دانشگاه سمنان

Semnan University
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دانشکده مهندسی مکانیک

دانشکده مهندسی مکانیک

درس طراحی سیستم های شاسی خودرو

VEHICLE CHASSIS SYSTEMS DESIGN

Chapter 6 – Cornering

Class Lecture

- ***Contents:***

INTRODUCTION

- The cornering behavior of a motor vehicle is an important performance mode often equated with handling.

- "Handling" is a loosely used term meant to imply the responsiveness of a vehicle to driver input, or the ease of control.

- As such, handling is an overall measure of the vehicle-driver combination. The driver and vehicle is a "closed-loop“.

- For purposes of characterizing only the vehicle, "open-loop" behavior is used.

Introduction

- Open loop refers to vehicle response to specific steering inputs, and is more precisely defined as "directional response" behavior.

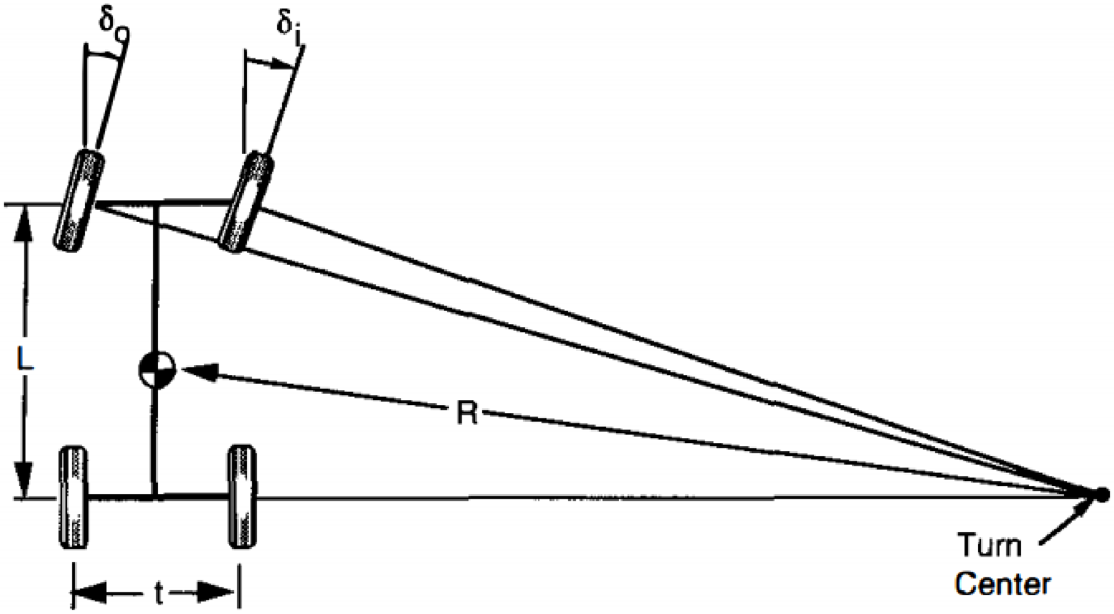
- The most commonly used measure of open-loop response is the understeer gradient (a measure of performance under steady-state conditions).

- We first analyze turning behavior at low speed, and then consider the differences that arise under high-speed conditions.

LOW-SPEED TURNING

- At low speed (parking lot maneuvers) the tires need not develop lateral forces. Thus they roll with no slip angle.

- Ackerman Geometry:



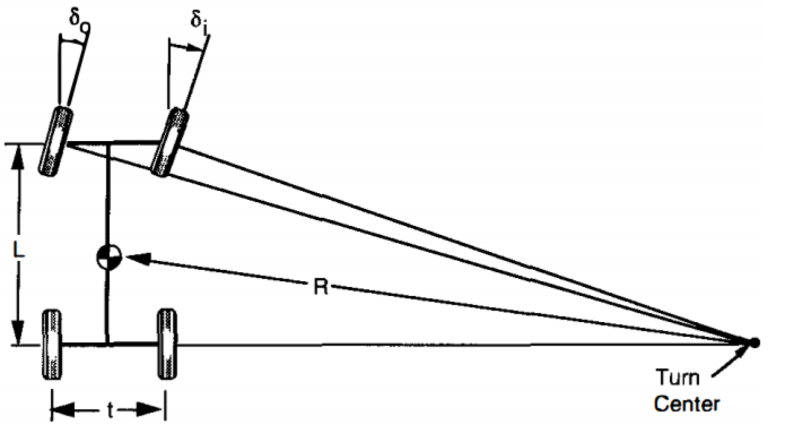
LOW-SPEED TURNING

- Center of Turn:
  - The perpendicular from each wheel should pass through the same point (the center of turn)
  - If they do not pass through the same point, the front tires will "fight" each other in the tum, with each experiencing some scrub (sideslip) in the turn.

- The ideal turning angles:

$$\delta_o \cong \frac{L}{(R + t/2)}$$

$$\delta_i \cong \frac{L}{(R - t/2)}$$



- The average angle of the front wheels: the Ackerman Angle $\delta \doteq L/R$

Low-Speed Turning

- Errors, or deviations, from the Ackerman in the left-right steer angles can have a significant influence on front tire wear.

- Errors do not have a significant influence on directional response; however, they do affect the centering torques in the steering system.

- With correct Ackerman geometry, the steering torques tend to increase consistently with steer angle, thus providing the driver with a natural feel in the feedback through the steering wheel.

Low-Speed Turning

- The rear wheels off-tracking distance, Δ

$$\Delta = R[1 - \cos(L/R)]$$

$$\cos z = 1 - \frac{z^2}{2!} + \frac{z^4}{4!} - \frac{z^6}{6!} \cdots\cdots \quad \longrightarrow \quad \Delta \cong \frac{L^2}{2R}$$

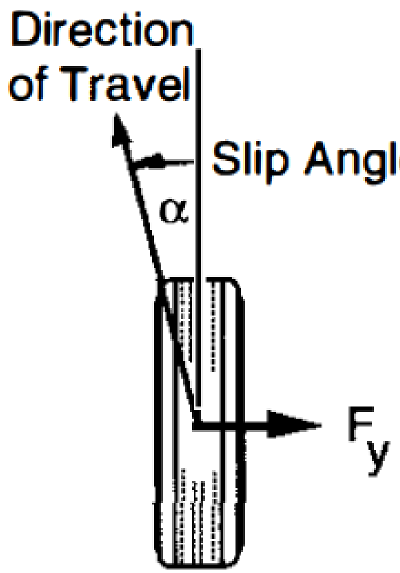
  - off-tracking is primarily of concern with long wheelbase vehicles such as trucks and buses.

High-Speed Cornering

- At high speed, the turning equations differ because lateral acceleration will be present.

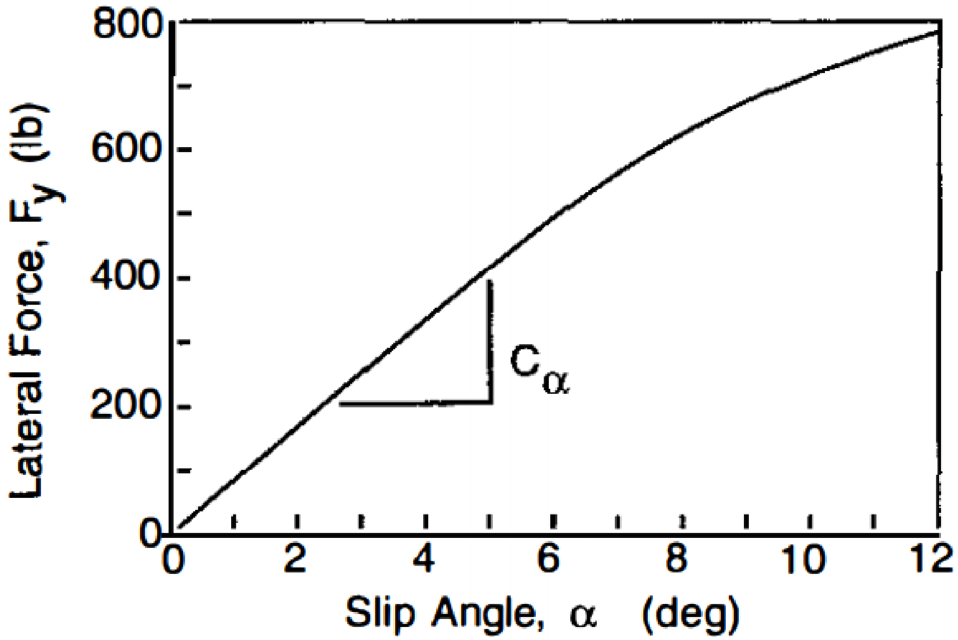
- To counteract the lateral acceleration the tires must develop lateral forces, and slip angles will be present at each wheel.

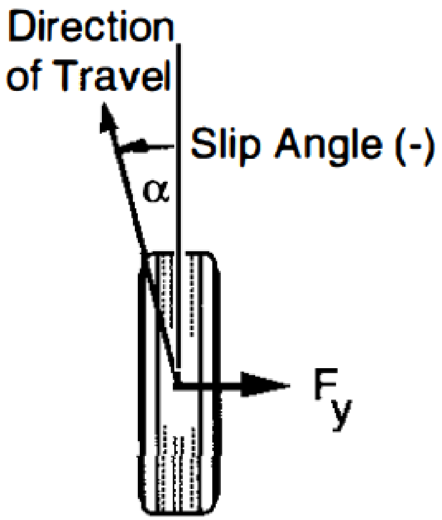
- The angle between its direction of heading and its direction of travel is known as slip angle, α.



High-Speed Cornering

- Tire Cornering Forces
  - The lateral force, denoted by F_y , is called the “cornering force”.





HIGH-SPEED CORNERING

- Tire Cornering Forces
  - At low slip angles (5 degrees or less) the relationship is linear:

$$\longrightarrow \quad F_y = C_\alpha \alpha$$

    - $C\alpha$: Cornering Stiffness

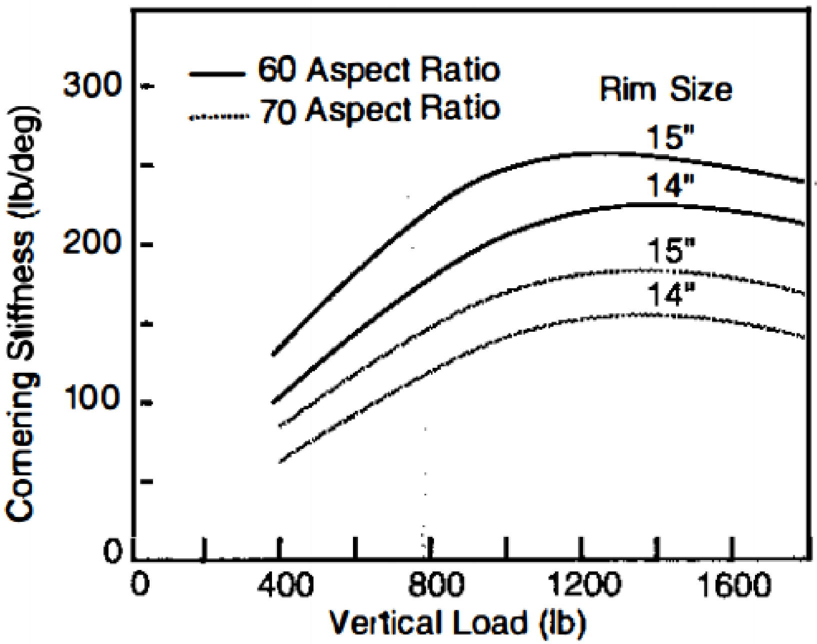
  - The cornering stiffness is dependent on many variables:
    - Tire size and Type (radial- versus bias-ply construction), Number of plies, Cord angles, wheel width, tread, The load and inflation

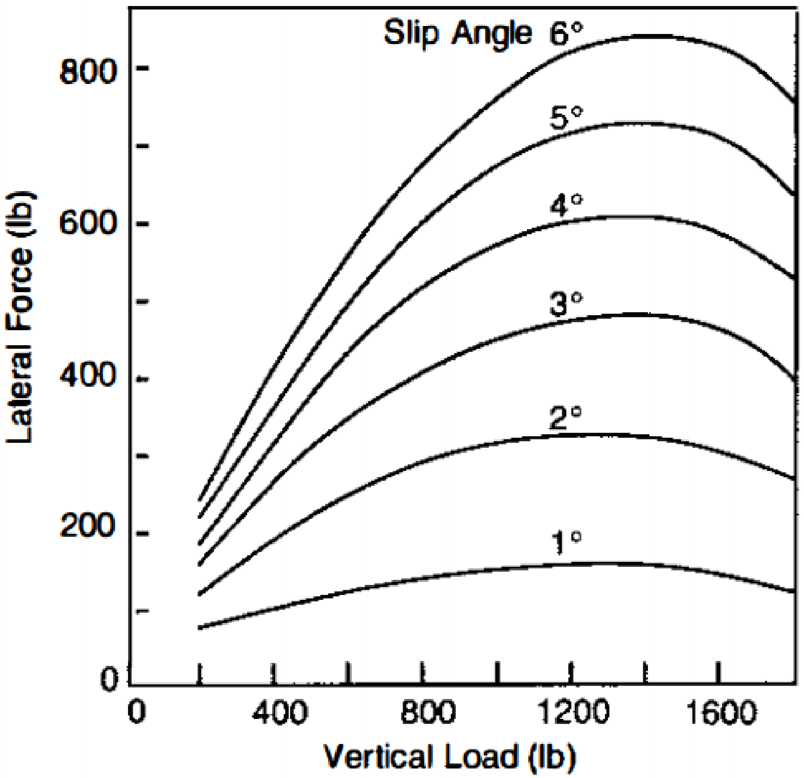
  - Because of the strong dependence of cornering force on load, tire cornering properties may also be described by the “cornering coefficient”: (Typ.=0.2)

$$\longrightarrow \quad CC_\alpha = C_\alpha / F_z$$

High-Speed Cornering

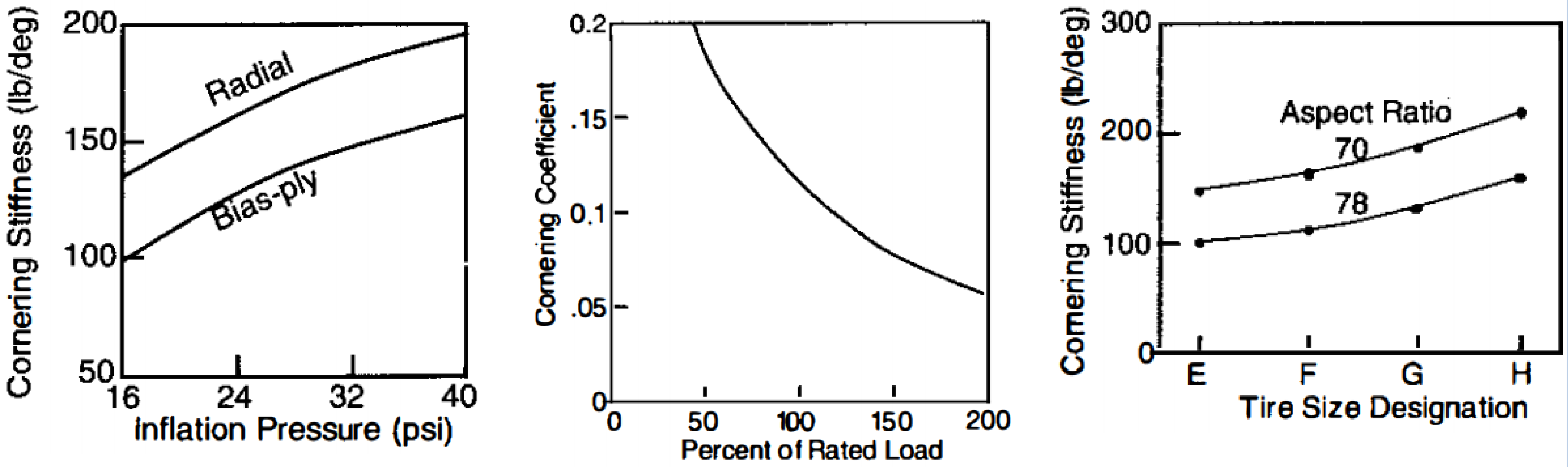
- Tire Cornering Forces





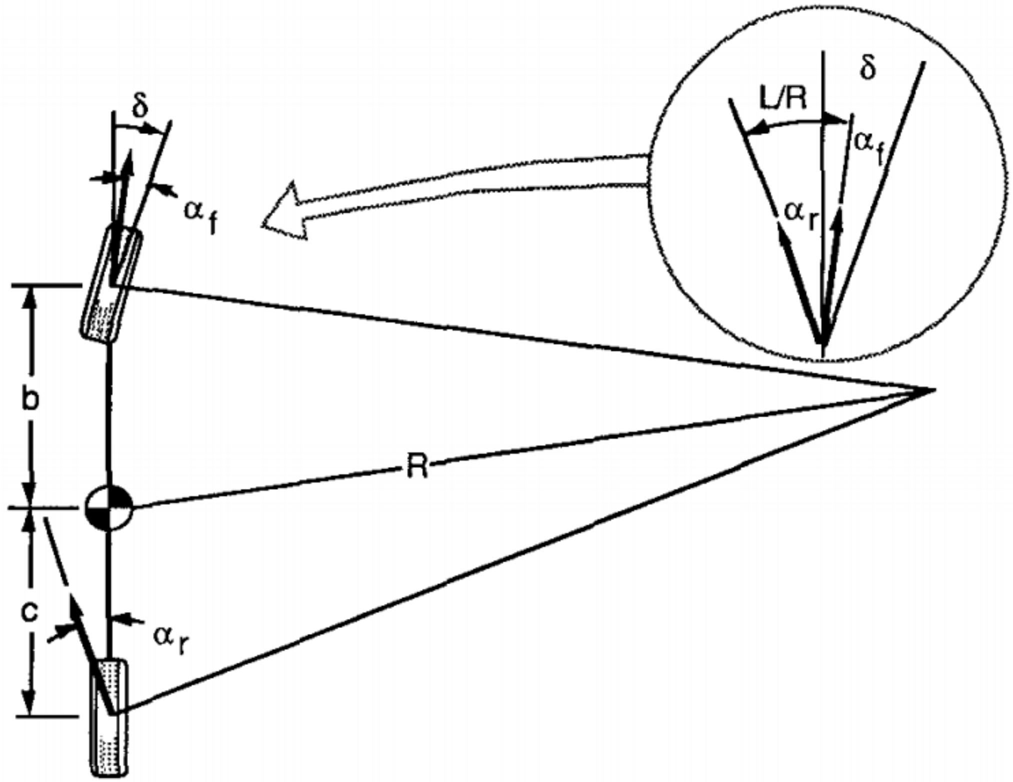
High-Speed Cornering

- Tire Cornering Forces



HIGH-SPEED CORNERING

- Cornering Equations
  - The Bicycle Model
    - ✓ At high speeds the radius of turn is much larger than the wheelbase of the vehicle.
    - ✓ Then small angles can be assumed, and the difference between steer angles on the outside and inside front wheels is negligible.



High-Speed Cornering

- Cornering Equations
  - Sum of the forces in the lateral direction

$$\Sigma F_y = F_{yf} + F_{yr} = M\, V^2/R$$

F_{yf} = Lateral (cornering) force at the front axle
F_{yr} = Lateral (cornering) force at the rear axle
M = Mass of the vehicle
V = Forward velocity
R = Radius of the turn

  - Moment equilibrium about the center of gravity

$$F_{yf}\, b - F_{yr}\, c = 0 \longrightarrow F_{yf} = F_{yr}\, c/b$$

HIGH-SPEED CORNERING

- Cornering Equations

$$M\,V^2/R = F_{yr}\,(c/b + 1) = F_{yr}\,(b + c)/b = F_{yr}\,L/b$$

$$F_{yr} = M\,b/L\,(V^2/R)$$

- ✓ Proportional to the vehicle mass carried on the rear axle and lateral acceleration.
- ✓ Same for front axle

High-Speed Cornering

- Cornering Equations
  - The Slip Angles at the front and rear wheels:

$$\alpha_f = W_f V^2/(C_{\alpha f}\, g\, R)$$

$$\alpha_r = W_r V^2/(C_{\alpha r}\, g\, R)$$

  - From the geometry of the vehicle in the turn:

$$\delta = 57.3\, L/R + \alpha_f - \alpha_r$$

$$\rightarrow \quad \delta = 57.3 \frac{L}{R} + \frac{W_f V^2}{C_{\alpha f}\, g\, R} - \frac{W_r V^2}{C_{\alpha r}\, g\, R}$$

HIGH-SPEED CORNERING

- Cornering Equations

$$\delta = 57.3\frac{L}{R} + \left(\frac{W_f}{C_{\alpha f}} - \frac{W_r}{C_{\alpha r}}\right)\frac{V^2}{gR}$$

δ	= Steer angle at the front wheels (deg)
L	= Wheelbase (ft)
R	= Radius of turn (ft)
V	= Forward speed (ft/sec)
g	= Gravitational acceleration constant = 32.2 ft/sec^2
W_f	= Load on the front axle (lb)
W_r	= Load on the rear axle (lb)
$C_{\alpha f}$	= Cornering stiffness of the front tires (lb_y/deg)
$C_{\alpha r}$	= Cornering stiffness of the rear tires (lb_y/deg)

HIGH-SPEED CORNERING

- Understeer Gradient
  - The equation in a shorthand form:

$$\delta = 57.3\, L/R + K\, a_y$$

K = Understeer gradient (deg/g)
a_y = Lateral acceleration (g)

- ✓ It describes how the steer angle of the vehicle must be changed with the radius of turn or the lateral acceleration.

- ✓ Understeer Gradient: $K = \left(\frac{W_f}{C_{\alpha f}} - \frac{W_r}{C_{\alpha r}}\right)$

High-Speed Cornering

- Understeer Gradient → $\delta = 57.3\,L/R + K\,a_y$
  - 1) Neutral Steer:

$$W_f / C_{\alpha f} = W_r / C_{\alpha r} \rightarrow K = 0 \rightarrow \alpha_f = \alpha_r$$

- ✓ On a constant-radius turn, no change in steer angle will be required as the speed is varied.
- ✓ Steer angle required to make the turn will be equivalent to the Ackerman Angle.
- ✓ Physically, the neutral steer case corresponds to a balance on the vehicle such that the "force" of the lateral acceleration at the CG causes an identical increase in slip angle at both the front and rear wheels

HIGH-SPEED CORNERING

- Understeer Gradient → $\delta = 57.3\, L/R + K\, a_y$
  - 2) Understeer:

$$W_f / C_{\alpha f} > W_r / C_{\alpha r} \rightarrow K > 0 \rightarrow \alpha_f > \alpha_r$$

- ✓ On a constant-radius turn, the steer angle will have to increase with speed.
- ✓ It increases linearly with the lateral acceleration and with the square of the speed.
- ✓ In the understeer case, the lateral acceleration at the CG causes the front wheels to slip sideways to a greater extent than at the rear wheels. Thus to develop the lateral force at the front wheels necessary to maintain the radius of turn, the front wheels must be steered to a greater angle.

High-Speed Cornering

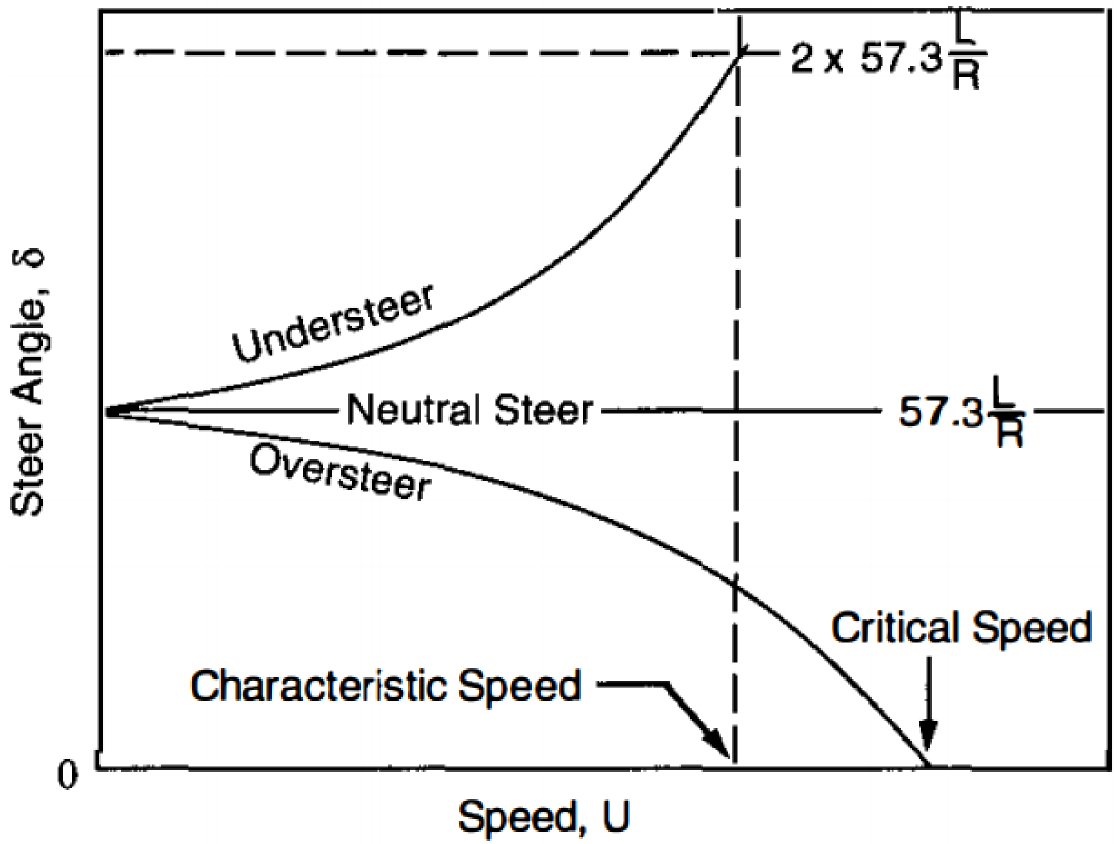
- Understeer Gradient → $\delta = 57.3\, L/R + K\, a_y$
  - 3) Oversteer:

$$W_f / C_{\alpha f} < W_r / C_{\alpha r} \rightarrow K < 0 \rightarrow \alpha_f < \alpha_r$$

- ✓ On a constant-radius turn, the steer angle will have to decrease as the speed (and lateral acceleration) is increased.
- ✓ In this case, the lateral acceleration at the CG causes the slip angle on the rear wheels to increase more than at the front. The outward drift at the rear of the vehicle turns the front wheels inward, thus diminishing the radius of turn.
- ✓ The increase in lateral acceleration that follows causes the rear to drift out even further and the process continues unless the steer angle is reduced to maintain the radius of turn.

HIGH-SPEED CORNERING

- Understeer Gradient
  - The steer angle changes with speed on a constant-radius tum



High-Speed Cornering

- Characteristic Speed
  - The speed at which the steer angle required to negotiate any turn is twice the Ackerman Angle.

$$K\,a_y = 57.3\,L/R$$

$$\rightarrow V_{char} = \sqrt{57.3\,L\,g/K}$$

HIGH-SPEED CORNERING

- Critical Speed
  - In the oversteer case, a critical speed will exist above which the vehicle will be unstable.

$$V_{crit} = \sqrt{-57.3\ L\ g/K}$$

- ✓ The critical speed is dependent on the wheelbase of the vehicle; for a given level of oversteer, long-wheelbase vehicles have a higher critical speed than short wheelbase vehicles.

- ✓ An oversteer vehicle can be driven at speeds less than the critical, but becomes directionally unstable at and above the critical speed.

High-Speed Cornering

- Lateral Acceleration Gain
  - The purpose for steering a vehicle is to produce lateral acceleration.
  - Ratio of lateral acceleration to the steering angle

$$\rightarrow \quad \frac{a_y}{\delta} = \frac{\dfrac{V^2}{57.3\, L\, g}}{1 + \dfrac{K\, V^2}{57.3\, L\, g}} \qquad \text{(deg/sec)}$$

    - ✓ When K is zero (neutral steer), the lateral acceleration gain is determined only by the numerator and is directly proportional to speed squared.
    - ✓ When K is positive (understeer), the gain is diminished by the second term in the denominator, and is always less than that of a neutral steer vehicle.

HIGH-SPEED CORNERING

- Lateral Acceleration Gain

$$\frac{a_y}{\delta} = \frac{\dfrac{V^2}{57.3\,L\,g}}{1 + \dfrac{K\,V^2}{57.3\,L\,g}} \qquad \text{(deg/sec)}$$

- ✓ When K is negative (oversteer), the second term in the denominator subtracts from 1, increasing the lateral acceleration gain.
- ✓ The magnitude of the term is dependent on the square of the speed, and goes to the value of 1 when the speed reaches the critical speed.
- ✓ Thus the critical speed corresponds to the denominator becoming zero (infinite gain) in the above equation.

High-Speed Cornering

- Yaw Velocity Gain
  - A second reason for steering a vehicle is to change the heading angle by developing a yaw velocity (sometimes called "yaw rate").
  - The yaw velocity, r:

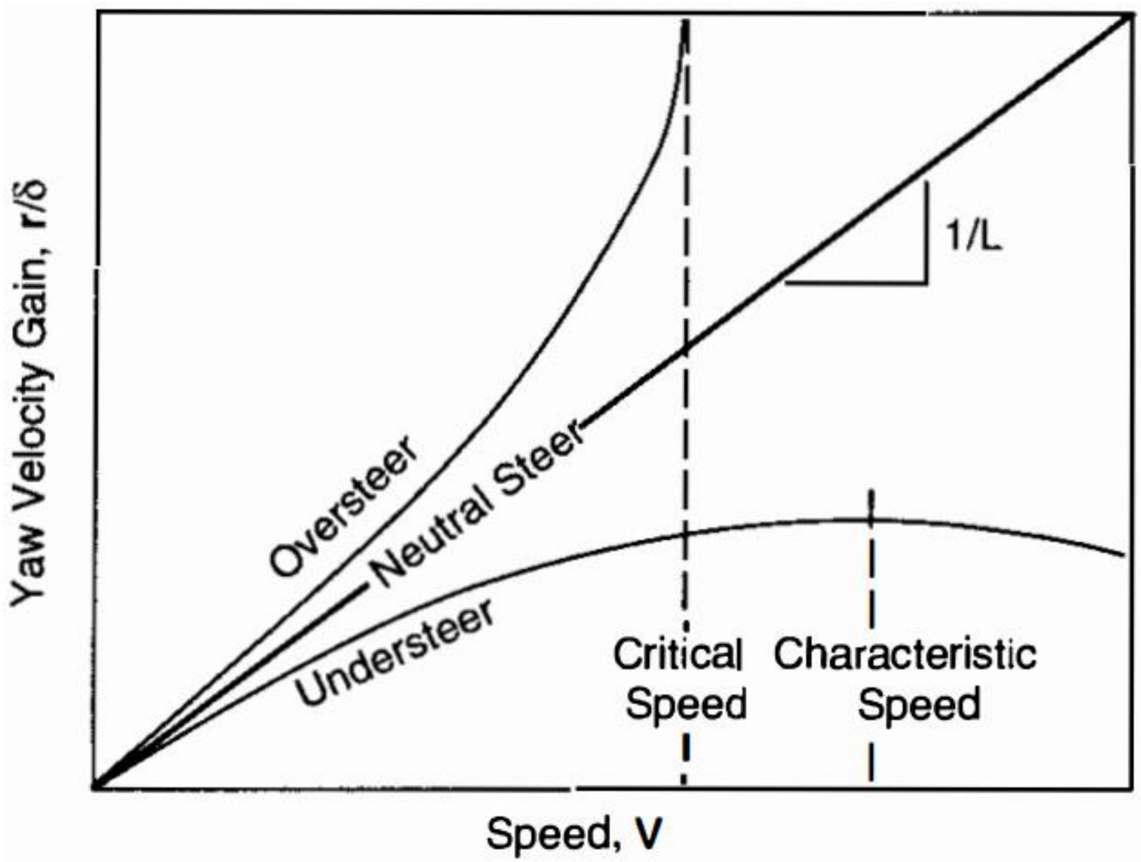
$$r = 57.3\ V/R \qquad \text{(deg/sec)}$$

  - The ratio of yaw velocity to steering angle:

$$\rightarrow \quad \frac{r}{\delta} = \frac{V/L}{1 + \dfrac{K\,V^2}{57.3\,L\,g}}$$

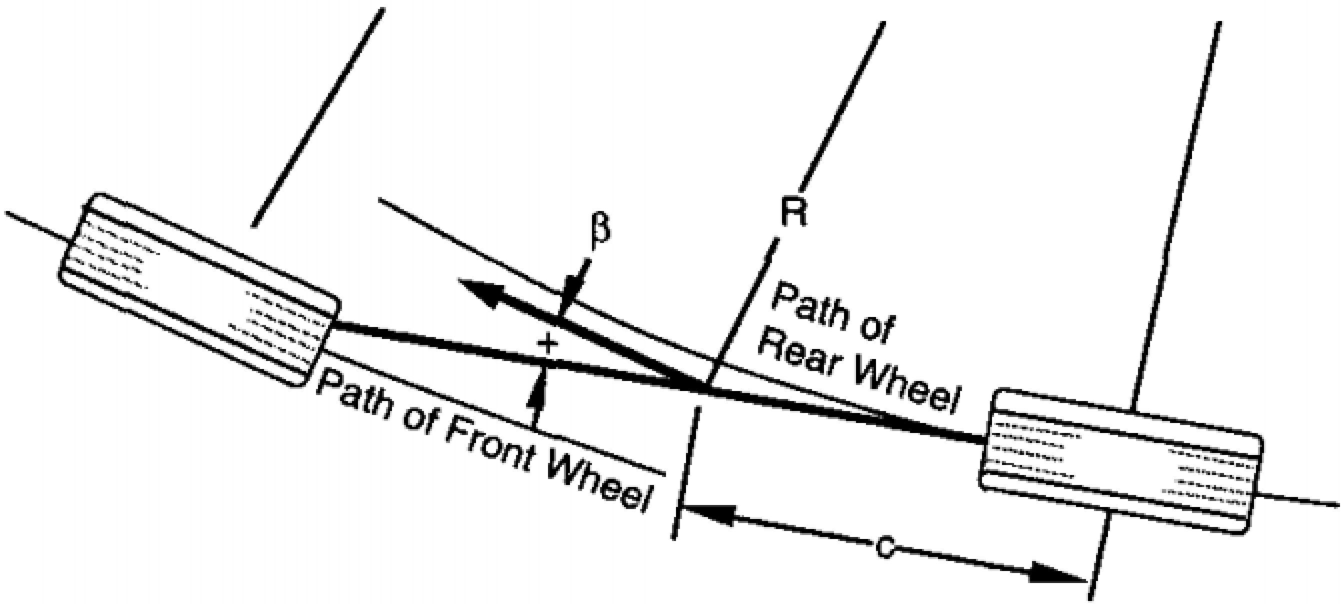
High-Speed Cornering

- Yaw Velocity Gain



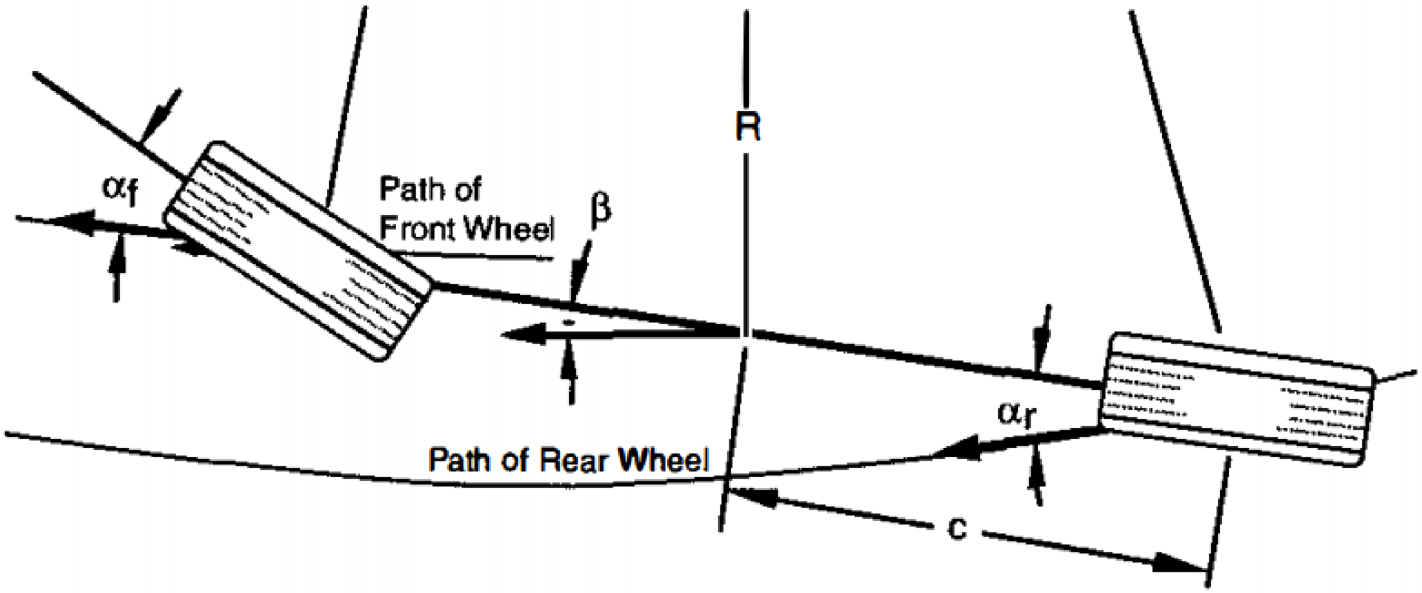
HIGH-SPEED CORNERING

- Sideslip Angle
  - At low speed turn



High-Speed Cornering

- Sideslip Angle
  - At high speed turn



HIGH-SPEED CORNERING

- Sideslip Angle
  - For any speed the sideslip angle, β at the CG will be:

$$\beta = 57.3\ c/R - \alpha_r$$
$$= 57.3\ c/R - W_r V^2/(C_{\alpha r}\ g\ R)$$

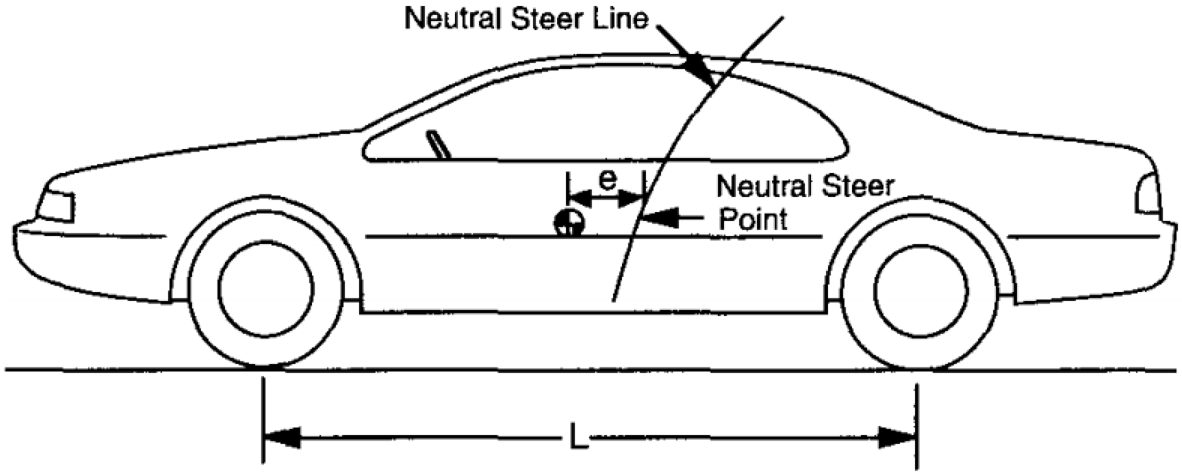
  - The speed at which the sideslip angle becomes zero:

$$V_{\beta=0} = \sqrt{57.3\ g\ c\ C_{\alpha r}/W_r}$$

    - ✓ It is independent of the radius of turn.

High-Speed Cornering

- Static Margin
  - A term often used in discussions of handling is the static margin and, like understeer coefficient or characteristic speed, provides a measure of the steady state handling behavior.
  - Static margin is determined by the point on the vehicle where a side force will produce no steady-state yaw velocity (i.e., the neutral steer point).



High-Speed Cornering

- Static Margin
  - The static margin is defined as the distance the neutral steer point falls behind the CG, normalized by the wheelbase:

$$\text{Static Margin} = e/L$$

- ✓ Behind CG → Static margin is positive → Understeer
- ✓ At the CG → Static margin is zero → Neutral steer
- ✓ Ahead of CG → Static margin is negative → Oversteer.

  - On typical vehicles the static margin falls in the range of 0.05 to 0.07 behind the CG.

Suspension Effects on Cornering

- The analysis of turning has shown that the behavior is dependent on the ratios of load/cornering coefficient on the front and rear axles.

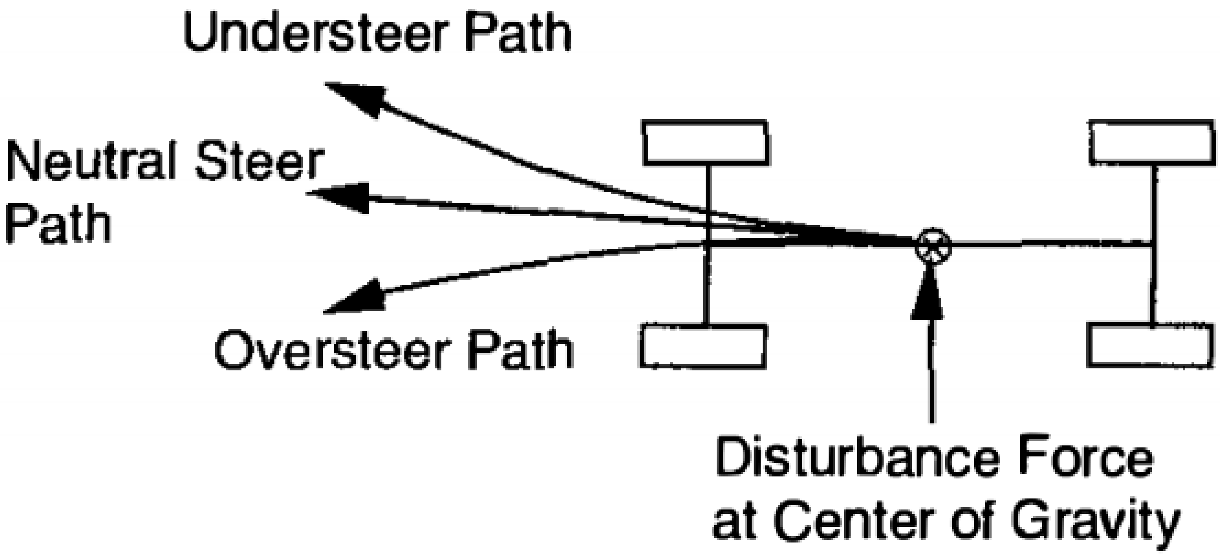
$$K = \left(\frac{W_f}{C_{\alpha f}} - \frac{W_r}{C_{\alpha r}}\right)$$

- The ratios have the engineering units of deg/g, and have been called the "cornering compliance“.

- The lateral force in a turn is actually a "D' Alembert" force at the CG, it is distributed at the axles in exact proportions to the weight (as the gravitational force is distributed).

SUSPENSION EFFECTS ON CORNERING

- Although the understeer gradient was derived for the case of a vehicle in a turn, it can be shown that the gradient determines vehicle response to disturbances in straight-ahead driving



Suspension Effects on Cornering

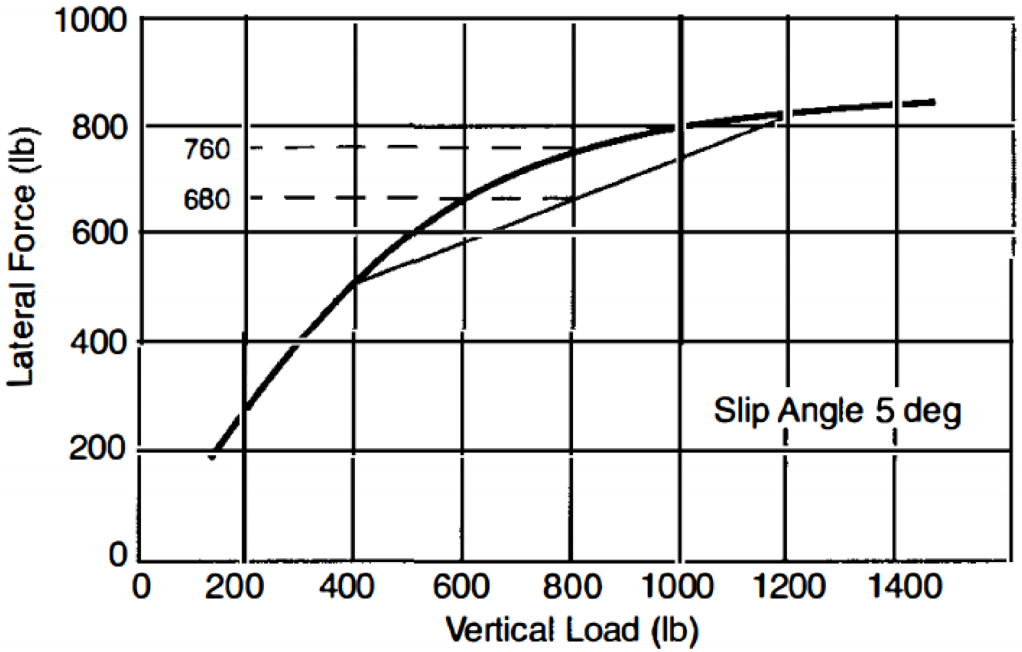
- Although tire cornering stiffness was used as the basis for developing the equations for understeer/oversteer, there are multiple factors in vehicle design that may influence the cornering forces developed in the presence of a lateral acceleration.

- Any design factor that influences the cornering force developed at a wheel will have a direct effect on directional response.

- The suspensions and steering system are the primary sources of these influences.

Suspension Effects on Cornering

- Roll Moment Distribution
  - For virtually all pneumatic tires the cornering forces are dependent on, and nonlinear with, load.
  - This is important because load is transferred in the lateral direction in cornering due to the elevation of the vehicle CG above the ground plane.



Suspension Effects on Cornering

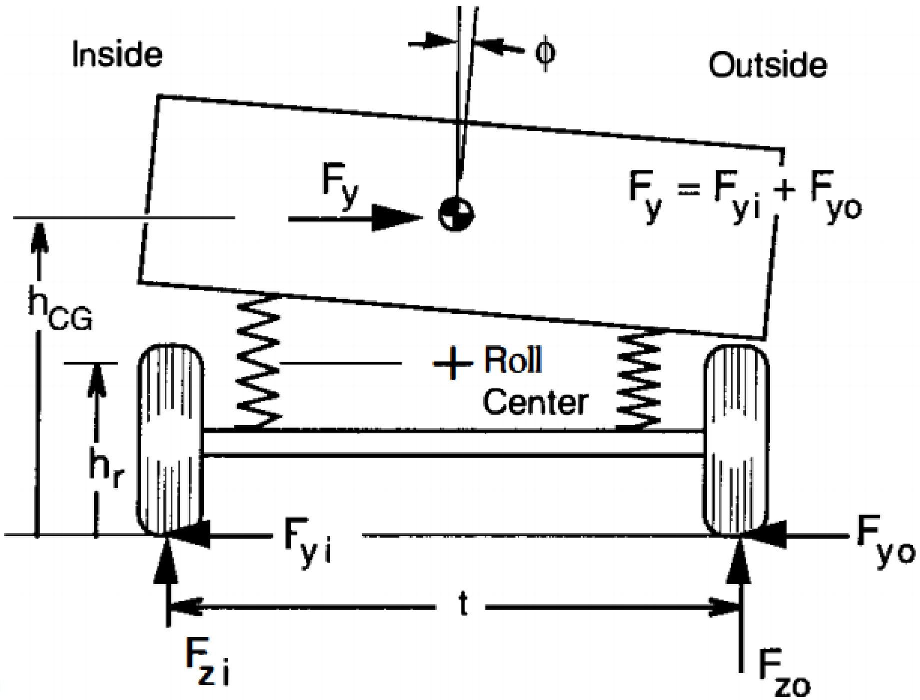
- Roll Moment Distribution
  - For a vehicle at 800 lb load on each wheel, about 760 lb of lateral force will be developed by each wheel at the 5-degree slip angle.
  - In hard cornering, the loads might typically change to 400 lb on the inside wheel and 1200 lb on the outside.
  - Then the average lateral force from both tires will be reduced to about 680 lb.
  - Consequently, the tires will have to assume a greater slip angle to maintain the lateral force necessary for the turn.
  - Actually, this mechanism is at work on both axles of all vehicles.
  - Whether it contributes to understeer or oversteer depends on the balance of roll moments distributed on the front and rear axles.

Suspension Effects on Cornering

- Roll Moment Distribution
  - More roll moment on the front axle contributes to understeer, whereas more roll moment on the rear axle contributes to oversteer.
  - Auxiliary roll stiffeners (stabilizer bars) alter handling performance primarily through this mechanism (applied to the front axle for understeer, and to the rear for oversteer).

SUSPENSION EFFECTS ON CORNERING

- Roll Moment Distribution
  - The roll moment



$$K_\phi = 0.5\, K_s\, s^2$$

K_ϕ = Roll stiffness of the suspension
K_s = Vertical rate of each of the left and right springs
s = Lateral separation between the springs

Suspension Effects on Cornering

- Roll Moment Distribution
  - The suspension is further characterized by a "roll center," the point at which the lateral forces are transferred from the axle to the sprung mass.
  - The roll center can also be thought of as the point on the body at which a lateral force application will produce no roll angle.
  - It is also the point around which the axle rolls when subjected to a pure roll moment.

Suspension Effects on Cornering

- Roll Moment Distribution
  - By writing Newton's Second Law for moments on the axle, we can determine the relationship between wheel loads and the lateral force and roll angle.

$$F_{zo} - F_{zi} = 2\,F_y\,h_r/t + 2\,K_\phi\,\phi/t = 2\,\Delta F_z$$

F_{zo} = Load on the outside wheel in the turn
F_{zi} = Load on the inside wheel in the turn
F_y = Lateral force = $F_{yi} + F_{yo}$
h_r = Roll center height
t = Tread (track width)
K_ϕ = Roll stiffness of the suspension
ϕ = Roll angle of the body

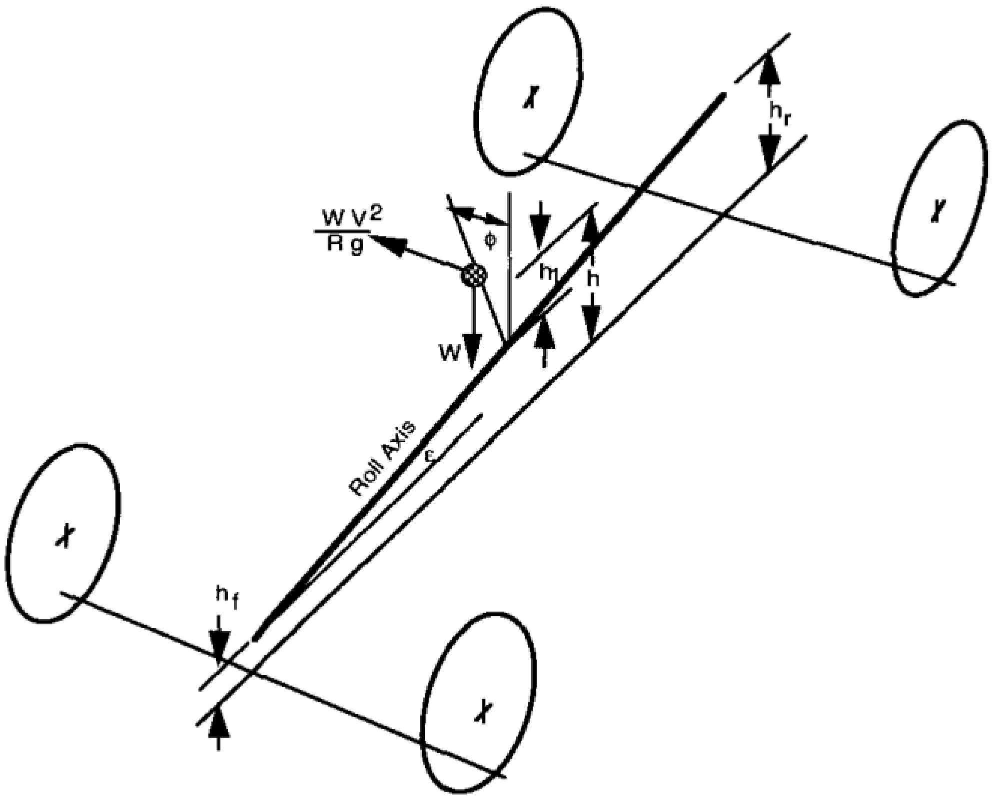
Suspension Effects on Cornering

- Roll Moment Distribution
  - Lateral load transfer arises from two mechanisms:
    - ✓ 1) Lateral load transfer due to cornering forces
    - ✓ 2) Lateral load transfer due to vehicle roll

$$\rightarrow \quad F_{zo} - F_{zi} = 2\, F_y\, h_r/t + 2\, K_\phi\, \phi/t = 2\, \Delta F_z$$

Suspension Effects on Cornering

- Roll Moment Distribution
  - The total vehicle must be considered to obtain the expression for the roll moment distribution on the front and rear axles.



SUSPENSION EFFECTS ON CORNERING

- Roll Moment Distribution
  - In this case, we define a roll axis as the line connecting the roll centers of the front and rear suspensions.
  - The moment about the roll axis:

$$M_\phi = [W\ h_1 \sin\phi + W\ V^2/(R\ g)\ h_1 \cos\phi] \cos\varepsilon$$

  - For small angles:

$$\rightarrow \quad M_\phi = W\ h_1\ [V^2/(R\ g) + \phi]$$

SUSPENSION EFFECTS ON CORNERING

- Roll Moment Distribution
  - Writing for front and rear axes: $M_\phi = M_{\phi f} + M_{\phi r} = (K_{\phi f} + K_{\phi r})\,\phi$
  - So: → $$\phi = \frac{W\,h_1\,V^2/(R\,g)}{K_{\phi f} + K_{\phi r} - W\,h_1}$$
  - The derivative of this expression with respect to the lateral acceleration produces an expression for the roll rate of the vehicle:

→ $$R_\phi = d\phi/da_y = W\,h_1/[K_{\phi f} + K_{\phi r} - W\,h_1]$$

✓ The roll rate is usually in the range of 3 to 7 degrees/g on typical passenger cars.

Suspension Effects on Cornering

- Roll Moment Distribution
  - Combining the expressions allows solution for the roll moments on the front and rear axles:

$$M'_{\phi f} = K_{\phi f} \frac{W h_1 V^2/(R g)}{K_{\phi f} + K_{\phi r} - W h_1} + W_f h_f V^2/(R g) = \Delta F_{zf} t_f$$

$$M'_{\phi r} = K_{\phi r} \frac{W h_1 V^2/(R g)}{K_{\phi f} + K_{\phi r} - W h_1} + W_r h_r V^2/(R g) = \Delta F_{zr} t_r$$

$$\Delta F_{zf} = F_{zfo} - W_f/2 = -(F_{zfi} - W_f/2)$$
$$\Delta F_{zr} = F_{zro} - W_r/2 = -(F_{zri} - W_r/2)$$

Suspension Effects on Cornering

- Roll Moment Distribution
  - In general, the roll moment distribution on vehicles tends to be biased toward the front wheels due to a number of factors:
    - ✓ 1) Relative to load, the front spring rate is usually slightly lower than that at the rear (for flat ride)
    - ✓ 2) Designers usually strive for higher front roll stiffness to ensure understeer in the limit of cornering.
    - ✓ 3) Stabilizer bars are often used on the front axle to obtain higher front roll stiffness.
  - The difference in load between the left and right wheels on the axle can be calculated…

SUSPENSION EFFECTS ON CORNERING

- Roll Moment Distribution
  - The steer angle necessary to maintain a turn: $\delta = 57.3\, L/R + \alpha_f - \alpha_r$

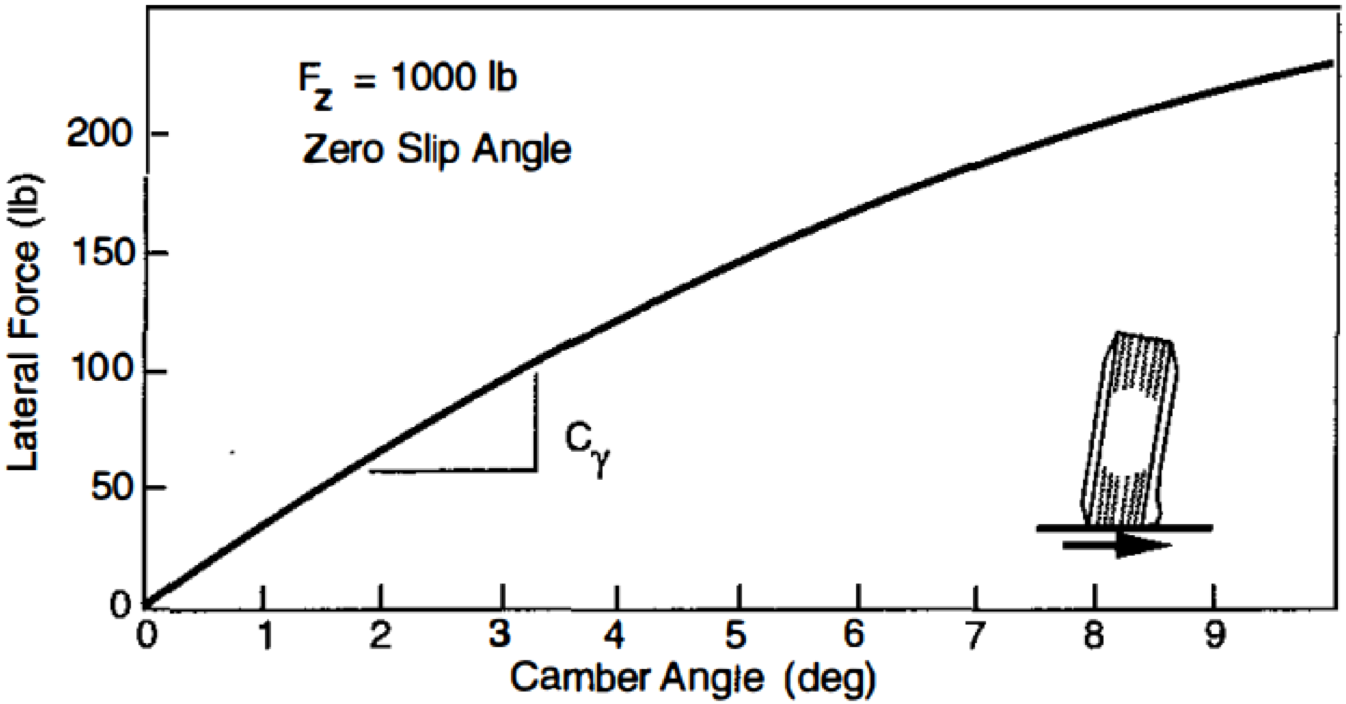
$$\delta = 57.3 \frac{L}{R} + \left[\left(\frac{W_f}{C_{\alpha f}} - \frac{W_r}{C_{\alpha r}} \right) + \left(\frac{W_f}{C_{\alpha f}} \frac{2\, b\, \Delta F_{zf}^2}{C_{\alpha f}} - \frac{W_r}{C_{\alpha r}} \frac{2\, b\, \Delta F_{zr}^2}{C_{\alpha r}} \right) \right] \frac{V^2}{R\, g}$$

    - ✓ The first term is simply the understeer gradient arising from the nominal cornering stiffness of the tires.
    - ✓ The second term represents the understeer gradient arising from lateral load transfer.

  - Since all the variables in the above equation are positive, the contribution from the front axle is always understeer; that from the rear axle is always negative, meaning it is an oversteer effect.

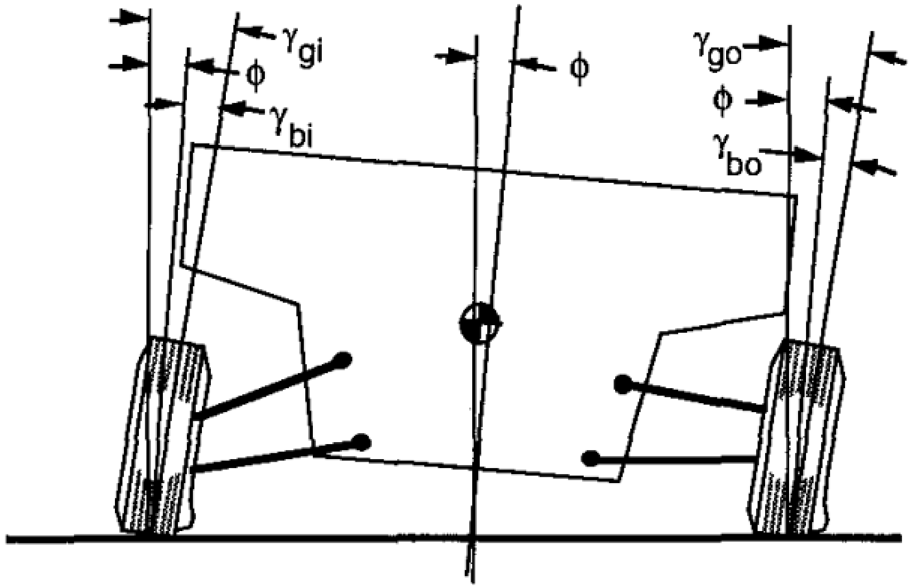
Suspension Effects on Cornering

- Camber Change
  - The inclination of a wheel outward from the body is known as the camber angle.
  - Camber on a wheel will produce a lateral force known as "camber thrust“.
  - Camber angle produces much less lateral force than slip angle.



Suspension Effects on Cornering

- Camber Change

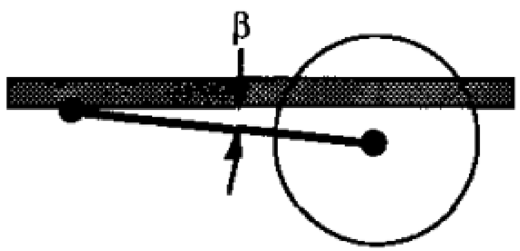


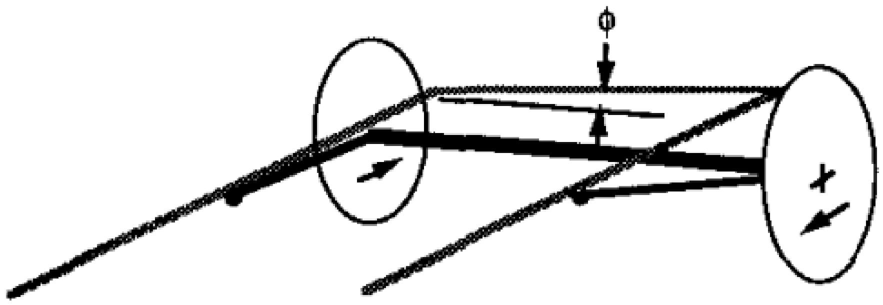
$$\delta = 57.3\frac{L}{R} + \left[\left(\frac{W_f}{C_{\alpha f}} - \frac{W_r}{C_{\alpha r}}\right) + \left(\frac{C_{\gamma f}}{C_{\alpha f}}\frac{\partial \gamma_f}{\partial \phi} - \frac{C_{\gamma r}}{C_{\alpha r}}\frac{\partial \gamma_r}{\partial \phi}\right)\frac{\partial \phi}{\partial a_y}\right]\frac{V^2}{R\,g}$$

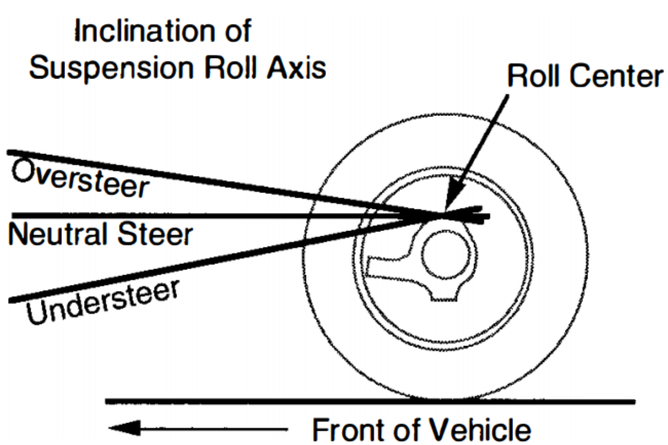
$$\longrightarrow \quad K_{camber} = \left(\frac{C_{\gamma f}}{C_{\alpha f}}\frac{\partial \gamma_f}{\partial \phi} - \frac{C_{\gamma r}}{C_{\alpha r}}\frac{\partial \gamma_r}{\partial \phi}\right)\frac{\partial \phi}{\partial a_y}$$

Suspension Effects on Cornering

- Roll Steer
  - When a vehicle rolls in cornering, the suspension kinematics may be such that the wheels steer.

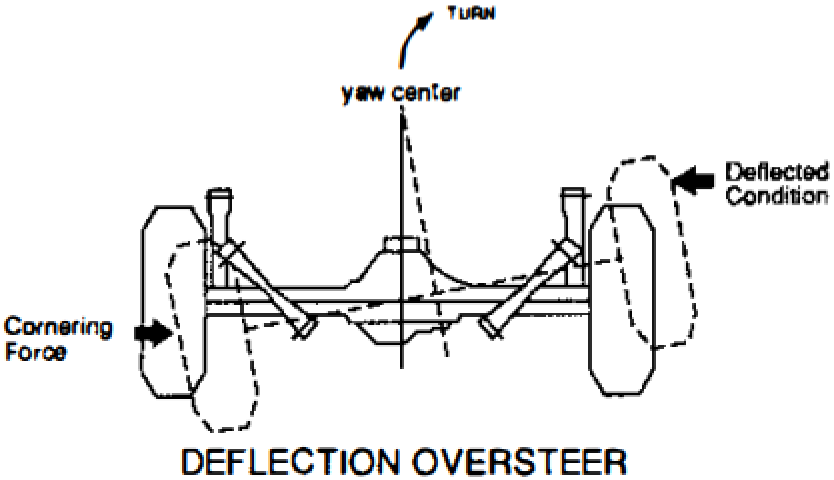


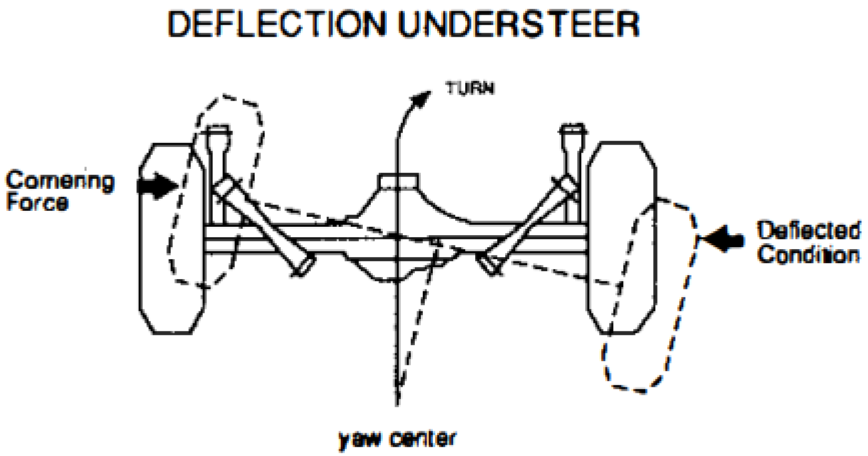




Suspension Effects on Cornering

- Lateral Force Compliance Steer
  - With the soft bushings used in suspension linkages, there is the possibility of steer arising from lateral compliance in the suspension.





Suspension Effects on Cornering

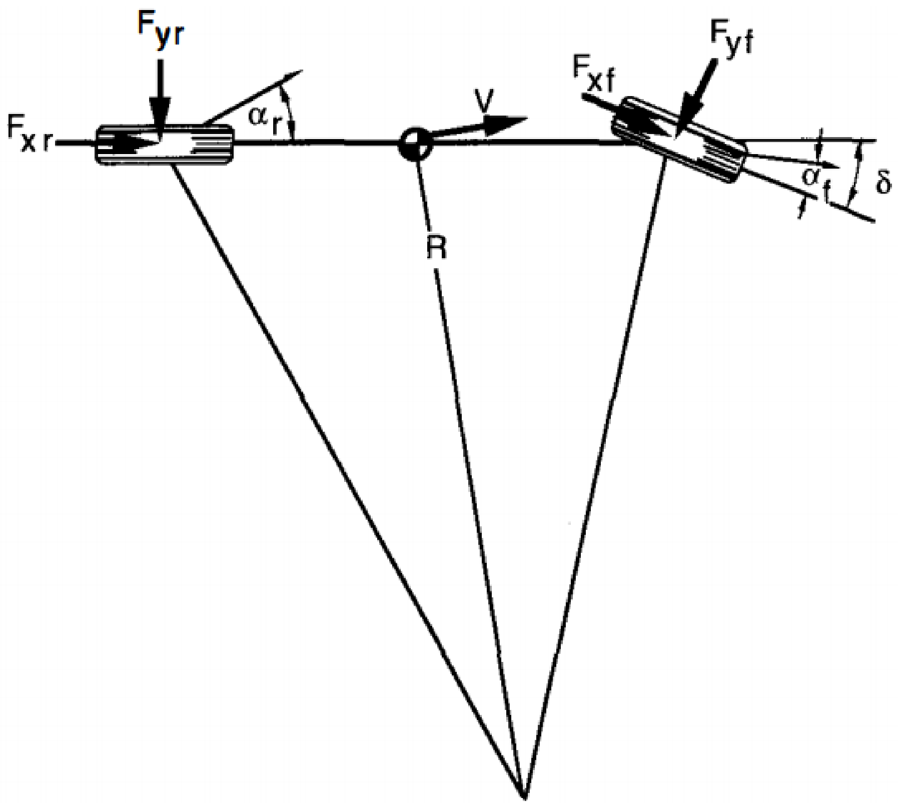
- Aligning Torque
  - The aligning torque experienced by the tires on a vehicle always resists the attempted tum, thus it is the source of an understeer effect.
  - Aligning torque is the manifestation of the fact that the lateral forces are developed by a tire at a point behind the tire center.
  - By deriving the turning equations with the assumption that the lateral forces are developed not at the wheels, but at a distance "p" behind each wheel, the understeer term obtained is as:

$$\longrightarrow K_{at} = W \frac{p}{L} \frac{C_{\alpha f} + C_{\alpha r}}{C_{\alpha f} C_{\alpha r}}$$

    - ✓ Because the Cα values are positive, the aligning torque effect is positive (understeer) and cannot ever be negative (oversteer).

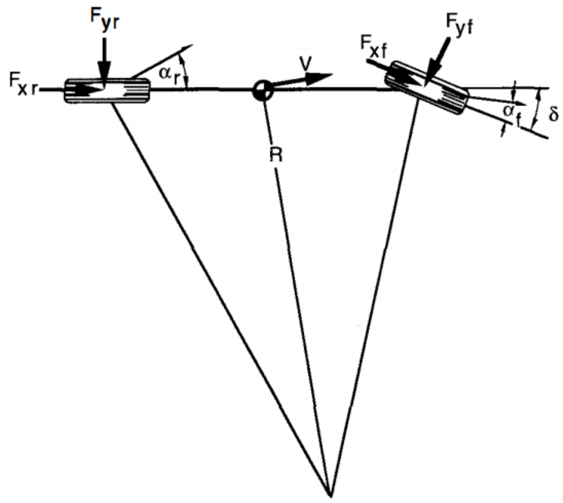
SUSPENSION EFFECTS ON CORNERING

- Effect of Tractive Forces on Cornering
  - Consider the case that drive forces present at front and rear wheels.



Suspension Effects on Cornering

- Effect of Tractive Forces on Cornering
  - The bicycle model for turning:



$$W_f V^2/(R g) = F_{yf} \cos(\alpha_f + \delta) + F_{xf} \sin(\alpha_f + \delta)$$

$$W_r V^2/(R g) = F_{yr} \cos \alpha_r + F_{xr} \sin \alpha_r$$

W_f, W_r = Load on the front and rear axles
V = Forward speed
R = Radius of turn
F_{yf}, F_{yr} = Cornering forces on front and rear axles
F_{xf}, F_{xr} = Tractive forces on the front and rear axles
α_f, α_r = Slip angles at front and rear wheels

Summary of Understeer Effects

- Effect of Tractive Forces on Cornering

$$\delta = \frac{57.3\ L/R}{1 + F_{xf}/C_{\alpha f}} + \left[\left(\frac{W_f}{C_{\alpha f}} - \frac{W_r}{C_{\alpha r}}\right) - \left(\frac{W_f}{C_{\alpha f}}\frac{F_{xf}}{C_{\alpha f}} - \frac{W_r}{C_{\alpha r}}\frac{F_{xr}}{C_{\alpha r}}\right)\right]\frac{V^2}{R\,g}$$

- 1) Ackerman steer angle altered by the tractive force on the front axle.
- 2) Understeer gradient
- 3) Effect of tractive forces on the understeer behavior of the vehicle

Summary of Understeer Effects

UNDERSTEER COMPONENT	SOURCE
$K_{tires} = \frac{W_f}{C_{\alpha f}} - \frac{W_r}{C_{\alpha r}}$	Tire cornering stiffness
$K_{camber} = \left(\frac{C_{\gamma f}}{C_{\alpha f}} \frac{\partial \gamma_f}{\partial \phi} - \frac{C_{\gamma r}}{C_{\alpha r}} \frac{\partial \gamma_r}{\partial \phi}\right) \frac{\partial \phi}{\partial a_y}$	Camber thrust
$K_{roll\ steer} = (\varepsilon_f - \varepsilon_r)\, d\phi/da_y$	Roll steer
$K_{lfcs} = A_f W_f - A_r W_r$	Lateral force compliance steer
$K_{at} = W \frac{p}{L} \frac{C_{\alpha f} + C_{\alpha r}}{C_{\alpha f} C_{\alpha r}}$	Aligning torque
$K_{llt} = \frac{W_f}{C_{\alpha f}} \frac{2b\, \Delta F_{zf}^2}{C_{\alpha f}} - \frac{W_r}{C_{\alpha r}} \frac{2b\, \Delta F_{zr}^2}{C_{\alpha r}}$	Lateral load transfer
$K_{strg} = W_f \frac{r\nu + p}{K_{ss}}$	Steering system

Experimental Measurement of Understeer Gradient

- Understeer gradient is defined by the SAE as:
  - "The quantity obtained by subtracting the Ackerman steer angle gradient from the ratio of the steering wheel angle gradient to the overall steering ratio”

$$\rightarrow \quad \delta = 57.3\, L/R + K\, a_y$$

- Experimental Measurement
  - Constant Radius Method
  - Constant Speed Method