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درس دینامیک

Engineering Mechanics Dynamics

Meriam, Kraige & Bolton

9th Edition

Chapter 4: **Kinetics of Systems of Particles**

❑ Contents:

CHAPTER **4**

Kinetics of Systems of Particles


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The forces of interaction between the rotating blades of this jet engine and the air which passes over them is a subject which is introduced in this chapter.

CHAPTER OUTLINE

4/1 Introduction

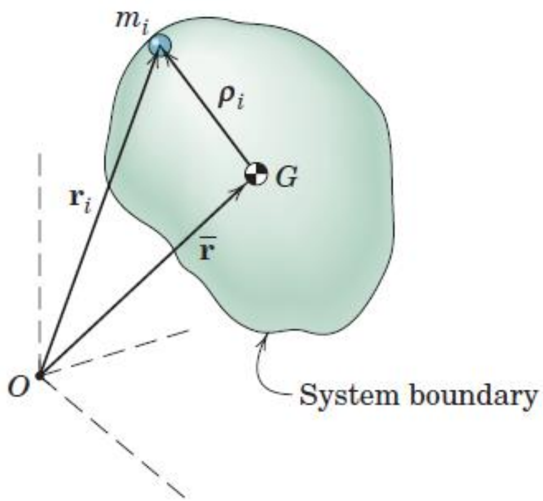
- In the previous two chapters, we have applied the principles of dynamics to the motion of a particle.

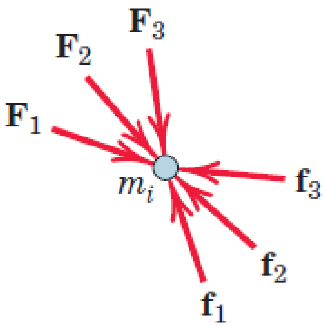
- Our next major step in the development of dynamics is to extend these principles, which we applied to a single particle, to describe the motion of a general system of particles.

- Recall that a rigid body is a solid system of particles wherein the distances between particles remain essentially unchanged.

4/2 Generalized Newton's Second Law

- Considering n mass particles bounded by a closed surface in space





- Forces F1, F2, F3, . . . acting on m_i from sources external to the envelop
- Forces f1, f2, f3, . . . acting on m_i from sources internal to the system boundary

4/2 Generalized Newton's Second Law

- The center of mass G of the isolated system of particles

$$m\bar{\mathbf{r}} = \Sigma m_i \mathbf{r}_i \qquad m = \Sigma m_i$$

- Newton's second law when applied to m_i gives:

$$\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \cdots + \mathbf{f}_1 + \mathbf{f}_2 + \mathbf{f}_3 + \cdots = m_i \ddot{\mathbf{r}}_i$$

$$\rightarrow \quad \Sigma\mathbf{F} + \Sigma\mathbf{f} = \Sigma m_i \ddot{\mathbf{r}}_i$$

- Substitution into the summation of the equations of motion gives:

$$\Sigma\mathbf{F} = m\ddot{\bar{\mathbf{r}}} \quad \text{or} \quad \mathbf{F} = m\bar{\mathbf{a}}$$

$$\Sigma F_x = m\bar{a}_x \qquad \Sigma F_y = m\bar{a}_y \qquad \Sigma F_z = m\bar{a}_z$$

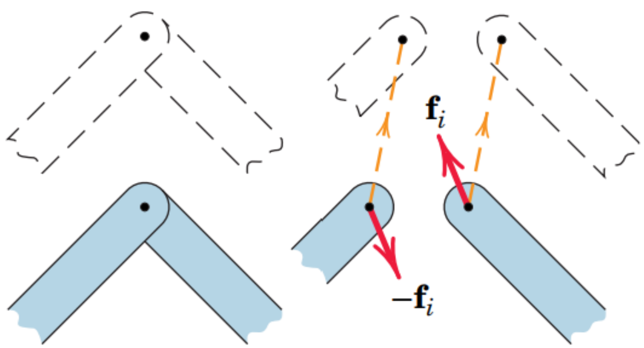
4/3 Work-Energy

Work-Energy Relation

$$(U_{1\text{-}2})_i = \Delta T_i \quad \rightarrow \quad U_{1\text{-}2} = \Delta T \quad \text{or} \quad T_1 + U_{1\text{-}2} = T_2$$

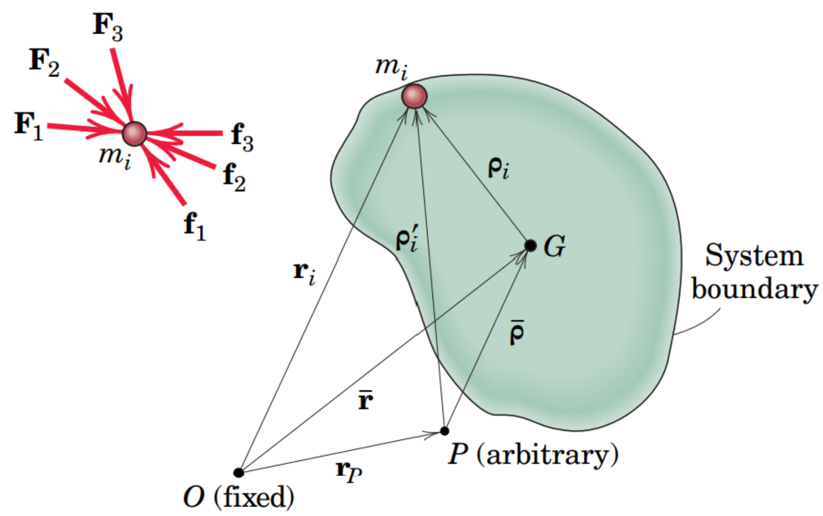
$$U'_{1\text{-}2} = \Delta T + \Delta V \qquad T_1 + V_1 + U'_{1\text{-}2} = T_2 + V_2$$

- For a rigid body or a system of rigid bodies joined by ideal frictionless connections, no net work is done by the internal interacting forces or moments in the connections.



4/3 Work-Energy

Kinetic Energy Expression



$$T = \Sigma \frac{1}{2} m_i v_i^2$$

$$\mathbf{v}_i = \bar{\mathbf{v}} + \dot{\boldsymbol{\rho}}_i$$

$$T = \Sigma \frac{1}{2} m_i \mathbf{v}_i \cdot \mathbf{v}_i = \Sigma \frac{1}{2} m_i (\bar{\mathbf{v}} + \dot{\boldsymbol{\rho}}_i) \cdot (\bar{\mathbf{v}} + \dot{\boldsymbol{\rho}}_i)$$
$$= \Sigma \frac{1}{2} m_i \bar{v}^2 + \Sigma \frac{1}{2} m_i |\dot{\boldsymbol{\rho}}_i|^2 + \Sigma m_i \bar{\mathbf{v}} \cdot \dot{\boldsymbol{\rho}}_i$$

Because $\boldsymbol{\rho}_i$ is measured from the mass center, $\Sigma m_i \boldsymbol{\rho}_i = \mathbf{0}$

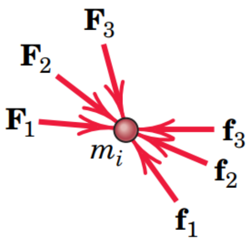
$$T = \frac{1}{2} m \bar{v}^2 + \Sigma \frac{1}{2} m_i |\dot{\boldsymbol{\rho}}_i|^2$$

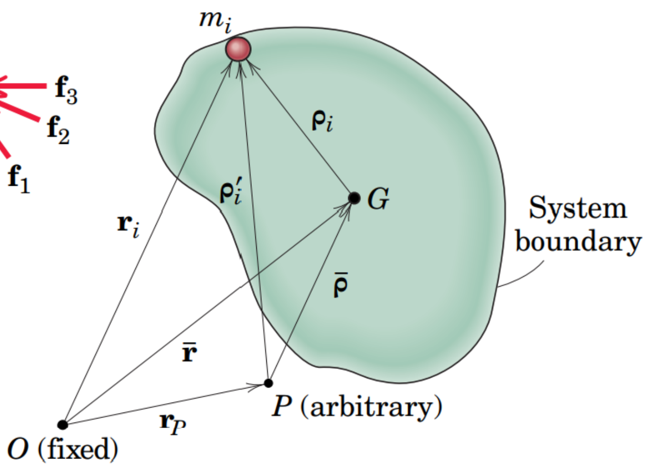
4/4 Impulse-Momentum

Linear Momentum

$$\mathbf{G} = \Sigma m_i(\bar{\mathbf{v}} + \dot{\boldsymbol{\rho}}_i) = \Sigma m_i \bar{\mathbf{v}} + \frac{d}{dt}\Sigma m_i \boldsymbol{\rho}_i$$

$$= \bar{\mathbf{v}}\,\Sigma m_i + \frac{d}{dt}(\mathbf{0})$$





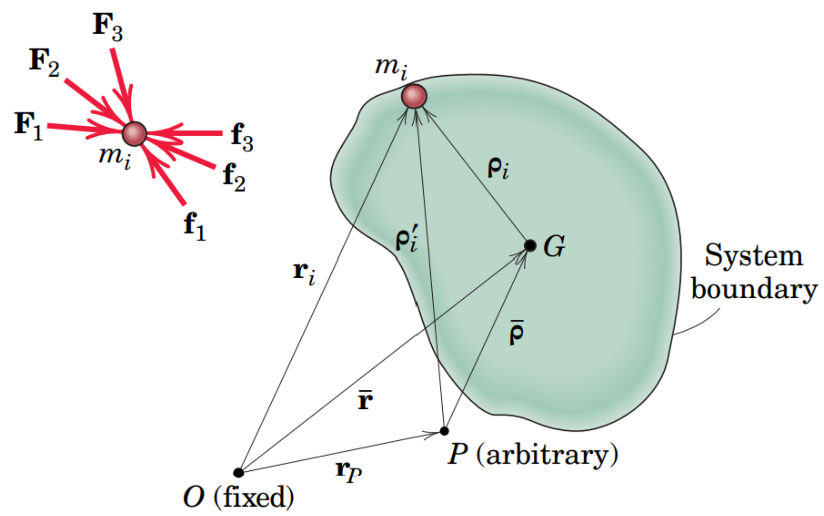
$$\mathbf{G} = m\bar{\mathbf{v}}$$

$$\Sigma\mathbf{F} = \dot{\mathbf{G}}$$

4/4 Impulse-Momentum

Angular Momentum

About a Fixed Point O.



$$\mathbf{H}_O = \Sigma(\mathbf{r}_i \times m_i\mathbf{v}_i)$$

$$\dot{\mathbf{H}}_O = \Sigma(\dot{\mathbf{r}}_i \times m_i\mathbf{v}_i) + \Sigma(\mathbf{r}_i \times m_i\dot{\mathbf{v}}_i)$$

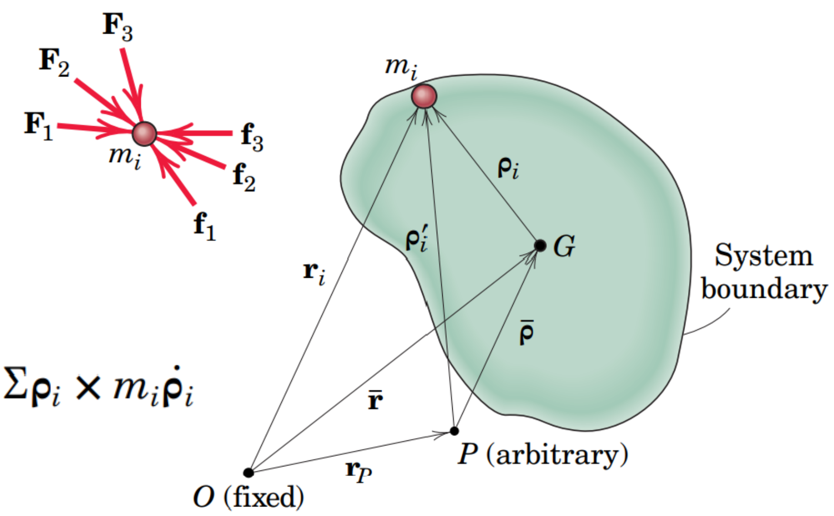
$$\Sigma(\mathbf{r}_i \times m_i\mathbf{a}_i) = \Sigma(\mathbf{r}_i \times \mathbf{F}_i)$$

$$\Sigma\mathbf{M}_O = \dot{\mathbf{H}}_O$$

4/4 Impulse-Momentum

Angular Momentum

About the Mass Center G.



$$\mathbf{H}_G = \Sigma \boldsymbol{\rho}_i \times m_i \dot{\mathbf{r}}_i$$

$$\mathbf{H}_G = \Sigma \boldsymbol{\rho}_i \times m_i(\dot{\bar{\mathbf{r}}} + \dot{\boldsymbol{\rho}}_i) = \Sigma \boldsymbol{\rho}_i \times m_i \dot{\bar{\mathbf{r}}} + \Sigma \boldsymbol{\rho}_i \times m_i \dot{\boldsymbol{\rho}}_i$$

↓

$$-\dot{\bar{\mathbf{r}}} \times \Sigma m_i \boldsymbol{\rho}_i$$

→ $$\mathbf{H}_G = \Sigma \boldsymbol{\rho}_i \times m_i \dot{\boldsymbol{\rho}}_i$$

$$\dot{\mathbf{H}}_G = \Sigma \dot{\boldsymbol{\rho}}_i \times m_i(\dot{\bar{\mathbf{r}}} + \dot{\boldsymbol{\rho}}_i) + \Sigma \boldsymbol{\rho}_i \times m_i \ddot{\mathbf{r}}_i$$

→ $$\boxed{\Sigma \mathbf{M}_G = \dot{\mathbf{H}}_G}$$

4/4 Impulse-Momentum

Angular Momentum

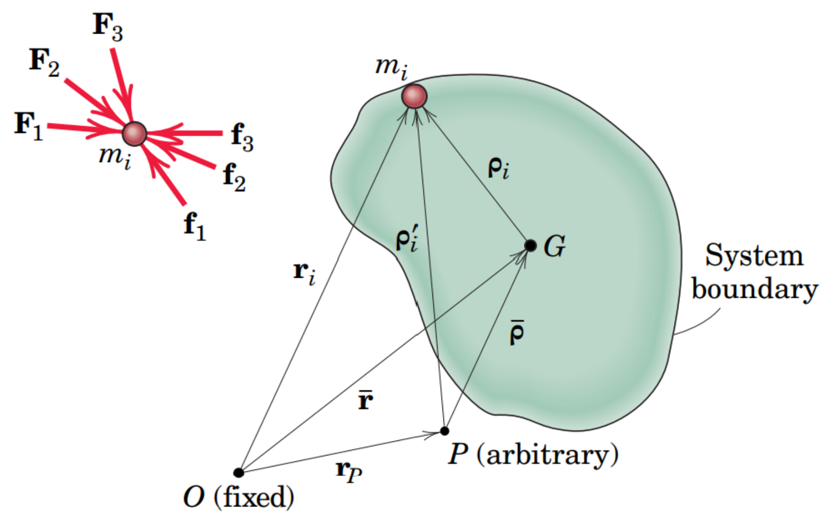
About an Arbitrary Point P.

$$\mathbf{H}_P = \Sigma\boldsymbol{\rho}'_i \times m_i\dot{\mathbf{r}}_i = \Sigma(\bar{\boldsymbol{\rho}} + \boldsymbol{\rho}_i) \times m_i\dot{\mathbf{r}}_i$$

$$\boxed{\mathbf{H}_P = \mathbf{H}_G + \bar{\boldsymbol{\rho}} \times m\bar{\mathbf{v}}}$$

$$\Sigma\mathbf{M}_P = \Sigma\mathbf{M}_G + \bar{\boldsymbol{\rho}} \times \Sigma\mathbf{F}$$

$$\boxed{\Sigma\mathbf{M}_P = \dot{\mathbf{H}}_G + \bar{\boldsymbol{\rho}} \times m\bar{\mathbf{a}}}$$



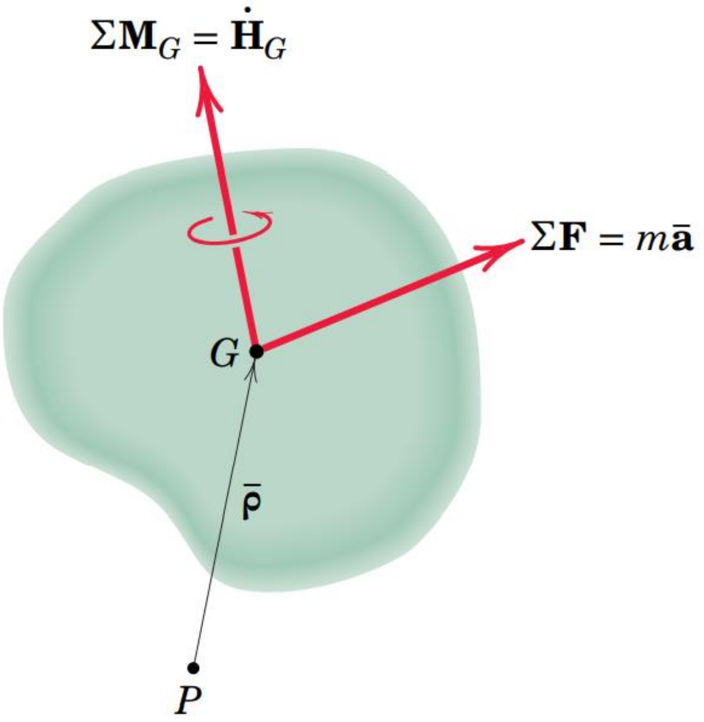
4/4 Impulse-Momentum

Angular Momentum

About an Arbitrary Point P.

$$\mathbf{H}_P = \mathbf{H}_G + \bar{\boldsymbol{\rho}} \times m\bar{\mathbf{v}}$$

$$\Sigma\mathbf{M}_P = \dot{\mathbf{H}}_G + \bar{\boldsymbol{\rho}} \times m\bar{\mathbf{a}}$$



4/5 Conservation of Energy and Momentum

Conservation of Energy

- A mass system is said to be conservative if it does not lose energy by virtue of internal friction forces which do negative work or by virtue of inelastic members which dissipate energy upon cycling.
- If no work is done on a conservative system during an interval of motion by external forces (other than gravity or other potential forces), then none of the energy of the system is lost.

$$\Delta T + \Delta V = 0$$

$$T_1 + V_1 = T_2 + V_2$$

4/5 Conservation of Energy and Momentum

Conservation of Momentum

- The principle of conservation of linear momentum

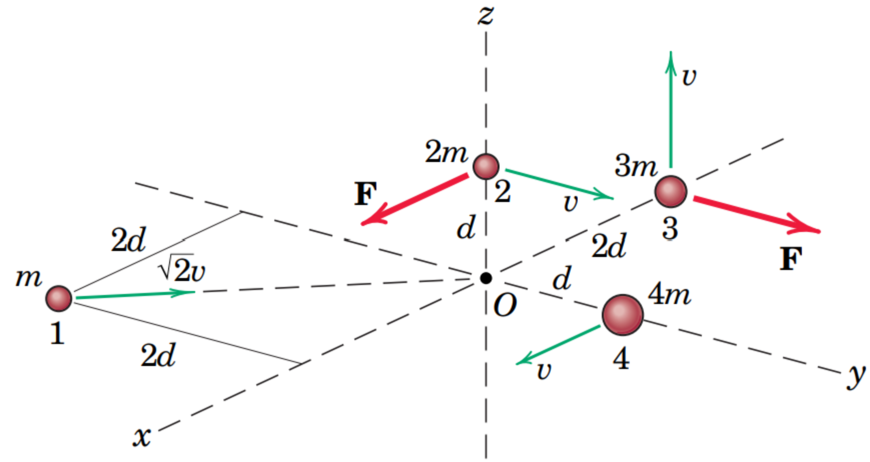
$$\mathbf{G}_1 = \mathbf{G}_2$$

- The principle of conservation of angular momentum

$$(\mathbf{H}_O)_1 = (\mathbf{H}_O)_2 \qquad \text{or} \qquad (\mathbf{H}_G)_1 = (\mathbf{H}_G)_2$$

SAMPLE PROBLEM 4/1

The system of four particles has the indicated particle masses, positions, velocities, and external forces. Determine $\bar{\mathbf{r}}$, $\dot{\bar{\mathbf{r}}}$, $\ddot{\bar{\mathbf{r}}}$, T, $\mathbf{G}$, $\mathbf{H}_O$, $\dot{\mathbf{H}}_O$, $\mathbf{H}_G$, and $\dot{\mathbf{H}}_G$.



$$\bar{\mathbf{r}} = \frac{\Sigma m_i \mathbf{r}_i}{\Sigma m_i} = \frac{m(2d\mathbf{i} - 2d\mathbf{j}) + 2m(d\mathbf{k}) + 3m(-2d\mathbf{i}) + 4m(d\mathbf{j})}{m + 2m + 3m + 4m}$$

$$\dot{\bar{\mathbf{r}}} = \frac{\Sigma m_i \dot{\mathbf{r}}_i}{\Sigma m_i} = \frac{m(-v\mathbf{i} + v\mathbf{j}) + 2m(v\mathbf{j}) + 3m(v\mathbf{k}) + 4m(v\mathbf{i})}{10m} = v(0.3\mathbf{i} + 0.3\mathbf{j} + 0.3\mathbf{k})$$

$$\ddot{\bar{\mathbf{r}}} = \frac{\Sigma \mathbf{F}}{\Sigma m_i} = \frac{F\mathbf{i} + F\mathbf{j}}{10m} = \frac{F}{10m}(\mathbf{i} + \mathbf{j})$$

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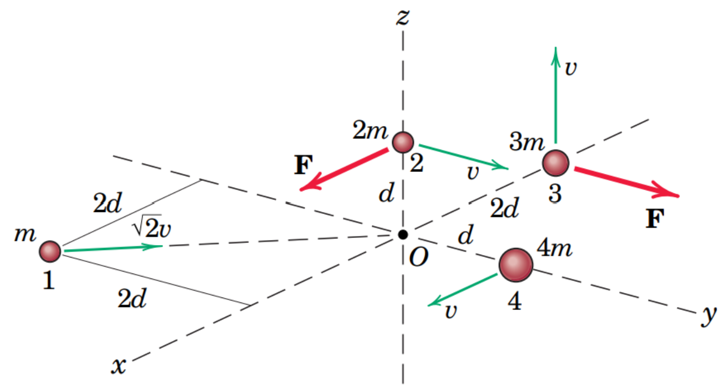


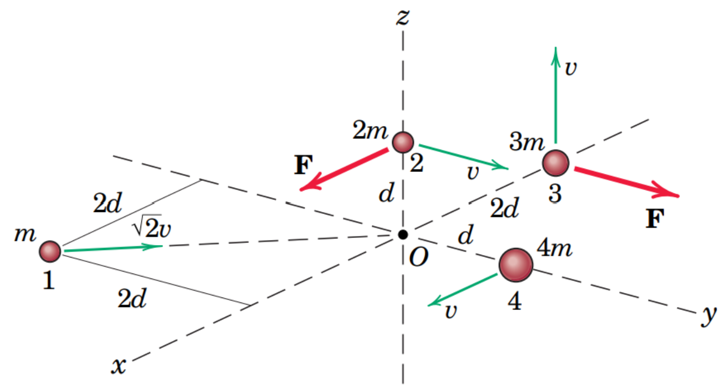
$$T = \Sigma \frac{1}{2} m_i {v_i}^2 = \frac{1}{2}\left[m(\sqrt{2}v)^2 + 2mv^2 + 3mv^2 + 4mv^2\right] = \frac{11}{2} mv^2$$

$$\mathbf{G} = (\Sigma m_i)\dot{\bar{\mathbf{r}}} = 10m(v)(0.3\mathbf{i} + 0.3\mathbf{j} + 0.3\mathbf{k}) = mv(3\mathbf{i} + 3\mathbf{j} + 3\mathbf{k})$$

$$\mathbf{H}_O = \Sigma \mathbf{r}_i \times m_i \dot{\mathbf{r}}_i = \mathbf{0} - 2mvd\mathbf{i} + 3mv(2d)\mathbf{j} - 4mvd\mathbf{k}$$

$$= mvd(-2\mathbf{i} + 6\mathbf{j} - 4\mathbf{k})$$

$$\dot{\mathbf{H}}_O = \Sigma \mathbf{M}_O = -2dF\mathbf{k} + Fd\mathbf{j} = Fd(\mathbf{j} - 2\mathbf{k})$$



$$[\mathbf{H}_G = \mathbf{H}_O + \bar{\boldsymbol{\rho}} \times m\bar{\mathbf{v}}]$$

$$\mathbf{H}_G = mvd(-2\mathbf{i} + 6\mathbf{j} - 4\mathbf{k}) - d(-0.4\mathbf{i} + 0.2\mathbf{j} + 0.2\mathbf{k}) \times$$

$$10mv(0.3\mathbf{i} + 0.3\mathbf{j} + 0.3\mathbf{k}) = mvd(-2\mathbf{i} + 4.2\mathbf{j} - 2.2\mathbf{k})$$

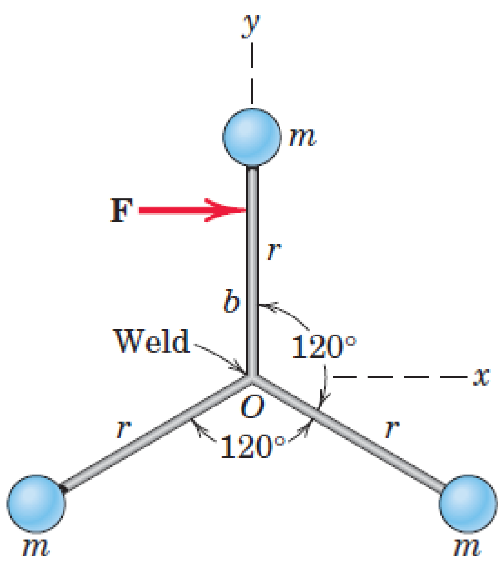
$$[\dot{\mathbf{H}}_G = \Sigma\mathbf{M}_O - \bar{\boldsymbol{\rho}} \times m\bar{\mathbf{a}}]$$

$$\dot{\mathbf{H}}_G = Fd(\mathbf{j} - 2\mathbf{k}) - d(-0.4\mathbf{i} + 0.2\mathbf{j} + 0.2\mathbf{k}) \times 10m\left(\frac{F}{10m}\right)(\mathbf{i} + \mathbf{j})$$

$$= Fd(0.2\mathbf{i} + 0.8\mathbf{j} - 1.4\mathbf{k})$$

SAMPLE PROBLEM 4/2

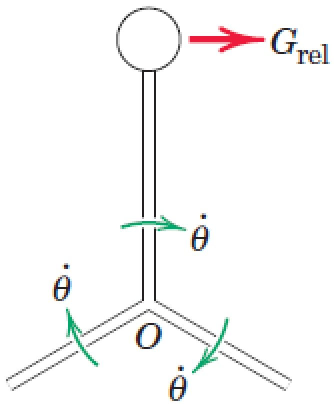
Each of the three balls has a mass m and is welded to the rigid equiangular frame of negligible mass. The assembly rests on a smooth horizontal surface. If a force **F** is suddenly applied to one bar as shown, determine (a) the acceleration of point O and (b) the angular acceleration $\ddot{\theta}$ of the frame.



$[\Sigma \mathbf{F} = m\bar{\mathbf{a}}]$ $\qquad F\mathbf{i} = 3m\bar{\mathbf{a}} \qquad \bar{\mathbf{a}} = \mathbf{a}_O = \dfrac{F}{3m}\mathbf{i}$

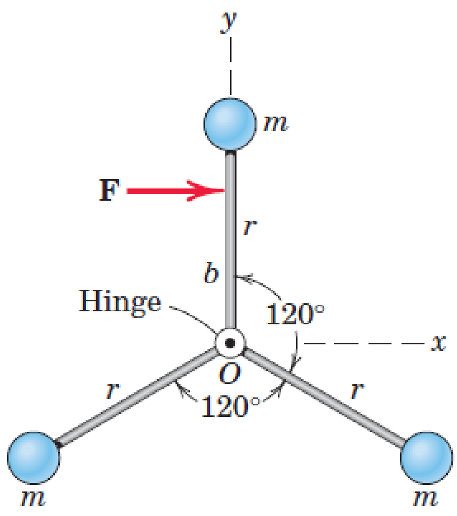
$H_O = H_G = 3(mr\dot{\theta})r = 3mr^2\dot{\theta}$

$[\Sigma \mathbf{M}_G = \dot{\mathbf{H}}_G] \qquad Fb = \dfrac{d}{dt}(3mr^2\dot{\theta}) = 3mr^2\ddot{\theta} \qquad \text{so} \qquad \ddot{\theta} = \dfrac{Fb}{3mr^2}$



SAMPLE PROBLEM 4/3

Consider the same conditions as for Sample Problem 4/2, except that the spokes are freely hinged at O and so do not constitute a rigid system. Explain the difference between the two problems.



Solution The generalized Newton's second law holds for any mass system, so that the acceleration $\bar{\mathbf{a}}$ of the mass center G is the same as with Sample Problem 4/2, namely,

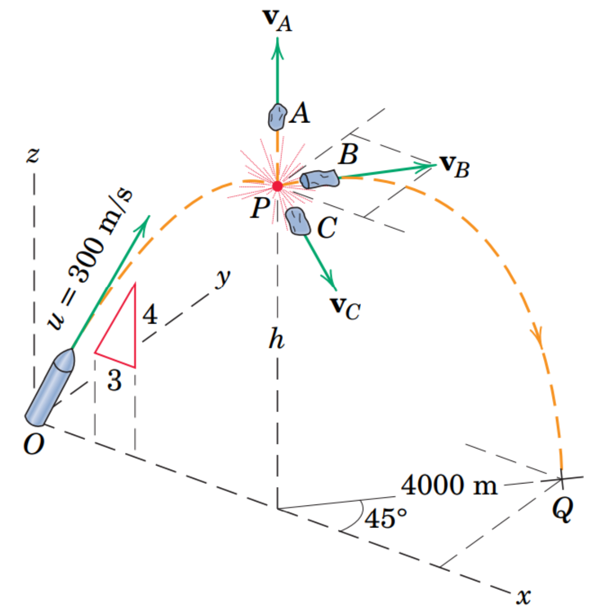
$$\bar{\mathbf{a}} = \frac{F}{3m}\mathbf{i}$$

Although G coincides with O at the instant represented, the motion of the hinge O is not the same as the motion of G since O will not remain the center of mass as the angles between the spokes change.

Both ΣM_G and $\dot{H}_G$ have the same values for the two problems at the instant represented. However, the angular motions of the spokes in this problem are all different and are not easily determined.

SAMPLE PROBLEM 4/4

A shell with a mass of 20 kg is fired from point O, with a velocity u = 300 m/s in the vertical x-z plane at the inclination shown. When it reaches the top of its trajectory at P, it explodes into three fragments A, B, and C. Immediately after the explosion, fragment A is observed to rise vertically a distance of 500 m above P, and fragment B is seen to have a horizontal velocity $\mathbf{v}_B$ and eventually lands at point Q. When recovered, the masses of the fragments A, B, and C are found to be 5, 9, and 6 kg, respectively. Calculate the velocity which fragment C has immediately after the explosion. Neglect atmospheric resistance.

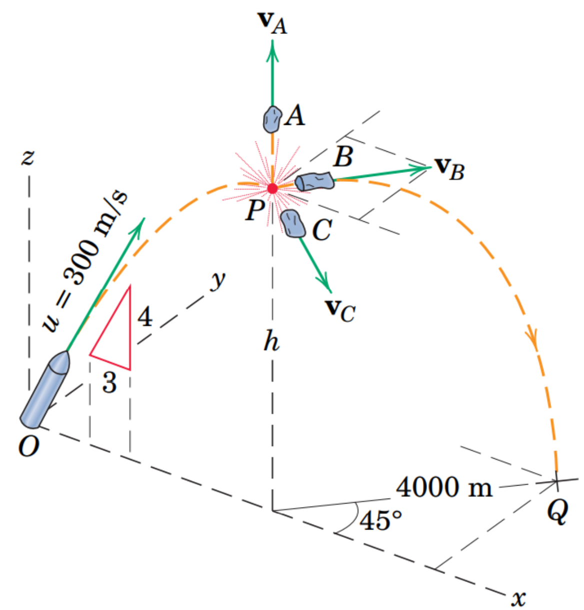


$$t = u_z/g = 300(4/5)/9.81 = 24.5 \text{ s}$$

$$h = \frac{u_z^{\,2}}{2g} = \frac{[(300)(4/5)]^2}{2(9.81)} = 2940 \text{ m}$$

$$v_A = \sqrt{2gh_A} = \sqrt{2(9.81)(500)} = 99.0 \text{ m/s}$$

$$v_B = s/t = 4000/24.5 = 163.5 \text{ m/s}$$



$$[\mathbf{G}_1 = \mathbf{G}_2] \qquad m\mathbf{v} = m_A\mathbf{v}_A + m_B\mathbf{v}_B + m_C\mathbf{v}_C$$

$$20(300)(\tfrac{3}{5})\mathbf{i} = 5(99.0\mathbf{k}) + 9(163.5)(\mathbf{i}\cos 45° + \mathbf{j}\sin 45°) + 6\mathbf{v}_C$$

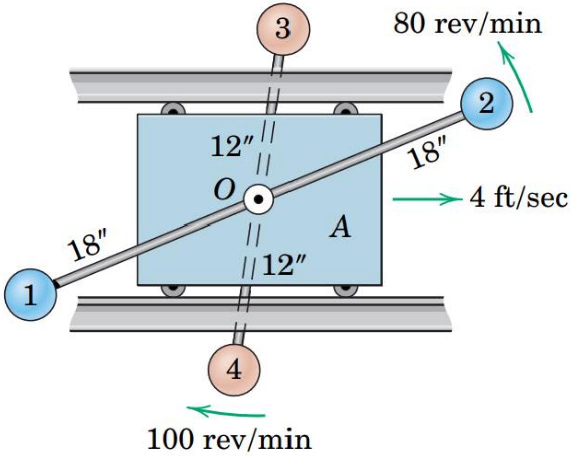
$$6\mathbf{v}_C = 2560\mathbf{i} - 1040\mathbf{j} - 495\mathbf{k}$$

$$\mathbf{v}_C = 427\mathbf{i} - 173.4\mathbf{j} - 82.5\mathbf{k} \text{ m/s}$$

$$v_C = \sqrt{(427)^2 + (173.4)^2 + (82.5)^2} = 468 \text{ m/s}$$

SAMPLE PROBLEM 4/5

The 32.2-lb carriage A moves horizontally in its guide with a speed of 4 ft/sec and carries two assemblies of balls and light rods which rotate about a shaft at O in the carriage. Each of the four balls weighs 3.22 lb. The assembly on the front face rotates counterclockwise at a speed of 80 rev/min, and the assembly on the back side rotates clockwise at a speed of 100 rev/min. For the entire system, calculate (*a*) the kinetic energy T, (*b*) the magnitude G of the linear momentum, and (*c*) the magnitude H_O of the angular momentum about point O.



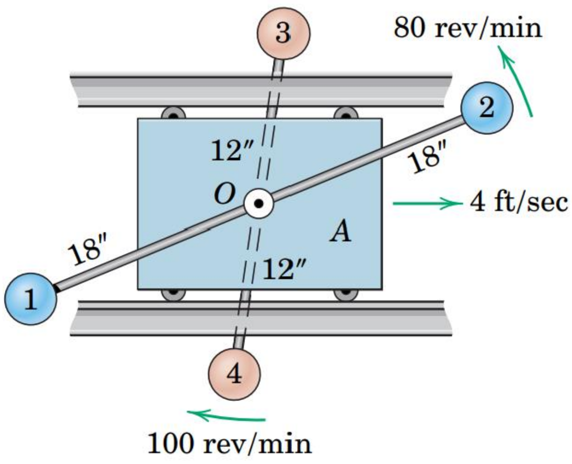
$$[|\dot{\rho}_i| = v_{rel} = r\dot{\theta}] \qquad (v_{rel})_{1,2} = \frac{18}{12}\frac{80(2\pi)}{60} = 12.57 \text{ ft/sec}$$

$$(v_{rel})_{3,4} = \frac{12}{12}\frac{100(2\pi)}{60} = 10.47 \text{ ft/sec}$$

$$\frac{1}{2}m\bar{v}^2 = \frac{1}{2}\left(\frac{32.2}{32.2} + 4\,\frac{3.22}{32.2}\right)(4^2) = 11.20 \text{ ft-lb}$$

$$\Sigma\frac{1}{2}m_i|\dot{\boldsymbol{\rho}}_i|^2 = 2\left[\frac{1}{2}\frac{3.22}{32.2}(12.57)^2\right]_{(1,2)} + 2\left[\frac{1}{2}\frac{3.22}{32.2}(10.47)^2\right]_{(3,4)}$$

$$= 15.80 + 10.96 = 26.8 \text{ ft-lb}$$



$$T = \frac{1}{2}m\bar{v}^2 + \Sigma\frac{1}{2}m_i|\dot{\boldsymbol{\rho}}_i|^2 = 11.20 + 26.8 = 38.0 \text{ ft-lb}$$

$[\mathbf{G} = m\bar{\mathbf{v}}]$ $$G = \left(\frac{32.2}{32.2} + 4\,\frac{3.22}{32.2}\right)(4) = 5.6 \text{ lb-sec}$$

$$H_O = \Sigma|\mathbf{r}_i \times m_i\mathbf{v}_i|$$

$$H_O = \left[2\left(\frac{3.22}{32.2}\right)\left(\frac{18}{12}\right)(12.57)\right]_{(1,2)} - \left[2\left(\frac{3.22}{32.2}\right)\left(\frac{12}{12}\right)(10.47)\right]_{(3,4)}$$

$$= 3.77 - 2.09 = 1.676 \text{ ft-lb-sec}$$

4/6 Steady Mass Flow

- The dynamics of mass flow is of great importance in the description of fluid machinery of all types including turbines, pumps, nozzles, air-breathing jet engines, and rockets.

- The treatment of mass flow in this article is not intended to take the place of a study of fluid mechanics, but merely to present the basic principles and equations of momentum.

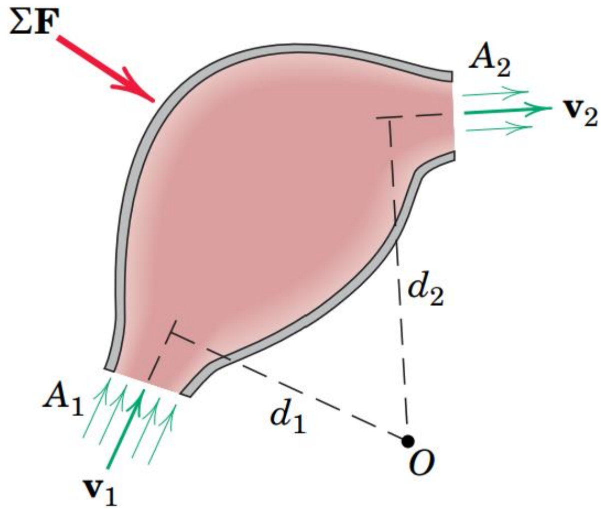
- One of the most important cases of mass fl ow occurs during steady-fl ow conditions where the rate at which mass enters a given volume equals the rate at which mass leaves the same volume.

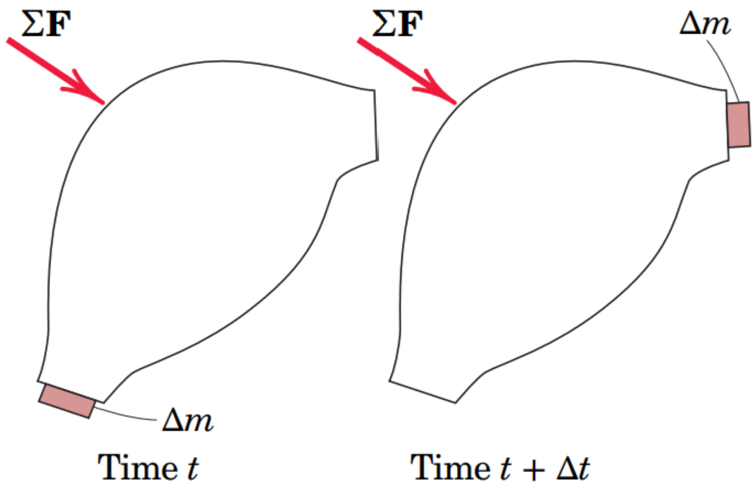
4/6 Steady Mass Flow

Analysis of Flow Through a Rigid Container

- Consider a rigid container, shown in section in Fig. 4/5a, into which mass flows in a steady stream at the rate m′
- Conservation of mass:

$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2 = m'$$

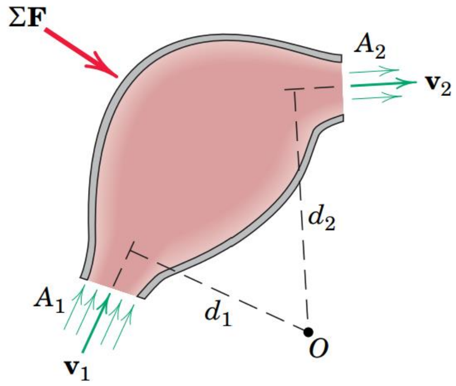




4/6 Steady Mass Flow

- To describe the forces which act, we isolate either the mass of fluid within the container or the entire container and the fluid within it.
  - ✓ We would use the first approach if the forces between the container and the fluid were to be described.
  - ✓ We would adopt the second approach when the forces external to the container are desired.

- The latter situation is our primary interest, in which case, the system isolated consists of the fixed structure of the container and the fluid within it at a particular instant of time.



4/6 Steady Mass Flow

Incremental Analysis

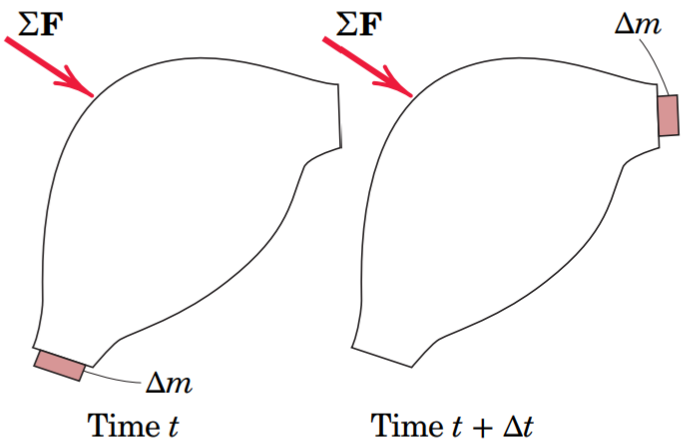
- The expression for $\dot{G}$:

$$\Delta \mathbf{G} = (\Delta m)\mathbf{v}_2 - (\Delta m)\mathbf{v}_1 = \Delta m(\mathbf{v}_2 - \mathbf{v}_1)$$

$$\dot{\mathbf{G}} = m'\Delta \mathbf{v}$$

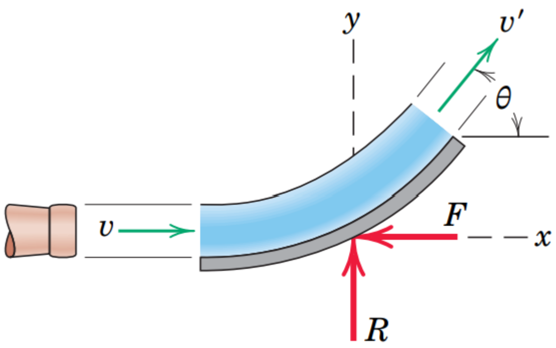
$$m' = \lim_{\Delta t \to 0} \left(\frac{\Delta m}{\Delta t} \right) = \frac{dm}{dt}$$

$$\boxed{\Sigma \mathbf{F} = m'\Delta \mathbf{v}}$$



SAMPLE PROBLEM 4/6

The smooth vane shown diverts the open stream of fluid of cross-sectional area A, mass density ρ, and velocity v. (*a*) Determine the force components R and F required to hold the vane in a fixed position. (*b*) Find the forces when the vane is given a constant velocity u less than v and in the direction of v.



Fixed vane

$$\Delta v_x = v' \cos\theta - v = -v(1 - \cos\theta)$$

$$\Delta v_y = v' \sin\theta - 0 = v\sin\theta$$

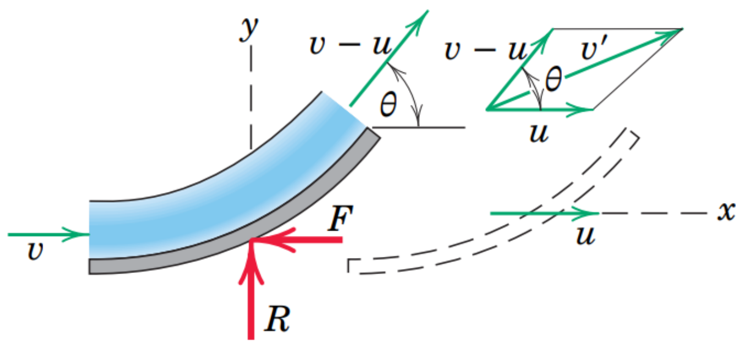
$$m' = \rho A v$$

$[\Sigma F_x = m'\Delta v_x]$ $\qquad -F = \rho A v[-v(1 - \cos\theta)]$

$$F = \rho A v^2(1 - \cos\theta)$$

$[\Sigma F_y = m'\Delta v_y]$ $\qquad R = \rho A v[v\sin\theta]$

$$R = \rho A v^2 \sin\theta$$



Moving vane

$$\Delta v_x = (v - u)\cos\theta + (u - v) = -(v - u)(1 - \cos\theta)$$

$$\Delta v_y = (v - u)\sin\theta$$

$$m' = \rho A(v - u)$$

$[\Sigma F_x = m'\Delta v_x]$ $\quad -F = \rho A(v - u)[-(v - u)(1 - \cos\theta)]$

$$F = \rho A(v - u)^2(1 - \cos\theta)$$

$[\Sigma F_y = m'\Delta v_y]$ $\quad R = \rho A(v - u)^2 \sin\theta$

4/7 Variable Mass

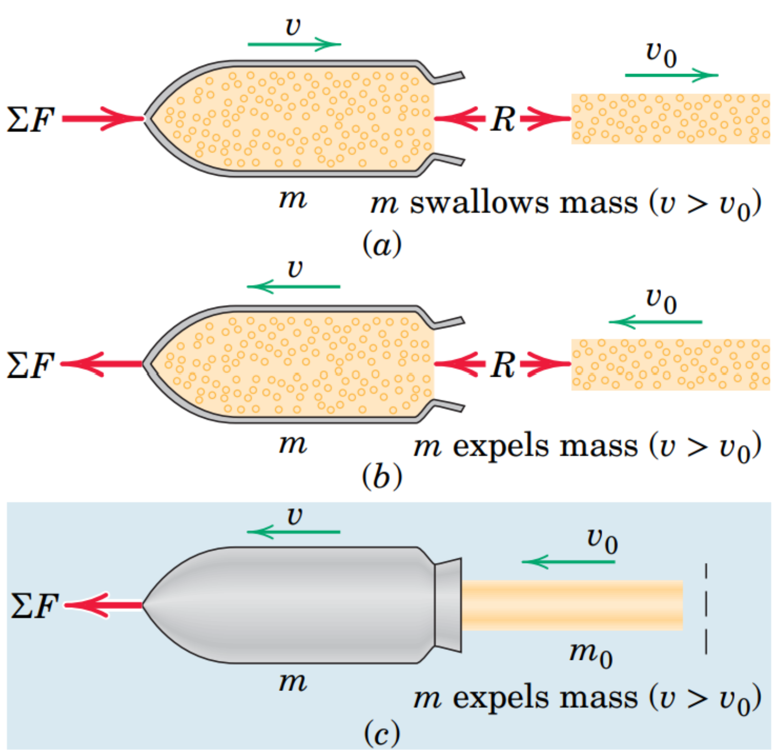
Equation of Motion

- The mass of the body and its velocity at any instant are m and v.
- The stream of matter is assumed to be moving with a constant velocity v_0 less than v.

$$R = m'(v - v_0) = \dot{m}u$$

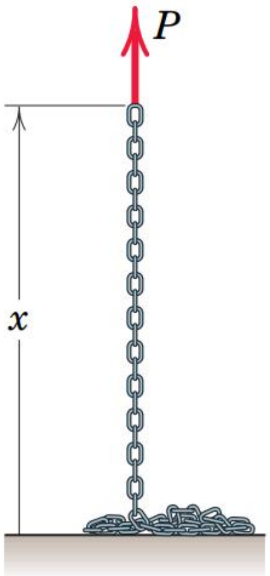
$$\Sigma F - R = m\dot{v}$$

$$\Sigma F = m\dot{v} + \dot{m}u$$

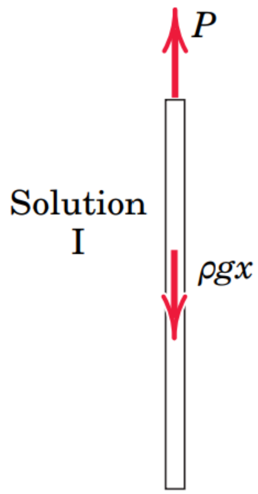


SAMPLE PROBLEM 4/10

The end of a chain of length L and mass ρ per unit length which is piled on a platform is lifted vertically with a constant velocity v by a variable force P. Find P as a function of the height x of the end above the platform. Also find the energy lost during the lifting of the chain.



Solution I (Variable-Mass Approach)



$$\Sigma F_x = P - \rho g x$$

$$\dot{v} = 0$$

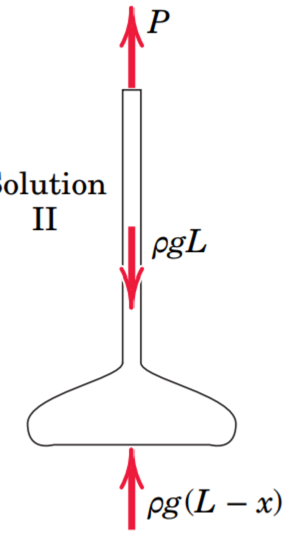
$$\dot{m} = \rho v$$

$$u = v - 0 = v$$

$$[\Sigma F = m\dot{v} + \dot{m}u] \qquad P - \rho g x = 0 + \rho v(v) \qquad P = \rho(gx + v^2)$$

Solution II (Constant-Mass Approach)

$$\left[\Sigma F_x = \frac{dG_x}{dt}\right] \quad P + \rho g(L - x) - \rho g L = \frac{d}{dt}(\rho x v) \quad P = \rho(gx + v^2)$$



Energy Loss

$$U'_{1\text{-}2} = \int P\,dx - \Delta E = \Delta T + \Delta V_g$$

$$\int P\,dx = \int_0^L (\rho g x + \rho v^2)dx = \tfrac{1}{2}\rho g L^2 + \rho v^2 L$$

$$\Delta T = \tfrac{1}{2}\rho L v^2 \qquad \Delta V_g = \rho g L \frac{L}{2} = \tfrac{1}{2}\rho g L^2$$

$$\rightarrow \quad \frac{1}{2}\rho g L^2 + \rho v^2 L - \Delta E = \frac{1}{2}\rho L v^2 + \frac{1}{2}\rho g L^2 \qquad \Delta E = \frac{1}{2}\rho L v^2$$