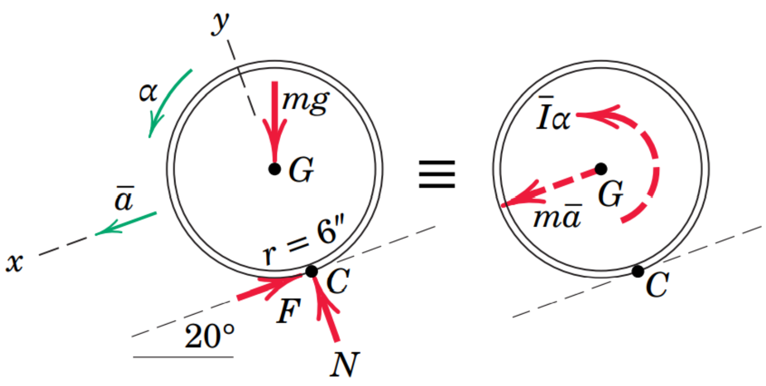
Assume that the hoop rolls without slipping

$$\bar{a} = r\alpha$$

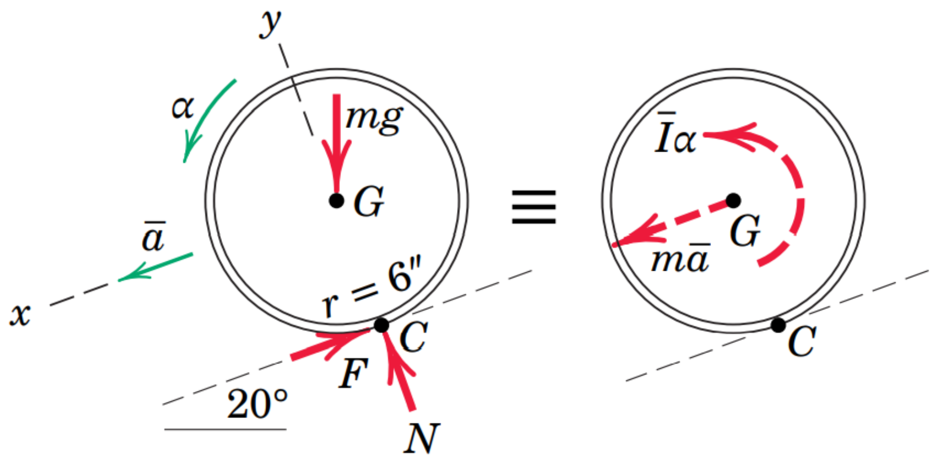
$$[\Sigma F_x = m\bar{a}_x] \qquad mg \sin 20° - F = m\bar{a}$$

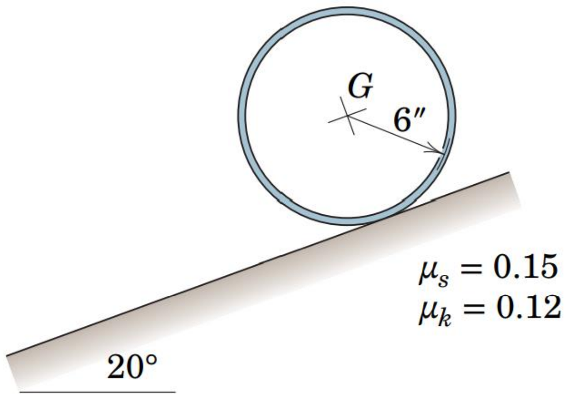
$$[\Sigma F_y = m\bar{a}_y = 0] \qquad N - mg \cos 20° = 0$$

$$[\Sigma M_G = \bar{I}\alpha] \qquad Fr = mr^2\alpha$$



$$\bar{a} = \frac{g}{2} \sin 20° = \frac{32.2}{2}(0.342) = 5.51 \text{ ft/sec}^2$$





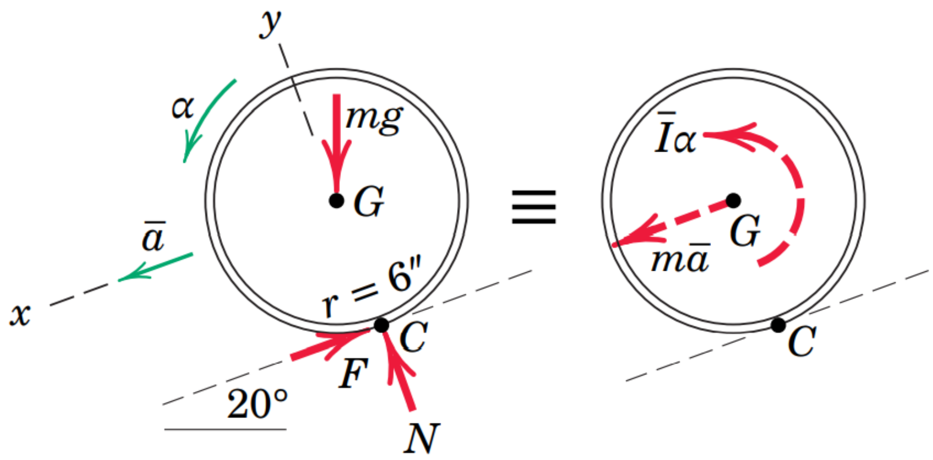
Alternatively: $[\Sigma M_C = \bar{I}\alpha + m\bar{a}d]$ $\quad mgr \sin 20° = mr^2 \dfrac{\bar{a}}{r} + m\bar{a}r$ $\quad \bar{a} = \dfrac{g}{2} \sin 20°$

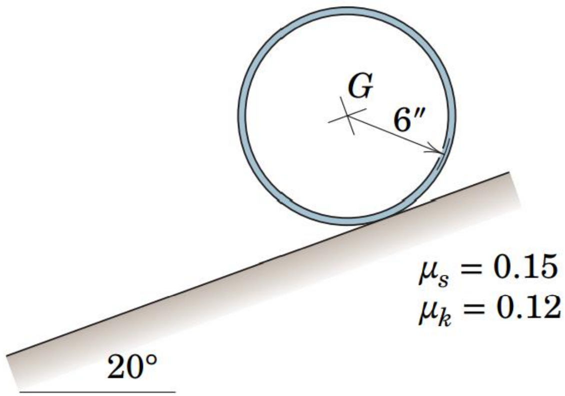
$$F = mg \sin 20° - m\frac{g}{2} \sin 20° = 0.1710mg$$

$$N = mg \cos 20° = 0.940mg$$

→ hoop slips as it rolls

$[F_{max} = \mu_s N]$ $\quad F_{max} = 0.15(0.940mg) = 0.1410mg$





$[F = \mu_k N]$ $\qquad F = 0.12(0.940mg) = 0.1128mg$

$[\Sigma F_x = m\bar{a}_x]$ $\qquad mg \sin 20° - 0.1128mg = m\bar{a}$

$$\bar{a} = 0.229(32.2) = 7.38 \text{ ft/sec}^2$$

$[\Sigma M_G = \bar{I}\alpha]$ $\qquad 0.1128mg(r) = mr^2\alpha$

$$\alpha = \frac{0.1128(32.2)}{6/12} = 7.26 \text{ rad/sec}^2$$

$[x = \frac{1}{2}at^2]$ → $t = \sqrt{\frac{2x}{a}} = \sqrt{\frac{2(10)}{7.38}} = 1.646 \text{ sec}$

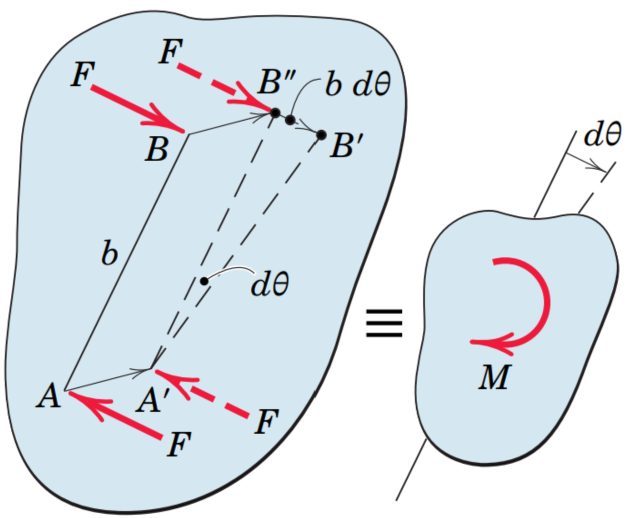
SECTION B Work and Energy

6/6 Work-Energy Relations

Work of Forces and Couples

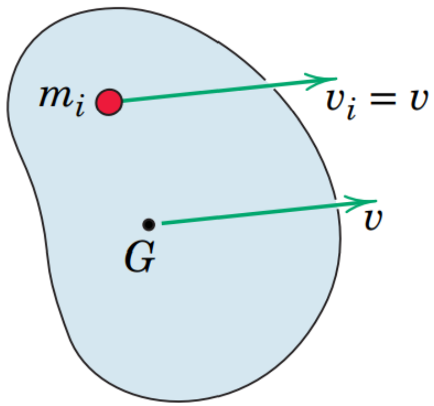
$$U = \int \mathbf{F} \cdot d\mathbf{r} \qquad \text{or} \qquad U = \int (F \cos \alpha)\, ds$$

$$U = \int M \, d\theta$$



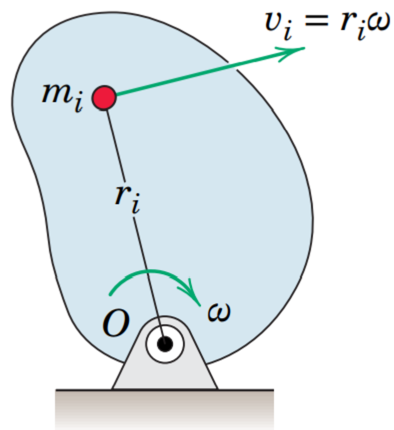
6/6 Work-Energy Relations

Kinetic Energy



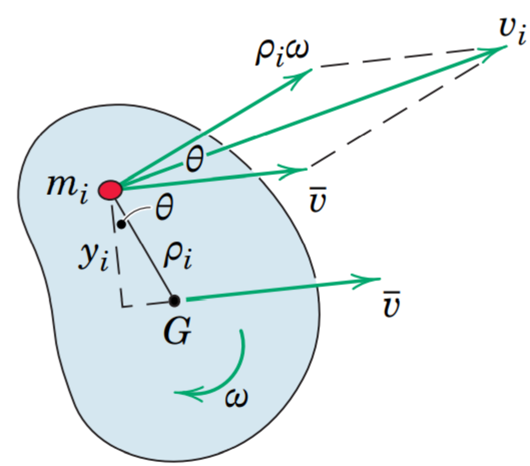
(a) Translation

$$T = \frac{1}{2}mv^2$$



(b) Fixed-Axis Rotation

$$T = \frac{1}{2}I_O\omega^2$$



(c) General Plane Motion

$$T = \frac{1}{2}m\bar{v}^2 + \frac{1}{2}\bar{I}\omega^2$$

$$T = \frac{1}{2}I_C\omega^2$$

C: instantaneous center of zero velocity

6/6 Work-Energy Relations

Potential Energy and the Work-Energy Equation

- Gravitational potential energy V_g and elastic potential energy V_e

$$T_1 + U_{1\text{-}2} = T_2$$

$$T_1 + V_1 + U'_{1\text{-}2} = T_2 + V_2$$

Power

$$P = \frac{dU}{dt} = \frac{\mathbf{F} \cdot d\mathbf{r}}{dt} = \mathbf{F} \cdot \mathbf{v}$$

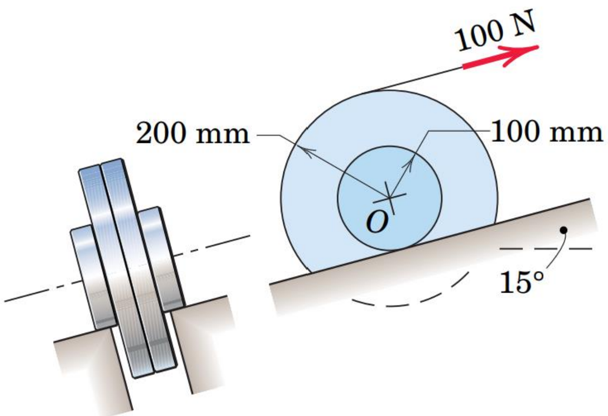
$$P = \frac{dU}{dt} = \frac{M\,d\theta}{dt} = M\omega$$

→

$$P = \mathbf{F} \cdot \mathbf{v} + M\omega$$

SAMPLE PROBLEM 6/9

The wheel rolls up the incline on its hubs without slipping and is pulled by the 100-N force applied to the cord wrapped around its outer rim. If the wheel starts from rest, compute its angular velocity ω after its center has moved a distance of 3 m up the incline. The wheel has a mass of 40 kg with center of mass at O and has a centroidal radius of gyration of 150 mm. Determine the power input from the 100-N force at the end of the 3-m motion interval.



$$U_{1\text{-}2} = 100\,\frac{200 + 100}{100}(3) - (392 \sin 15°)(3) = 595 \text{ J}$$

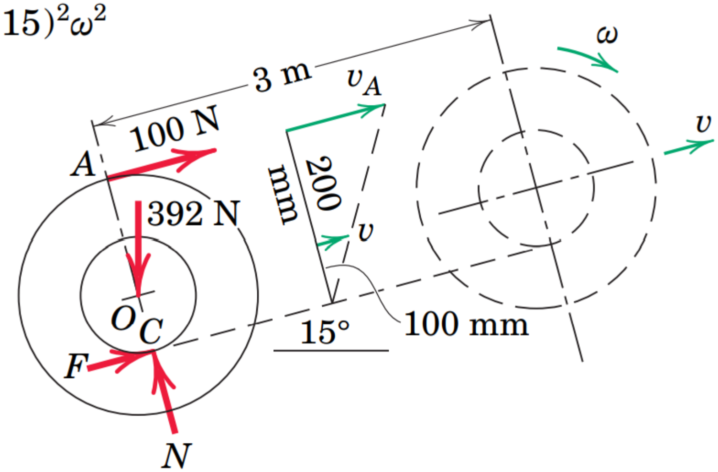
$$\left[T = \tfrac{1}{2}m\bar{v}^2 + \tfrac{1}{2}\bar{I}\omega^2\right] \qquad T_1 = 0 \qquad T_2 = \tfrac{1}{2}40(0.10\omega)^2 + \tfrac{1}{2}40(0.15)^2\omega^2$$

$$= 0.650\omega^2$$

$$\left[T = \tfrac{1}{2}I_C\omega^2\right] \qquad T = \tfrac{1}{2}40[(0.15)^2 + (0.10)^2]\omega^2 = 0.650\omega^2$$

$$[T_1 + U_{1\text{-}2} = T_2] \qquad 0 + 595 = 0.650\omega^2 \qquad \omega = 30.3 \text{ rad/s}$$

$$[P = \mathbf{F}\cdot\mathbf{v}] \qquad P_{100} = 100(0.3)(30.3) = 908 \text{ W}$$



SECTION C Impulse and Momentum

6/8 Impulse-Momentum Equations

Linear Momentum

$$\mathbf{G} = m\bar{\mathbf{v}}$$

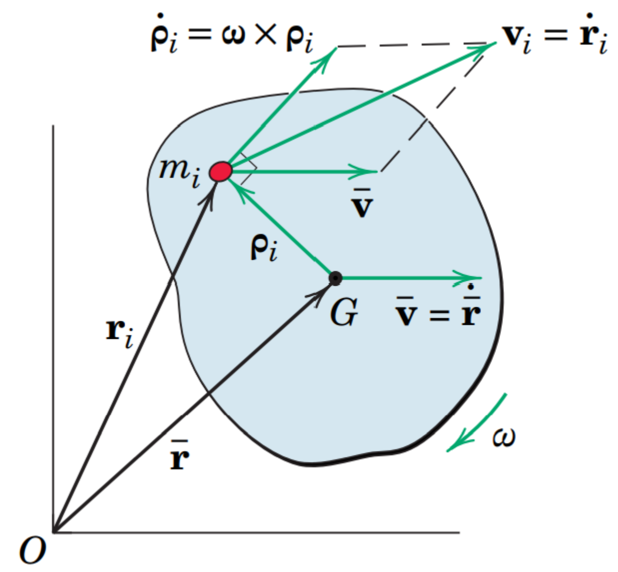
$$\Sigma \mathbf{F} = \dot{\mathbf{G}}$$

$$\mathbf{G}_1 + \int_{t_1}^{t_2} \Sigma \mathbf{F}\, dt = \mathbf{G}_2$$

$$\Sigma F_x = \dot{G}_x \qquad \Sigma F_y = \dot{G}_y$$

$$(G_x)_1 + \int_{t_1}^{t_2} \Sigma F_x\, dt = (G_x)_2$$

$$(G_y)_1 + \int_{t_1}^{t_2} \Sigma F_y\, dt = (G_y)_2$$



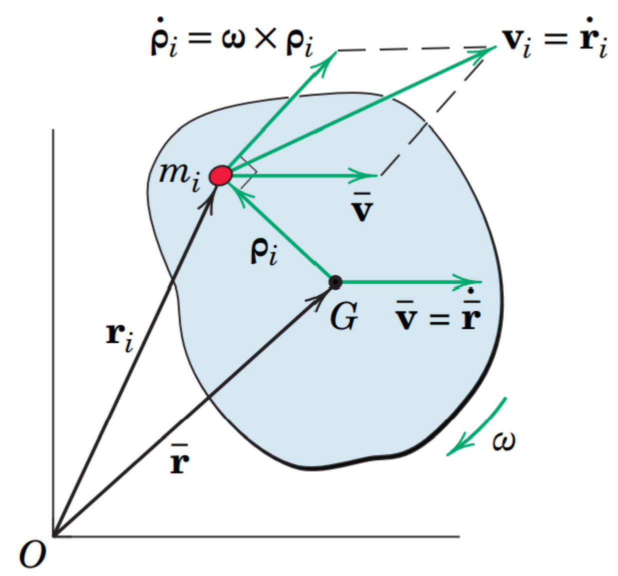
6/8 Impulse-Momentum Equations

Angular Momentum

$$H_G = \bar{I}\omega$$

$$\Sigma M_G = \dot{H}_G$$

$$(H_G)_1 + \int_{t_1}^{t_2} \Sigma M_G \, dt = (H_G)_2$$



6/8 Impulse-Momentum Equations

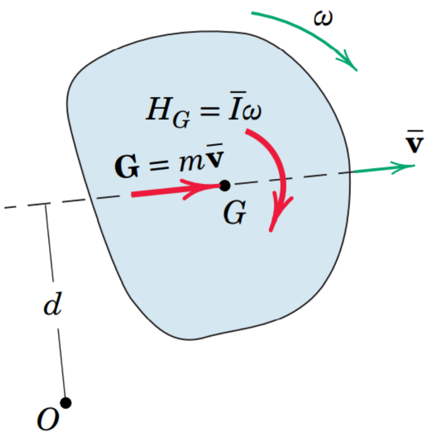
Angular Momentum

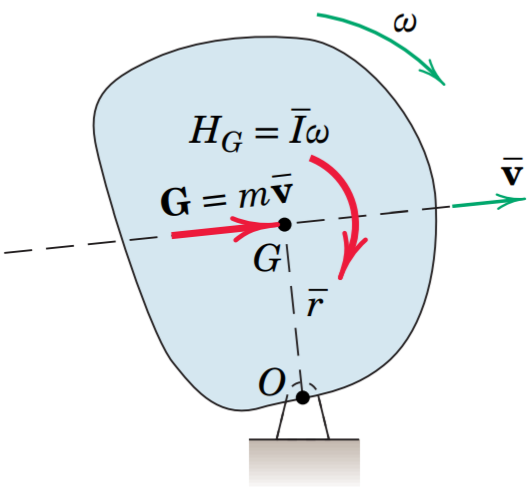
$$H_O = \bar{I}\omega + m\bar{v}d$$

$$H_O = I_O\omega$$

$$\Sigma M_O = \dot{H}_O$$

$$(H_O)_1 + \int_{t_1}^{t_2} \Sigma M_O \, dt = (H_O)_2$$





6/8 Impulse-Momentum Equations

Conservation of Momentum

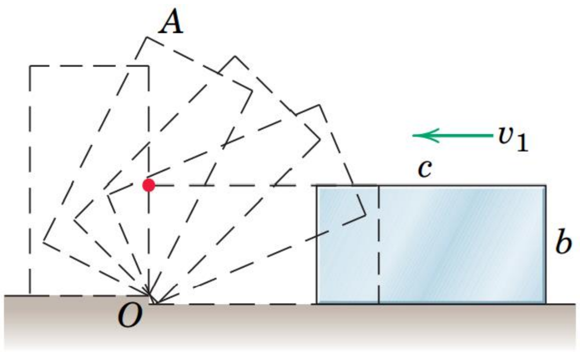
$$\mathbf{G}_1 = \mathbf{G}_2$$

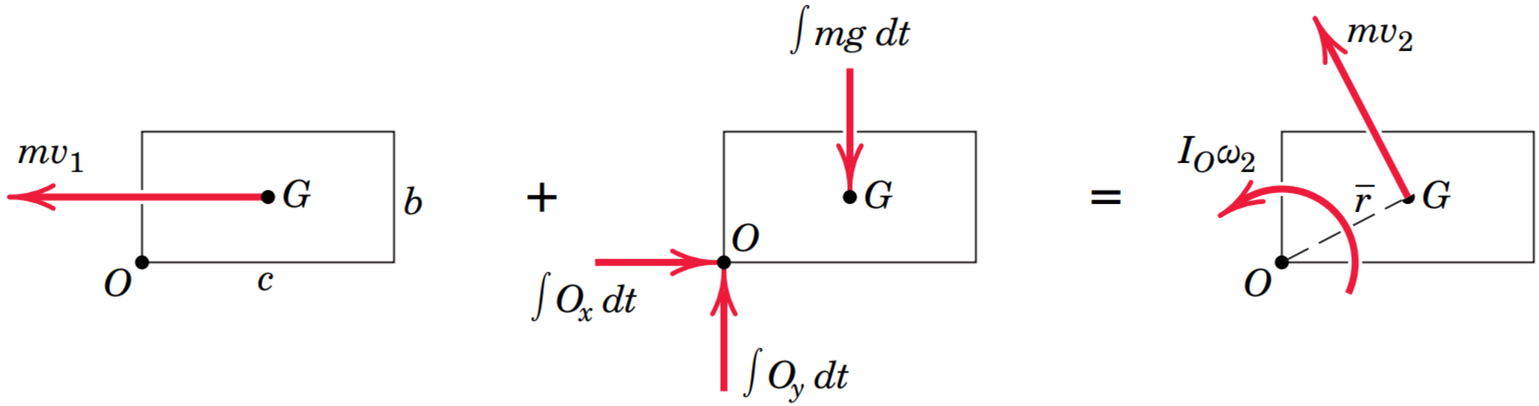
$$(\mathbf{H}_O)_1 = (\mathbf{H}_O)_2$$

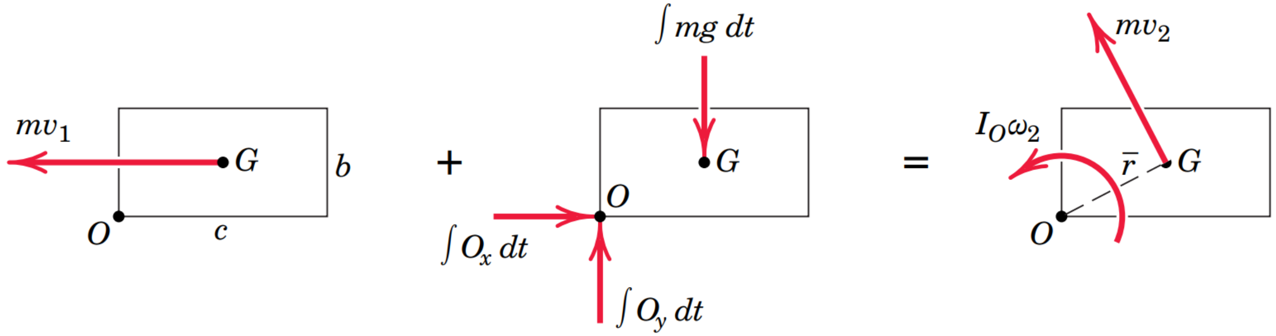
$$(\mathbf{H}_G)_1 = (\mathbf{H}_G)_2$$

SAMPLE PROBLEM 6/16

The uniform rectangular block of dimensions shown is sliding to the left on the horizontal surface with a velocity v_1 when it strikes the small step at O. Assume negligible rebound at the step and compute the minimum value of v_1 which will permit the block to pivot freely about O and just reach the standing position A with no velocity. Compute the percentage energy loss n for $b = c$.





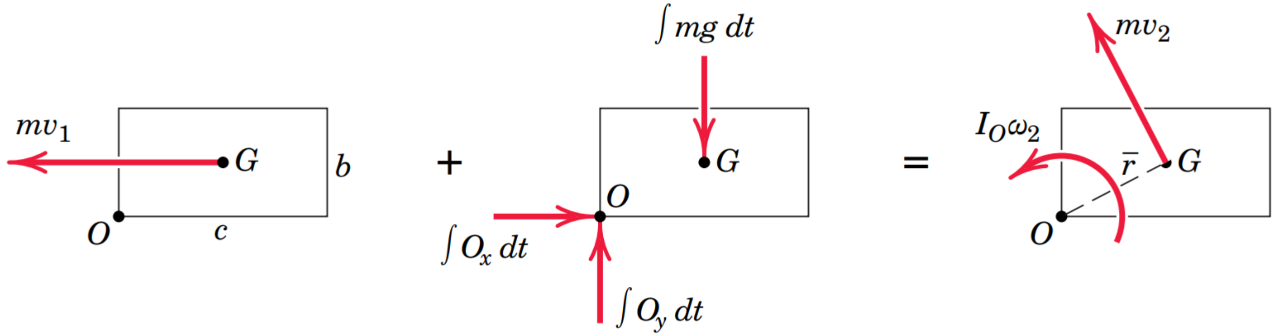


I. Collision

$$[H_O = I_O\omega] \qquad (H_O)_2 = \left\{\frac{1}{12}m(b^2+c^2) + m\left[\left(\frac{c}{2}\right)^2 + \left(\frac{b}{2}\right)^2\right]\right\}\omega_2$$

$$= \frac{m}{3}(b^2+c^2)\omega_2$$

$$[(H_O)_1 = (H_O)_2] \qquad mv_1\frac{b}{2} = \frac{m}{3}(b^2+c^2)\omega_2 \qquad \omega_2 = \frac{3v_1b}{2(b^2+\ c^2)}$$



II. Rotation about *O*

$$[T_2 + V_2 = T_3 + V_3] \qquad \frac{1}{2} I_O {\omega_2}^2 + 0 = 0 + mg\left[\sqrt{\left(\frac{b}{2}\right)^2 + \left(\frac{c}{2}\right)^2} - \frac{b}{2}\right]$$

$$\frac{1}{2}\frac{m}{3}(b^2 + c^2)\left[\frac{3v_1 b}{2(b^2 + c^2)}\right]^2 = \frac{mg}{2}\left(\sqrt{b^2 + c^2} - b\right)$$

$$v_1 = 2\sqrt{\frac{g}{3}\left(1 + \frac{c^2}{b^2}\right)\left(\sqrt{b^2 + c^2} - b\right)}$$

$$n = \frac{|\Delta E|}{E} = \frac{\frac{1}{2}m{v_1}^2 - \frac{1}{2}I_O{\omega_2}^2}{\frac{1}{2}m{v_1}^2} = 1 - \frac{k_O^2 {\omega_2}^2}{{v_1}^2} = 1 - \left(\frac{b^2 + c^2}{3}\right)\left[\frac{3b}{2(b^2 + c^2)}\right]^2$$

$$= 1 - \frac{3}{4\left(1 + \frac{c^2}{b^2}\right)} \qquad\qquad n = 62.5\% \text{ for } b = c$$