دانشگاه سمنان

Semnan University
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دانشکده مهندسی مکانیک

دانشکده مهندسی مکانیک

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ENGINEERING MECHANICS
DYNAMICS

MERIAM, KRAIGE & BOLTON

9TH EDITION

Chapter 5: **Plane Kinematics of Rigid Bodies**

❑ *CONTENTS:*

دانشگاه سمنان

PART II

Dynamics of Rigid Bodies

CHAPTER 5

Plane Kinematics of Rigid Bodies

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Rigid-body kinematics describes the relationships between the linear and angular motions of bodies without regard to the forces and moments associated with such motions. The designs of gears, cams, connecting links, and many other moving machine parts are largely kinematic problems.

CHAPTER OUTLINE

5/1 Introduction

- We need to describe the motion of rigid bodies for two important reasons.
  - First, we frequently need to generate, transmit, or control certain motions by the use of cams, gears, and linkages of various types.
  - Second, we must often determine the motion of a rigid body caused by the forces applied to it.
- This chapter covers the kinematics of rigid-body motion which may be analyzed as occurring in a single plane.
- In Chapter 7 we will present an introduction to the kinematics of motion in three dimensions.

5/1 Introduction

Rigid-Body Assumption

- A rigid body:
  - A system of particles for which the distances between the particles remain unchanged.
  - If the movements associated with the changes in shape are very small compared with the movements of the body as a whole, then the assumption of rigidity is usually acceptab

5/1 Introduction

Plane Motion

- A rigid body executes plane motion when all parts of the body move in parallel planes.

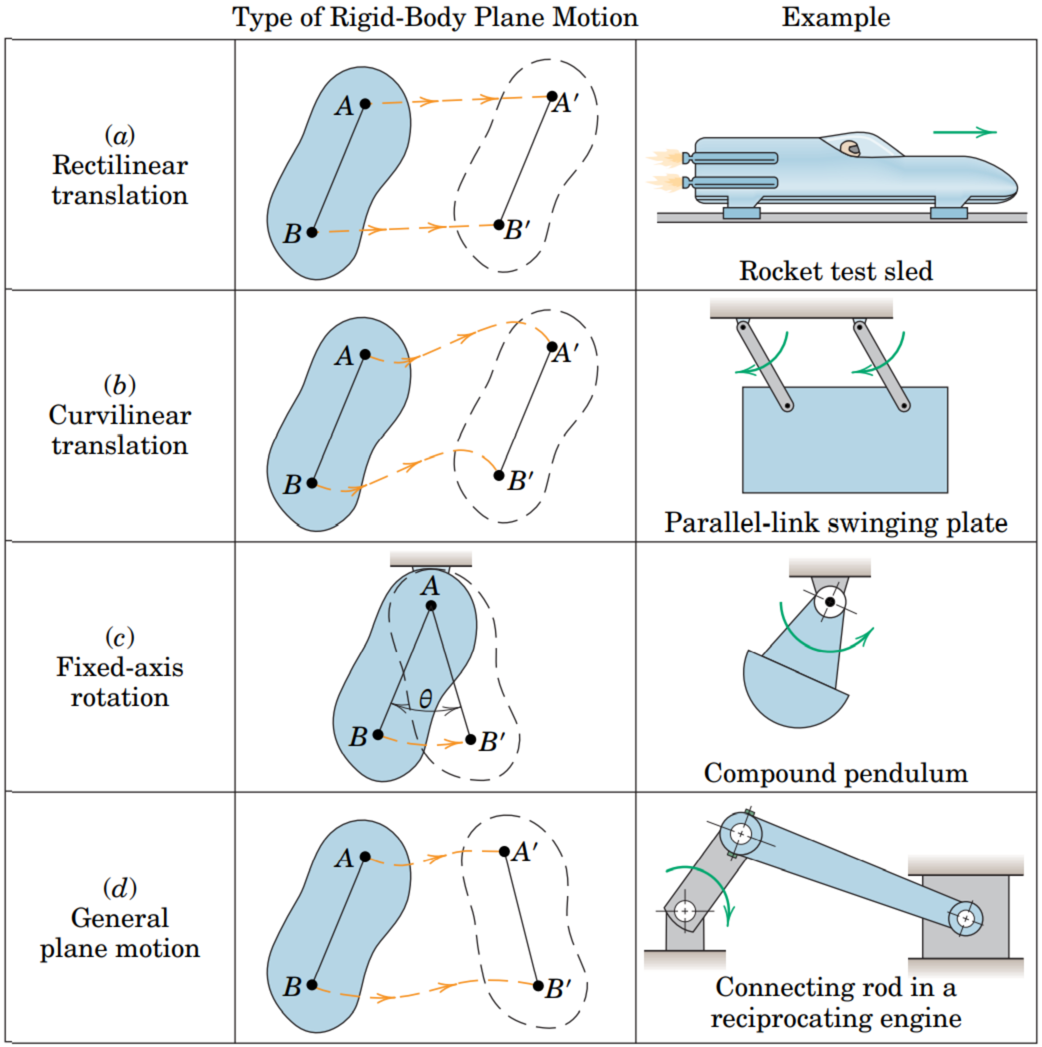
- For convenience, we generally consider the plane of motion to be the plane which contains the mass center, and we treat the body as a thin slab whose motion is confined to the plane of the slab.

- This idealization adequately describes a very large category of rigid-body motions encountered in engineering.

5/1 Introduction

Plane Motion

- Translation
- Rotation
- General plane motion



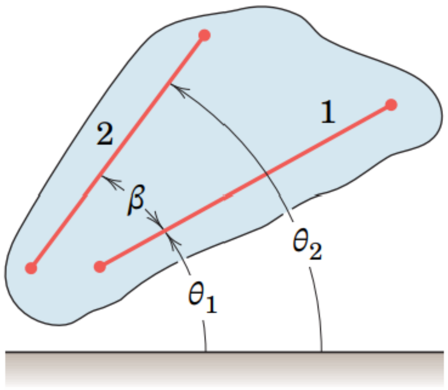
5/2 Rotation

- The rotation of a rigid body is described by its angular motion.

$$\theta_2 = \theta_1 + \beta$$

$$\dot{\theta}_2 = \dot{\theta}_1$$

$$\ddot{\theta}_2 = \ddot{\theta}_1$$



- All lines on a rigid body in its plane of motion have the same angular displacement, the same angular velocity, and the same angular acceleration

5/2 Rotation

Angular-Motion Relations

The angular velocity ω and angular acceleration α

$$\omega = \frac{d\theta}{dt} = \dot{\theta}$$

$$\alpha = \frac{d\omega}{dt} = \dot{\omega} \qquad \text{or} \qquad \alpha = \frac{d^2\theta}{dt^2} = \ddot{\theta}$$

$$\omega \, d\omega = \alpha \, d\theta \qquad \text{or} \qquad \dot{\theta} \, d\dot{\theta} = \ddot{\theta} \, d\theta$$

- For rotation with constant angular acceleration:

$$\omega = \omega_0 + \alpha t$$

$$\omega^2 = {\omega_0}^2 + 2\alpha(\theta - \theta_0)$$

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

5/2 Rotation

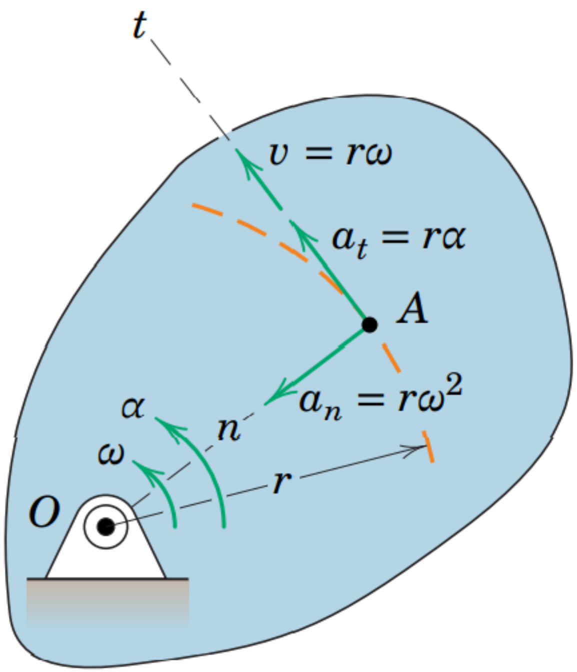
Angular-Motion Relations

Rotation about a Fixed Axis

$$v = r\omega$$

$$a_n = r\omega^2 = v^2/r = v\omega$$

$$a_t = r\alpha$$



5/2 Rotation

Angular-Motion Relations

- Using the cross-product relationship

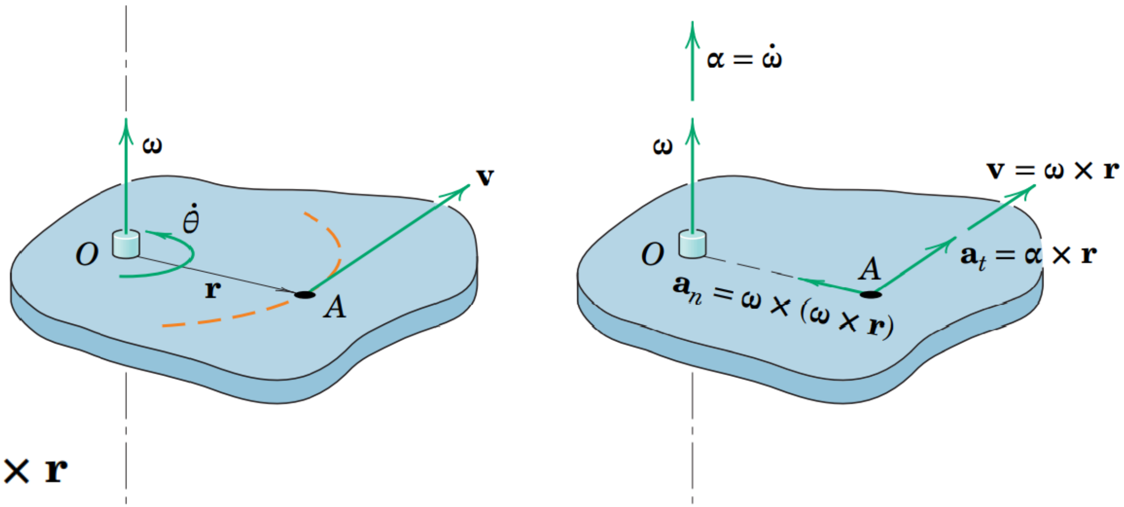
$$\mathbf{v} = \dot{\mathbf{r}} = \boldsymbol{\omega} \times \mathbf{r}$$

$$\mathbf{r} \times \boldsymbol{\omega} = -\mathbf{v}$$

$$\mathbf{a} = \dot{\mathbf{v}} = \boldsymbol{\omega} \times \dot{\mathbf{r}} + \dot{\boldsymbol{\omega}} \times \mathbf{r}$$

$$= \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) + \dot{\boldsymbol{\omega}} \times \mathbf{r}$$

$$= \boldsymbol{\omega} \times \mathbf{v} + \boldsymbol{\alpha} \times \mathbf{r}$$



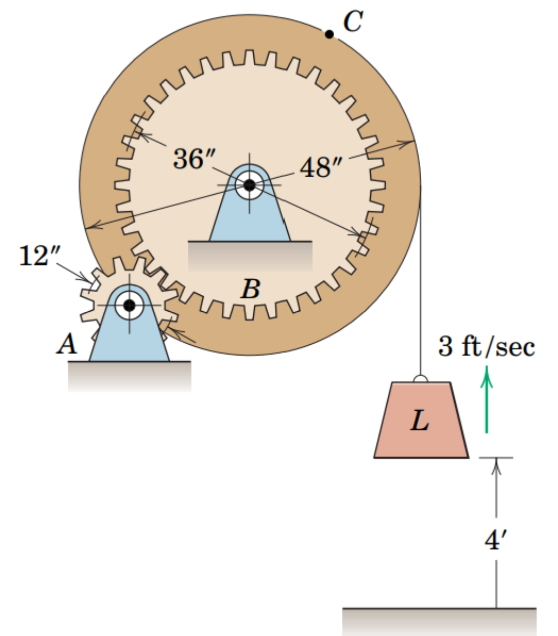
$$\mathbf{v} = \boldsymbol{\omega} \times \mathbf{r}$$

$$\mathbf{a}_n = \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r})$$

$$\mathbf{a}_t = \boldsymbol{\alpha} \times \mathbf{r}$$

SAMPLE PROBLEM 5/2

The pinion A of the hoist motor drives gear B, which is attached to the hoisting drum. The load L is lifted from its rest position and acquires an upward velocity of 3 ft/sec in a vertical rise of 4 ft with constant acceleration. As the load passes this position, compute (*a*) the acceleration of point C on the cable in contact with the drum and (*b*) the angular velocity and angular acceleration of the pinion A.



$[v^2 = 2as]$ $\quad a = a_t = v^2/2s = 3^2/[2(4)] = 1.125 \text{ ft/sec}^2$

$[a_n = v^2/r]$ $\quad a_n = 3^2/(24/12) = 4.5 \text{ ft/sec}^2$

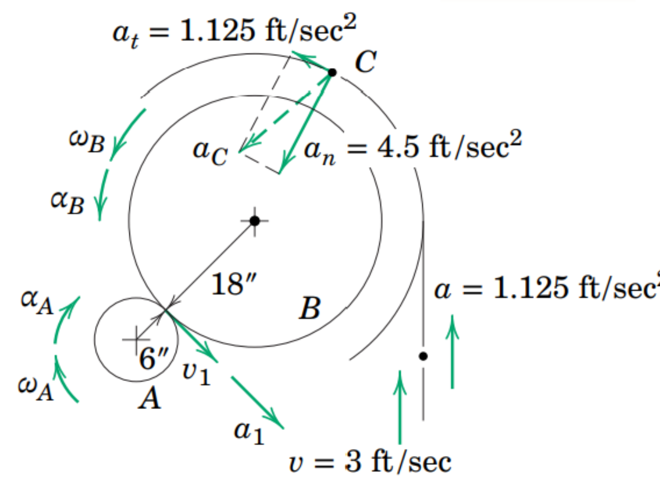
$[a = \sqrt{a_n{}^2 + a_t{}^2}]$ $\quad a_C = \sqrt{(4.5)^2 + (1.125)^2} = 4.64 \text{ ft/sec}^2$

$[v = r\omega]$ $\quad \omega_B = v/r = 3/(24/12) = 1.5 \text{ rad/sec}$

$[a_t = r\alpha]$ $\quad \alpha_B = a_t/r = 1.125/(24/12) = 0.562 \text{ rad/sec}^2$

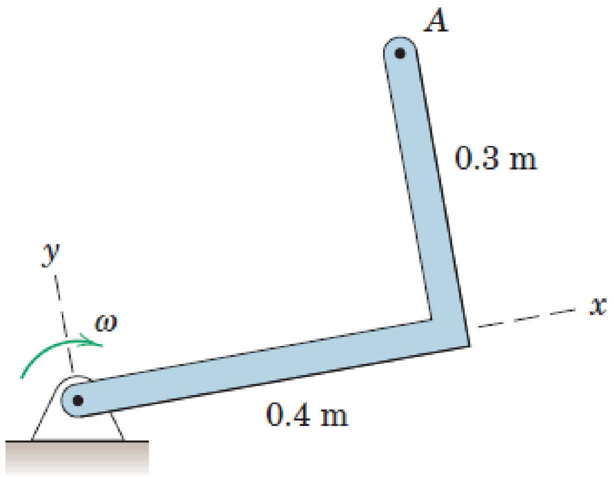
$$\omega_A = \frac{r_B}{r_A}\omega_B = \frac{18/12}{6/12}1.5 = 4.5 \text{ rad/sec CW}$$

$$\alpha_A = \frac{r_B}{r_A}\alpha_B = \frac{18/12}{6/12}0.562 = 1.688 \text{ rad/sec}^2 \text{ CW}$$



SAMPLE PROBLEM 5/3

The right-angle bar rotates clockwise with an angular velocity which is decreasing at the rate of 4 rad/s^2. Write the vector expressions for the velocity and acceleration of point A when $\omega = 2$ rad/s.



$\boldsymbol{\omega} = -2\mathbf{k}$ rad/s and $\boldsymbol{\alpha} = +4\mathbf{k}$ rad/s^2

$[\mathbf{v} = \boldsymbol{\omega} \times \mathbf{r}]$	$\mathbf{v} = -2\mathbf{k} \times (0.4\mathbf{i} + 0.3\mathbf{j}) = 0.6\mathbf{i} - 0.8\mathbf{j}$ m/s
$[\mathbf{a}_n = \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r})]$	$\mathbf{a}_n = -2\mathbf{k} \times (0.6\mathbf{i} - 0.8\mathbf{j}) = -1.6\mathbf{i} - 1.2\mathbf{j}$ m/s^2
$[\mathbf{a}_t = \boldsymbol{\alpha} \times \mathbf{r}]$	$\mathbf{a}_t = 4\mathbf{k} \times (0.4\mathbf{i} + 0.3\mathbf{j}) = -1.2\mathbf{i} + 1.6\mathbf{j}$ m/s^2
$[\mathbf{a} = \mathbf{a}_n + \mathbf{a}_t]$	$\mathbf{a} = -2.8\mathbf{i} + 0.4\mathbf{j}$ m/s^2

$v = \sqrt{0.6^2 + 0.8^2} = 1$ m/s and $a = \sqrt{2.8^2 + 0.4^2} = 2.83$ m/s^2

5/3 Absolute Motion

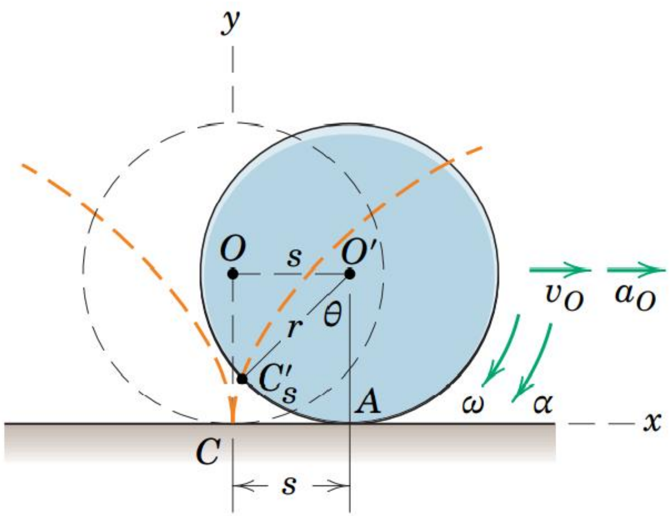
- We now develop the approach of absolute-motion analysis to describe the plane kinematics of rigid bodies.

- In this approach, we make use of the geometric relations which define the configuration of the body involved and then proceed to take the time derivatives of the defi ning geometric relations to obtain velocities and accelerations.

- The absolute-motion approach to rigid-body kinematics is quite straightforward, provided the configuration lends itself to a geometric description which is not overly complex. If the geometric configuration is awkward or complex, analysis by the principles of relative motion may be preferable.

SAMPLE PROBLEM 5/4

A wheel of radius r rolls on a flat surface without slipping. Determine the angular motion of the wheel in terms of the linear motion of its center O. Also determine the acceleration of a point on the rim of the wheel as the point comes into contact with the surface on which the wheel rolls.



$$s = r\theta$$

$$v_O = r\omega$$

$$a_O = r\alpha$$

$$x = s - r\sin\theta = r(\theta - \sin\theta) \qquad y = r - r\cos\theta = r(1 - \cos\theta)$$

$$\dot{x} = r\dot{\theta}(1 - \cos\theta) = v_O(1 - \cos\theta) \qquad \dot{y} = r\dot{\theta}\sin\theta = v_O\sin\theta$$

$$\ddot{x} = \dot{v}_O(1 - \cos\theta) + v_O\dot{\theta}\sin\theta \qquad \ddot{y} = \dot{v}_O\sin\theta + v_O\dot{\theta}\cos\theta$$

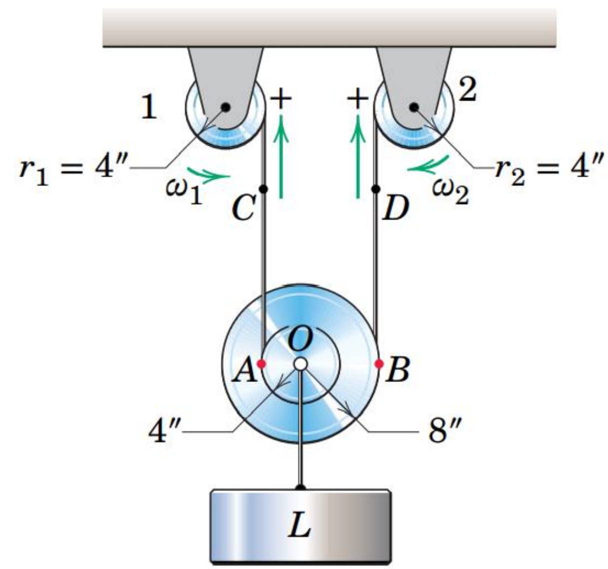
$$= a_O(1 - \cos\theta) + r\omega^2\sin\theta \qquad = a_O\sin\theta + r\omega^2\cos\theta$$

$$\theta = 0 \rightarrow \ddot{x} = 0 \quad \text{and} \quad \ddot{y} = r\omega^2$$

$$\mathbf{v} = \dot{x}\mathbf{i} + \dot{y}\mathbf{j} \qquad \mathbf{a} = \ddot{x}\mathbf{i} + \ddot{y}\mathbf{j}$$

SAMPLE PROBLEM 5/5

The load L is being hoisted by the pulley-and-cable arrangement shown. Each cable is wrapped securely around its respective pulley so it does not slip. The two pulleys to which L is attached are fastened together to form a single rigid body. Calculate the velocity and acceleration of the load L and the corresponding angular velocity ω and angular acceleration α of the double pulley under the following conditions:



Case (a) Pulley 1: $\omega_1 = \dot{\omega}_1 = 0$ (pulley at rest)

Pulley 2: $\omega_2 = 2$ rad/sec, $\alpha_2 = \dot{\omega}_2 = -3$ rad/sec^2

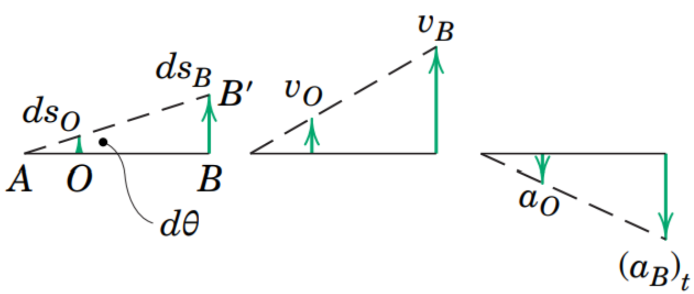
Case (b) Pulley 1: $\omega_1 = 1$ rad/sec, $\alpha_1 = \dot{\omega}_1 = 4$ rad/sec^2

Pulley 2: $\omega_2 = 2$ rad/sec, $\alpha_2 = \dot{\omega}_2 = -2$ rad/sec^2

Case (a)

$$ds_B = \overline{AB}\, d\theta \qquad v_B = \overline{AB}\omega \qquad (a_B)_t = \overline{AB}\alpha$$

$$ds_O = \overline{AO}\, d\theta \qquad v_O = \overline{AO}\omega \qquad a_O = \overline{AO}\alpha$$



Case (a)

$v_D = r_2\omega_2 = 4(2) = 8$ in./sec and $a_D = r_2\alpha_2 = 4(-3) = -12$ in./sec^2

$$\omega = v_B/\overline{AB} = v_D/\overline{AB} = 8/12 = 2/3 \text{ rad/sec(CCW)}$$

$$\alpha = (a_B)_t/\overline{AB} = a_D/\overline{AB} = -12/12 = -1 \text{ rad/sec}^2\text{(CW)}$$

$$v_O = \overline{AO}\omega = 4(2/3) = 8/3 \text{ in./sec}$$

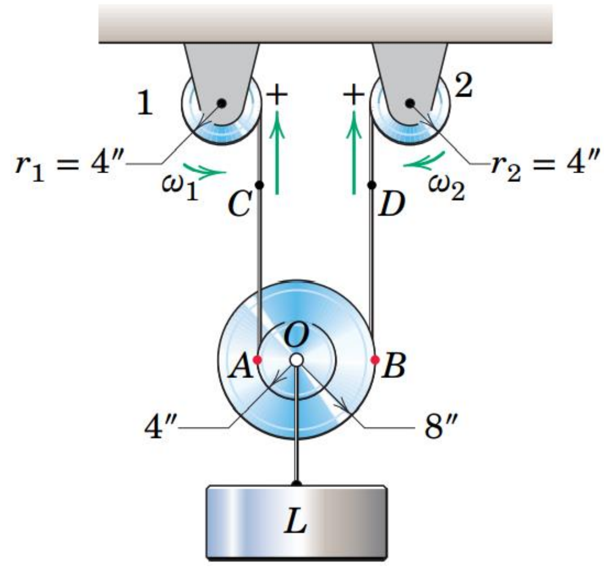
$$a_O = \overline{AO}\alpha = 4(-1) = -4 \text{ in./sec}^2$$

SAMPLE PROBLEM 5/5

The load L is being hoisted by the pulley-and-cable arrangement shown. Each cable is wrapped securely around its respective pulley so it does not slip. The two pulleys to which L is attached are fastened together to form a single rigid body. Calculate the velocity and acceleration of the load L and the corresponding angular velocity ω and angular acceleration α of the double pulley under the following conditions:

Case (a) Pulley 1: $\omega_1 = \dot{\omega}_1 = 0$ (pulley at rest)
Pulley 2: $\omega_2 = 2$ rad/sec, $\alpha_2 = \dot{\omega}_2 = -3$ rad/sec^2

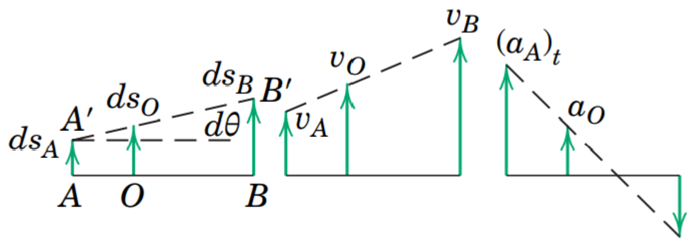
Case (b) Pulley 1: $\omega_1 = 1$ rad/sec, $\alpha_1 = \dot{\omega}_1 = 4$ rad/sec^2
Pulley 2: $\omega_2 = 2$ rad/sec, $\alpha_2 = \dot{\omega}_2 = -2$ rad/sec^2



Case (b)

$$ds_B - ds_A = \overline{AB}\, d\theta \qquad v_B - v_A = \overline{AB}\omega \qquad (a_B)_t - (a_A)_t = \overline{AB}\alpha$$

$$ds_O - ds_A = \overline{AO}\, d\theta \qquad v_O - v_A = \overline{AO}\omega \qquad a_O - (a_A)_t = \overline{AO}\alpha$$



Case (b)

$$v_C = r_1\omega_1 = 4(1) = 4 \text{ in./sec} \qquad v_D = r_2\omega_2 = 4(2) = 8 \text{ in./sec}$$

$$a_C = r_1\alpha_1 = 4(4) = 16 \text{ in./sec}^2 \qquad a_D = r_2\alpha_2 = 4(-2) = -8 \text{ in./sec}^2$$

$$\omega = \frac{v_B - v_A}{\overline{AB}} = \frac{v_D - v_C}{\overline{AB}} = \frac{8-4}{12} = 1/3 \text{ rad/sec (CCW)}$$

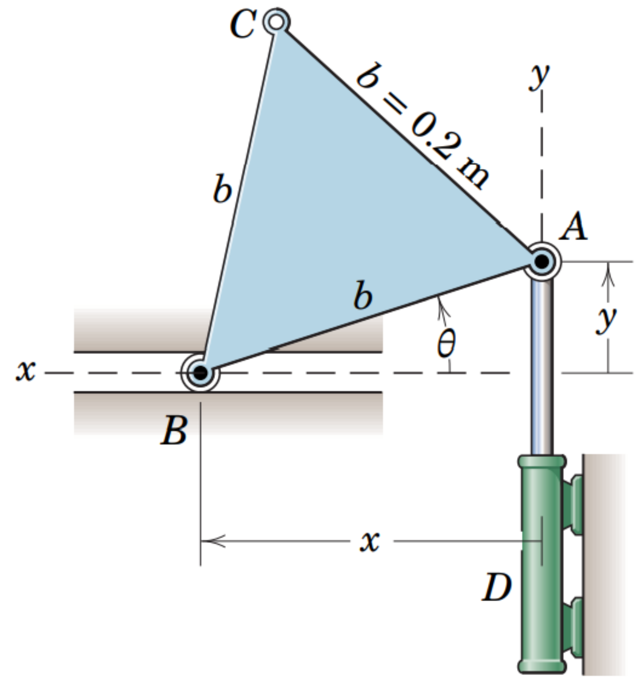
$$\alpha = \frac{(a_B)_t - (a_A)_t}{\overline{AB}} = \frac{a_D - a_C}{\overline{AB}} = \frac{-8 - 16}{12} = -2 \text{ rad/sec}^2\text{(CW)}$$

$$v_O = v_A + \overline{AO}\omega = v_C + \overline{AO}\omega = 4 + 4(1/3) = 16/3 \text{ in./sec}$$

$$a_O = (a_A)_t + \overline{AO}\alpha = a_C + \overline{AO}\alpha = 16 + 4(-2) = 8 \text{ in./sec}^2$$

SAMPLE PROBLEM 5/6

Motion of the equilateral triangular plate ABC in its plane is controlled by the hydraulic cylinder D. If the piston rod in the cylinder is moving upward at the constant rate of 0.3 m/s during an interval of its motion, calculate for the instant when $\theta = 30°$ the velocity and acceleration of the center of the roller B in the horizontal guide and the angular velocity and angular acceleration of edge CB.



$$v_A = \dot{y} = 0.3 \text{ m/s}$$

$$a_A = \ddot{y} = 0$$

$$x^2 + y^2 = b^2$$

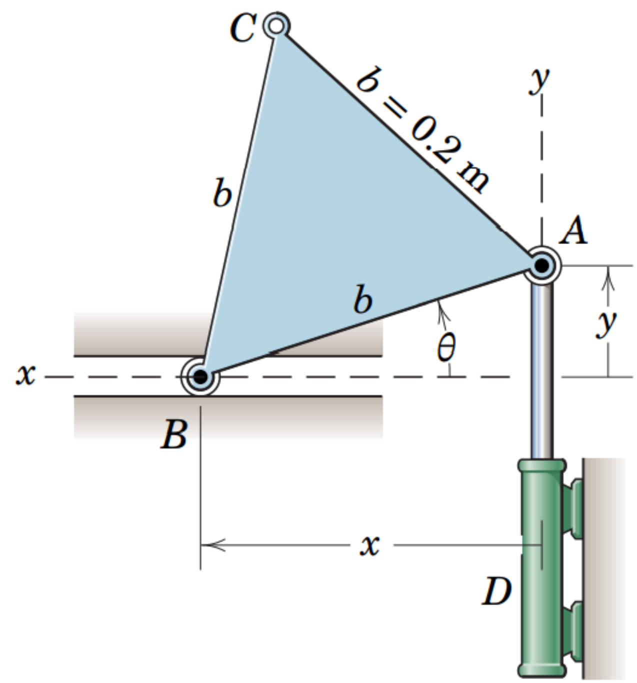
$$x\dot{x} + y\dot{y} = 0 \qquad \dot{x} = -\frac{y}{x}\dot{y}$$

$$x\ddot{x} + \dot{x}^2 + y\ddot{y} + \dot{y}^2 = 0 \qquad \ddot{x} = -\frac{\dot{x}^2 + \dot{y}^2}{x} - \frac{y}{x}\ddot{y}$$

$y = b \sin\theta,\ x = b \cos\theta, \text{ and } \ddot{y} = 0$

$$v_B = \dot{x} = -v_A \tan\theta$$

$$a_B = \ddot{x} = -\frac{{v_A}^2}{b} \sec^3\theta$$



$v_A = 0.3 \text{ m/s and } \theta = 30°$

$$v_B = -0.3\left(\frac{1}{\sqrt{3}}\right) = -0.1732 \text{ m/s}$$

$$a_B = -\frac{(0.3)^2(2/\sqrt{3})^3}{0.2} = -0.693 \text{ m/s}^2$$

$$\dot{y} = b\dot{\theta}\cos\theta \qquad \omega = \dot{\theta} = \frac{v_A}{b}\sec\theta \qquad \alpha = \dot{\omega} = \frac{v_A}{b}\dot{\theta}\sec\theta\tan\theta = \frac{{v_A}^2}{b^2}\sec^2\theta\tan\theta$$

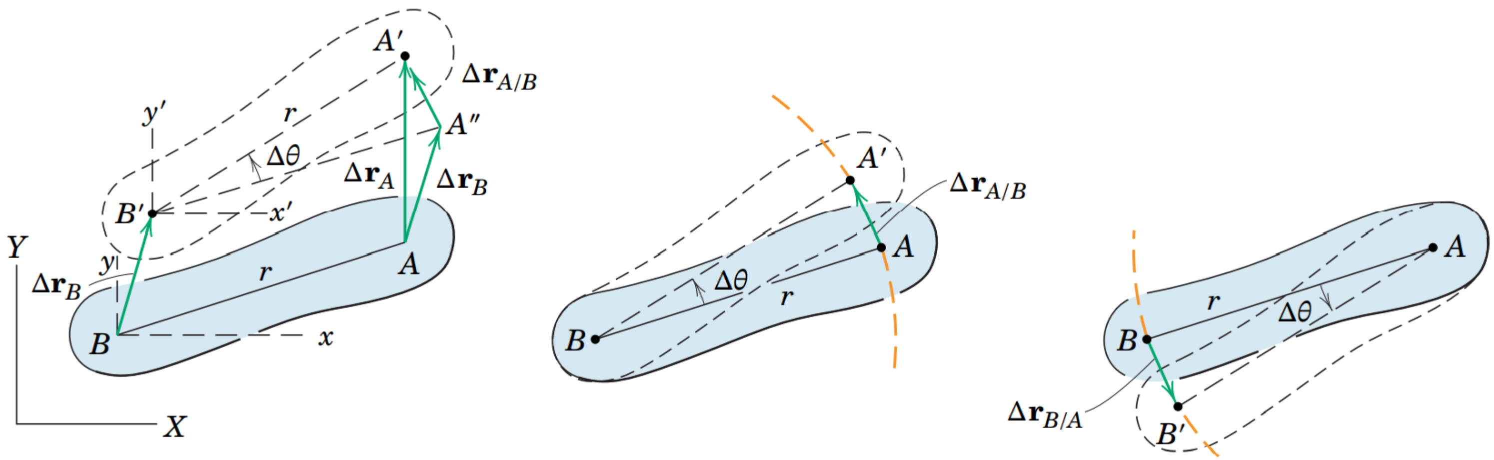
$$\omega = \frac{0.3}{0.2}\frac{2}{\sqrt{3}} = 1.732 \text{ rad/s}$$

$$\alpha = \frac{(0.3)^2}{(0.2)^2}\left(\frac{2}{\sqrt{3}}\right)^2\frac{1}{\sqrt{3}} = 1.732 \text{ rad/s}^2$$

5/4 Relative Velocity

- The principles of relative motion: $\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$

Relative Velocity Due to Rotation



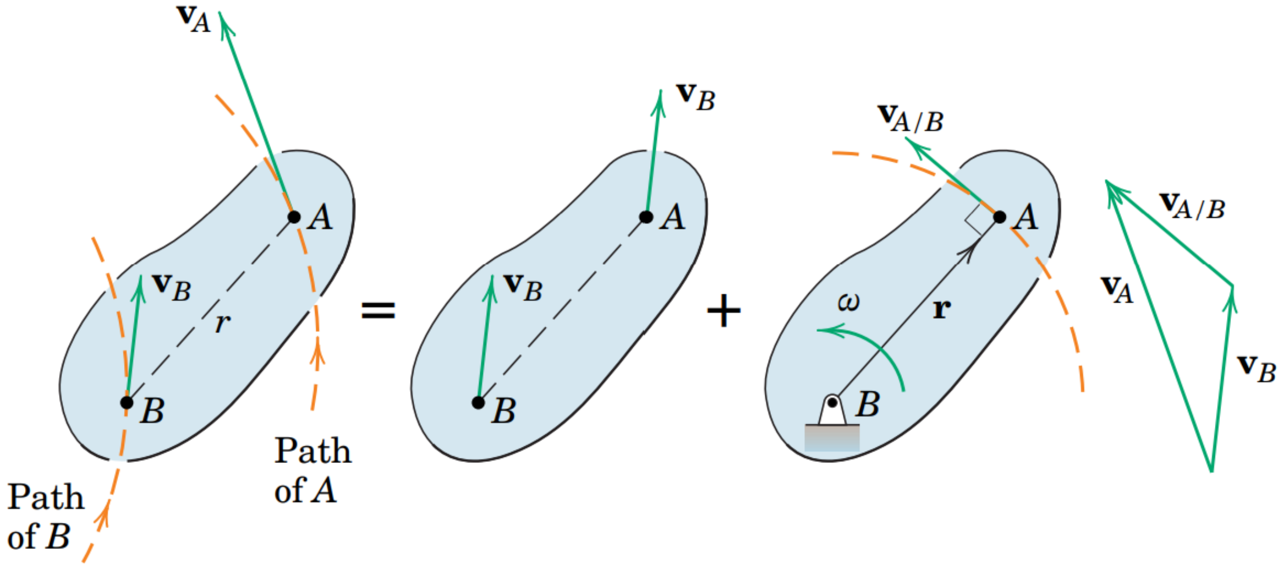
Motion relative to B — Motion relative to A

$$\Delta\mathbf{r}_A = \Delta\mathbf{r}_B + \Delta\mathbf{r}_{A/B} \quad\rightarrow\quad \mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B} \quad\rightarrow\quad v_{A/B} = r\omega$$

$$\mathbf{v}_{A/B} = \boldsymbol{\omega} \times \mathbf{r}$$

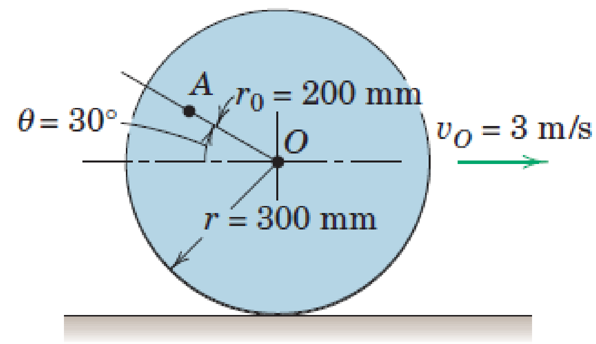
5/4 Relative Velocity

Interpretation of the Relative-Velocity Equation



SAMPLE PROBLEM 5/7

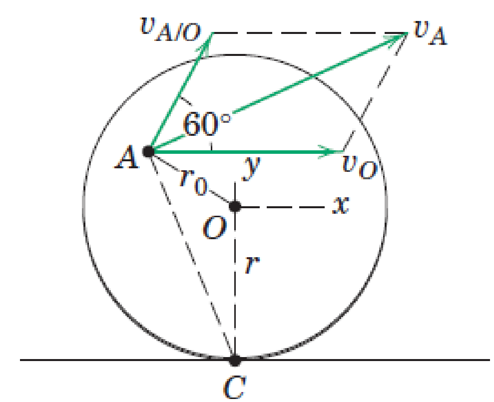
The wheel of radius $r = 300$ mm rolls to the right without slipping and has a velocity $v_O = 3$ m/s of its center O. Calculate the velocity of point A on the wheel for the instant represented.



Solution I (Scalar-Geometric)

$$\mathbf{v}_A = \mathbf{v}_O + \mathbf{v}_{A/O}$$

$$[v_{A/O} = r_0\dot{\theta}] \qquad v_{A/O} = 0.2(10) = 2 \text{ m/s}$$

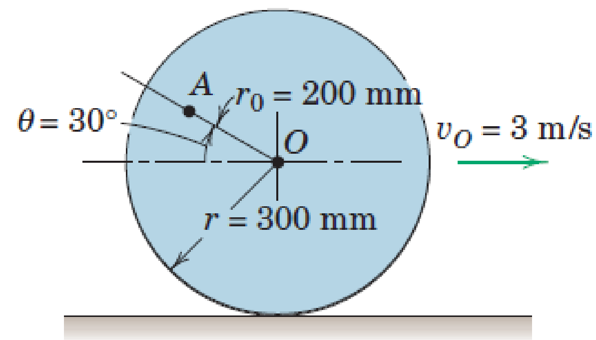


$$v_A{}^2 = 3^2 + 2^2 + 2(3)(2)\cos 60° = 19\ (\text{m/s})^2 \qquad v_A = 4.36 \text{ m/s}$$

$$v_{A/C} = \overline{AC}\omega = \frac{\overline{AC}}{\overline{OC}} v_O = \frac{0.436}{0.300}(3) = 4.36 \text{ m/s} \qquad v_A = v_{A/C} = 4.36 \text{ m/s}$$

SAMPLE PROBLEM 5/7

The wheel of radius $r = 300$ mm rolls to the right without slipping and has a velocity $v_O = 3$ m/s of its center O. Calculate the velocity of point A on the wheel for the instant represented.



Solution II (Vector)

$$\mathbf{v}_A = \mathbf{v}_O + \mathbf{v}_{A/O} = \mathbf{v}_O + \boldsymbol{\omega} \times \mathbf{r}_0$$

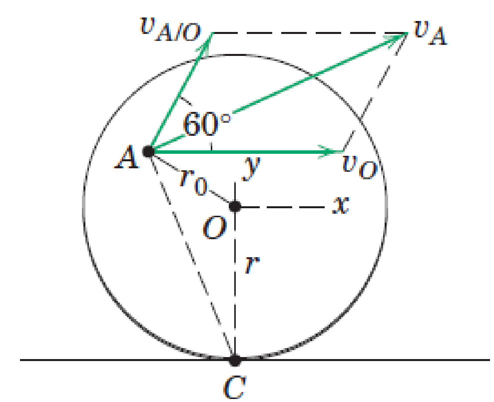
$$\boldsymbol{\omega} = -10\mathbf{k} \text{ rad/s}$$

$$\mathbf{r}_0 = 0.2(-\mathbf{i}\cos 30° + \mathbf{j}\sin 30°) = -0.1732\mathbf{i} + 0.1\mathbf{j} \text{ m}$$

$$\mathbf{v}_O = 3\mathbf{i} \text{ m/s}$$

$$\mathbf{v}_A = 3\mathbf{i} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & -10 \\ -0.1732 & 0.1 & 0 \end{vmatrix} = 3\mathbf{i} + 1.732\mathbf{j} + \mathbf{i}$$

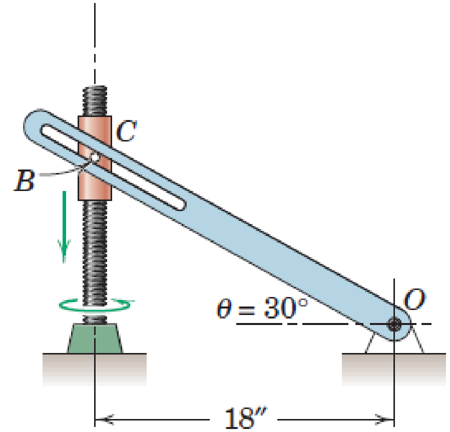
$$= 4\mathbf{i} + 1.732\mathbf{j} \text{ m/s}$$



$$v_A = \sqrt{4^2 + (1.732)^2} = \sqrt{19} = 4.36 \text{ m/s}$$

SAMPLE PROBLEM 5/10

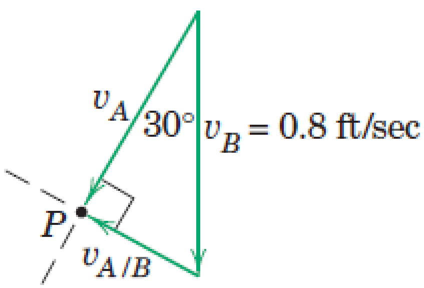
The power screw turns at a speed which gives the threaded collar C a velocity of 0.8 ft/sec vertically down. Determine the angular velocity of the slotted arm when $\theta = 30°$.

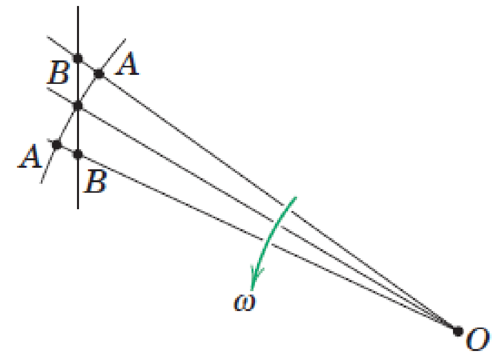


$$v_A = v_B \cos\theta = 0.8 \cos 30° = 0.693 \text{ ft/sec}$$

$$\omega = \frac{v_A}{\overline{OA}} = \frac{0.693}{\left(\frac{18}{12}\right)/\cos 30°}$$

$$= 0.400 \text{ rad/sec CCW}$$





5/5 Instantaneous Center of Zero Velocity

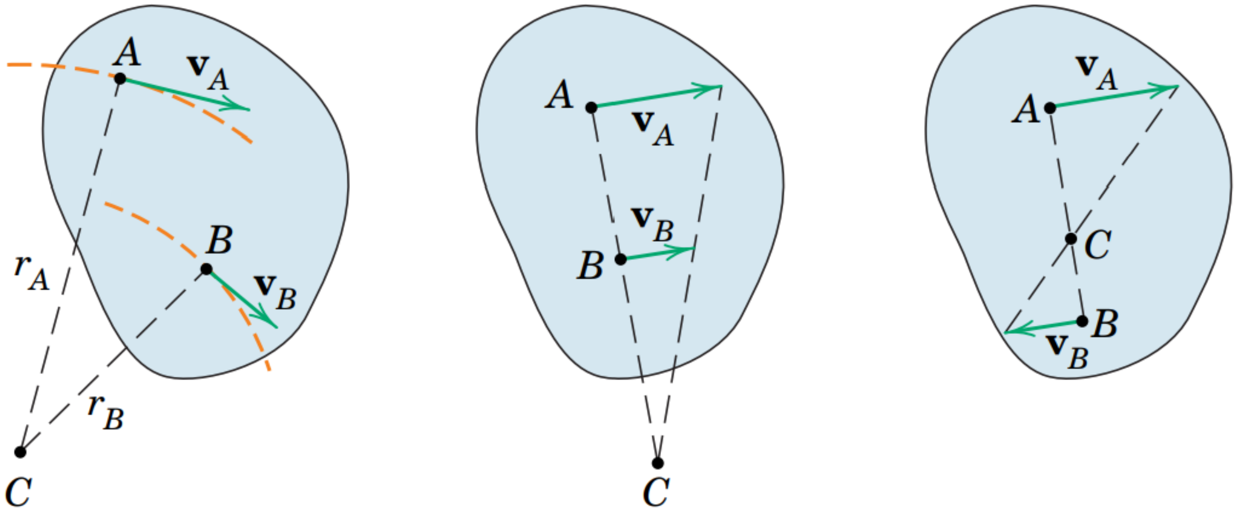
- We can solve the problem by choosing a unique reference point which momentarily has zero velocity.

- As far as velocities are concerned, the body may be considered to be in pure rotation about an axis, normal to the plane of motion, passing through this point.

- This axis is called the instantaneous axis of zero velocity, and the intersection of this axis with the plane of motion is known as the instantaneous center of zero velocity.

5/5 Instantaneous Center of Zero Velocity

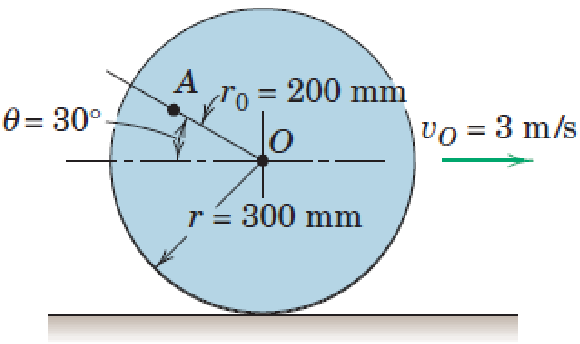
Locating the Instantaneous Center



$$\omega = \frac{v_A}{r_A}$$

SAMPLE PROBLEM 5/11

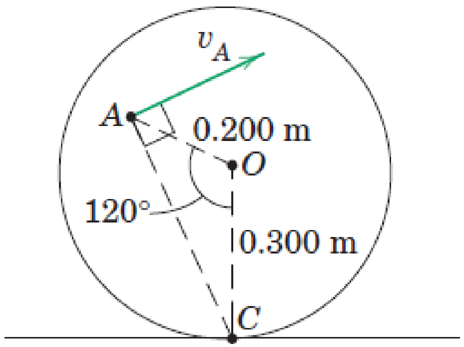
The wheel of Sample Problem 5/7, shown again here, rolls to the right without slipping, with its center O having a velocity $v_O = 3$ m/s. Locate the instantaneous center of zero velocity and use it to find the velocity of point A for the position indicated.



$[\omega = v/r]$ $\qquad \omega = v_O / \overline{OC} = 3/0.300 = 10$ rad/s

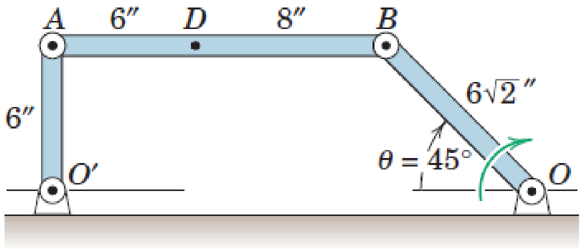
$$\overline{AC} = \sqrt{(0.300)^2 + (0.200)^2 - 2(0.300)(0.200)\cos 120^\circ} = 0.436 \text{ m}$$

$[v = r\omega]$ $\qquad v_A = \overline{AC}\omega = 0.436(10) = 4.36$ m/s



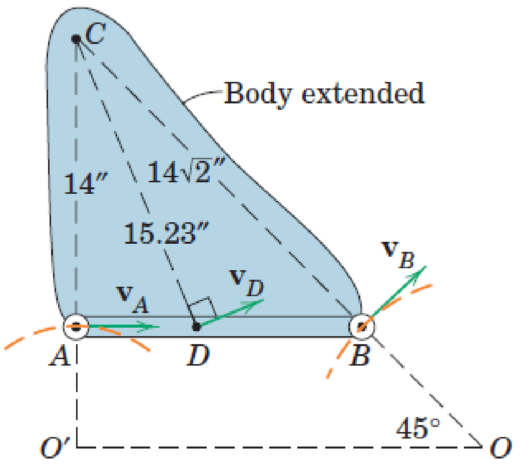
SAMPLE PROBLEM 5/12

Arm OB of the linkage has a clockwise angular velocity of 10 rad/sec in the position shown where $\theta = 45°$. Determine the velocity of A, the velocity of D, and the angular velocity of link AB for the position shown.



$[\omega = v/r]$ $$\omega_{BC} = \frac{v_B}{\overline{BC}} = \frac{\overline{OB}\omega_{OB}}{\overline{BC}} = \frac{6\sqrt{2}(10)}{14\sqrt{2}}$$

$$= 4.29 \text{ rad/sec CCW}$$



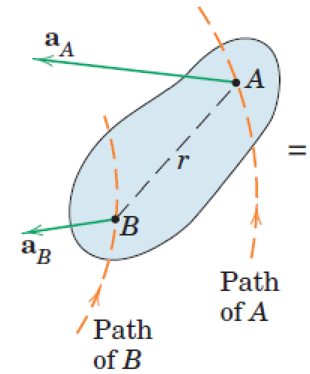
$[v = r\omega]$ $$v_A = \frac{14}{12}(4.29) = 5.00 \text{ ft/sec}$$

$$v_D = \frac{15.23}{12}(4.29) = 5.44 \text{ ft/sec}$$

5/6 Relative Acceleration

- The relative-acceleration equation:

$$\mathbf{a}_A = \mathbf{a}_B + \mathbf{a}_{A/B}$$

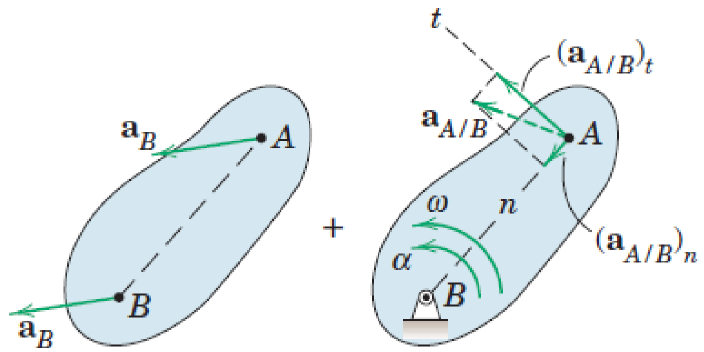


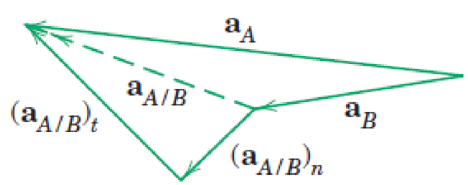
Relative Acceleration Due to Rotation

$$\mathbf{a}_A = \mathbf{a}_B + (\mathbf{a}_{A/B})_n + (\mathbf{a}_{A/B})_t$$

$$(a_{A/B})_n = v_{A/B}{}^2/r = r\omega^2$$
$$(a_{A/B})_t = \dot{v}_{A/B} = r\alpha$$

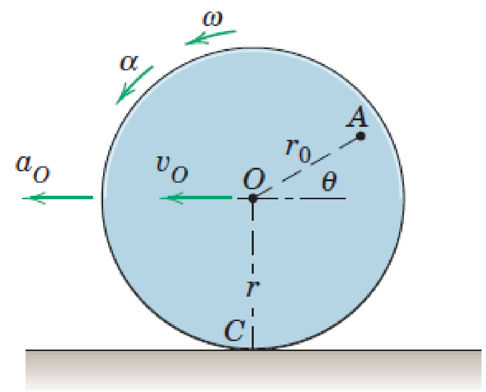
$$(\mathbf{a}_{A/B})_n = \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r})$$
$$(\mathbf{a}_{A/B})_t = \boldsymbol{\alpha} \times \mathbf{r}$$





SAMPLE PROBLEM 5/13

The wheel of radius r rolls to the left without slipping and, at the instant considered, the center O has a velocity $\mathbf{v}_O$ and an acceleration $\mathbf{a}_O$ to the left. Determine the acceleration of points A and C on the wheel for the instant considered.

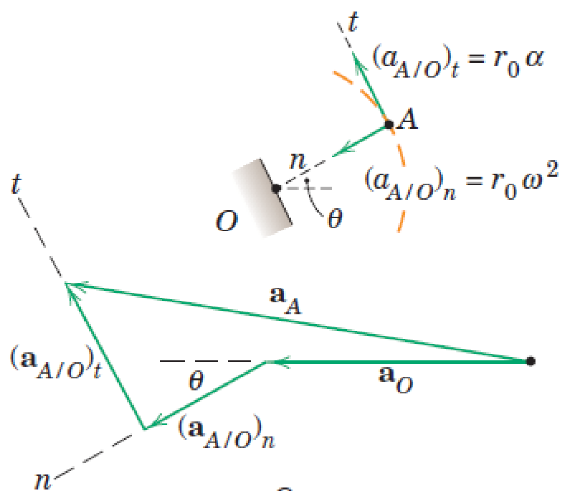


$$\omega = v_O/r \qquad \text{and} \qquad \alpha = a_O/r$$

$$\mathbf{a}_A = \mathbf{a}_O + \mathbf{a}_{A/O} = \mathbf{a}_O + (\mathbf{a}_{A/O})_n + (\mathbf{a}_{A/O})_t$$

$$(a_{A/O})_n = r_0\omega^2 = r_0\left(\frac{v_O}{r}\right)^2$$

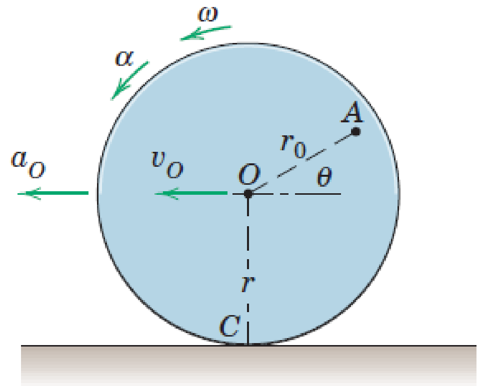
$$(a_{A/O})_t = r_0\alpha = r_0\left(\frac{a_O}{r}\right)$$



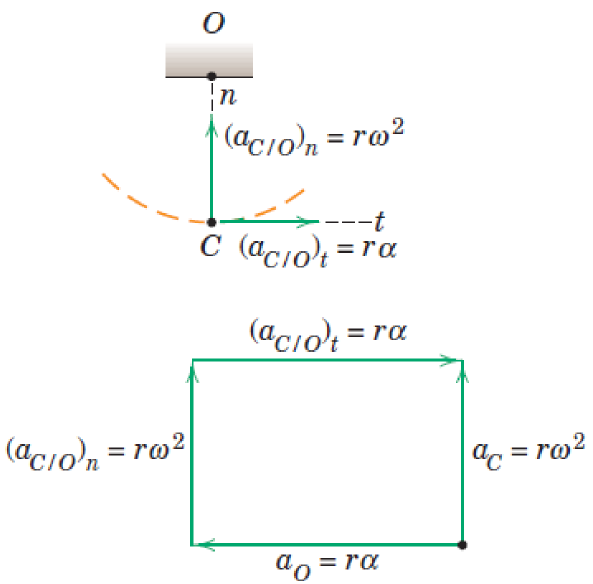
$$\begin{aligned} a_A &= \sqrt{(a_A)_n{}^2 + (a_A)_t{}^2} \\ &= \sqrt{[a_O\cos\theta + (a_{A/O})_n]^2 + [a_O\sin\theta + (a_{A/O})_t]^2} \\ &= \sqrt{(r\alpha\cos\theta + r_0\omega^2)^2 + (r\alpha\sin\theta + r_0\alpha)^2} \end{aligned}$$

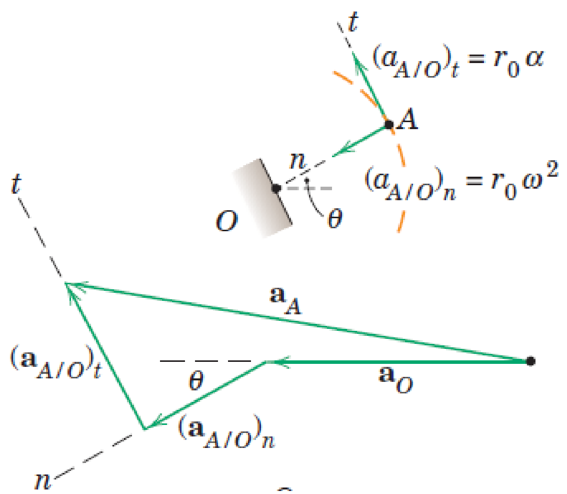
SAMPLE PROBLEM 5/13

The wheel of radius r rolls to the left without slipping and, at the instant considered, the center O has a velocity $\mathbf{v}_O$ and an acceleration $\mathbf{a}_O$ to the left. Determine the acceleration of points A and C on the wheel for the instant considered.



$$\mathbf{a}_C = \mathbf{a}_O + \mathbf{a}_{C/O} \quad \Rightarrow \quad a_C = r\omega^2$$

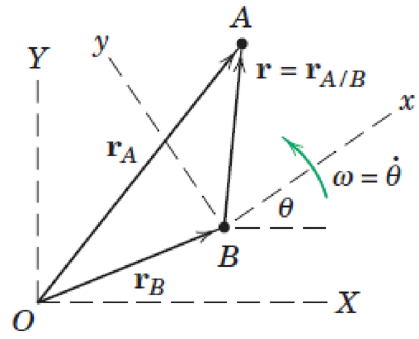




5/7 Motion Relative to Rotating Axes

- The absolute position vector of A:

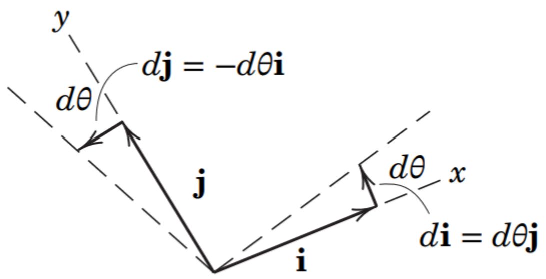
$$\mathbf{r}_A = \mathbf{r}_B + \mathbf{r} = \mathbf{r}_B + (x\mathbf{i} + y\mathbf{j})$$

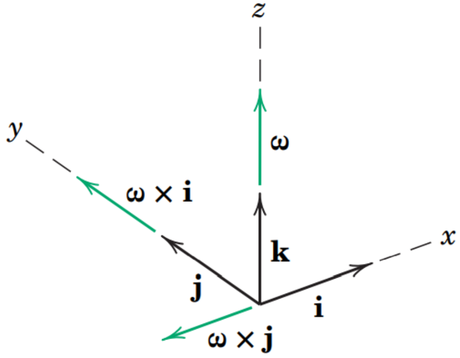


Time Derivatives of Unit Vectors

$$\dot{\mathbf{i}} = \omega\mathbf{j} \quad \text{and} \quad \dot{\mathbf{j}} = -\omega\mathbf{i}$$

$$\dot{\mathbf{i}} = \boldsymbol{\omega} \times \mathbf{i} \quad \text{and} \quad \dot{\mathbf{j}} = \boldsymbol{\omega} \times \mathbf{j}$$





5/7 Motion Relative to Rotating Axes

Relative Velocity

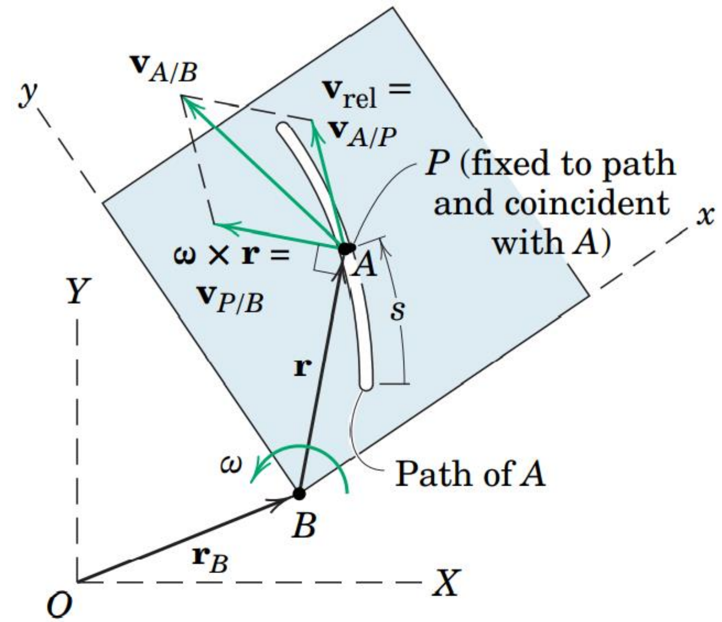
$$\dot{\mathbf{r}}_A = \dot{\mathbf{r}}_B + \frac{d}{dt}(x\mathbf{i} + y\mathbf{j})$$

$$= \dot{\mathbf{r}}_B + (x\dot{\mathbf{i}} + y\dot{\mathbf{j}}) + (\dot{x}\mathbf{i} + \dot{y}\mathbf{j})$$

$$x\dot{\mathbf{i}} + y\dot{\mathbf{j}} = \boldsymbol{\omega} \times x\mathbf{i} + \boldsymbol{\omega} \times y\mathbf{j} = \boldsymbol{\omega} \times (x\mathbf{i} + y\mathbf{j}) = \boldsymbol{\omega} \times \mathbf{r}$$

$$\dot{x}\mathbf{i} + \dot{y}\mathbf{j} = \mathbf{v}_{\text{rel}}$$

$$\mathbf{v}_A = \mathbf{v}_B + \boldsymbol{\omega} \times \mathbf{r} + \mathbf{v}_{\text{rel}}$$



$$\mathbf{v}_A = \mathbf{v}_B + \boldsymbol{\omega} \times \mathbf{r} + \mathbf{v}_{\text{rel}}$$

$$\mathbf{v}_A = \underbrace{\mathbf{v}_B + \mathbf{v}_{P/B}}_{} + \mathbf{v}_{A/P}$$

$$\mathbf{v}_A = \mathbf{v}_P \underbrace{\quad + \mathbf{v}_{A/P}}_{}$$

$$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$$

5/7 Motion Relative to Rotating Axes

Transformation of a Time Derivative

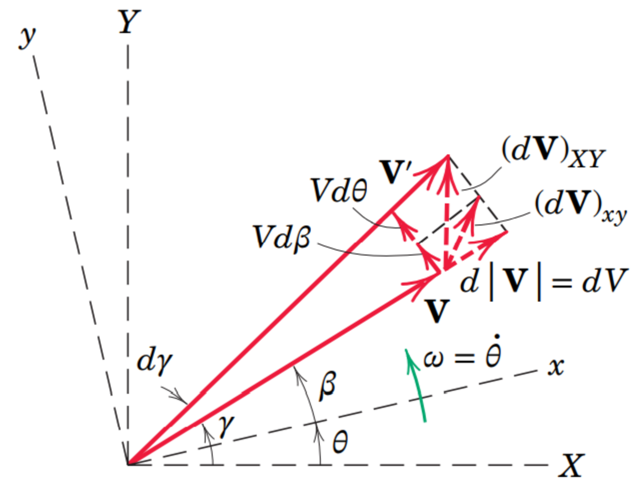
- Transformation of the time derivative of the position vector between rotating and nonrotating axes:

$$\mathbf{V} = V_x\mathbf{i} + V_y\mathbf{j}$$

$$\left(\frac{d\mathbf{V}}{dt}\right)_{XY} = (\dot{V}_x\mathbf{i} + \dot{V}_y\mathbf{j}) + (V_x\dot{\mathbf{i}} + V_y\dot{\mathbf{j}})$$

$$\left(\frac{d\mathbf{V}}{dt}\right)_{XY} = \left(\frac{d\mathbf{V}}{dt}\right)_{xy} + \boldsymbol{\omega} \times \mathbf{V}$$

$$(\dot{\mathbf{V}})_{XY} = (\dot{\mathbf{V}})_{xy} + \boldsymbol{\omega} \times \mathbf{V}$$



5/7 Motion Relative to Rotating Axes

Relative Acceleration

$$\mathbf{a}_A = \mathbf{a}_B + \dot{\omega} \times \mathbf{r} + \omega \times \dot{\mathbf{r}} + \dot{\mathbf{v}}_{\text{rel}}$$

$$\dot{\mathbf{r}} = \frac{d}{dt}(x\mathbf{i} + y\mathbf{j}) = (x\dot{\mathbf{i}} + y\dot{\mathbf{j}}) + (\dot{x}\mathbf{i} + \dot{y}\mathbf{j}) = \omega \times \mathbf{r} + \mathbf{v}_{\text{rel}}$$

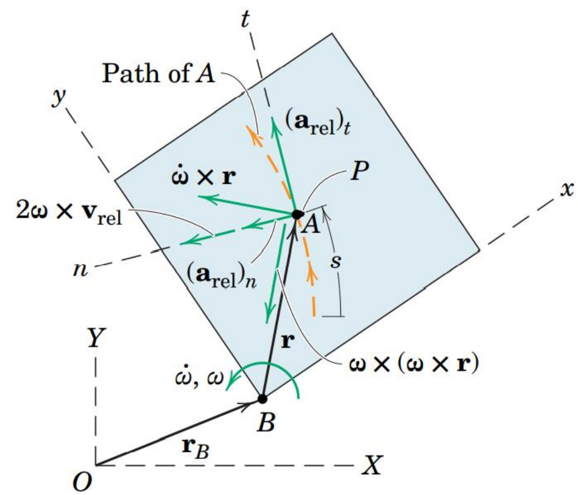
$$\omega \times \dot{\mathbf{r}} = \omega \times (\omega \times \mathbf{r} + \mathbf{v}_{\text{rel}}) = \omega \times (\omega \times \mathbf{r}) + \omega \times \mathbf{v}_{\text{rel}}$$

$$\dot{\mathbf{v}}_{\text{rel}} = \frac{d}{dt}(\dot{x}\mathbf{i} + \dot{y}\mathbf{j}) = (\dot{x}\dot{\mathbf{i}} + \dot{y}\dot{\mathbf{j}}) + (\ddot{x}\mathbf{i} + \ddot{y}\mathbf{j})$$

$$= \omega \times (\dot{x}\mathbf{i} + \dot{y}\mathbf{j}) + (\ddot{x}\mathbf{i} + \ddot{y}\mathbf{j})$$

$$= \omega \times \mathbf{v}_{\text{rel}} + \mathbf{a}_{\text{rel}}$$

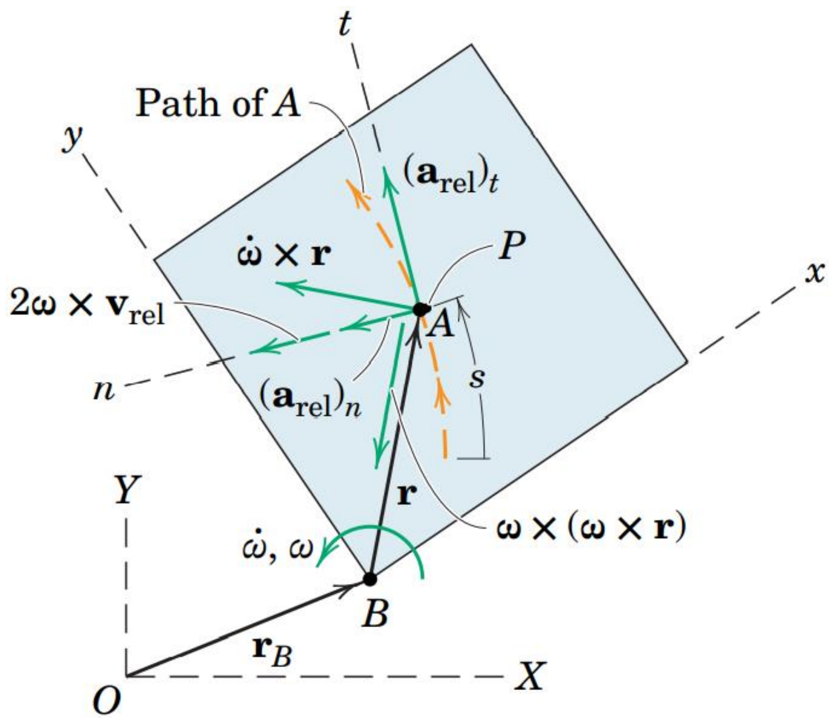
$$\mathbf{a}_A = \mathbf{a}_B + \dot{\omega} \times \mathbf{r} + \omega \times (\omega \times \mathbf{r}) + 2\omega \times \mathbf{v}_{\text{rel}} + \mathbf{a}_{\text{rel}}$$



5/7 Motion Relative to Rotating Axes

Relative Acceleration

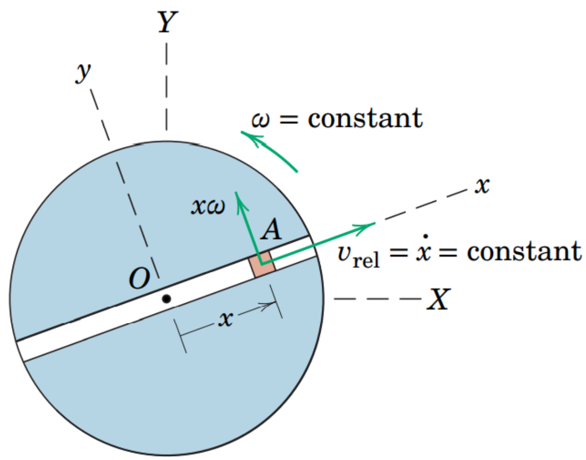
$$\mathbf{a}_A = \mathbf{a}_B + \dot{\boldsymbol{\omega}} \times \mathbf{r} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) + 2\boldsymbol{\omega} \times \mathbf{v}_{rel} + \mathbf{a}_{rel}$$

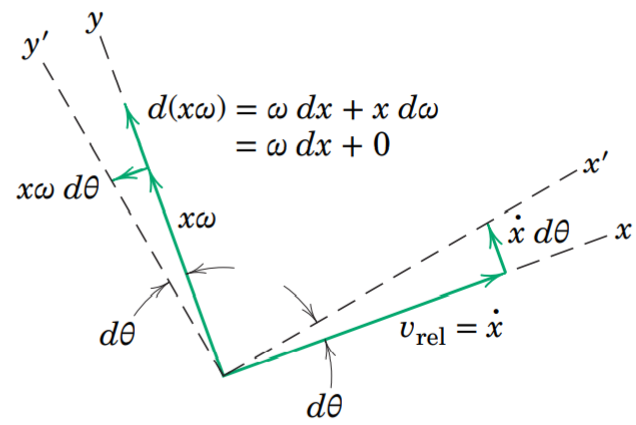


5/7 Motion Relative to Rotating Axes

Coriolis Acceleration

The term $2\boldsymbol{\omega} \times \mathbf{v}_{rel}$





$$\mathbf{a}_A = \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) + 2\boldsymbol{\omega} \times \mathbf{v}_{rel} \rightarrow \mathbf{a}_A = -x\omega^2\mathbf{i} + 2\dot{x}\omega\mathbf{j}$$

5/7 Motion Relative to Rotating Axes

Rotating versus Nonrotating Systems

$$\mathbf{a}_A = \mathbf{a}_B + \underbrace{\dot{\omega} \times \mathbf{r} + \omega \times (\omega \times \mathbf{r})} + \underbrace{2\omega \times \mathbf{v}_{\text{rel}} + \mathbf{a}_{\text{rel}}}$$

$$\mathbf{a}_A = \underbrace{\mathbf{a}_B + \qquad \mathbf{a}_{P/B}} \qquad + \qquad \mathbf{a}_{A/P}$$

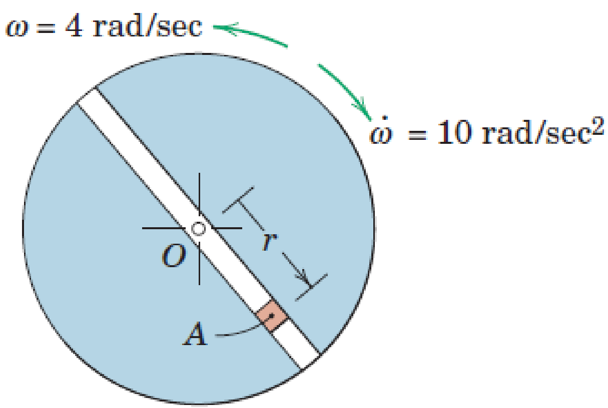
$$\mathbf{a}_A = \qquad \mathbf{a}_P \qquad \underbrace{\qquad + \qquad \mathbf{a}_{A/P}}$$

$$\mathbf{a}_A = \mathbf{a}_B + \qquad \mathbf{a}_{A/B}$$

$$\boxed{\mathbf{a}_A = \mathbf{a}_P + 2\omega \times \mathbf{v}_{\text{rel}} + \mathbf{a}_{\text{rel}}}$$

SAMPLE PROBLEM 5/16

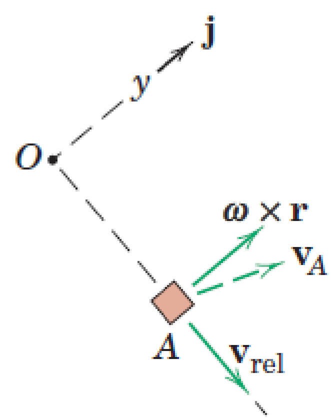
At the instant represented, the disk with the radial slot is rotating about O with a counterclockwise angular velocity of 4 rad/sec which is decreasing at the rate of 10 rad/sec^2. The motion of slider A is separately controlled, and at this instant, $r = 6$ in., $\dot{r} = 5$ in./sec, and $\ddot{r} = 81$ in./sec^2. Determine the absolute velocity and acceleration of A for this position.

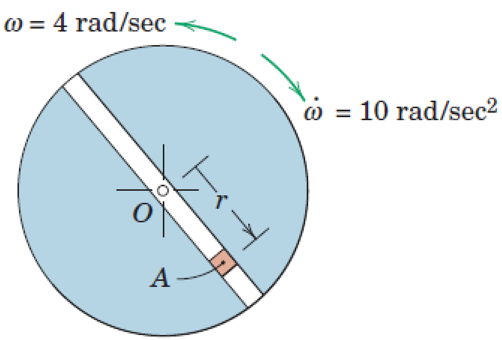


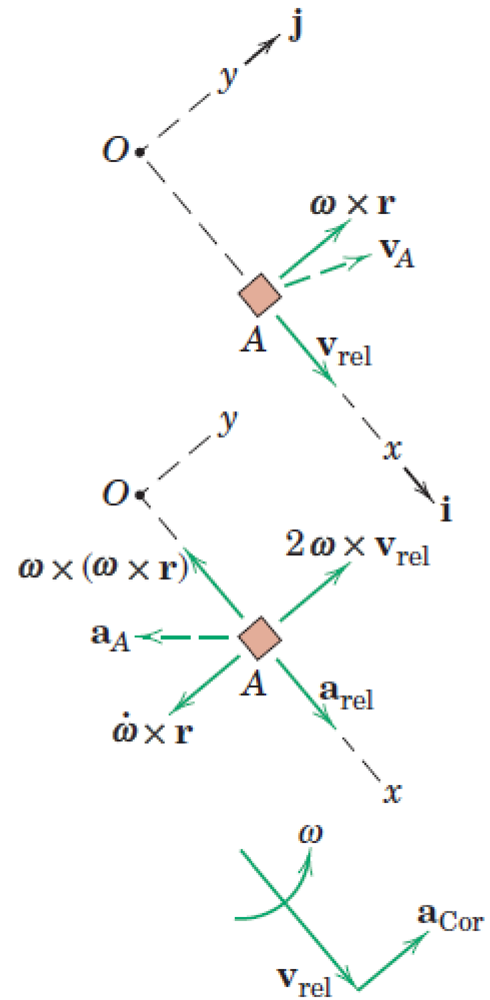
$$\mathbf{v}_A = \boldsymbol{\omega} \times \mathbf{r} + \mathbf{v}_{\text{rel}}$$

$$\mathbf{v}_A = 4\mathbf{k} \times 6\mathbf{i} + 5\mathbf{i} = 24\mathbf{j} + 5\mathbf{i} \text{ in./sec}$$

$$v_A = \sqrt{(24)^2 + (5)^2} = 24.5 \text{ in./sec}$$







$$\mathbf{a}_A = \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) + \dot{\boldsymbol{\omega}} \times \mathbf{r} + 2\boldsymbol{\omega} \times \mathbf{v}_{rel} + \mathbf{a}_{rel}$$

$$\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) = 4\mathbf{k} \times (4\mathbf{k} \times 6\mathbf{i}) = 4\mathbf{k} \times 24\mathbf{j} = -96\mathbf{i} \text{ in./sec}^2$$

$$\dot{\boldsymbol{\omega}} \times \mathbf{r} = -10\mathbf{k} \times 6\mathbf{i} = -60\mathbf{j} \text{ in./sec}^2$$

$$2\boldsymbol{\omega} \times \mathbf{v}_{rel} = 2(4\mathbf{k}) \times 5\mathbf{i} = 40\mathbf{j} \text{ in./sec}^2$$

$$\mathbf{a}_{rel} = 81\mathbf{i} \text{ in./sec}^2$$

$$\mathbf{a}_A = (81 - 96)\mathbf{i} + (40 - 60)\mathbf{j} = -15\mathbf{i} - 20\mathbf{j} \text{ in./sec}^2$$

$$a_A = \sqrt{(15)^2 + (20)^2} = 25 \text{ in./sec}^2$$