دانشگاه سمنان

Semnan University
Faculty of Mechanical Engineering

دانشکده مهندسی مکانیک

دانشکده مهندسی مکانیک

درس مکاترونیک ۱

MECHATRONICS 1

Section 2:

Statics Review

<u>Reference:</u>

Engineering Mechanics

Statics

8th Edition

Meriam, Kraige & Bolton

Chapters 1 - 6

❑ ***CONTENTS:***

1.1 MECHANICS

- Mechanics is the physical science which deals with the effects of forces on objects
- The principles of mechanics are central to research and development in many fields…
- The subject of mechanics is logically divided into two parts:
  - ***Statics***, which concerns the equilibrium of bodies under action of forces
  - ***Dynamics***, which concerns the motion of bodies

1.2 BASIC CONCEPTS

- ***Space*** is the geometric region occupied by bodies whose positions are described by linear and angular measurements relative to a coordinate system.

- ***Time*** is the measure of the succession of events and is a basic quantity in dynamics.
  - Time is not directly involved in the analysis of statics problems.

1.2 BASIC CONCEPTS

- ***Mass*** is a measure of the inertia of a body, which is its resistance to a change of velocity.

- ***Force*** is the action of one body on another.
  - A force tends to move a body in the direction of its action.
  - The action of a force is characterized by its *magnitude*, by the *direction* of its action, and by its *point of application*.
  - Thus force is a vector quantity.

1.2 BASIC CONCEPTS

- A ***particle*** is a body of negligible dimensions. In the mathematical sense, a particle is a body whose dimensions are considered to be near zero.

- ***Rigid body.*** A body is considered rigid when the change in distance between any two of its points is negligible for the purpose at hand.

1.3 SCALARS AND VECTORS

- ***Scalar quantities***: only a magnitude is associated.
  - Examples: time, volume, density, speed, energy, and mass.

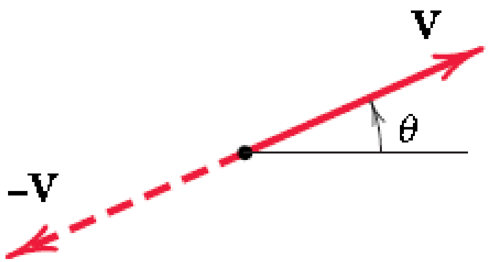
- ***Vector quantities***: possess direction as well as magnitude
  - Obey the parallelogram law of addition.
  - Examples: displacement, velocity, acceleration, force, moment, momentum

  - Vectors representing physical quantities can be classified as:
    - ✓ Free
    - ✓ Sliding
    - ✓ Fixed

1.3 SCALARS AND VECTORS

- Conventions for Equations and Diagrams

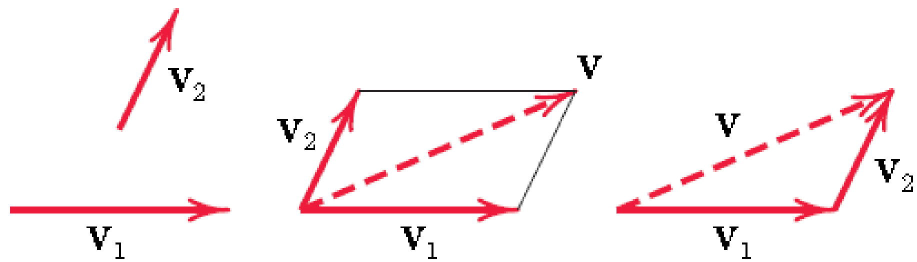
- A vector quantity **V** is represented by a line segment
  - Direction of the vector
  - Magnitude of the vector |**V**|



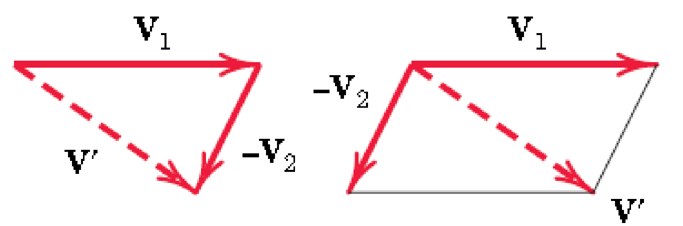
1.3 SCALARS AND VECTORS

- Vector Summation:

$\mathbf{V} = \mathbf{V}_1 + \mathbf{V}_2$

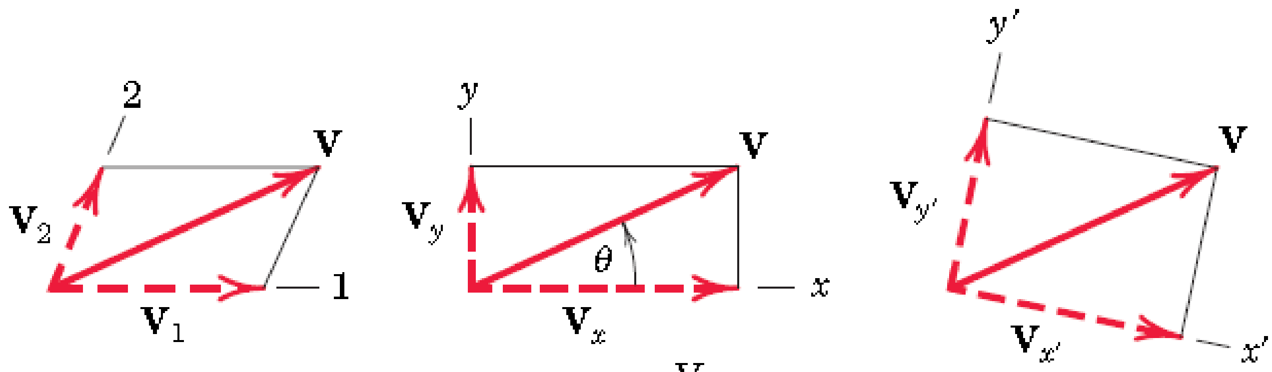


$\mathbf{V}' = \mathbf{V}_1 - \mathbf{V}_2$



1.3 SCALARS AND VECTORS

- *Components* of vector



$$\theta = \tan^{-1}\frac{V_y}{V_x}$$

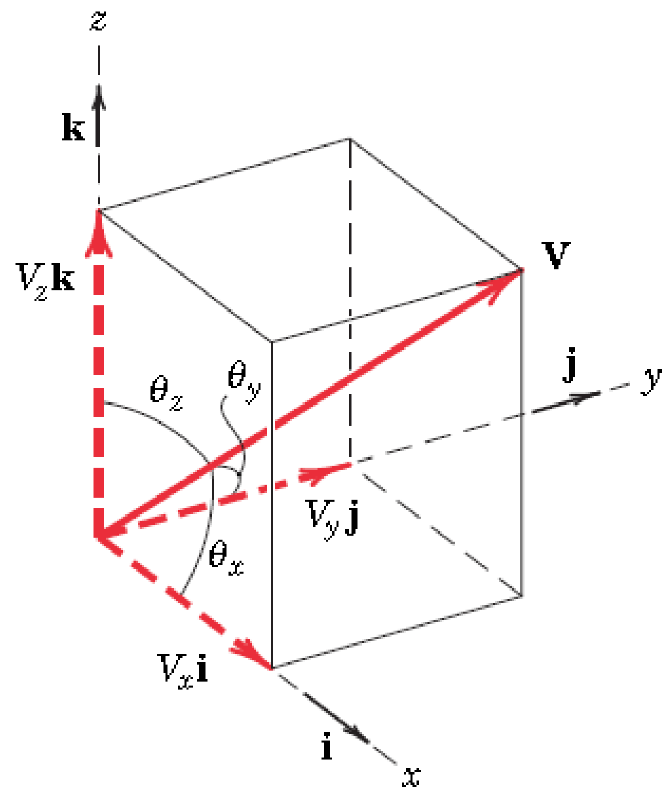
- Unit vector **n**:

$$\mathbf{V} = V\mathbf{n}$$

1.3 SCALARS AND VECTORS

- Three-dimensional vectors:
  - Unit vectors **i**, **j**, and **k** , which are vectors in the *x*-, *y*-, and *z*-directions

$$\mathbf{V} = V_x\mathbf{i} + V_y\mathbf{j} + V_z\mathbf{k}$$



$$l = \cos\theta_x \qquad m = \cos\theta_y \qquad n = \cos\theta_z$$

$$V_x = lV \qquad V_y = mV \qquad V_z = nV$$

$$V^2 = V_x^{\,2} + V_y^{\,2} + V_z^{\,2}$$

1.4 NEWTON’S LAWS

- **<u>Law I</u>**. A particle remains at rest or continues to move with uniform velocity (in a straight line with a constant speed) if there is no unbalanced force acting on it.

  - Newton’s first law contains the principle of the equilibrium of forces, which is the main topic of concern in statics

1.4 NEWTON’S LAWS

- **Law II**. The acceleration of a particle is proportional to the vector sum of forces acting on it and is in the direction of this vector sum.

$$\mathbf{F} = m\mathbf{a}$$

- **Law III**. The forces of action and reaction between interacting bodies are equal in magnitude, opposite in direction, and collinear (they lie on the same line)

1.5 UNITS

- In mechanics we use four fundamental quantities called dimensions.
  - These are length, mass, force, and time.
  - The units used to measure these quantities cannot all be chosen independently

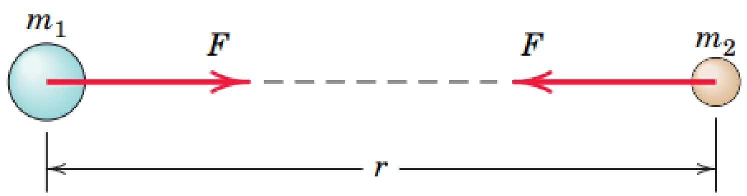
QUANTITY	DIMENSIONAL SYMBOL	SI UNITS			U.S. CUSTOMARY UNITS		
			UNIT	SYMBOL		UNIT	SYMBOL
Mass	M	Base units	kilogram	kg		slug	—
Length	L	Base units	meter	m	Base units	foot	ft
Time	T	Base units	second	s	Base units	second	sec
Force	F		newton	N	Base units	pound	lb

1.6 LAW OF GRAVITATION

- To compute the weight of a body: the gravitational force acting on it

- Law of gravitation:

$$F = G\frac{m_1 m_2}{r^2}$$



where F = the mutual force of attraction between two particles

G = a universal constant known as the *constant of gravitation* $\quad G = 6.673(10^{-11})\ \mathrm{m^3/(kg\cdot s^2)}$

m_1, m_2 = the masses of the two particles

r = the distance between the centers of the particles

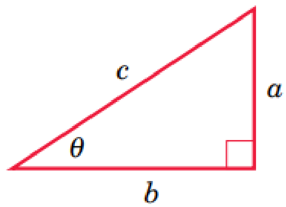
- Gravitational Attraction of the Earth: $W = mg$

Appendix C - Trigonometry

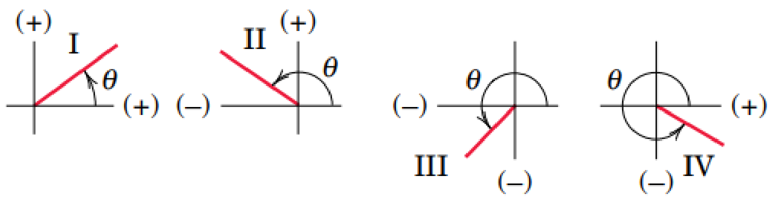
C/6 Trigonometry

1. Definitions

$\sin\theta = a/c$ $\quad \csc\theta = c/a$
$\cos\theta = b/c$ $\quad \sec\theta = c/b$
$\tan\theta = a/b$ $\quad \cot\theta = b/a$



2. Signs in the four quadrants



	I	II	III	IV
$\sin\theta$	+	+	–	–
$\cos\theta$	+	–	–	+
$\tan\theta$	+	–	+	–
$\csc\theta$	+	+	–	–
$\sec\theta$	+	–	–	+
$\cot\theta$	+	–	+	–

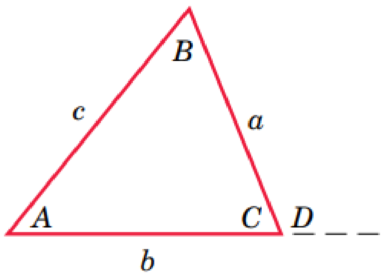
APPENDIX C - TRIGONOMETRY

3. Miscellaneous relations

$$\sin^2\theta + \cos^2\theta = 1$$
$$1 + \tan^2\theta = \sec^2\theta$$
$$1 + \cot^2\theta = \csc^2\theta$$
$$\sin\frac{\theta}{2} = \sqrt{\frac{1}{2}(1 - \cos\theta)}$$
$$\cos\frac{\theta}{2} = \sqrt{\frac{1}{2}(1 - \cos\theta)}$$
$$\sin 2\theta = 2\sin\theta\cos\theta$$
$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$
$$\sin(a \pm b) = \sin a\cos b \pm \cos a\sin b$$
$$\cos(a \pm b) = \cos a\cos b \mp \sin a\sin b$$

4. Law of sines

$$\frac{a}{b} = \frac{\sin A}{\sin B}$$



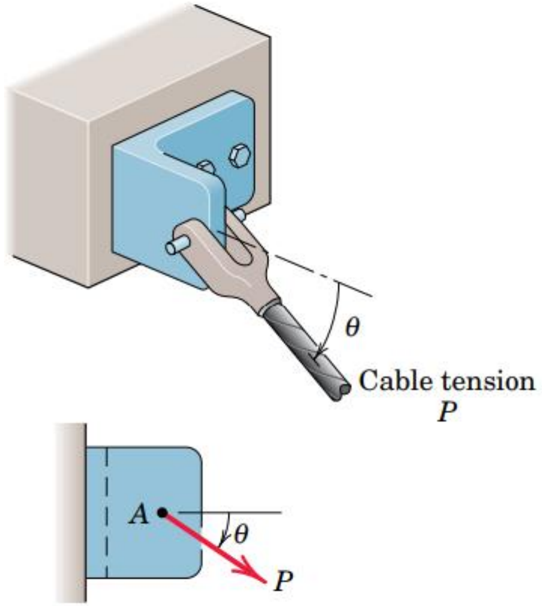
5. Law of cosines

$$c^2 = a^2 + b^2 - 2ab\cos C$$
$$c^2 = a^2 + b^2 + 2ab\cos D$$

❑ ***CONTENTS:***

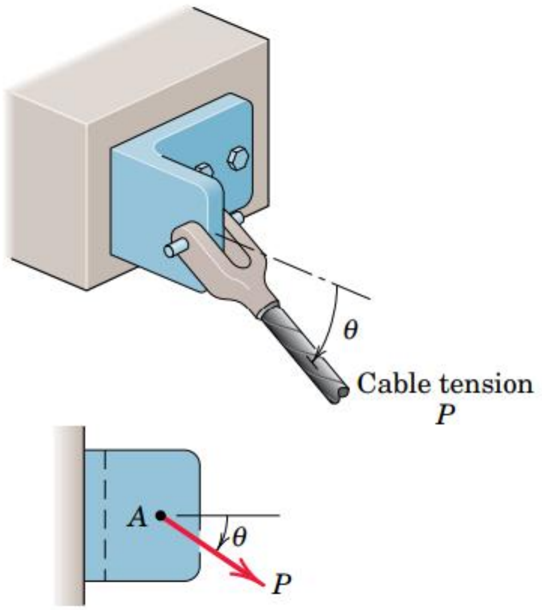
2.2 Force

- Properties of a single force:
  - Action of one body on another
  - Action which tends to cause acceleration
  - Vector quantity (Magnitude and Direction)
  - Forces may be combined by vector addition



2.2 FORCE

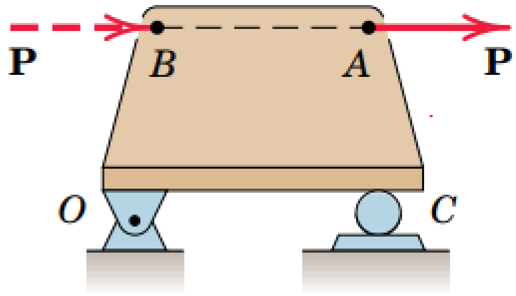
- Complete specification of the action of this force must include:
  - Magnitude
  - Direction
  - Point of application
    - ✓ We must treat it as a fixed vector



- External and Internal Effects
  - External Forces:
    - ✓ Applied forces
    - ✓ Reactive forces
  - External forces lead to creation of internal forces

2.2 FORCE

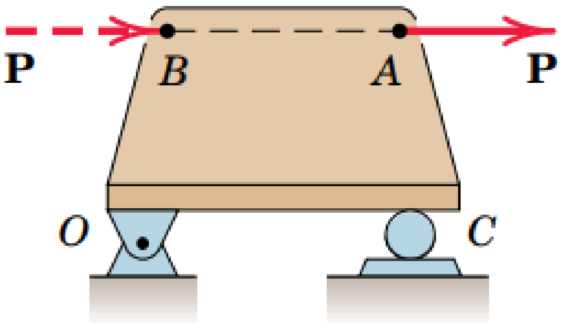
- Principle of Transmissibility
  - When dealing with the mechanics of a rigid body, we ignore deformations in the body
  - The external effects of the exerted force should be same
  - So it is not necessary to restrict the action of an applied force to a given point
  - For example:
    - ✓ Force P may be applied at A or at B or at any other point on its line of action
    - ✓ External effects: bearing support at O and roller support at C



2.2 FORCE

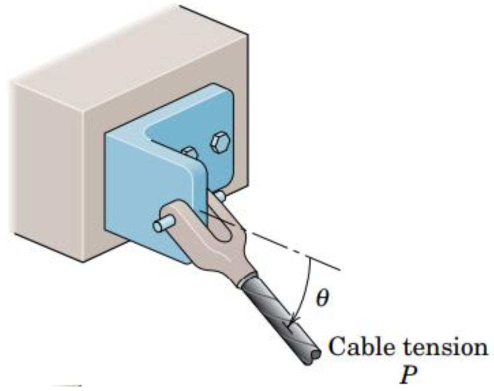
- Principle of Transmissibility:
  - A force may be applied at any point on its given line of action without altering the resultant effects of the force external to the rigid body on which it acts.

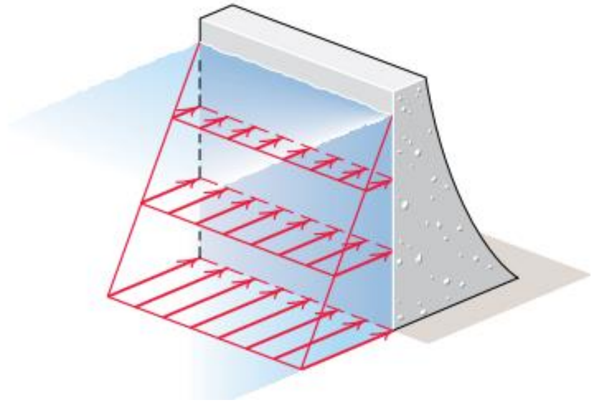
  - The force may be treated as a sliding vector:
    - ✓ Magnitude
    - ✓ Direction
    - ✓ Line of action



2.2 FORCE

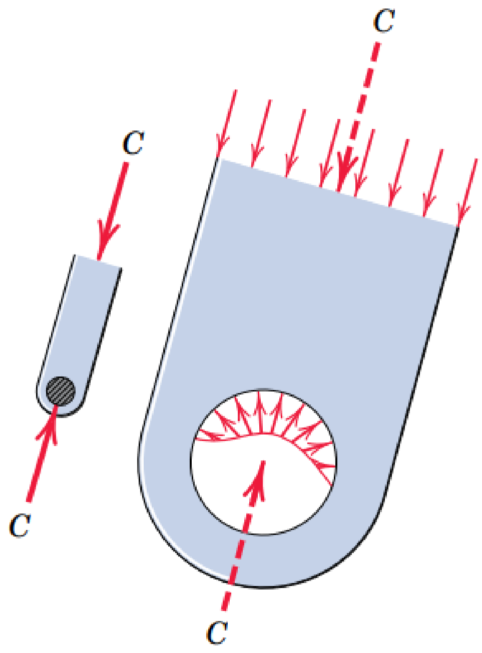
- Force Classification
  - Contact or Body forces:
    - ✓ A contact force is produced by direct physical contact
    - ✓ A body force is generated by virtue of the position of a body within a force field (such as a gravitational)





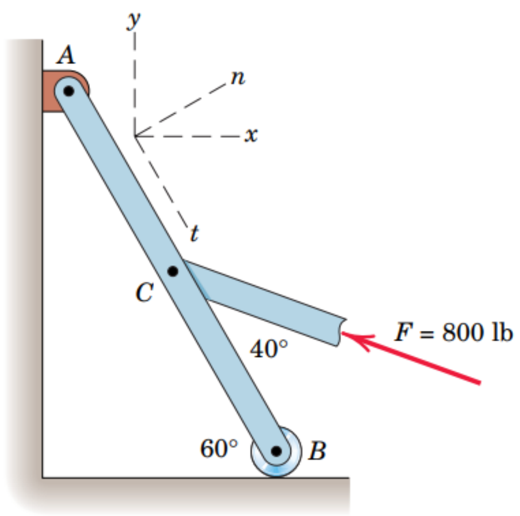
2.2 FORCE

- Force Classification
  - Concentrated or Distributed forces
    - ✓ Actually, almost all forces are distributed forces.
    - ✓ When the dimensions of the area are very small compared with the other dimensions of the body, we may consider the force to be concentrated



2.2 FORCE

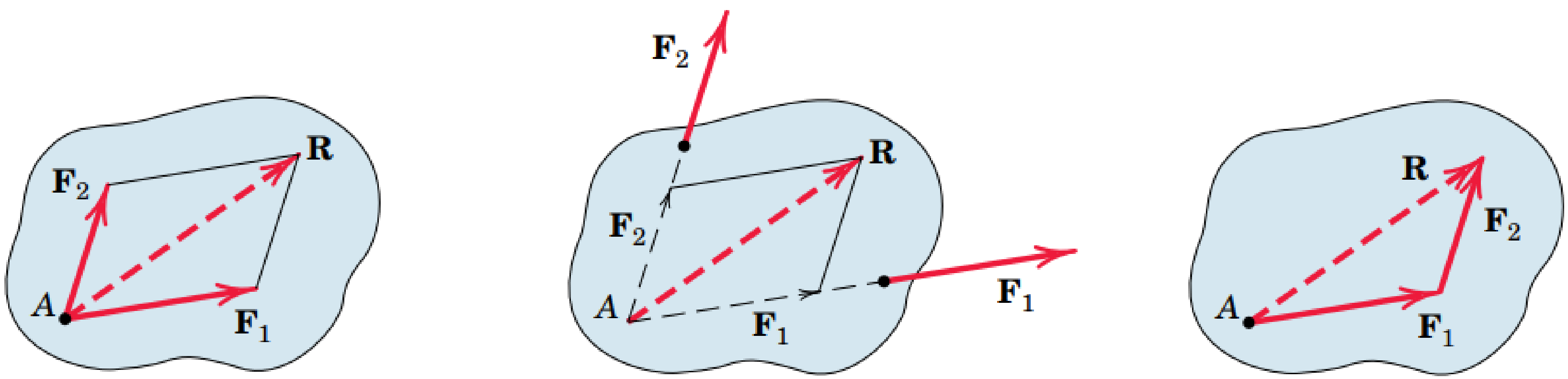
- Action and Reaction
  - According to Newton's third law, the action of a force is always accompanied by an equal and opposite reaction



2.2 Force

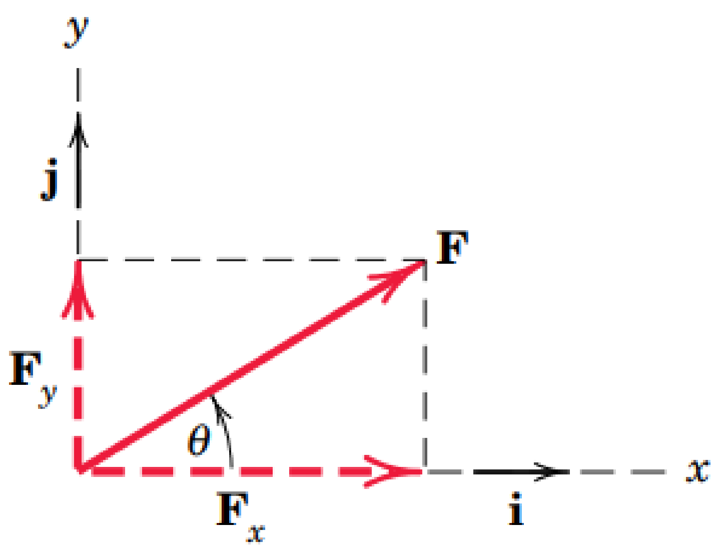
- Concurrent Forces
  - Their lines of action intersect at that point
  - They can be added using the parallelogram law in their common plane

$$\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2$$



2.3 Rectangular Components

- The most common two-dimensional resolution of a force vector: Rectangular Components



$$\mathbf{F} = \mathbf{F}_x + \mathbf{F}_y$$

$$\mathbf{F} = F_x\mathbf{i} + F_y\mathbf{j}$$

$$F_x = F\cos\theta \qquad F = \sqrt{{F_x}^2 + {F_y}^2} = |\mathbf{F}|$$

$$F_y = F\sin\theta \qquad \theta = \tan^{-1}\frac{F_y}{F_x}$$

2.3 Rectangular Components

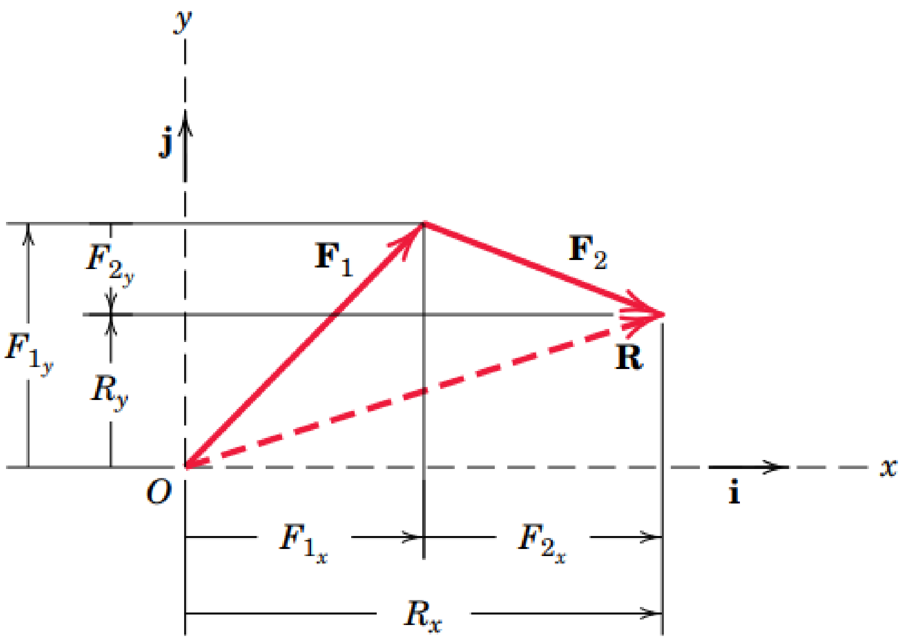
- Finding the sum or resultant R of two forces (which are concurrent)
  - Summing each component separately

$$\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2 = (F_{1_x}\mathbf{i} + F_{1_y}\mathbf{j}) + (F_{2_x}\mathbf{i} + F_{2_y}\mathbf{j})$$

$$R_x\mathbf{i} + R_y\mathbf{j} = (F_{1_x} + F_{2_x})\mathbf{i} + (F_{1_y} + F_{2_y})\,\mathbf{j}$$

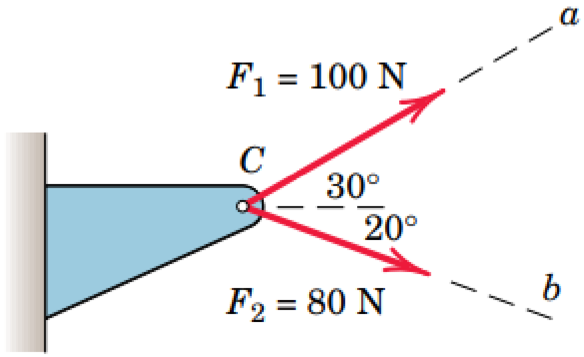
$$R_x = F_{1_x} + F_{2_x} = \Sigma F_x$$

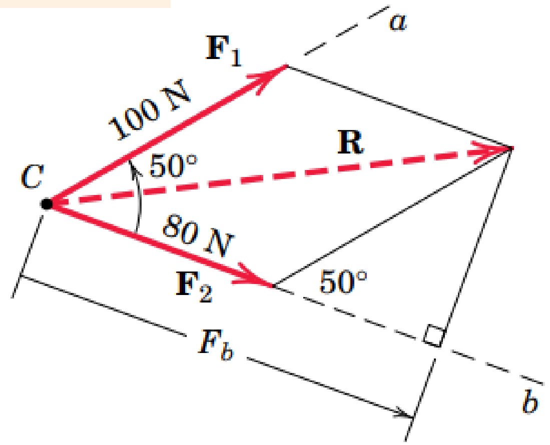
$$R_y = F_{1_y} + F_{2_y} = \Sigma F_y$$



Sample Problem 2/4

Forces $\mathbf{F}_1$ and $\mathbf{F}_2$ act on the bracket as shown. Determine the projection F_b of their resultant $\mathbf{R}$ onto the b-axis.





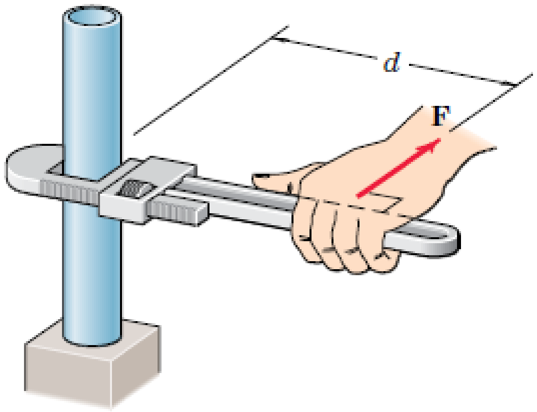
$$R^2 = (80)^2 + (100)^2 - 2(80)(100)\cos 130^\circ \qquad R = 163.4 \text{ N}$$

$$F_b = 80 + 100 \cos 50^\circ = 144.3 \text{ N}$$

2.4 MOMENTS

- A force can also tend to rotate a body about an axis
- Moment is also referred to as *torque*

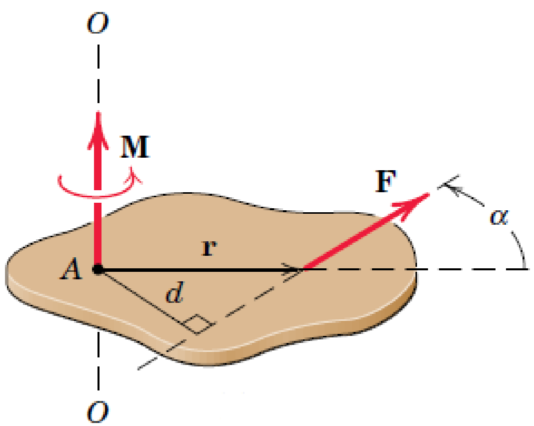
- The magnitude of this tendency depends on:
  - Magnitude *F* of the force
  - Effective length *d* of the wrench handle

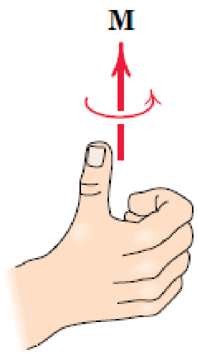


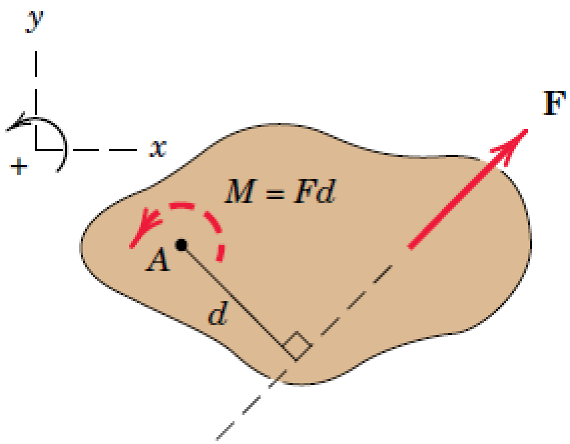
2.5 Moments

- Moment about a Point
  - Plus sign for counterclockwise moments
  - Minus sign for clockwise moments
  - Sign consistency within a given problem is essential.

$$M = Fd$$





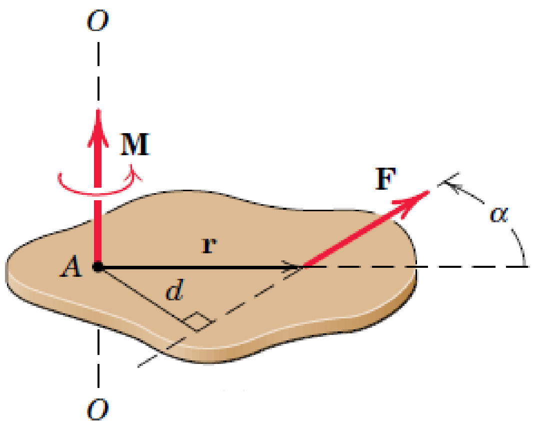


2.5 MOMENTS

- The Cross Product

$$\mathbf{M} = \mathbf{r} \times \mathbf{F}$$

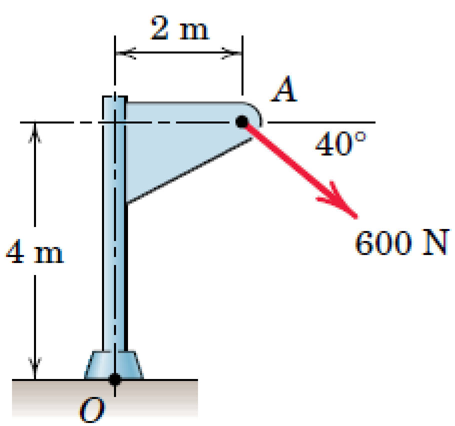
- The moment of **F** about point *A*
- **r** is a position vector which runs from the moment reference point *A* to *any* point on the line of action of **F**
- We must maintain the sequence **r** x **F**, because the sequence **F** x **r** would produce a vector with a sense opposite to that of the correct moment.



$$M = Fr \sin \alpha = Fd$$

Sample Problem 2/5

Calculate the magnitude of the moment about the base point O of the 600-N force in five different ways.

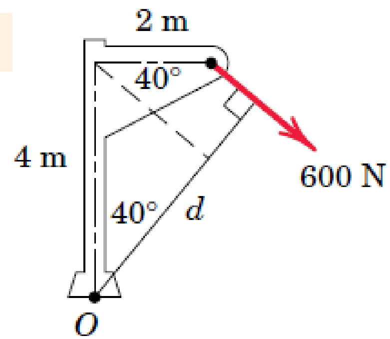


Solution. (I)

$$d = 4\cos 40^\circ + 2\sin 40^\circ = 4.35 \text{ m}$$

$$M = Fd$$

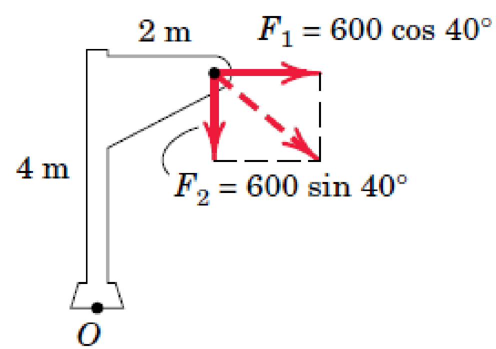
$$M_O = 600(4.35) = 2610 \text{ N·m}$$



Solution. (II)

$$F_1 = 600\cos 40^\circ = 460 \text{ N}, \qquad F_2 = 600\sin 40^\circ = 386 \text{ N}$$

$$M_O = 460(4) + 386(2) = 2610 \text{ N·m}$$



Solution. (III)

$d_1 = 4 + 2 \tan 40° = 5.68$ m

$M_O = 460(5.68) = 2610$ N·m

Solution. (IV)

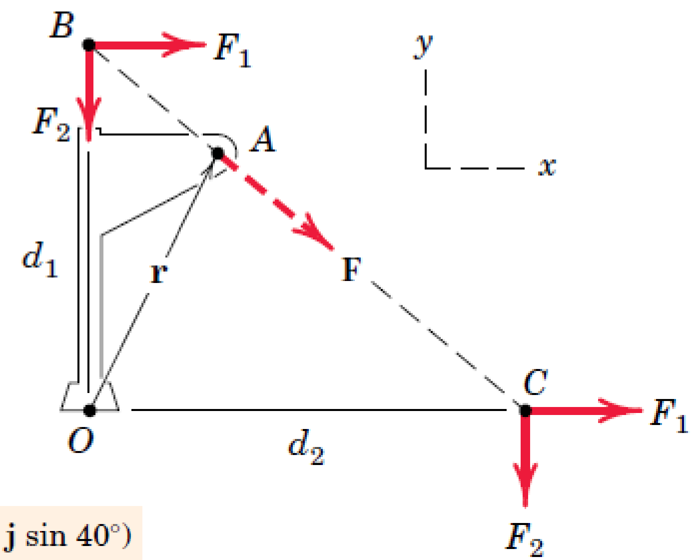
$d_2 = 2 + 4 \cot 40° = 6.77$ m

$M_O = 386(6.77) = 2610$ N·m

Solution. (V)

$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F} = (2\mathbf{i} + 4\mathbf{j}) \times 600(\mathbf{i} \cos 40° - \mathbf{j} \sin 40°)$$
$$= -2610\mathbf{k} \text{ N·m}$$

$M_O = 2610$ N·m



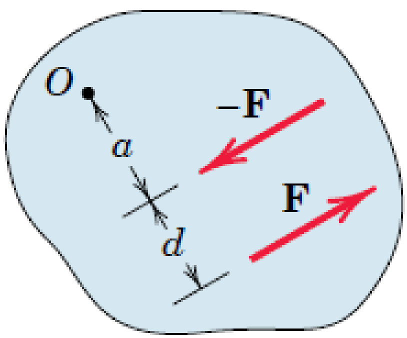
2.5 COUPLE

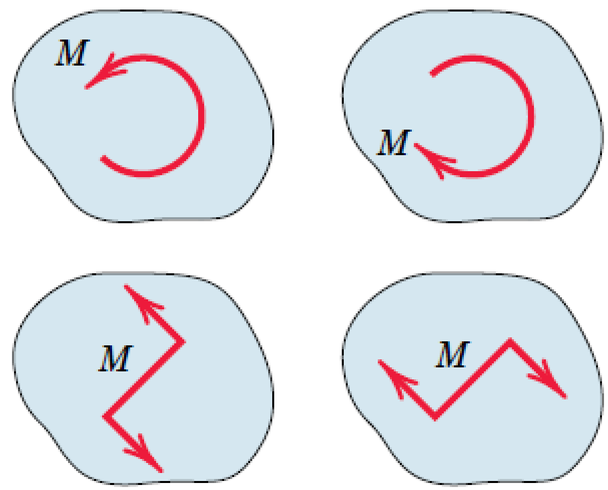
- The moment produced by two equal, opposite, and noncollinear forces is called a *couple*.

- The forces only effect is to produce a tendency of rotation

$$M = F(a + d) - Fa$$

$$\rightarrow \quad M = Fd$$





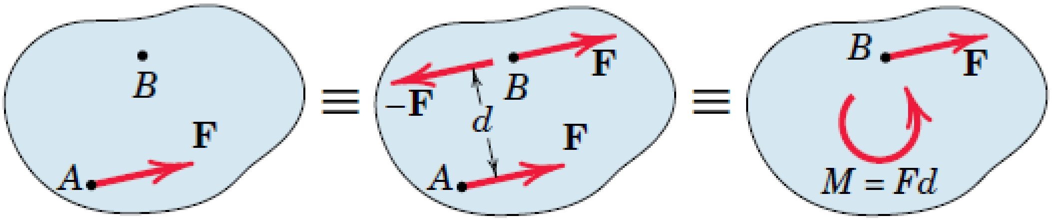
Counterclockwise couple

Clockwise couple

2.5 COUPLE

❑ Force–Couple Systems

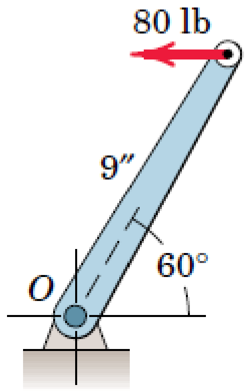
- The effect of a force acting on a body:
  - ✓ Push or pull the body in the direction of the force
  - ✓ Rotate the body about any fixed axis which does not intersect the line of the force



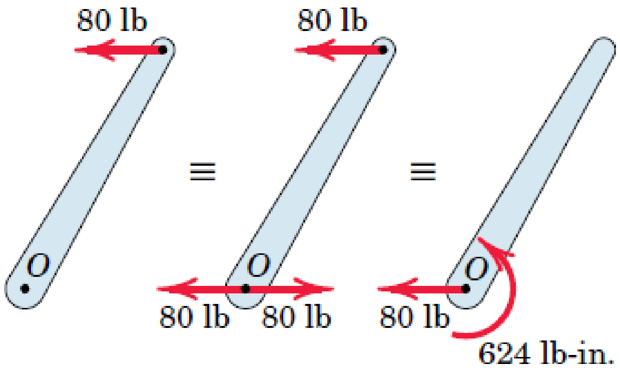
- By reversing this process, we can combine a given couple and a force which lies in the plane of the couple (normal to the couple vector) to produce a single, equivalent force.

Sample Problem 2/8

Replace the horizontal 80-lb force acting on the lever by an equivalent system consisting of a force at O and a couple.



$[M = Fd]$ $\quad M = 80(9 \sin 60°) = 624$ lb-in.



2.6 Resultant

- The *resultant* of a system of forces:
  - The simplest force combination which can replace the original forces without altering the external effect on the rigid body to which the forces are applied

- *Equilibrium* of a body:
  - The condition in which the resultant of all forces acting on the body is zero.

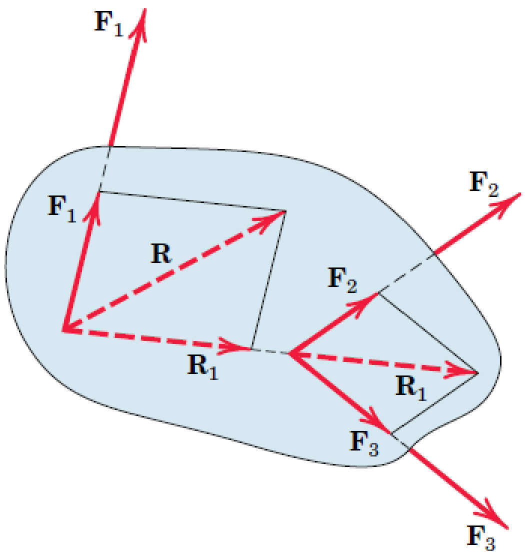
2.6 Resultant

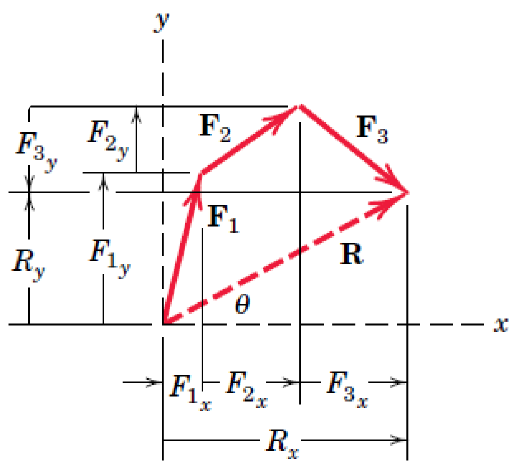
- The *resultant* of a system of forces

$$\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \cdots = \Sigma\mathbf{F}$$

$$R_x = \Sigma F_x \qquad R_y = \Sigma F_y \qquad R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$

$$\theta = \tan^{-1}\frac{R_y}{R_x} = \tan^{-1}\frac{\Sigma F_y}{\Sigma F_x}$$

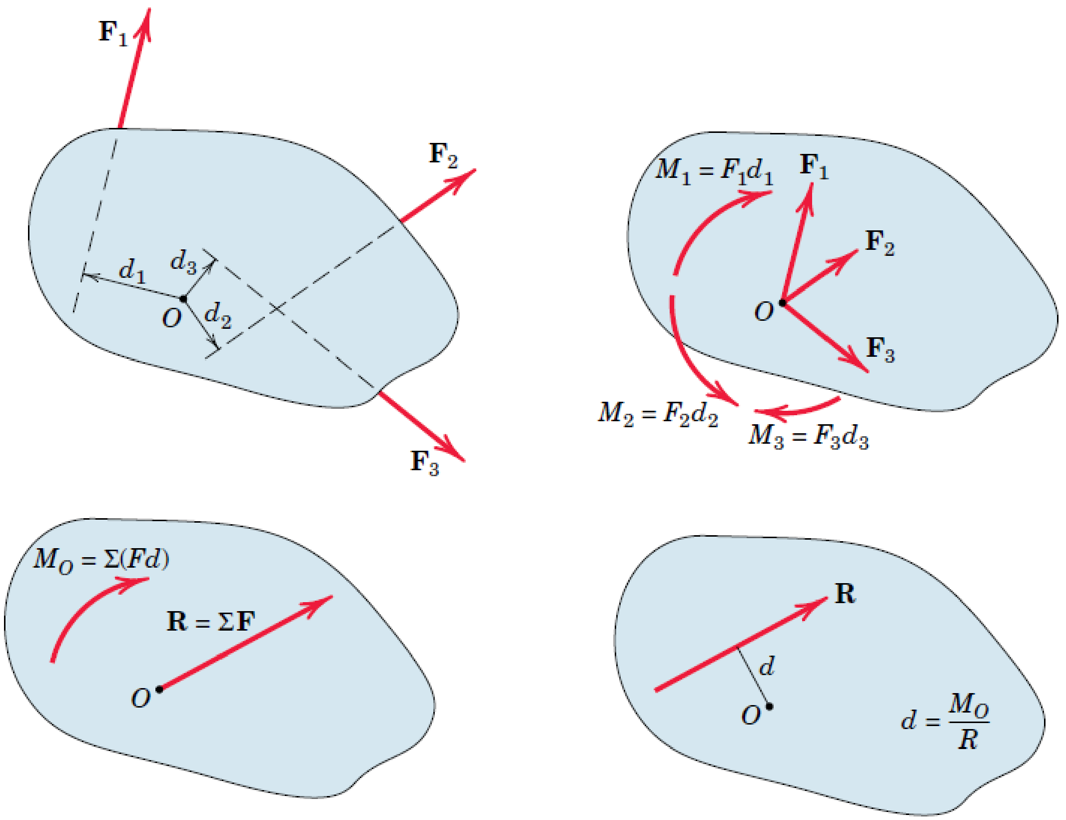




دانشگاه سمنان

2.6 RESULTANT

- Algebraic Method



2.6 RESULTANT

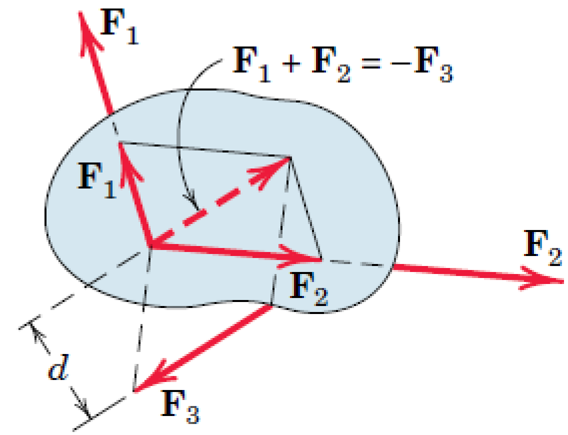
- Principle of Moments

$$\mathbf{R} = \Sigma \mathbf{F}$$

$$M_O = \Sigma M = \Sigma (Fd)$$

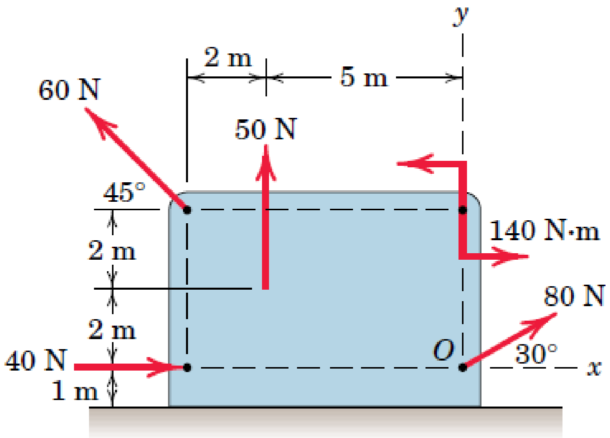
$$Rd = M_O$$

- The three forces have a zero resultant force but have a resultant clockwise couple ($M = F_3 d$)



Sample Problem 2/9

Determine the resultant of the four forces and one couple which act on the plate shown.



Solution. Point O is selected as a convenient reference point

$[R_x = \Sigma F_x]$ $\quad R_x = 40 + 80 \cos 30° - 60 \cos 45° = 66.9 \text{ N}$

$[R_y = \Sigma F_y]$ $\quad R_y = 50 + 80 \sin 30° + 60 \cos 45° = 132.4 \text{ N}$

$[R = \sqrt{R_x^2 + R_y^2}]$ $\quad R = \sqrt{(66.9)^2 + (132.4)^2} = 148.3 \text{ N}$

$\left[\theta = \tan^{-1} \frac{R_y}{R_x}\right]$ $\quad \theta = \tan^{-1} \frac{132.4}{66.9} = 63.2°$

$[M_O = \Sigma(Fd)]$ $\quad M_O = 140 - 50(5) + 60 \cos 45°(4) - 60 \sin 45°(7)$

$= -237 \text{ N·m}$

$[Rd = |M_O|]$ $\quad 148.3d = 237 \qquad d = 1.600 \text{ m}$

$$R_y b = |M_O| \qquad \text{and} \qquad b = \frac{237}{132.4} = 1.792 \text{ m}$$

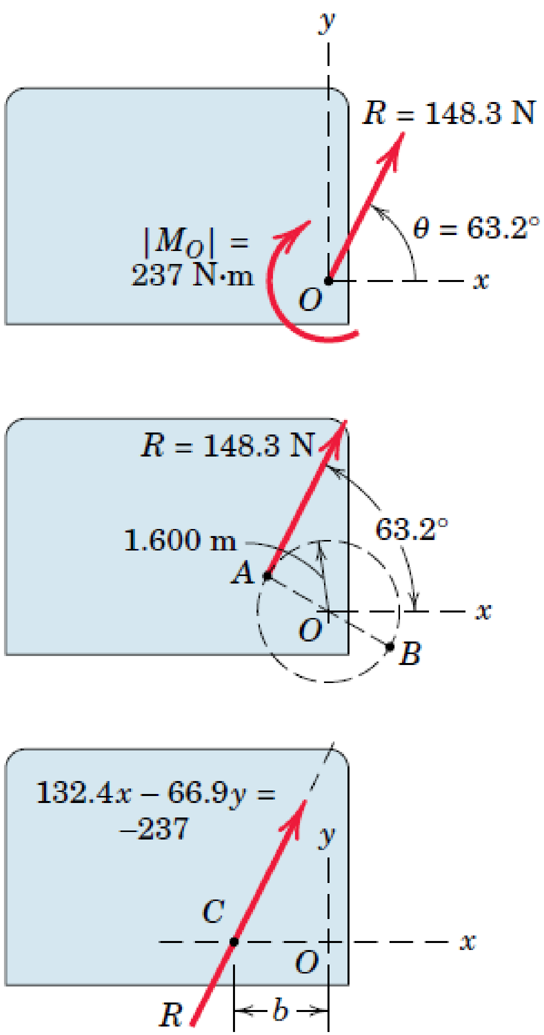
$$\mathbf{r} \times \mathbf{R} = \mathbf{M}_O$$

$$(x\mathbf{i} + y\mathbf{j}) \times (66.9\mathbf{i} + 132.4\mathbf{j}) = -237\mathbf{k}$$

$$(132.4x - 66.9y)\mathbf{k} = -237\mathbf{k}$$

$$132.4x - 66.9y = -237$$

By setting $y = 0$, we obtain $x = -1.792$ m



- ***Contents:***

3.1 INTRODUCTION

- **Statics:**
  - Description of the force conditions necessary and sufficient to maintain the equilibrium

- Procedures developed here form the basis for solving problems in both statics and dynamics

- When a body is in equilibrium, the resultant of *all* forces acting on it is zero.

$$\mathbf{R} = \Sigma\mathbf{F} = 0 \qquad \mathbf{M} = \Sigma\mathbf{M} = 0$$

3.2 System Isolation and the Free-Body Diagram

- First, Define the system to be analyzed and represent all forces acting on the body

  - A *mechanical system* is defined as a body or group of bodies which can be conceptually isolated from all other bodies

- ***Free-Body Diagram* (FBD)**:
  - A diagrammatic representation of the isolated system treated as a single body which shows all forces applied to the system containing

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3.2 System Isolation and the Free-Body Diagram

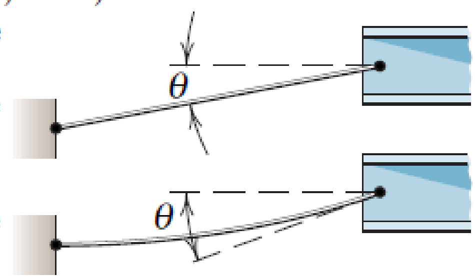
- Modeling the Action of Forces

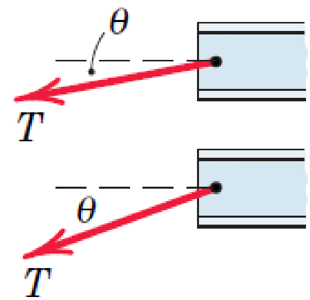
Type of Contact and Force Origin — Action on Body to Be Isolated

1. Flexible cable, belt, chain, or rope

Weight of cable negligible

Weight of cable not negligible





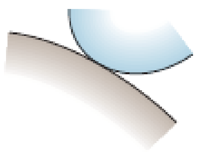
Force exerted by a flexible cable is always a tension away from the body in the direction of the cable.

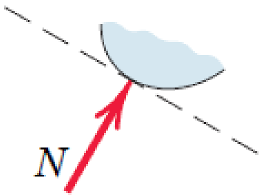
3.2 System Isolation and the Free-Body Diagram

- Modeling the Action of Forces

Type of Contact and Force Origin | Action on Body to Be Isolated

2. Smooth surfaces





Contact force is compressive and is normal to the surface.

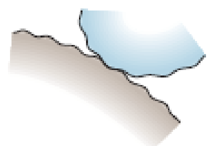
3.2 System Isolation and the Free-Body Diagram

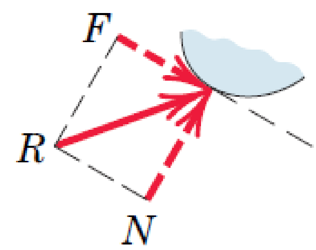
- Modeling the Action of Forces

Type of Contact and Force Origin Isolated

Action on Body to Be

3. Rough surfaces





Rough surfaces are capable of supporting a tangential component F (frictional force) as well as a normal component N of the resultant contact force R.

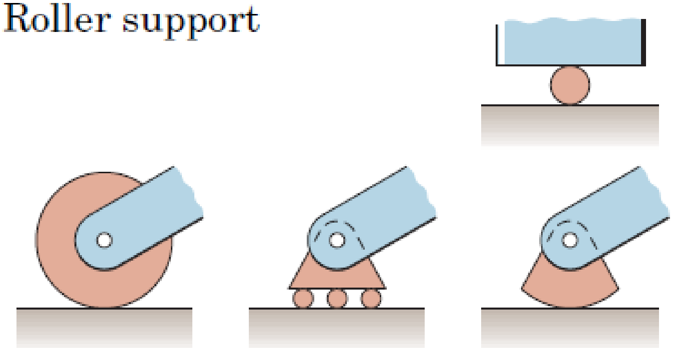
3.2 System Isolation and the Free-Body Diagram

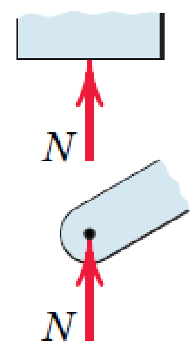
- Modeling the Action of Forces

Type of Contact and Force Origin

Action on Body to Be Isolated

4. Roller support





Roller, rocker, or ball support transmits a compressive force normal to the supporting surface.

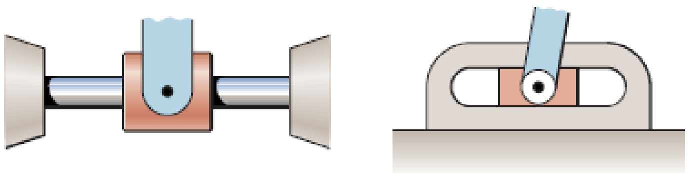
3.2 System Isolation and the Free-Body Diagram

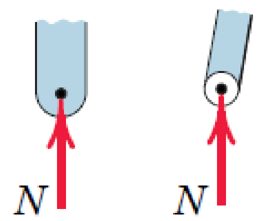
- Modeling the Action of Forces

Type of Contact and Force Origin Isolated

Action on Body to Be

5. Freely sliding guide





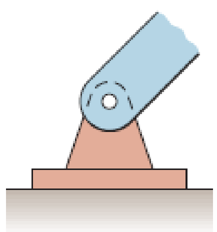
Collar or slider free to move along smooth guides; can support force normal to guide only.

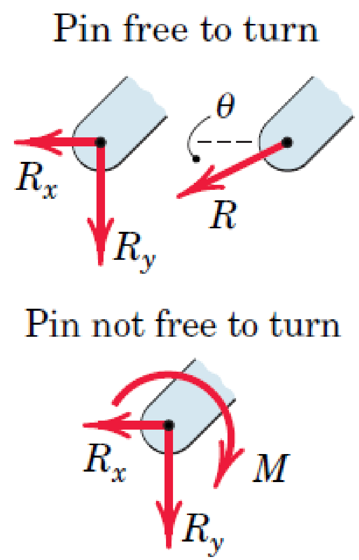
3.2 System Isolation and the Free-Body Diagram

- Modeling the Action of Forces

Type of Contact and Force Origin | Action on Body to Be Isolated

6. Pin connection





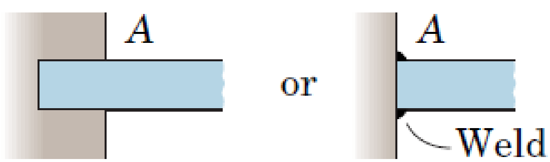
A freely hinged pin connection is capable of supporting a force in any direction in the plane normal to the pin axis. We may either show two components R_x and R_y or a magnitude R and direction θ. A pin not free to turn also supports a couple M.

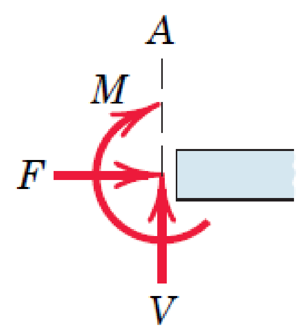
3.2 System Isolation and the Free-Body Diagram

- Modeling the Action of Forces

Type of Contact and Force Origin | Action on Body to Be Isolated

7. Built-in or fixed support





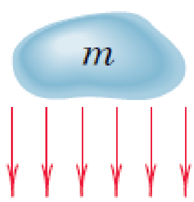
A built-in or fixed support is capable of supporting an axial force F, a transverse force V (shear force), and a couple M (bending moment) to prevent rotation.

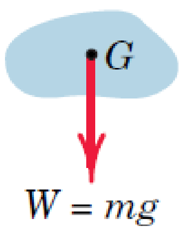
3.2 System Isolation and the Free-Body Diagram

- Modeling the Action of Forces

Type of Contact and Force Origin Action on Body to Be Isolated

8. Gravitational attraction





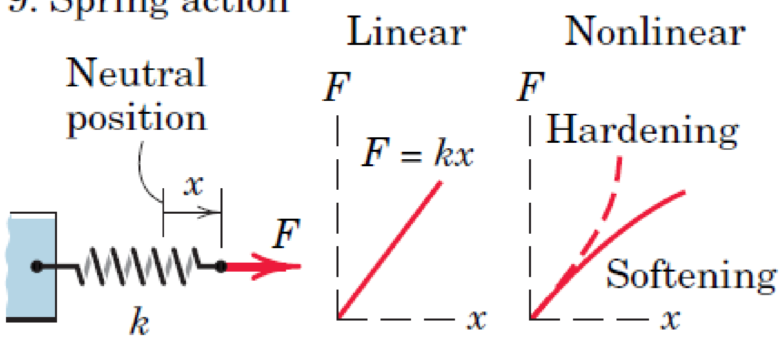
The resultant of gravitational attraction on all elements of a body of mass m is the weight $W = mg$ and acts toward the center of the earth through the center of gravity G.

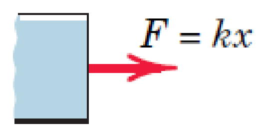
3.2 SYSTEM ISOLATION AND THE FREE-BODY DIAGRAM

- Modeling the Action of Forces

Type of Contact and Force Origin | Action on Body to Be Isolated

9. Spring action





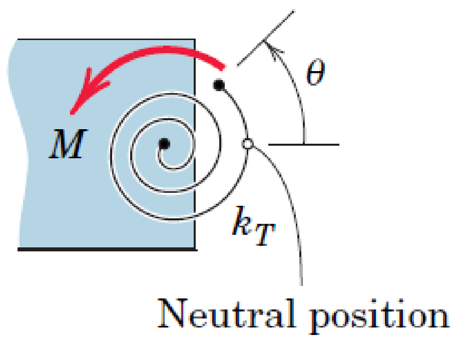
Spring force is tensile if the spring is stretched and compressive if compressed. For a linearly elastic spring the stiffness k is the force required to deform the spring a unit distance.

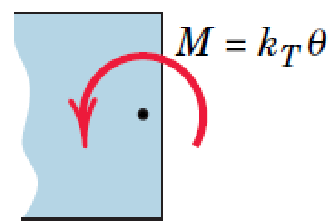
3.2 System Isolation and the Free-Body Diagram

- Modeling the Action of Forces

Type of Contact and Force Origin | Action on Body to Be Isolated

10. Torsional spring action





For a linear torsional spring, the applied moment M is proportional to the angular deflection θ from the neutral position. The stiffness k_T is the moment required to deform the spring one radian.

3.2 System Isolation and the Free-Body Diagram

Construction of Free-Body Diagrams

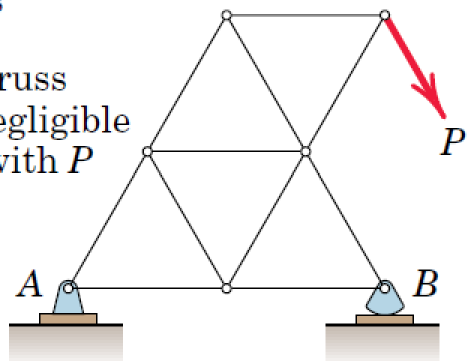
- **Step1.** Decide which system to isolate. (involve one or more of the desired unknown quantities)
- **Step2.** Isolate the system by drawing a diagram which represents its complete external boundary
- **Step 3.** Identify all forces acting as applied by the removed contacting and attracting bodies
- **Step 4.** Show the choice of coordinate axes directly on the diagram

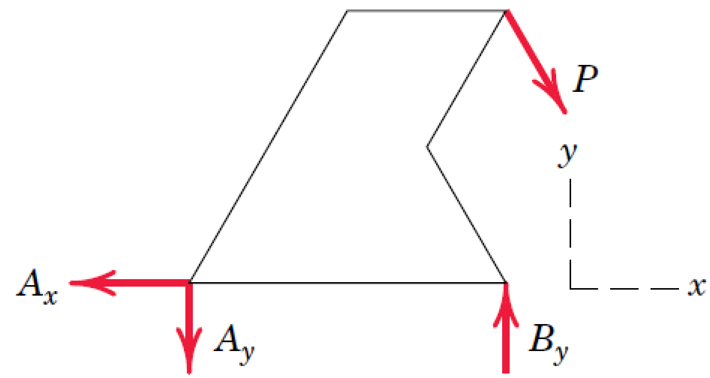
3.2 SYSTEM ISOLATION AND THE FREE-BODY DIAGRAM

- Examples of Free-Body Diagrams

1. Plane truss

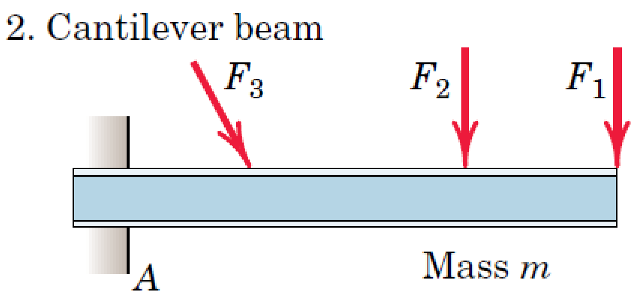
Weight of truss assumed negligible compared with P

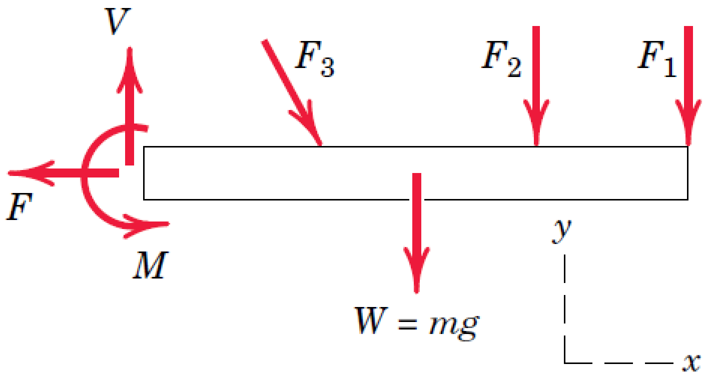




3.2 SYSTEM ISOLATION AND THE FREE-BODY DIAGRAM

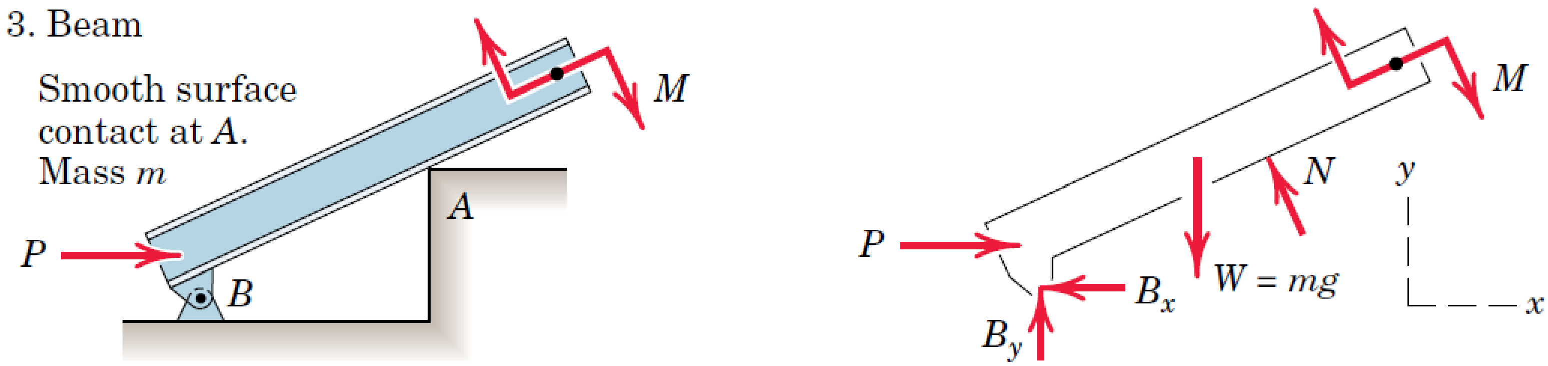
- Examples of Free-Body Diagrams





3.2 System Isolation and the Free-Body Diagram

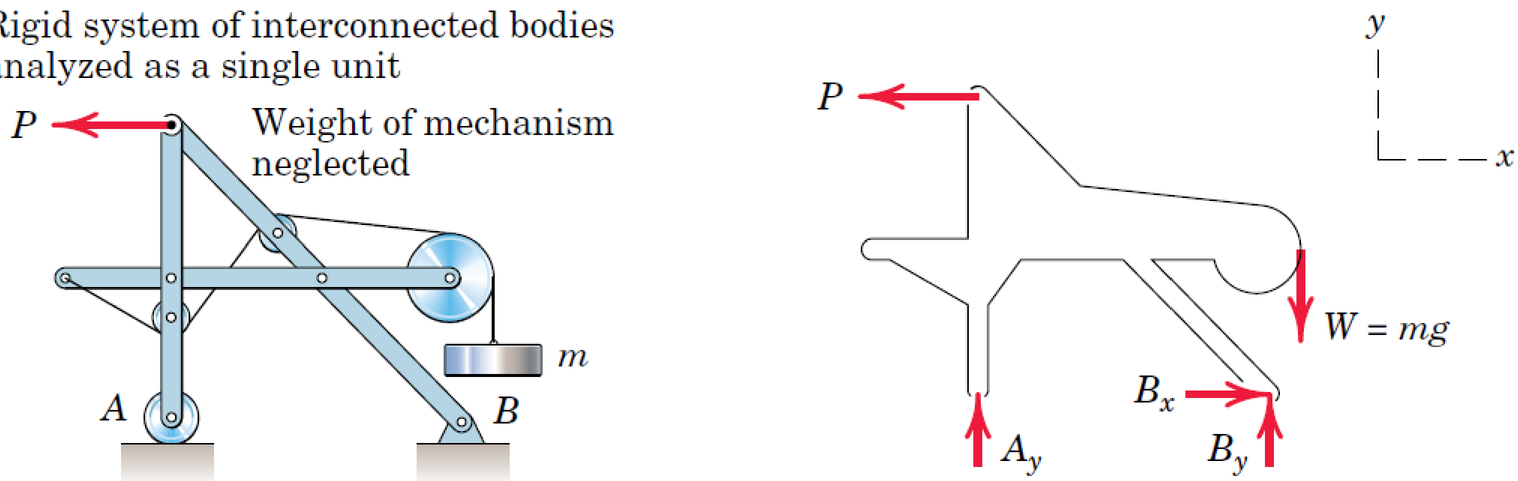
- Examples of Free-Body Diagrams



3.2 System Isolation and the Free-Body Diagram

- Examples of Free-Body Diagrams

4. Rigid system of interconnected bodies analyzed as a single unit



3.3 EQUILIBRIUM CONDITIONS

- **Equilibrium:**
  - The condition in which the resultant of all forces and moments acting on a body is zero.
  - A body is in equilibrium if all forces and moments applied to it are in balance.
  - Vector Equation:

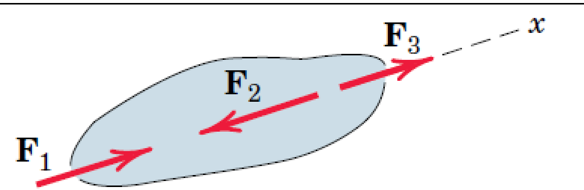
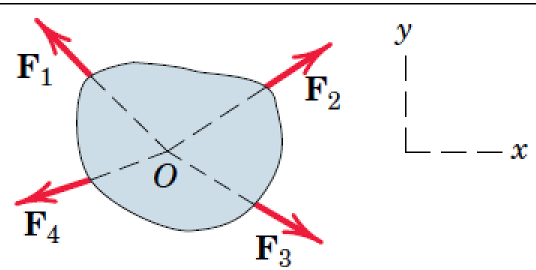
$$\mathbf{R} = \Sigma\mathbf{F} = \mathbf{0} \qquad \mathbf{M} = \Sigma\mathbf{M} = \mathbf{0}$$

  - Components Equation:
    - ✓ about any point O on or off the body

$$\Sigma F_x = 0 \qquad \Sigma F_y = 0 \qquad \Sigma M_O = 0$$

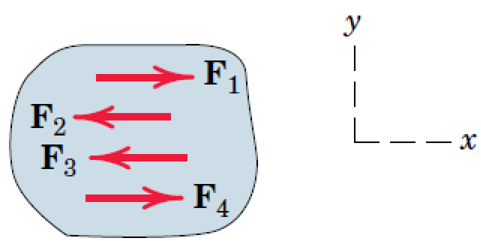
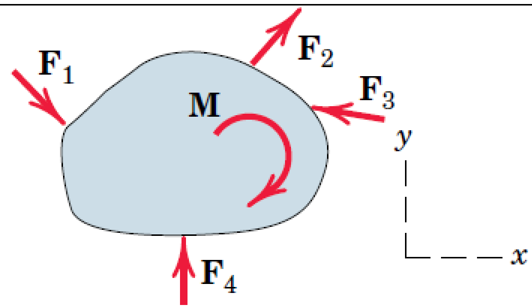
3.3 Equilibrium Conditions

- Categories of Equilibrium

Force System	Free-Body Diagram	Independent Equations
1. Collinear		$\Sigma F_x = 0$
2. Concurrent at a point		$\Sigma F_x = 0$ $\Sigma F_y = 0$

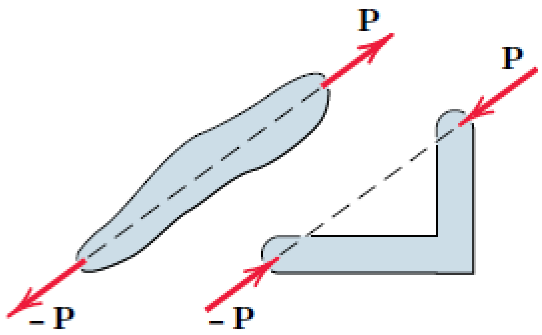
3.3 Equilibrium Conditions

- Categories of Equilibrium

Force System	Free-Body Diagram	Independent Equations
3. Parallel		$\Sigma F_x = 0 \quad \Sigma M_z = 0$
4. General		$\Sigma F_x = 0 \quad \Sigma M_z = 0$ $\Sigma F_y = 0$

3.3 Equilibrium Conditions

- Two-Force Members in equilibrium:
  - The forces must be:
    - ✓ *Equal*
    - ✓ *Opposite*
    - ✓ *Collinear*

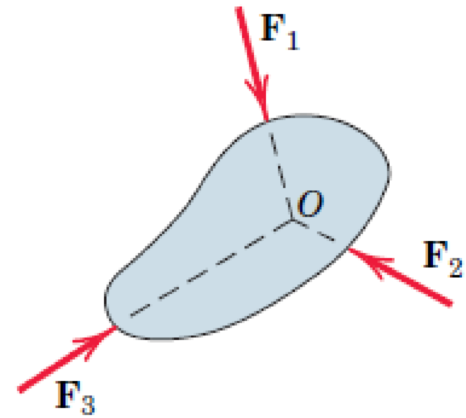


    - ✓ The shape of the member does not affect, the weights of the member is negligible

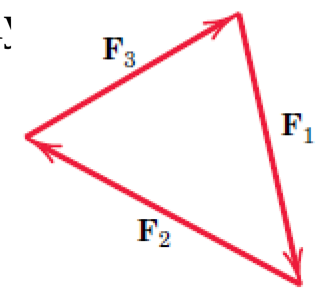
3.3 Equilibrium Conditions

- Three-Force Members in equilibrium:
  - Lines of action of the three forces to be *concurrent*

    (Except for 3 parallel forces)



- ✓ Three forces make closed pol



Closed polygon satisfies $\Sigma \mathbf{F} = 0$

3.3 Equilibrium Conditions

- Alternative Equilibrium Equations

  - ✓ The two points *A* and *B* must not lie on a line perpendicular to the *x*-direction.

$$\Sigma F_x = 0 \qquad \Sigma M_A = 0 \qquad \Sigma M_B = 0$$

  - ✓ *A*, *B*, and *C* are any three points not on the same straight line.

$$\Sigma M_A = 0 \qquad \Sigma M_B = 0 \qquad \Sigma M_C = 0$$

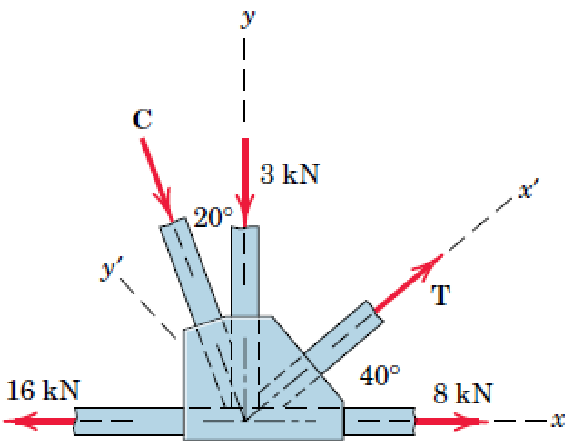
3.3 EQUILIBRIUM CONDITIONS

Approach to Solving Problems

1) Identify clearly the quantities which are known and unknown.
2) Choose body (or system of connected bodies) to be isolated.
3) Choose a convenient set of reference axes.
4) Identify and state the applicable force and moment principles or equations.
5) Match the number of independent equations with the number of unknowns.
6) Carry out the solution and check the results.

Sample Problem 3/1

Determine the magnitudes of the forces **C** and **T**, which, along with the other three forces shown, act on the bridge-truss joint.



Solution I (scalar algebra).

$[\Sigma F_x = 0]$ $$8 + T\cos 40^\circ + C\sin 20^\circ - 16 = 0$$

$$0.766T + 0.342C = 8$$

$[\Sigma F_y = 0]$ $$T\sin 40^\circ - C\cos 20^\circ - 3 = 0$$

$$0.643T - 0.940C = 3$$

$$T = 9.09\text{ kN} \qquad C = 3.03\text{ kN}$$

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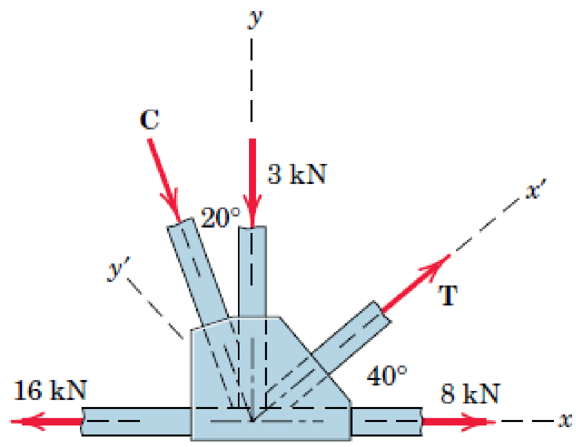
Sample Problem 3/1

Determine the magnitudes of the forces **C** and **T**, which, along with the other three forces shown, act on the bridge-truss joint.



Solution II (scalar algebra).

$[\Sigma F_{y'} = 0]$ $\quad -C \cos 20° - 3 \cos 40° - 8 \sin 40° + 16 \sin 40° = 0$

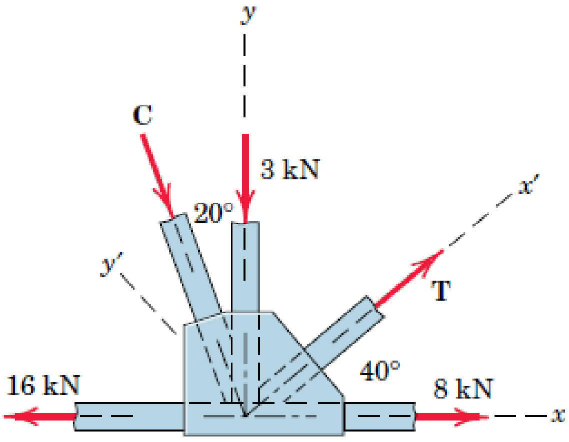
$C = 3.03 \text{ kN}$

$[\Sigma F_{x'} = 0]$ $\quad T + 8 \cos 40° - 16 \cos 40° - 3 \sin 40° - 3.03 \sin 20° = 0$

$T = 9.09 \text{ kN}$

Sample Problem 3/1

Determine the magnitudes of the forces **C** and **T**, which, along with the other three forces shown, act on the bridge-truss joint.



Solution III (vector algebra).

$$[\Sigma \mathbf{F} = \mathbf{0}] \qquad 8\mathbf{i} + (T\cos 40^\circ)\mathbf{i} + (T\sin 40^\circ)\mathbf{j} - 3\mathbf{j} + (C\sin 20^\circ)\mathbf{i} - (C\cos 20^\circ)\mathbf{j} - 16\mathbf{i} = \mathbf{0}$$

$$8 + T\cos 40^\circ + C\sin 20^\circ - 16 = 0$$

$$T\sin 40^\circ - 3 - C\cos 20^\circ = 0$$

$$T = 9.09 \text{ kN} \qquad C = 3.03 \text{ kN}$$

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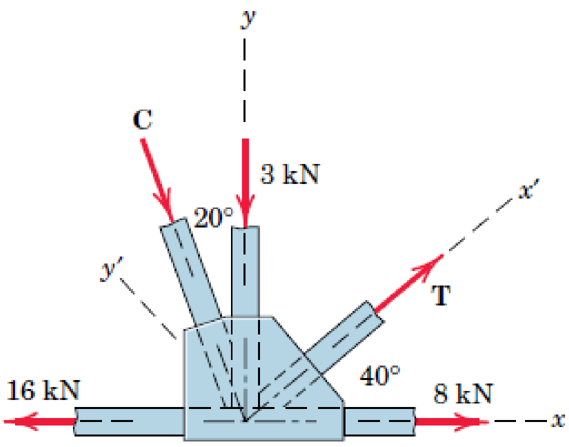
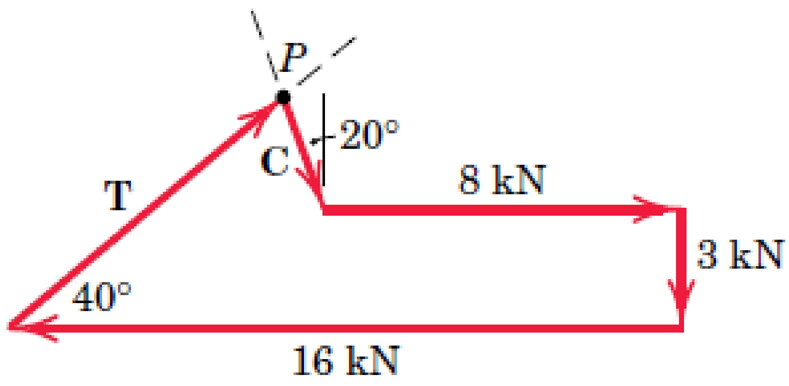
Sample Problem 3/1

Determine the magnitudes of the forces C and T, which, along with the other three forces shown, act on the bridge-truss joint.

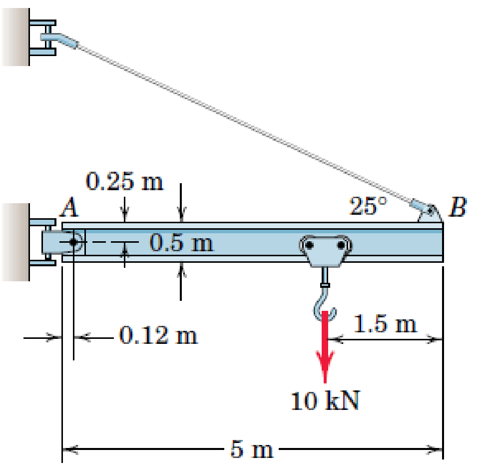


Solution IV (geometric).



Sample Problem 3/4

Determine the magnitude T of the tension in the supporting cable and the magnitude of the force on the pin at A for the jib crane shown. The beam AB is a standard 0.5-m I-beam with a mass of 95 kg per meter of length.



The weight of the beam is $95(10^{-3})(5)9.81 = 4.66$ kN

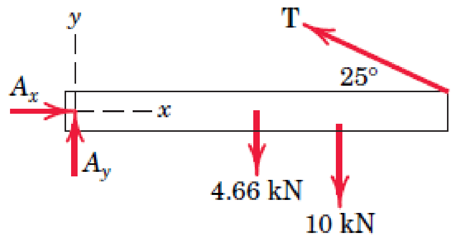
$[\Sigma M_A = 0]$ $\quad (T \cos 25°)0.25 + (T \sin 25°)(5 - 0.12) - 10(5 - 1.5 - 0.12) - 4.66(2.5 - 0.12) = 0$

$$T = 19.61 \text{ kN}$$

$[\Sigma F_x = 0]$ $\quad A_x - 19.61 \cos 25° = 0 \qquad A_x = 17.77 \text{ kN}$

$[\Sigma F_y = 0]$ $\quad A_y + 19.61 \sin 25° - 4.66 - 10 = 0 \qquad A_y = 6.37 \text{ kN}$

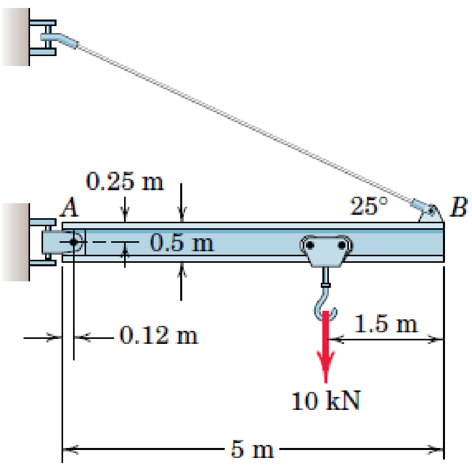
$$[A = \sqrt{A_x^2 + A_y^2}] \quad A = \sqrt{(17.77)^2 + (6.37)^2} = 18.88 \text{ kN}$$

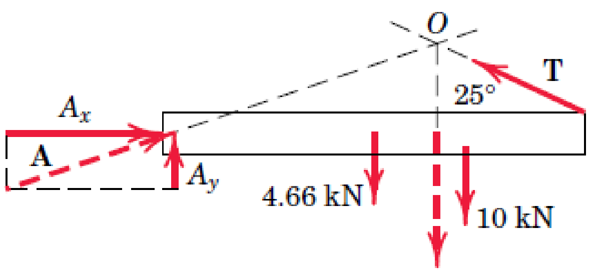


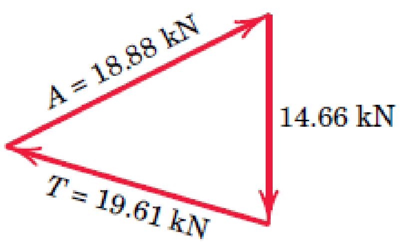
Free-body diagram

Sample Problem 3/4

Determine the magnitude T of the tension in the supporting cable and the magnitude of the force on the pin at A for the jib crane shown. The beam AB is a standard 0.5-m I-beam with a mass of 95 kg per meter of length.



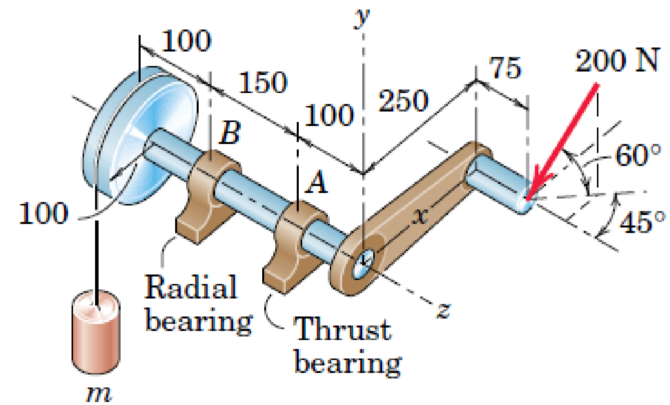




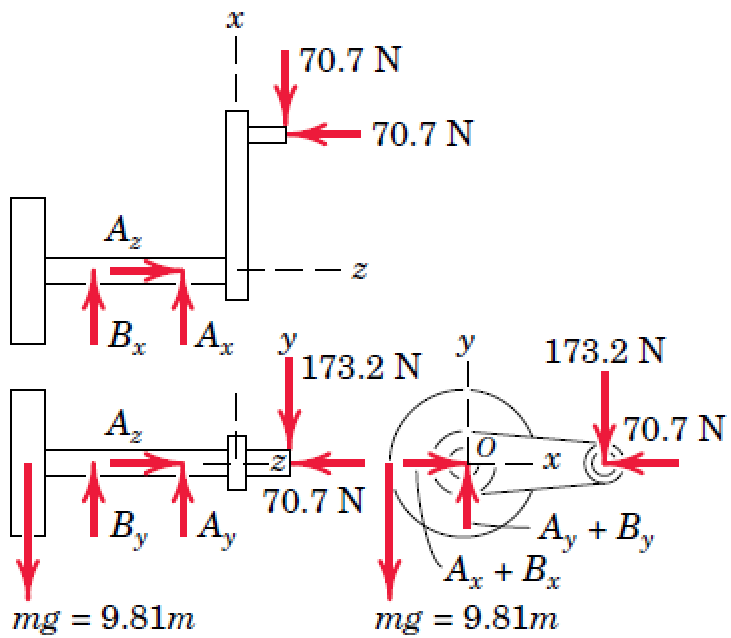
Graphical solution

Sample Problem 3/6

A 200-N force is applied to the handle of the hoist in the direction shown. The bearing A supports the thrust (force in the direction of the shaft axis), while bearing B supports only radial load (load normal to the shaft axis). Determine the mass m which can be supported and the total radial force exerted on the shaft by each bearing. Assume neither bearing to be capable of supporting a moment about a line normal to the shaft axis.



Dimensions in millimeters



$$[\Sigma M_O = 0] \quad 100(9.81m) - 250(173.2) = 0 \quad m = 44.1 \text{ kg}$$

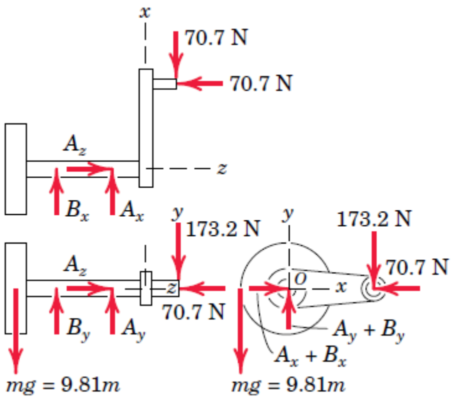
$$[\Sigma M_A = 0] \quad 150B_x + 175(70.7) - 250(70.7) = 0 \quad B_x = 35.4 \text{ N}$$

$$[\Sigma F_x = 0] \quad A_x + 35.4 - 70.7 = 0 \quad A_x = 35.4 \text{ N}$$

$$[\Sigma M_A = 0] \quad 150B_y + 175(173.2) - 250(44.1)(9.81) = 0 \quad B_y = 520 \text{ N}$$

$$[\Sigma F_y = 0] \quad A_y + 520 - 173.2 - (44.1)(9.81) = 0 \quad A_y = 86.8 \text{ N}$$

$$[\Sigma F_z = 0] \quad A_z = 70.7 \text{ N}$$



$$[A_r = \sqrt{A_x^2 + A_y^2}] \quad A_r = \sqrt{(35.4)^2 + (86.8)^2} = 93.5 \text{ N}$$

$$[B = \sqrt{B_x^2 + B_y^2}] \quad B = \sqrt{(35.4)^2 + (520)^2} = 521 \text{ N}$$

❑ ***Contents:***

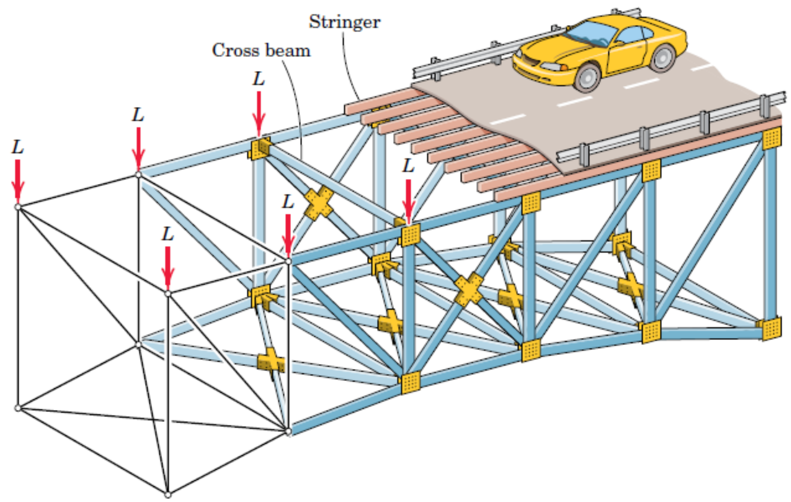
4.1 INTRODUCTION

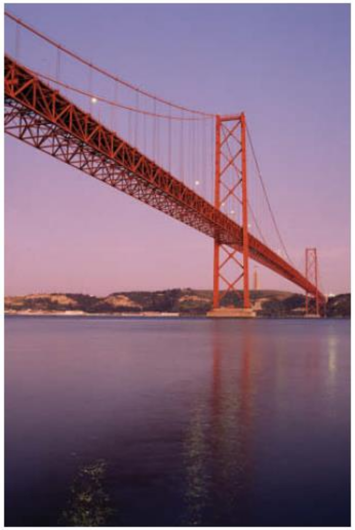
- An engineering structure:
  - Any connected system of members built to support or transfer forces and to safely withstand the loads applied to it.

- To determine the forces internal to an engineering structure;
  - dismember the structure and analyze separate free body diagrams of individual members or combinations of members.

4.2 Plane Trusses

- A framework composed of members joined at their ends to form a rigid structure is called a *truss*.
  - Bridges, roof supports, and other such structures are common examples of trusses.





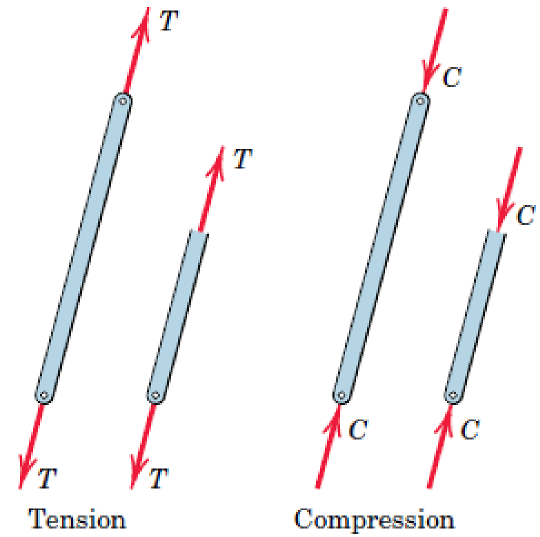
4.2 PLANE TRUSSES

- To design a truss:
  - Determine the forces in the various members
  - Select appropriate sizes and structural shapes to withstand the forces.

- Several assumptions are made in the force analysis of simple trusses:
  - All members to be *two-force members*.
    - ✓ A two-force member is one in equilibrium under the action of two forces only
  - Each member is normally a straight link joining the two points of application of force.
  - The two forces are applied at the ends and are necessarily equal, opposite, and *collinear*.
  - The weight of the member is small compared with the force it supports.

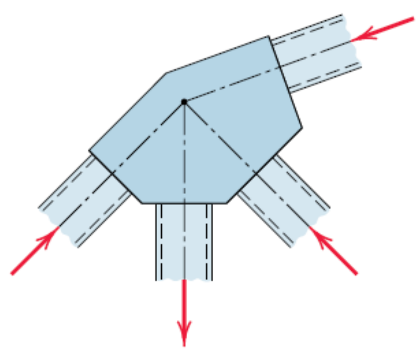
4.2 PLANE TRUSSES

- The member may be in tension or compression:



Two-Force Members

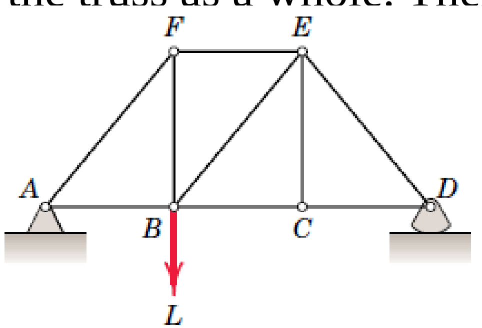
- Truss Connections and Supports:
  - ✓ When welded or riveted connections are used to join structural members, we may usually assume that the connection is a pin joint if the centerlines of the members are concurrent at the joint

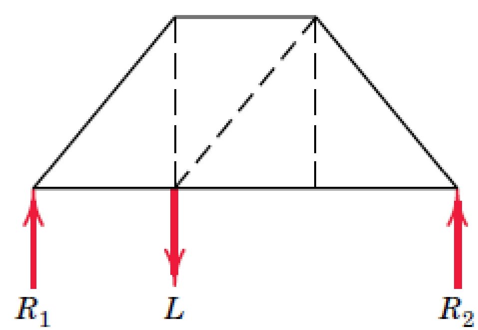


4.2 PLANE TRUSSES

- Two methods for the force analysis of simple trusses will be given:
  - ✓ *Method of Joints*
  - ✓ *Method of Sections*

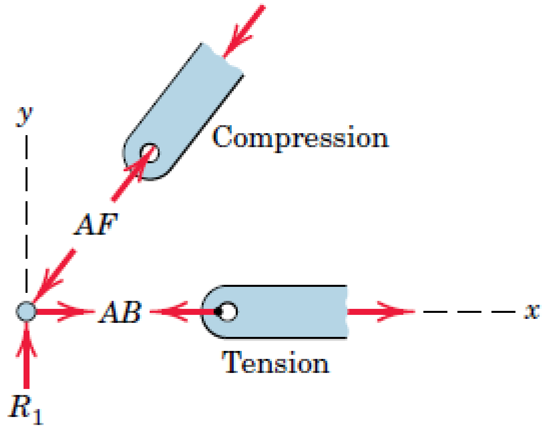
- ❖ The external reactions are usually determined first, by applying the equilibrium equations to the truss as a whole. Then the force analysis of the remainder of the truss is perf

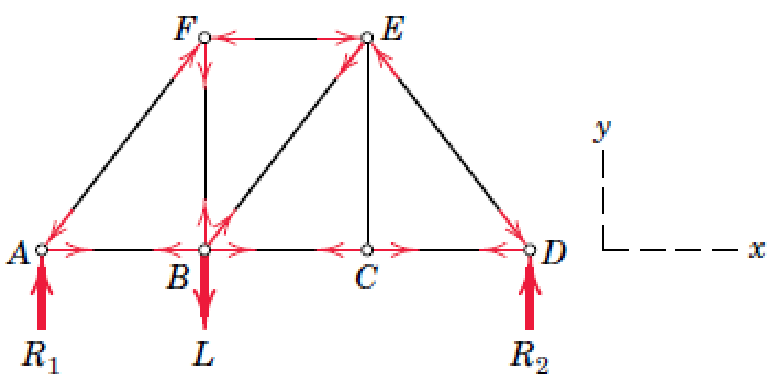




4.3 METHOD OF JOINTS

- Satisfying the conditions of equilibrium for the forces acting on the connecting pin of each joint.
  - The method therefore deals with the equilibrium of concurrent forces, and only two independent equilibrium equations are involved.

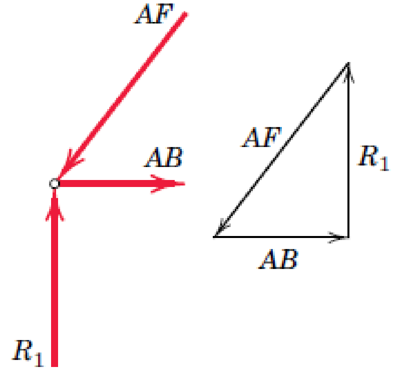




4.3 METHOD OF JOINTS

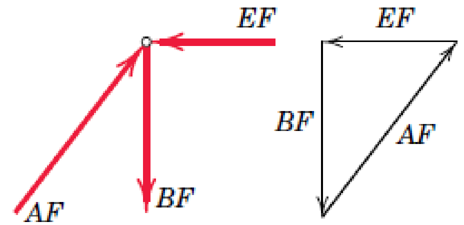
- Analyzing the Truss

1



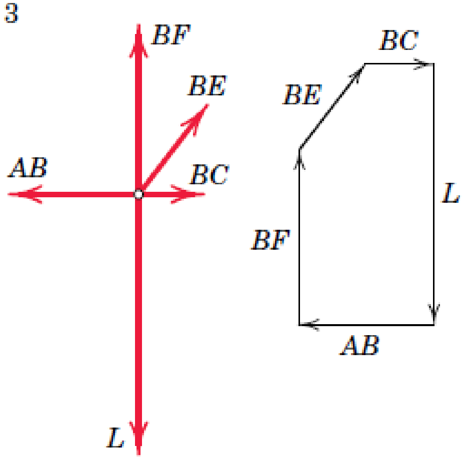
Joint A

2

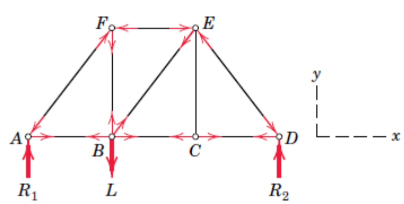


Joint F

3



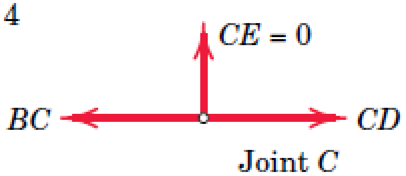
Joint B



4.3 METHOD OF JOINTS

- Analyzing the Truss

4



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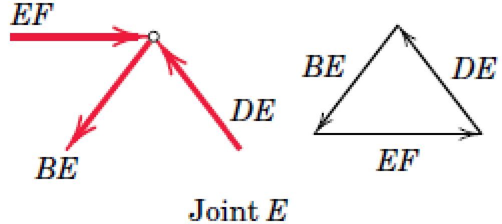

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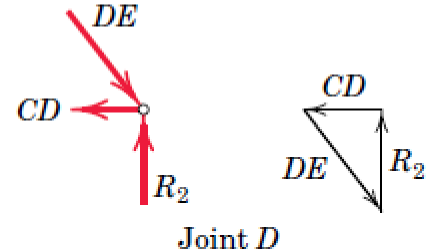
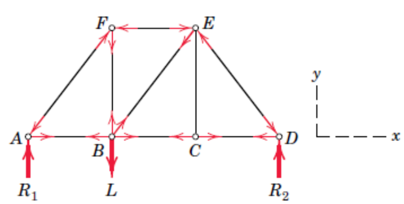




4.4 Method of Sections

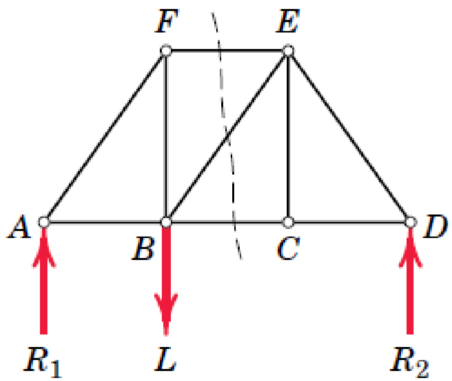
- We can take advantage of the third or moment equation of equilibrium by selecting an entire section of the truss for the free body in equilibrium.

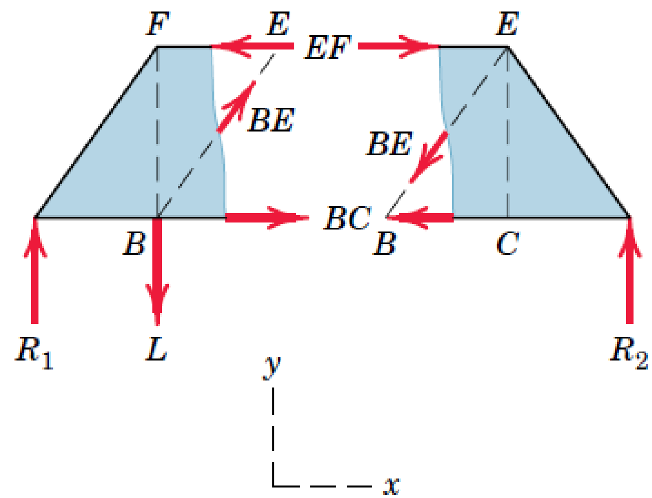
- The force in almost any desired member may be found directly from an analysis of a section which has cut that member.

- In choosing a section of the truss, in general, not more than three members whose forces are unknown should be cut.

4.4 METHOD OF SECTIONS

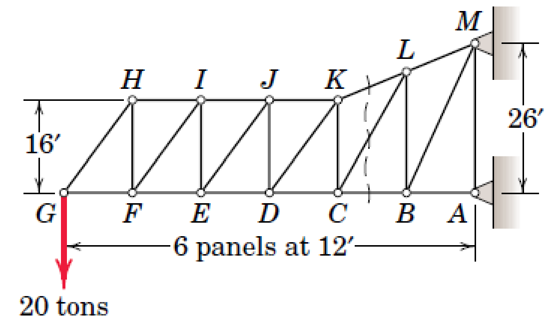
- Illustration of the Method of Sections





Sample Problem 4/3

Calculate the forces induced in members KL, CL, and CB by the 20-ton load on the cantilever truss.



$\overline{BL} = 16 + (26 - 16)/2 = 21 \text{ ft}$

$[\Sigma M_L = 0] \qquad 20(5)(12) - CB(21) = 0 \qquad CB = 57.1 \text{ tons } C$

$\theta = \tan^{-1}(5/12)$ → $\cos\theta = 12/13$

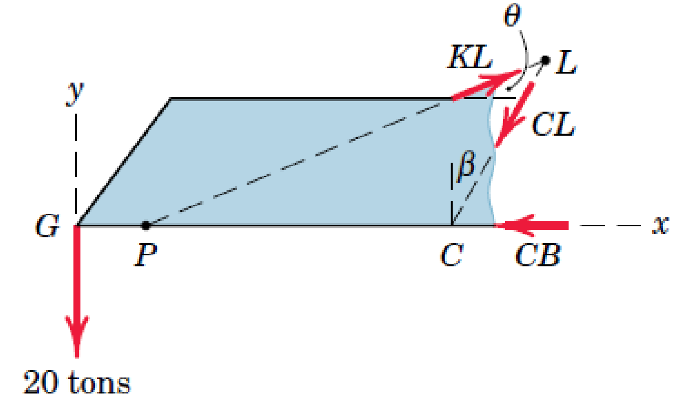
$[\Sigma M_C = 0] \qquad 20(4)(12) - \frac{12}{13}KL(16) = 0 \qquad KL = 65 \text{ tons } T$

$\overline{PC}/16 = 24/(26 - 16) = 38.4 \text{ ft}$

$\beta = \tan^{-1}(\overline{CB}/\overline{BL}) = \tan^{-1}(12/21) = 29.7°$ → $\cos\beta = 0.868$

→ $[\Sigma M_P = 0] \qquad 20(48 - 38.4) - CL(0.868)(38.4) = 0$

$CL = 5.76 \text{ tons } C$



4.6 FRAMES AND MACHINES

- *Frame* or *Machine:*
  - A structure which at least one of its individual members is a *multiforce member*.

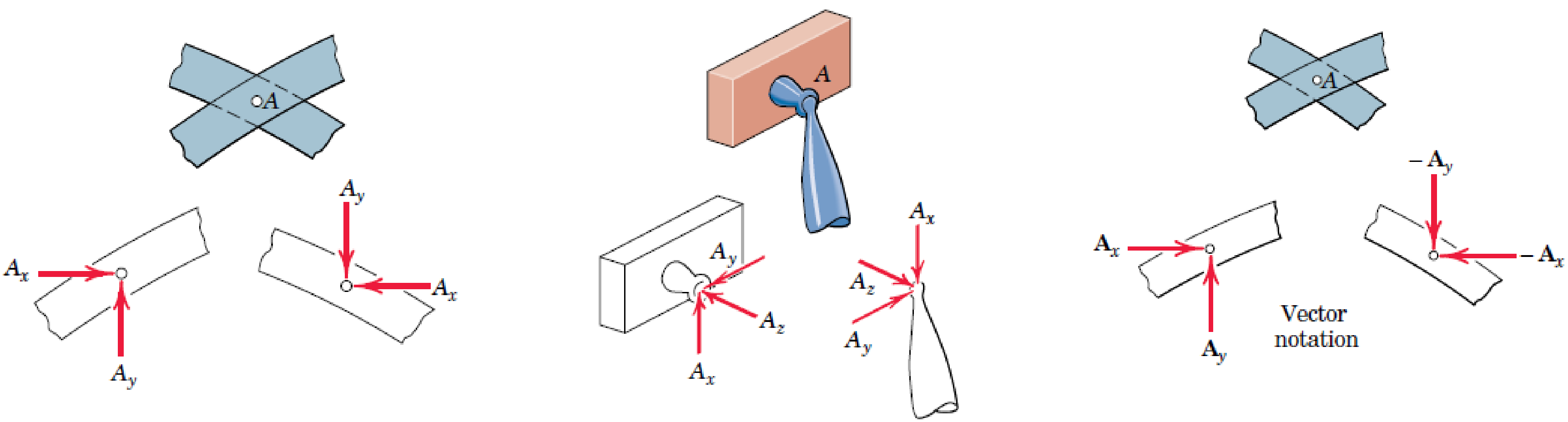
    - ✓ A multiforce member is defined as one with three or more forces acting on it, or one with two or more forces and one or more couples acting on it.
    - ✓ Frames are structures which are designed to support applied loads and are usually fixed in position.
    - ✓ Machines are structures which contain moving parts and are designed to transmit input forces or couples to output forces or couples.
    - ✓ Because frames and machines contain multiforce members, the forces in these members in general will *not* be in the directions of the members.
    - ✓ Therefore, we cannot analyze these structures by the methods developed.

4.6 FRAMES AND MACHINES

- Interconnected Rigid Bodies with Multiforce Members
  - The forces acting on each member of a connected system are found by isolating the member with a free-body diagram and applying the equations of equilibrium.
  - The *principle of action and reaction* must be carefully observed.

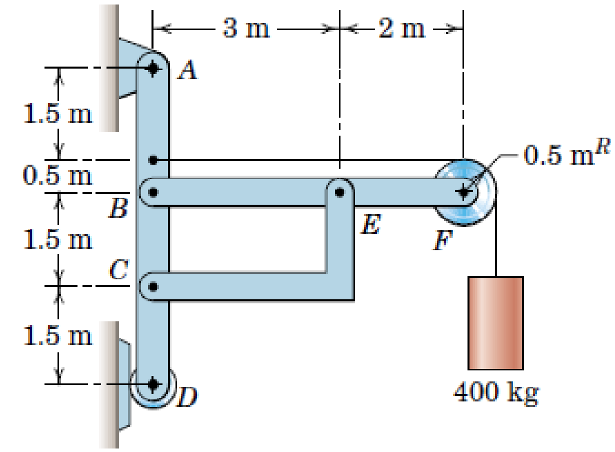
4.6 FRAMES AND MACHINES

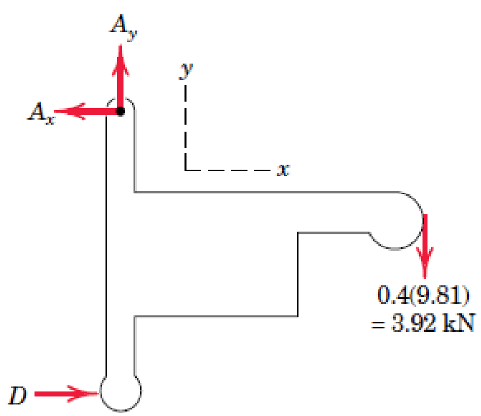
- Force Representation and Free-Body Diagrams



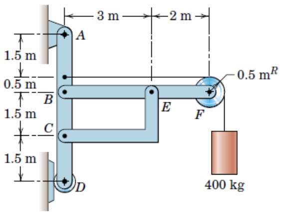
Sample Problem 4/6

The frame supports the 400-kg load in the manner shown. Neglect the weights of the members compared with the forces induced by the load and compute the horizontal and vertical components of all forces acting on each of the members.





$[\Sigma M_A = 0]$	$5.5(0.4)(9.81) - 5D = 0$	$D = 4.32\text{ kN}$
$[\Sigma F_x = 0]$	$A_x - 4.32 = 0$	$A_x = 4.32\text{ kN}$
$[\Sigma F_y = 0]$	$A_y - 3.92 = 0$	$A_y = 3.92\text{ kN}$

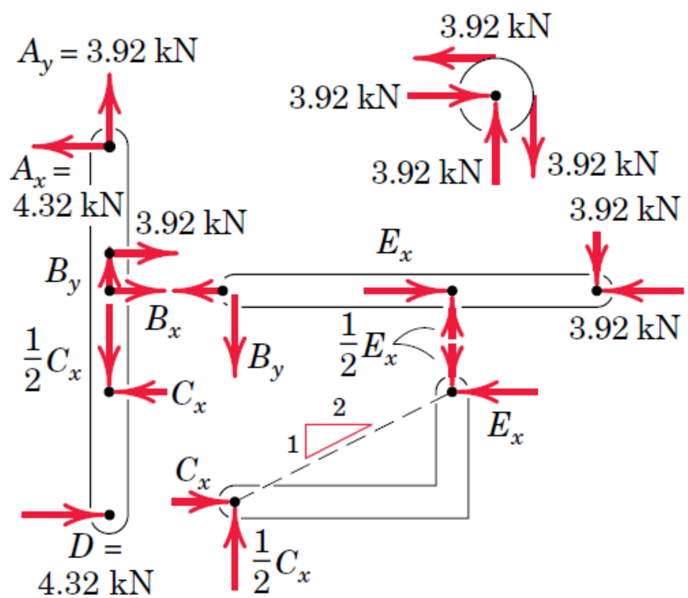


- Member BF:

$[\Sigma M_B = 0]$	$3.92(5) - \frac{1}{2}E_x(3) = 0$	$E_x = 13.08 \text{ kN}$
$[\Sigma F_y = 0]$	$B_y + 3.92 - 13.08/2 = 0$	$B_y = 2.62 \text{ kN}$
$[\Sigma F_x = 0]$	$B_x + 3.92 - 13.08 = 0$	$B_x = 9.15 \text{ kN}$

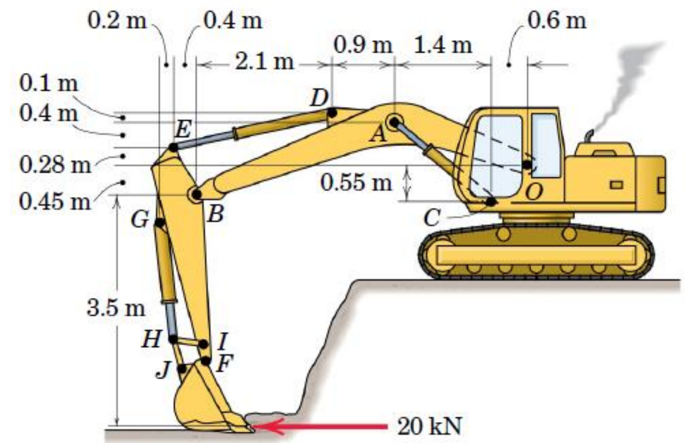
- Member AD:

$[\Sigma M_C = 0]$	$4.32(3.5) + 4.32(1.5) - 3.92(2) - 9.15(1.5) = 0$
$[\Sigma F_x = 0]$	$4.32 - 13.08 + 9.15 + 3.92 + 4.32 = 0$
$[\Sigma F_y = 0]$	$-13.08/2 + 2.62 + 3.92 = 0$



Sample Problem 4/9

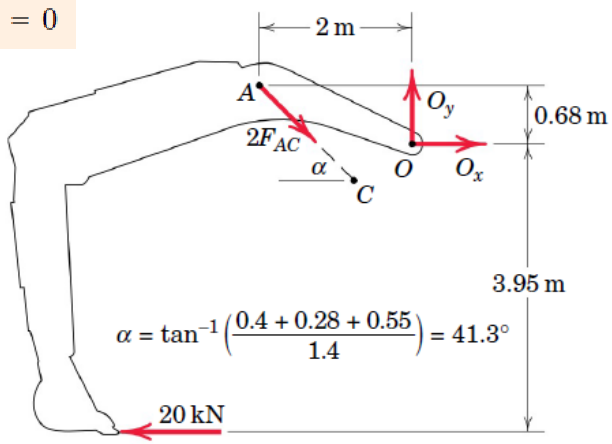
In the particular position shown, the excavator applies a 20-kN force parallel to the ground. There are two hydraulic cylinders *AC* to control the arm *OAB* and a single cylinder *DE* to control arm *EBIF*. (*a*) Determine the force in the hydraulic cylinders *AC* and the pressure p_{AC} against their pistons, which have an effective diameter of 95 mm. (*b*) Also determine the force in hydraulic cylinder *DE* and the pressure p_{DE} against its 105-mm-diameter piston. Neglect the weights of the members compared with the effects of the 20-kN force.



$$[\Sigma M_O = 0] \qquad -20\,000(3.95) - 2F_{AC}\cos 41.3°(0.68) + 2F_{AC}\sin 41.3°(2) = 0$$

$$F_{AC} = 48\,800 \text{ N or } 48.8 \text{ kN}$$

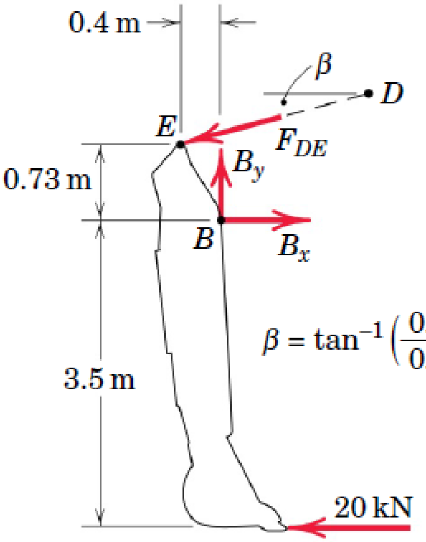
$$p_{AC} = \frac{F_{AC}}{A_{AC}} = \frac{48\,800}{\left(\pi \dfrac{0.095^2}{4}\right)} = 6.89(10^6) \text{ Pa or } 6.89 \text{ MPa}$$



$$[\Sigma M_B = 0] \qquad -20\,000(3.5) + F_{DE}\cos 11.31°(0.73) + F_{DE}\sin 11.31°(0.4) = 0$$

$$F_{DE} = 88\,100 \text{ N or } 88.1 \text{ kN}$$

$$p_{DE} = \frac{F_{DE}}{A_{DE}} = \frac{88\,100}{\left(\pi\dfrac{0.105^2}{4}\right)} = 10.18(10^6) \text{ Pa or } 10.18 \text{ MPa}$$



$$\beta = \tan^{-1}\left(\frac{0.1+0.4}{0.4+2.1}\right) = 11.31°$$

دانشگاه سمنان

Semnan University
Faculty of Mechanical Engineering

دانشکده مهندسی مکانیک

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درس استاتیک

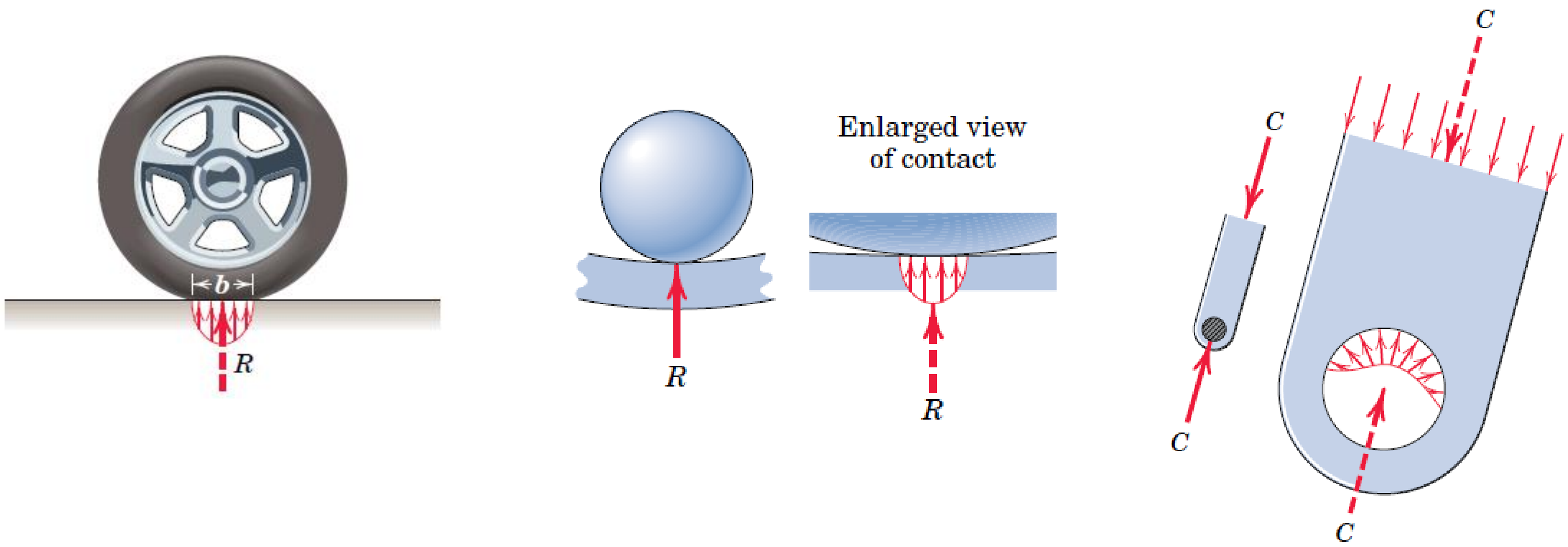
STATICS

Chapter 5 - Distributed Forces
Class Lecture

❑ CONTENTS:

5.1 Introduction

- Actually, “concentrated” forces do not exist in the exact sense, since every external force applied mechanically to a body is distributed over a finite contact area, however small.

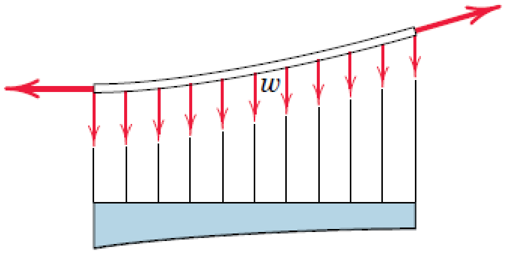


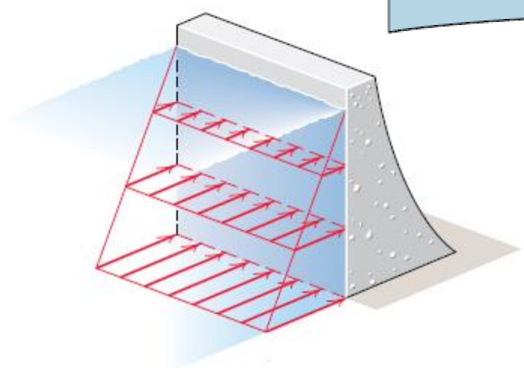
5.1 INTRODUCTION

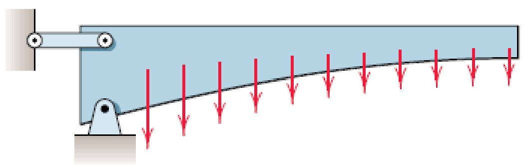
- When forces are applied over a region whose dimensions are not negligible compared with other pertinent dimensions, then we must account for the actual manner in which the force is distributed.
- We do this by summing the effects of the distributed force over the entire region using mathematical integration.
- This requires that we know the intensity of the force at any location.

5.1 INTRODUCTION

- There are three categories:
  - (1) Line Distribution
  - (2) Area Distribution
  - (3) Volume Distribution

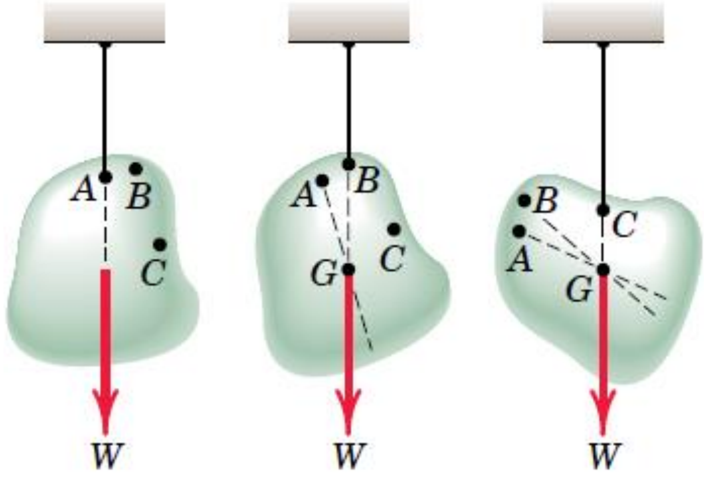






5.2 Center of Mass

- If we suspend the body from any point the body will be in equilibrium under the action of the cord tension and the resultant *W* of the gravitational forces acting on all particles of the body.
- If we repeat for other points, the center of gravity (CG) will be determined by intersection of these lines.

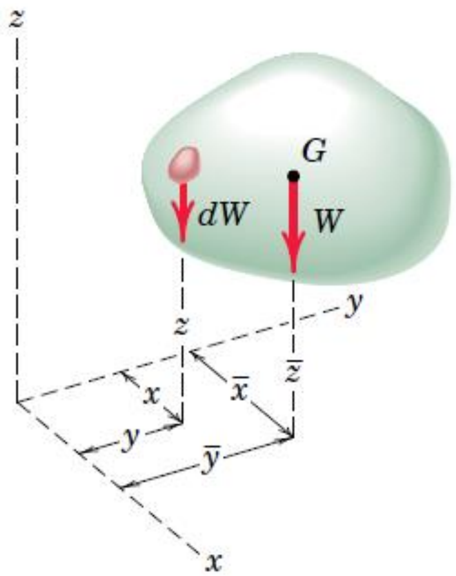


5.2 Center of Mass

- Determining the Center of Gravity
  - The moment of the resultant gravitational force *W* about any axis equals the sum of the moments about the same axis of the gravitational forces *dW* acting on all particles.

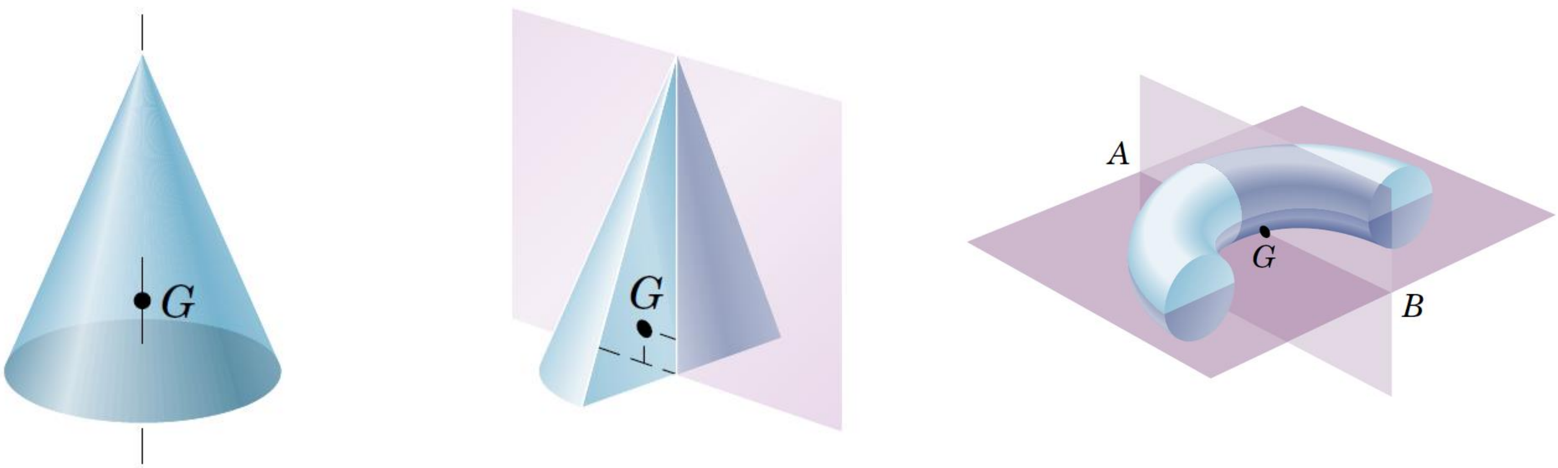
$$\bar{x} = \frac{\int x\, dW}{W} \qquad \bar{y} = \frac{\int y\, dW}{W} \qquad \bar{z} = \frac{\int z\, dW}{W}$$

$$\bar{x} = \frac{\int x\, dm}{m} \qquad \bar{y} = \frac{\int y\, dm}{m} \qquad \bar{z} = \frac{\int z\, dm}{m}$$



5.2 Center of Mass

- Using symmetry in CG determination

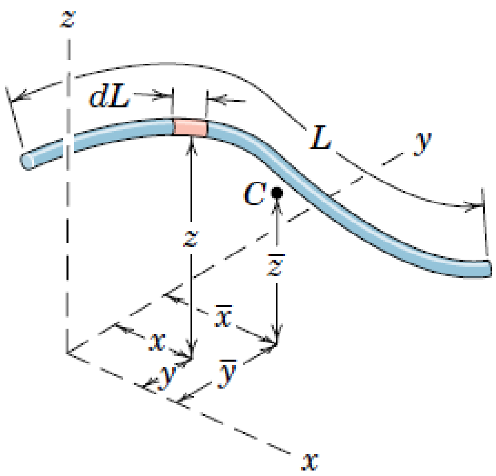


5.3 Centroids of Lines, Areas, and Volumes

- (1) Lines

$$dm = \rho A \, dL$$

$$\bar{x} = \frac{\int x \, dL}{L} \qquad \bar{y} = \frac{\int y \, dL}{L} \qquad \bar{z} = \frac{\int z \, dL}{L}$$

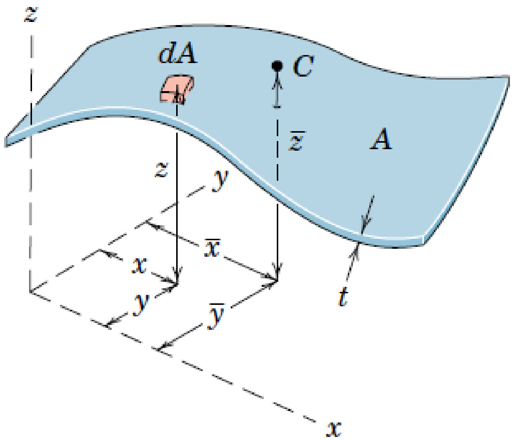


5.3 Centroids of Lines, Areas, and Volumes

- (2) Areas

$$dm = \rho t \, dA$$

$$\bar{x} = \frac{\int x \, dA}{A} \qquad \bar{y} = \frac{\int y \, dA}{A} \qquad \bar{z} = \frac{\int z \, dA}{A}$$

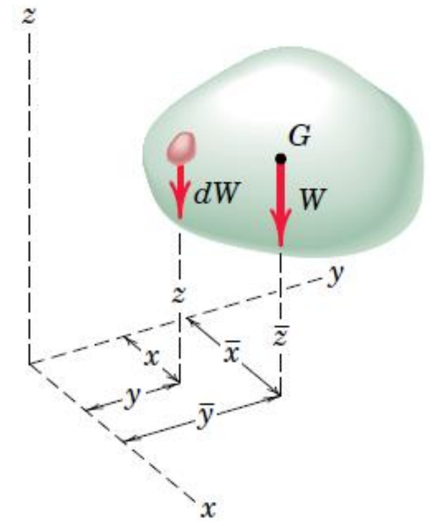


5.3 Centroids of Lines, Areas, and Volumes

- (3) Volumes

$$dm = \rho\, dV$$

$$\bar{x} = \frac{\int x\, dV}{V} \qquad \bar{y} = \frac{\int y\, dV}{V} \qquad \bar{z} = \frac{\int z\, dV}{V}$$

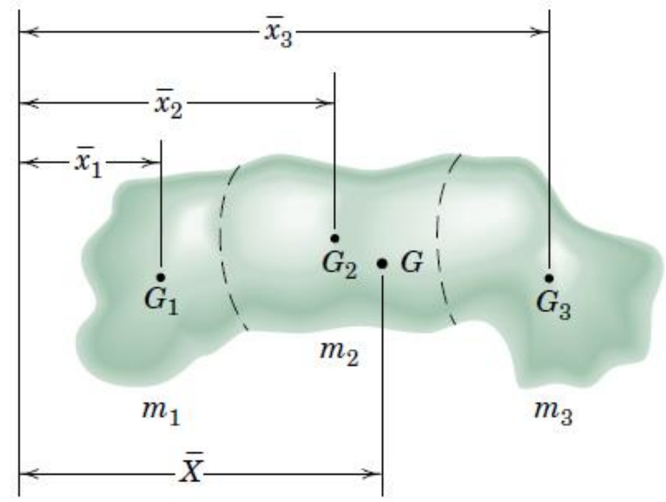


5.5 Composite Bodies and Figures; Approximations

- When a body or figure can be conveniently divided into several parts whose mass centers are easily determined, we use the principle of moments and treat each part as a finite element of the whole.

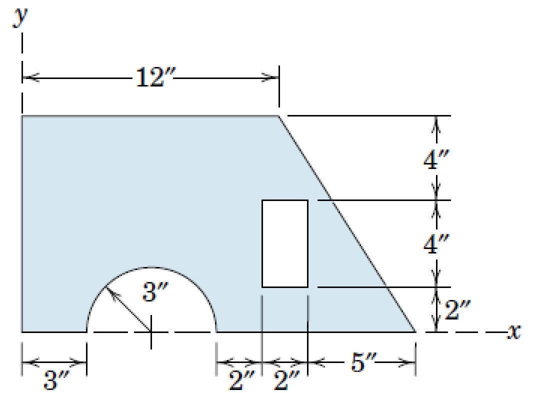
$$(m_1 + m_2 + m_3)\overline{X} = m_1\bar{x}_1 + m_2\bar{x}_2 + m_3\bar{x}_3$$

$$\overline{X} = \frac{\Sigma m\bar{x}}{\Sigma m} \qquad \overline{Y} = \frac{\Sigma m\bar{y}}{\Sigma m} \qquad \overline{Z} = \frac{\Sigma m\bar{z}}{\Sigma m}$$

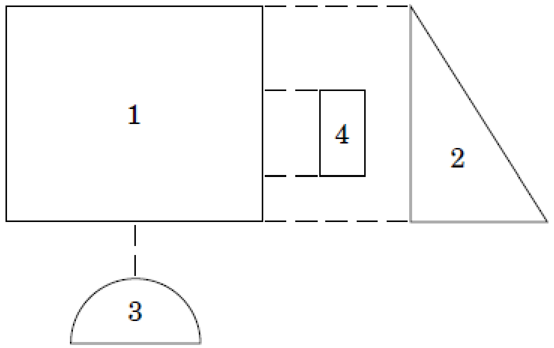


Sample Problem 5/6

Locate the centroid of the shaded area.



PART	A in.^{2}	$\bar{x}$ in.	$\bar{y}$ in.	$\bar{x}A$ in.^{3}	$\bar{y}A$ in.^{3}
1	120	6	5	720	600
2	30	14	10/3	420	100
3	−14.14	6	1.273	−84.8	−18
4	−8	12	4	−96	−32
TOTALS	127.9			959	650



$$\left[\bar{X} = \frac{\Sigma A\bar{x}}{\Sigma A}\right] \qquad \bar{X} = \frac{959}{127.9} = 7.50 \text{ in.}$$

$$\left[\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A}\right] \qquad \bar{Y} = \frac{650}{127.9} = 5.08 \text{ in.}$$

5.6 BEAMS - EXTERNAL EFFECTS

- *Beams* are structural members which offer resistance to bending due to applied loads.
- Beams are undoubtedly the most important of all structural members, so it is important to understand the basic theory underlying their design.

- We must:
  - First, establish the equilibrium requirements of the beam as a whole and any portion of it considered separately.
  - Second, we must establish the relations between the resulting forces and the accompanying internal resistance of the beam to support these forces.

5.6 Beams - External Effects

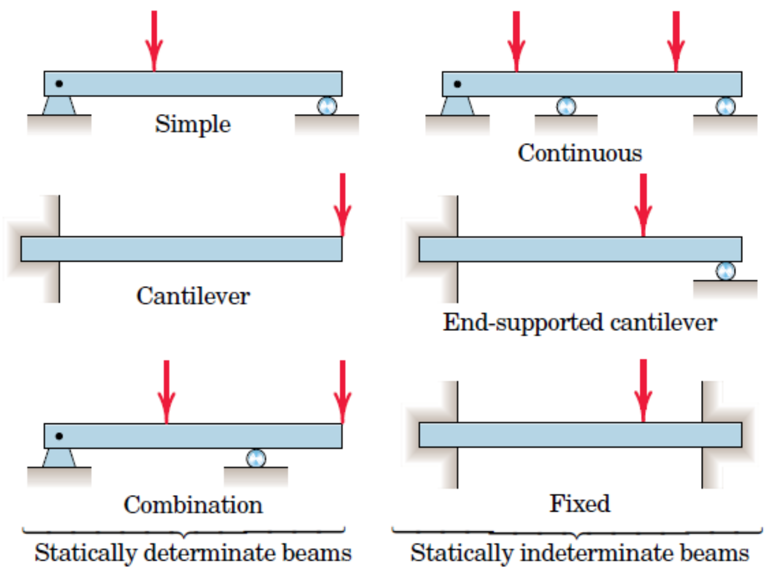
- Types of Beams:
  - *Statically determinate beams*

✓External support reactions can be calculated by the methods of statics alone are called.

  - *Statically indeterminate beams*

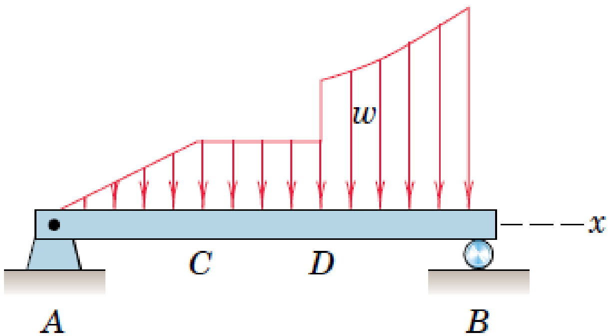
✓Has more supports than needed to provide equilibrium

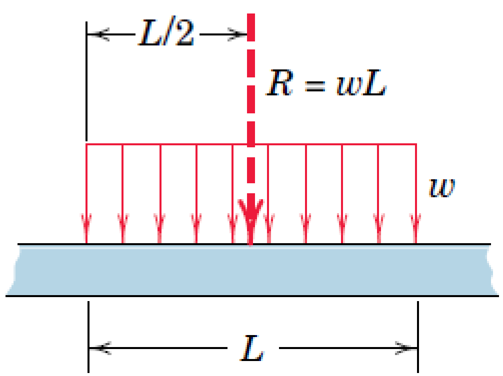
✓Load-deformation properties should be considered to calculate external support reactions

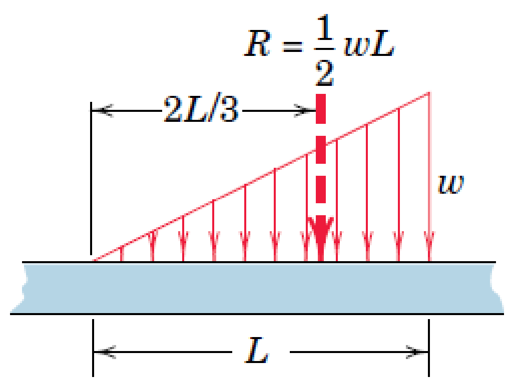


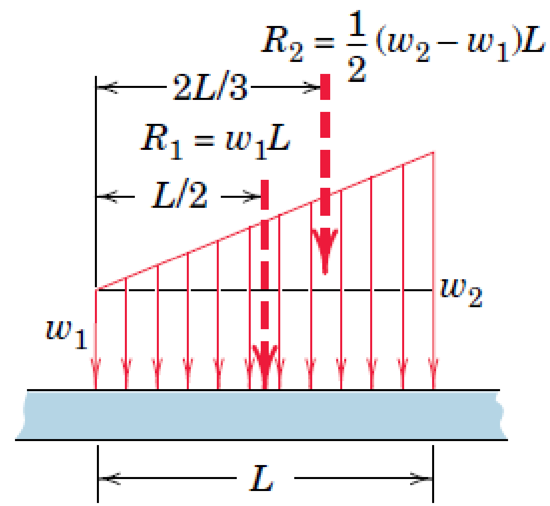
5.6 BEAMS - EXTERNAL EFFECTS

- Distributed Loads
  - Broking to simple cases
    - ✓ Constant
    - ✓ Rectangular
    - ✓ Triangular







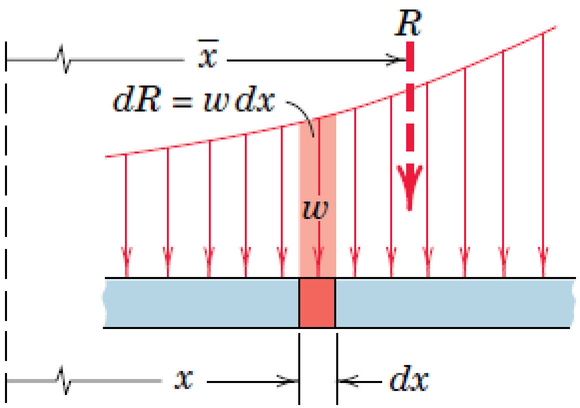


5.6 Beams - External Effects

- Distributed Loads
  - General distribution

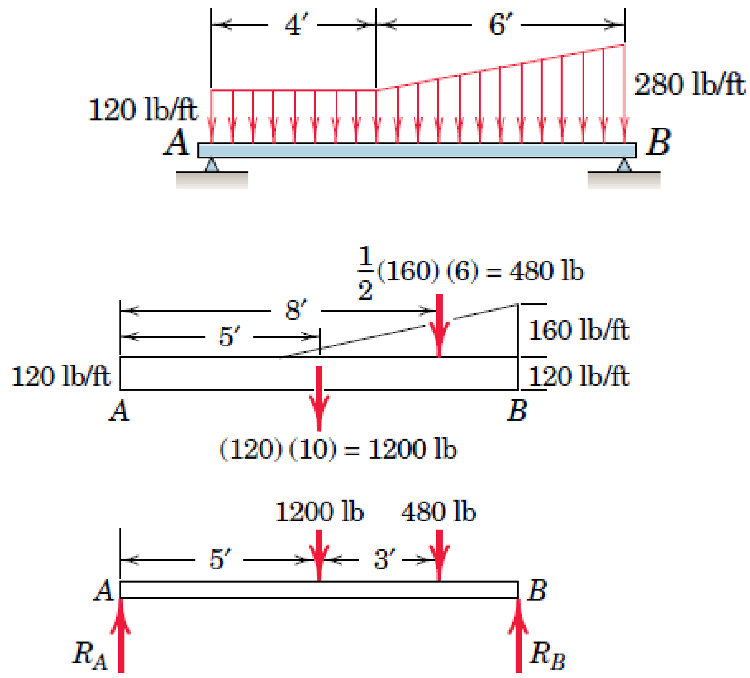
$$R = \int w \, dx$$

$$\bar{x} = \frac{\int xw \, dx}{R}$$



Sample Problem 5/11

Determine the equivalent concentrated load(s) and external reactions for the simply supported beam which is subjected to the distributed load shown.



$[\Sigma M_A = 0]$ $\quad 1200(5) + 480(8) - R_B(10) = 0$

$$R_B = 984 \text{ lb}$$

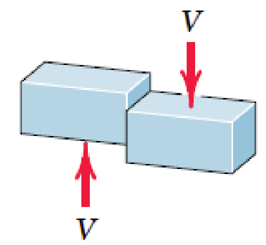
$[\Sigma M_B = 0]$ $\quad R_A(10) - 1200(5) - 480(2) = 0$

$$R_A = 696 \text{ lb}$$

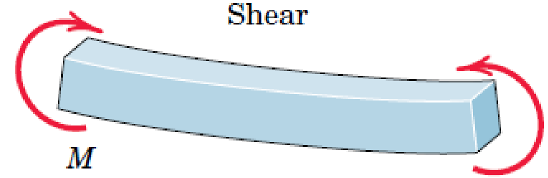
5.7 Beams - Internal Effects

- In addition to supporting tension or compression, a beam can resist:
  - Shear
  - Bending
  - Torsion

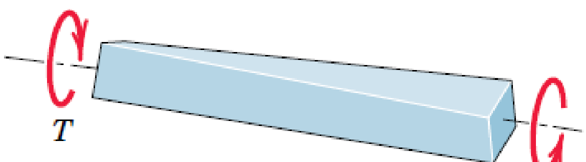
✓ These effects represent the vector components of the resultant of the forces acting on a transverse section of the beam.



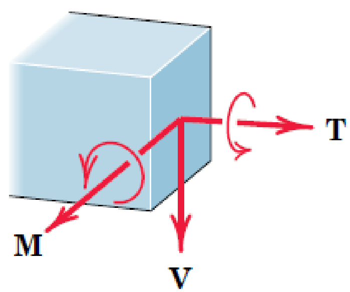
Shear



Bending



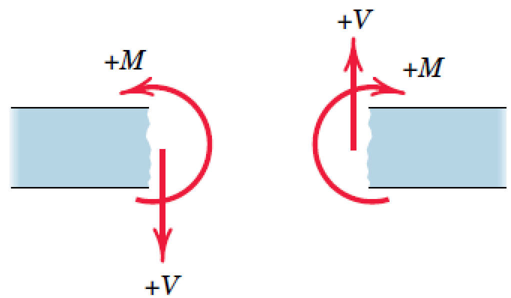
Torsion



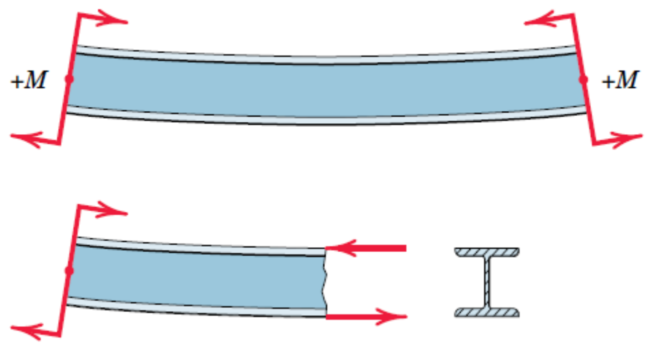
Combined loading

5.7 BEAMS - INTERNAL EFFECTS

- The conventions for positive values of shear *V* and bending moment *M* :

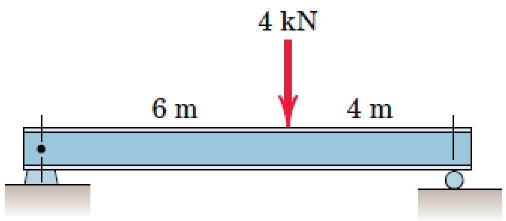


- Physical interpretation of the bending couple *M* :

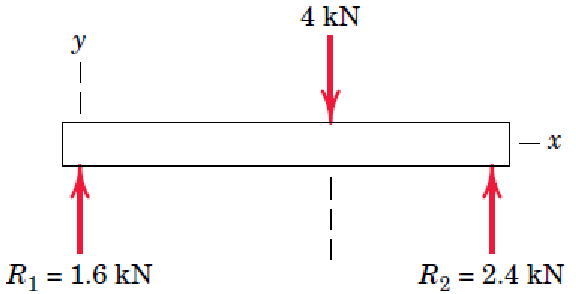


Sample Problem 5/13

Determine the shear and moment distributions produced in the simple beam by the 4-kN concentrated load.

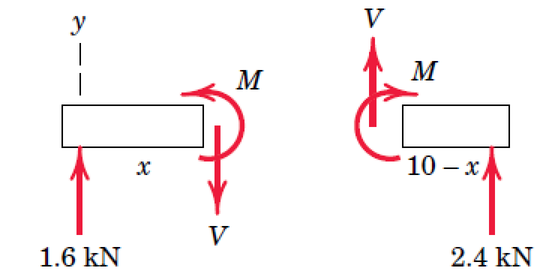


$R_1 = 1.6 \text{ kN} \qquad R_2 = 2.4 \text{ kN}$



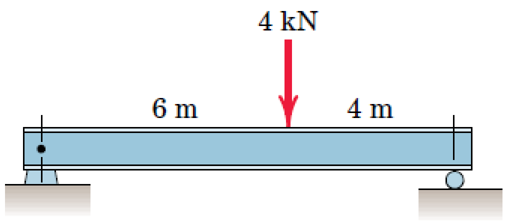
$[\Sigma F_y = 0]$	$1.6 - V = 0$	$V = 1.6 \text{ kN}$
$[\Sigma M_{R_1} = 0]$	$M - 1.6x = 0$	$M = 1.6x$

$[\Sigma F_y = 0]$	$V + 2.4 = 0$	$V = -2.4 \text{ kN}$
$[\Sigma M_{R_2} = 0]$	$-(2.4)(10 - x) + M = 0$	$M = 2.4(10 - x)$



Sample Problem 5/13

Determine the shear and moment distributions produced in the simple beam by the 4-kN concentrated load.



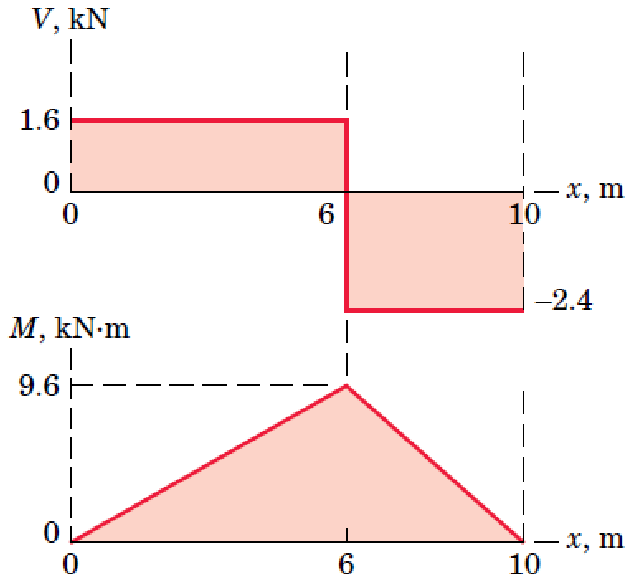
$R_1 = 1.6\ \text{kN} \qquad R_2 = 2.4\ \text{kN}$

$[\Sigma F_y = 0] \qquad 1.6 - V = 0 \qquad V = 1.6\ \text{kN}$

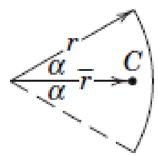
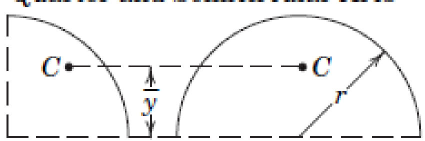
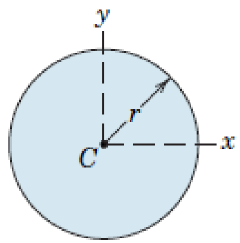
$[\Sigma M_{R_1} = 0] \qquad M - 1.6x = 0 \qquad M = 1.6x$

$[\Sigma F_y = 0] \qquad V + 2.4 = 0 \qquad V = -2.4\ \text{kN}$

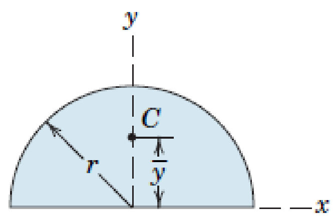
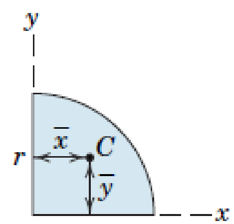
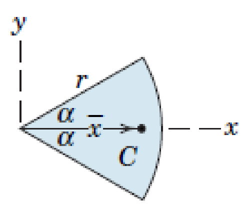
$[\Sigma M_{R_2} = 0] \qquad -(2.4)(10 - x) + M = 0 \qquad M = 2.4(10 - x)$



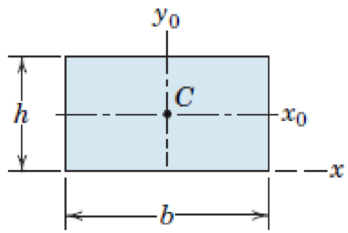
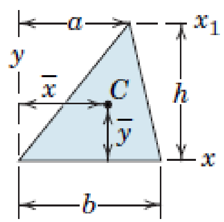
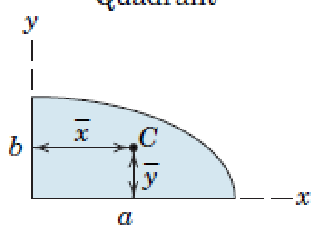
APPENDIX D

Arc Segment 	$\bar{r} = \frac{r \sin\alpha}{\alpha}$	—
Quarter and Semicircular Arcs 	$\bar{y} = \frac{2r}{\pi}$	—
Circular Area 	—	$I_x = I_y = \frac{\pi r^4}{4}$ $I_z = \frac{\pi r^4}{2}$

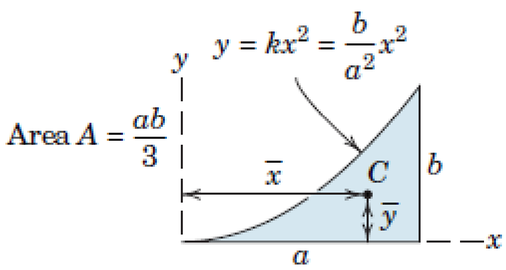
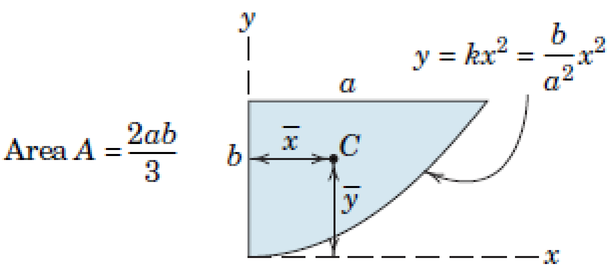
APPENDIX D

Semicircular Area 	$\bar{y} = \frac{4r}{3\pi}$	$I_x = I_y = \frac{\pi r^4}{8}$ $\bar{I}_x = \left(\frac{\pi}{8} - \frac{8}{9\pi}\right) r^4$ $I_z = \frac{\pi r^4}{4}$
Quarter-Circular Area 	$\bar{x} = \bar{y} = \frac{4r}{3\pi}$	$I_x = I_y = \frac{\pi r^4}{16}$ $\bar{I}_x = \bar{I}_y = \left(\frac{\pi}{16} - \frac{4}{9\pi}\right) r^4$ $I_z = \frac{\pi r^4}{8}$
Area of Circular Sector 	$\bar{x} = \frac{2}{3}\frac{r \sin\alpha}{\alpha}$	$I_x = \frac{r^4}{4}\left(\alpha - \frac{1}{2}\sin 2\alpha\right)$ $I_y = \frac{r^4}{4}\left(\alpha + \frac{1}{2}\sin 2\alpha\right)$ $I_z = \frac{1}{2} r^4 \alpha$

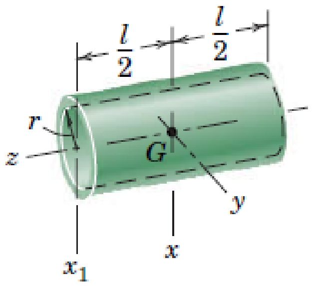
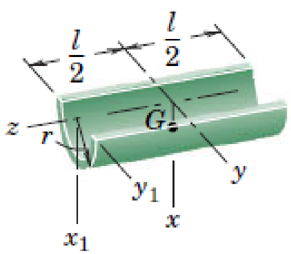
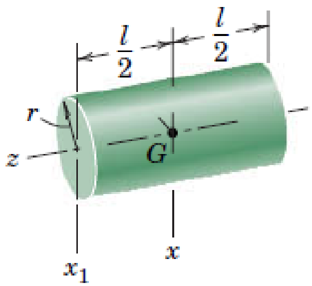
APPENDIX D

Rectangular Area 	—	$I_x = \frac{bh^3}{3}$ $\bar{I}_x = \frac{bh^3}{12}$ $\bar{I}_z = \frac{bh}{12}(b^2 + h^2)$
Triangular Area 	$\bar{x} = \frac{a+b}{3}$ $\bar{y} = \frac{h}{3}$	$I_x = \frac{bh^3}{12}$ $\bar{I}_x = \frac{bh^3}{36}$ $I_{x_1} = \frac{bh^3}{4}$
Area of Elliptical Quadrant 	$\bar{x} = \frac{4a}{3\pi}$ $\bar{y} = \frac{4b}{3\pi}$	$I_x = \frac{\pi ab^3}{16}, \quad \bar{I}_x = \left(\frac{\pi}{16} - \frac{4}{9\pi}\right)ab^3$ $I_y = \frac{\pi a^3 b}{16}, \quad \bar{I}_y = \left(\frac{\pi}{16} - \frac{4}{9\pi}\right)a^3 b$ $I_z = \frac{\pi ab}{16}(a^2 + b^2)$

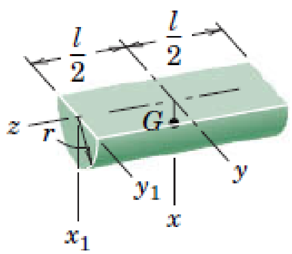
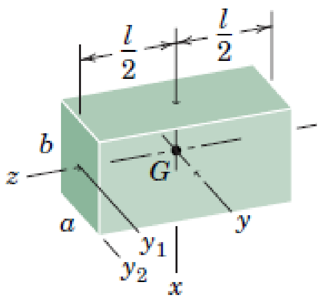
APPENDIX D

Subparabolic Area 	$\bar{x} = \frac{3a}{4}$ $\bar{y} = \frac{3b}{10}$	$I_x = \frac{ab^3}{21}$ $I_y = \frac{a^3b}{5}$ $I_z = ab\left(\frac{a^3}{5} + \frac{b^2}{21}\right)$
Parabolic Area 	$\bar{x} = \frac{3a}{8}$ $\bar{y} = \frac{3b}{5}$	$I_x = \frac{2ab^3}{7}$ $I_y = \frac{2a^3b}{15}$ $I_z = 2ab\left(\frac{a^2}{15} + \frac{b^2}{7}\right)$

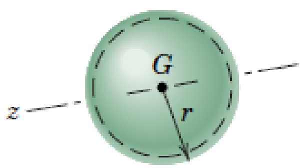
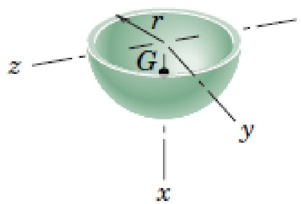
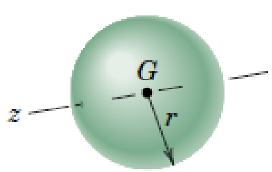
APPENDIX D

 Circular Cylindrical Shell	—	$I_{xx} = \frac{1}{2}mr^2 + \frac{1}{12}ml^2$ $I_{x_1x_1} = \frac{1}{2}mr^2 + \frac{1}{3}ml^2$ $I_{zz} = mr^2$
 Half Cylindrical Shell	$\bar{x} = \frac{2r}{\pi}$	$I_{xx} = I_{yy}$ $= \frac{1}{2}mr^2 + \frac{1}{12}ml^2$ $I_{x_1x_1} = I_{y_1y_1}$ $= \frac{1}{2}mr^2 + \frac{1}{3}ml^2$ $I_{zz} = mr^2$ $\bar{I}_{zz} = \left(1 - \frac{4}{\pi^2}\right) mr^2$
 Circular Cylinder	—	$I_{xx} = \frac{1}{4}mr^2 + \frac{1}{12}ml^2$ $I_{x_1x_1} = \frac{1}{4}mr^2 + \frac{1}{3}ml^2$ $I_{zz} = \frac{1}{2}mr^2$

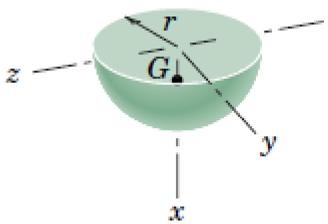
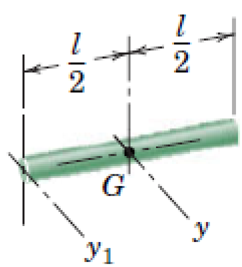
APPENDIX D

 Semicylinder	$\bar{x} = \frac{4r}{3\pi}$	$I_{xx} = I_{yy} = \frac{1}{4}mr^2 + \frac{1}{12}ml^2$ $I_{x_1x_1} = I_{y_1y_1} = \frac{1}{4}mr^2 + \frac{1}{3}ml^2$ $I_{zz} = \frac{1}{2}mr^2$ $\bar{I}_{zz} = \left(\frac{1}{2} - \frac{16}{9\pi^2}\right)mr^2$
 Rectangular Parallelepiped	—	$I_{xx} = \frac{1}{12}m(a^2 + l^2)$ $I_{yy} = \frac{1}{12}m(b^2 + l^2)$ $I_{zz} = \frac{1}{12}m(a^2 + b^2)$ $I_{y_1y_1} = \frac{1}{12}mb^2 + \frac{1}{3}ml^2$ $I_{y_2y_2} = \frac{1}{3}m(b^2 + l^2)$

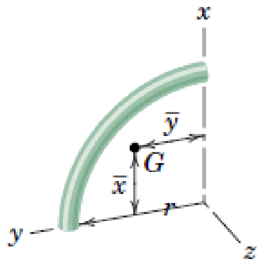
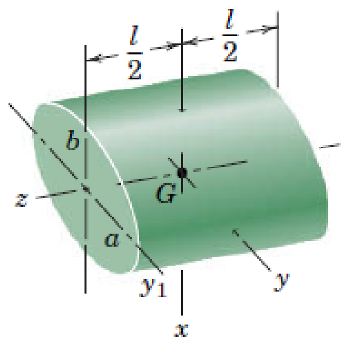
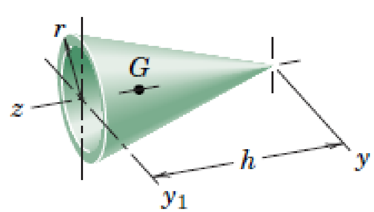
Appendix D

	Spherical Shell	—	$I_{zz} = \frac{2}{3}mr^2$
	Hemispherical Shell	$\bar{x} = \frac{r}{2}$	$I_{xx} = I_{yy} = I_{zz} = \frac{2}{3}mr^2$ $\bar{I}_{yy} = \bar{I}_{zz} = \frac{5}{12}mr^2$
	Sphere	—	$I_{zz} = \frac{2}{5}mr^2$

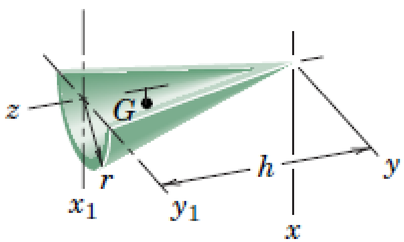
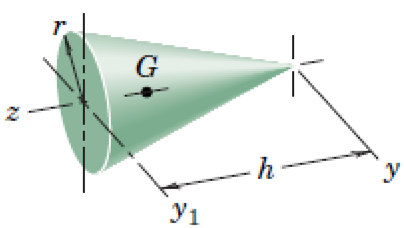
Appendix D

	Hemisphere	$\bar{x} = \frac{3r}{8}$	$I_{xx} = I_{yy} = I_{zz} = \frac{2}{5}mr^2$ $\bar{I}_{yy} = \bar{I}_{zz} = \frac{83}{320}mr^2$
	Uniform Slender Rod	—	$I_{yy} = \frac{1}{12}ml^2$ $I_{y_1y_1} = \frac{1}{3}ml^2$

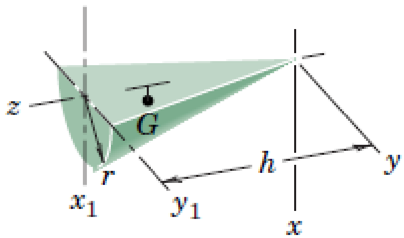
APPENDIX D

	Quarter-Circular Rod	$\bar{x} = \bar{y}$ $= \frac{2r}{\pi}$	$I_{xx} = I_{yy} = \frac{1}{2}mr^2$ $I_{zz} = mr^2$
	Elliptical Cylinder	—	$I_{xx} = \frac{1}{4}ma^2 + \frac{1}{12}ml^2$ $I_{yy} = \frac{1}{4}mb^2 + \frac{1}{12}ml^2$ $I_{zz} = \frac{1}{4}m(a^2 + b^2)$ $I_{y_1y_1} = \frac{1}{4}mb^2 + \frac{1}{3}ml^2$
	Conical Shell	$\bar{z} = \frac{2h}{3}$	$I_{yy} = \frac{1}{4}mr^2 + \frac{1}{2}mh^2$ $I_{y_1y_1} = \frac{1}{4}mr^2 + \frac{1}{6}mh^2$ $I_{zz} = \frac{1}{2}mr^2$ $\bar{I}_{yy} = \frac{1}{4}mr^2 + \frac{1}{18}mh^2$

Appendix D

	Half Conical Shell	$\bar{x} = \frac{4r}{3\pi}$ $\bar{z} = \frac{2h}{3}$	$I_{xx} = I_{yy} = \frac{1}{4}mr^2 + \frac{1}{2}mh^2$ $I_{x_1x_1} = I_{y_1y_1} = \frac{1}{4}mr^2 + \frac{1}{6}mh^2$ $I_{zz} = \frac{1}{2}mr^2$ $\bar{I}_{zz} = \left(\frac{1}{2} - \frac{16}{9\pi^2}\right)mr^2$
	Right Circular Cone	$\bar{z} = \frac{3h}{4}$	$I_{yy} = \frac{3}{20}mr^2 + \frac{3}{5}mh^2$ $I_{y_1y_1} = \frac{3}{20}mr^2 + \frac{1}{10}mh^2$ $I_{zz} = \frac{3}{10}mr^2$ $\bar{I}_{yy} = \frac{3}{20}mr^2 + \frac{3}{80}mh^2$

APPENDIX D

<table>
<tr>
<td>

Half Cone</td>
<td>$\bar{x} = \frac{r}{\pi}$
$\bar{z} = \frac{3h}{4}$</td>
<td>$I_{xx} = I_{yy}$
$= \frac{3}{20}mr^2 + \frac{3}{5}mh^2$
$I_{x_1x_1} = I_{y_1y_1}$
$= \frac{3}{20}mr^2 + \frac{1}{10}mh^2$
$I_{zz} = \frac{3}{10}mr^2$
$\bar{I}_{zz} = \left(\frac{3}{10} - \frac{1}{\pi^2}\right)mr^2$</td>
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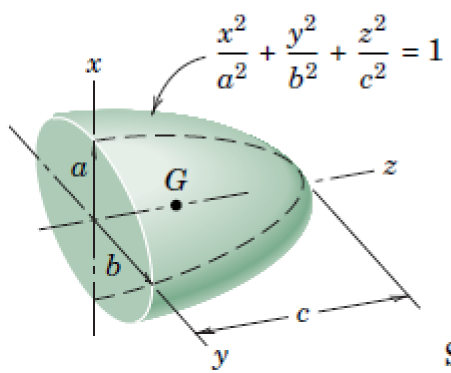
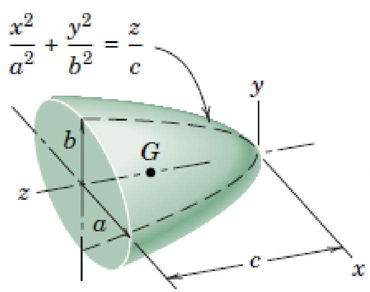
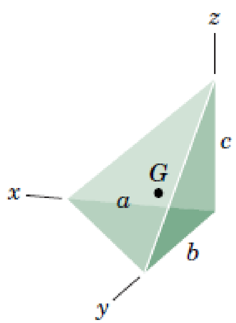
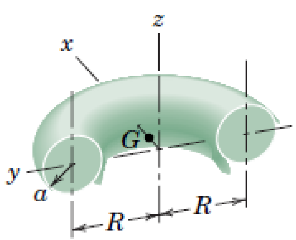
<tr>
<td>

Semiellipsoid</td>
<td>$\bar{z} = \frac{3c}{8}$</td>
<td>$I_{xx} = \frac{1}{5}m(b^2 + c^2)$
$I_{yy} = \frac{1}{5}m(a^2 + c^2)$
$I_{zz} = \frac{1}{5}m(a^2 + b^2)$
$\bar{I}_{xx} = \frac{1}{5}m(b^2 + \frac{19}{64}c^2)$
$\bar{I}_{yy} = \frac{1}{5}m(a^2 + \frac{19}{64}c^2)$</td>
</tr>
</table>

APPENDIX D

	Elliptic Paraboloid	$\bar{z} = \frac{2c}{3}$	$I_{xx} = \frac{1}{6}mb^2 + \frac{1}{2}mc^2$ $I_{yy} = \frac{1}{6}ma^2 + \frac{1}{2}mc^2$ $I_{zz} = \frac{1}{6}m(a^2 + b^2)$ $\bar{I}_{xx} = \frac{1}{6}m(b^2 + \frac{1}{3}c^2)$ $\bar{I}_{yy} = \frac{1}{6}m(a^2 + \frac{1}{3}c^2)$
	Rectangular Tetrahedron	$\bar{x} = \frac{a}{4}$ $\bar{y} = \frac{b}{4}$ $\bar{z} = \frac{c}{4}$	$I_{xx} = \frac{1}{10}m(b^2 + c^2)$ $I_{yy} = \frac{1}{10}m(a^2 + c^2)$ $I_{zz} = \frac{1}{10}m(a^2 + b^2)$ $\bar{I}_{xx} = \frac{3}{80}m(b^2 + c^2)$ $\bar{I}_{yy} = \frac{3}{80}m(a^2 + c^2)$ $\bar{I}_{zz} = \frac{3}{80}m(a^2 + b^2)$
	Half Torus	$\bar{x} = \frac{a^2 + 4R^2}{2\pi R}$	$I_{xx} = I_{yy} = \frac{1}{2}mR^2 + \frac{5}{8}ma^2$ $I_{zz} = mR^2 + \frac{3}{4}ma^2$

❑ ***CONTENTS:***

6.1 INTRODUCTION

- We have usually assumed that the forces of action and reaction between contacting surfaces act normal to the surfaces. This assumption characterizes the interaction between smooth surfaces.
- Although this ideal assumption often involves only a relatively small error, there are many problems in which we must consider the ability of contacting surfaces to support tangential as well as normal forces.
- Tangential forces generated between contacting surfaces are called friction forces and occur to some degree in the interaction between all real surfaces.
- Whenever a tendency exists for one contacting surface to slide along another surface, the friction forces developed are always in a direction to oppose this tendency

6.1 INTRODUCTION

- Friction forces are present throughout nature and exist in all machines no matter how accurately constructed or carefully lubricated.
- A machine or process in which friction is small enough to be neglected is said to be ideal. When friction must be taken into account, the machine or process is termed real.
- In all cases where there is sliding motion between parts, the friction forces result in a loss of energy which is dissipated in the form of heat.
- Wear is another effect of friction.

6.2 TYPES OF FRICTION

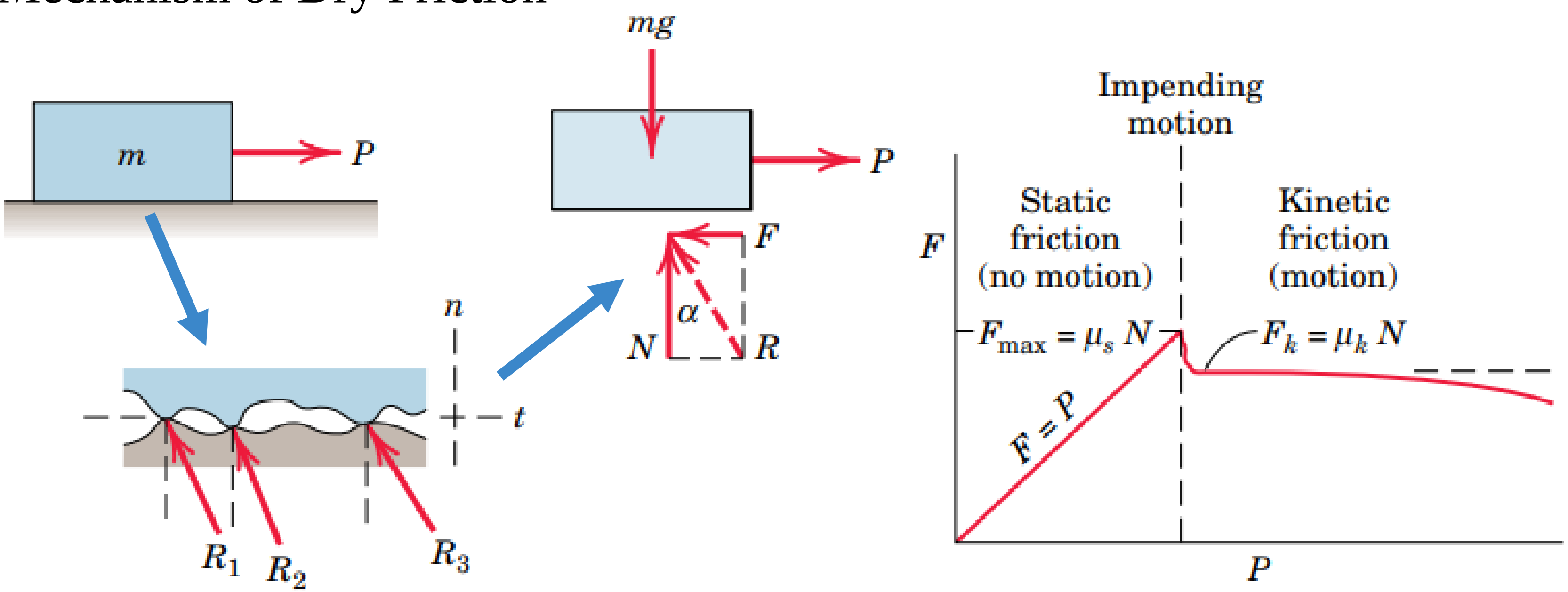
- (a) Dry Friction:
  - Dry friction occurs when the unlubricated surfaces of two solids are in contact under a condition of sliding or a tendency to slide. This type of friction is also called **Coulomb friction**.

- (b) Fluid Friction:
  - Fluid friction occurs when adjacent layers in a fluid (liquid or gas) are moving at different velocities.

- (c) Internal Friction:
  - Internal friction occurs in all solid materials which are subjected to cyclical loading.

6.3 Dry Friction

- Mechanism of Dry Friction



- ✓ μ_s : the coefficient of static friction

$$F_{max} = \mu_s N$$

$$F < \mu_s N$$

6.3 Dry Friction

- Kinetic Friction
  - After slippage occurs, a condition of kinetic friction accompanies the ensuing motion.
  - Kinetic friction force is usually somewhat less than the maximum static friction force.
  - The kinetic friction force F_k is also proportional to the normal force.

✓ μ_k : the coefficient of kinetic friction → $F_k = \mu_k N$

6.3 DRY FRICTION

- Some typical values of coefficients

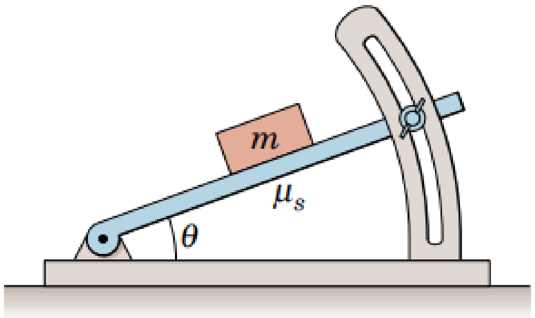
CONTACTING SURFACE	STATIC, μ_s	KINETIC, μ_k
Steel on steel (dry)	0.6	0.4
Steel on steel (greasy)	0.1	0.05
Teflon on steel	0.04	0.04
Steel on babbitt (dry)	0.4	0.3
Steel on babbitt (greasy)	0.1	0.07
Brass on steel (dry)	0.5	0.4
Brake lining on cast iron	0.4	0.3
Rubber tires on smooth pavement (dry)	0.9	0.8
Wire rope on iron pulley (dry)	0.2	0.15
Hemp rope on metal	0.3	0.2
Metal on ice		0.02

6.3 Dry Friction

- Types of Friction Problems
  - 1) The condition of impending motion is known to exist.
    - ✓ Body is in equilibrium and on the verge of slipping
    - ✓ Friction force equals the limiting static friction: $F_{max} = \mu_s N$
  - 2) Neither the condition of impending motion nor the condition of motion is known to exist.
    - ✓ First assume static equilibrium and then solve for the friction force F
      - (a) $F < (F_{max}=\mu_s N)$: The body is in static equilibrium as assumed.
      - (b) $F = (F_{max}=\mu_s N)$: Motion impends, the assumption of static equilibrium is valid.
      - (c) $F > (F_{max}=\mu_s N)$: The assumption of equilibrium is therefore invalid, and motion occurs.
  - 3) Relative motion is known to exist between the contacting surfaces
    - ✓ Kinetic coefficient of friction clearly applies $F = \mu_k N$

Sample Problem 6/1

Determine the maximum angle θ which the adjustable incline may have with the horizontal before the block of mass m begins to slip. The coefficient of static friction between the block and the inclined surface is μ_s.



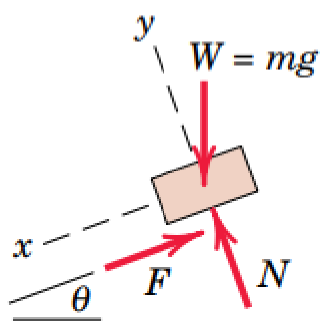
$[\Sigma F_x = 0]$	$mg \sin\theta - F = 0$	$F = mg \sin\theta$
$[\Sigma F_y = 0]$	$-mg \cos\theta + N = 0$	$N = mg \cos\theta$

$$F/N = \tan\theta$$

$$F = F_{max} = \mu_s N$$

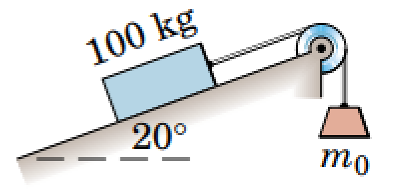
$$\mu_s = \tan\theta_{max}$$

$$\theta_{max} = \tan^{-1}\mu_s$$



Sample Problem 6/2

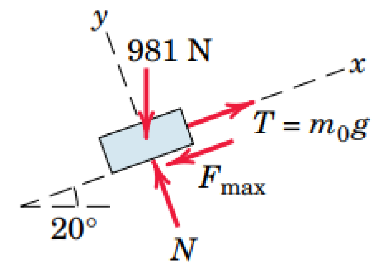
Determine the range of values which the mass m_0 may have so that the 100-kg block shown in the figure will neither start moving up the plane nor slip down the plane. The coefficient of static friction for the contact surfaces is 0.30.



$$[\Sigma F_y = 0] \qquad N - 981 \cos 20° = 0 \qquad N = 922 \text{ N}$$

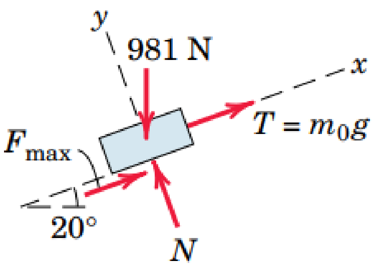
$$[F_{max} = \mu_s N] \qquad F_{max} = 0.30(922) = 277 \text{ N}$$

$$[\Sigma F_x = 0] \qquad m_0(9.81) - 277 - 981 \sin 20° = 0 \qquad m_0 = 62.4 \text{ kg}$$



Case I

$$[\Sigma F_x = 0] \qquad m_0(9.81) + 277 - 981 \sin 20° = 0 \qquad m_0 = 6.01 \text{ kg}$$



Case II