دانشگاه سمنان

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دانشکده مهندسی مکانیک

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درس دینامیک

Engineering Mechanics Dynamics

Meriam, Kraige & Bolton
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Chapter 3: **Kinetics of Particles**

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CHAPTER 3

Kinetics of Particles

CHAPTER OUTLINE

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The designers of amusement-park rides such as this roller coaster must not rely upon the principles of equilibrium alone as they develop specifications for the cars and the supporting structure. The particle kinetics of each car must be considered in estimating the involved forces so that a safe system can be designed.

CHAPTER 3

Kinetics of Particles

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The designers of amusement-park rides such as this roller coaster must not rely upon the principles of equilibrium alone as they develop specifications for the cars and the supporting structure. The particle kinetics of each car must be considered in estimating the involved forces so that a safe system can be designed.

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3/1 Introduction

- According to Newton's second law, a particle will accelerate when it is subjected to unbalanced forces.
- Kinetics is the study of the relations between unbalanced forces and the resulting changes in motion.
- We combine our knowledge of the properties of forces, which we developed in statics, and the kinematics of particle motion, and solve engineering problems involving force, mass, and motion.

3/1 Introduction

- General approaches to the solution of kinetics problems:
  - (A) Direct application of Newton's second law

    (called the force-mass-acceleration method)
  - (B) Use of work and energy principles
  - (C) Solution by impulse and momentum methods.

SECTION A Force, Mass, and Acceleration

3/2 Newton's Second Law

- The ratios of applied force to corresponding acceleration all equal the same number, provided the units used for measurement are not changed in the experiments.

$$\frac{F_1}{a_1} = \frac{F_2}{a_2} = \cdots = \frac{F}{a} = C, \qquad \text{a constant}$$

- We conclude that the constant C is a measure of some invariable property of the particle. This property is the inertia of the particle, which is its resistance to rate of change of velocity.

3/3 Equation of Motion and Solution of Problems

- A particle of mass m is subjected to the action of concurrent forces:

$$\Sigma \mathbf{F} = m\mathbf{a}$$

- We usually express it in scalar component form with the use of one of the coordinate systems.

- The choice of an appropriate coordinate system depends on the type of motion involved.

3/3 Equation of Motion and Solution of Problems

Two Types of Dynamics Problems

- First type:
  - The acceleration of the particle is either specified or can be determined directly from known kinematic conditions.

- Second type:
  - The forces acting on the particle are specified and we must determine the resulting motion.

3/3 Equation of Motion and Solution of Problems

Constrained and Unconstrained Motion

- Unconstrained motion:
  - The particle is free of mechanical guides and follows a path determined by its initial motion and by the forces which are applied to it from external sources.

- Constrained motion:
  - The path of the particle is partially or totally determined by restraining guides.

3/3 Equation of Motion and Solution of Problems

Free-Body Diagram

- The only reliable way to account accurately and consistently for every force is to isolate the particle under consideration from all contacting and influencing bodies and replace the bodies removed by the forces they exert on the particle isolated.

- The resulting free body diagram is the means by which every force, known and unknown, which acts on the particle is represented and thus accounted for.

- Only after this vital step has been completed should you write the appropriate equation or equations of motion.

3/4 Rectilinear Motion

- If we choose the x-direction, for example, as the direction of the rectilinear motion of a particle:

$$\Sigma F_x = ma_x$$

$$\Sigma F_y = 0$$

$$\Sigma F_z = 0$$

- For cases where we are not free to choose a coordinate direction along the motion:

$$\Sigma F_x = ma_x$$

$$\Sigma F_y = ma_y$$

$$\Sigma F_z = ma_z$$

3/4 Rectilinear Motion

- Acceleration and resultant force:

$$\mathbf{a} = a_x\mathbf{i} + a_y\mathbf{j} + a_z\mathbf{k}$$

$$a = \sqrt{a_x^{\ 2} + a_y^{\ 2} + a_z^{\ 2}}$$

$$\Sigma\mathbf{F} = \Sigma F_x\mathbf{i} + \Sigma F_y\mathbf{j} + \Sigma F_z\mathbf{k}$$

$$|\Sigma\mathbf{F}| = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2 + (\Sigma F_z)^2}$$

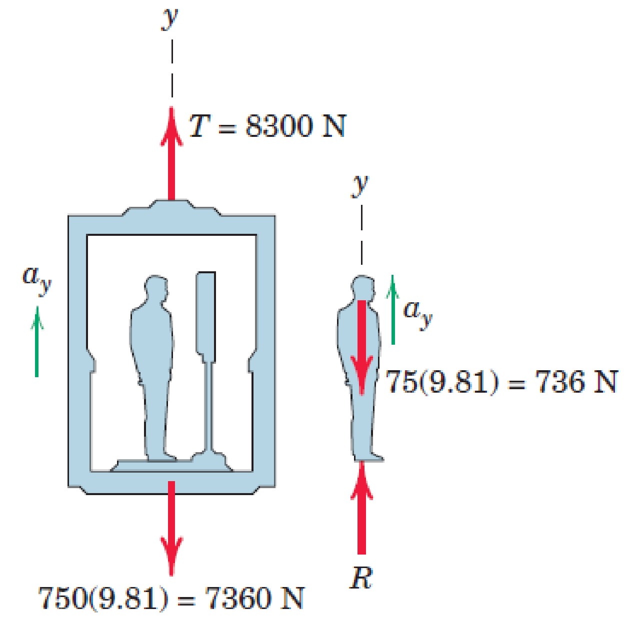
SAMPLE PROBLEM 3/1

A 75-kg man stands on a spring scale in an elevator. During the first 3 seconds of motion from rest, the tension T in the hoisting cable is 8300 N. Find the reading R of the scale in newtons during this interval and the upward velocity v of the elevator at the end of the 3 seconds. The total mass of the elevator, man, and scale is 750 kg.

$[\Sigma F_y = ma_y]$ $\quad 8300 - 7360 = 750a_y \quad a_y = 1.257 \text{ m/s}^2$

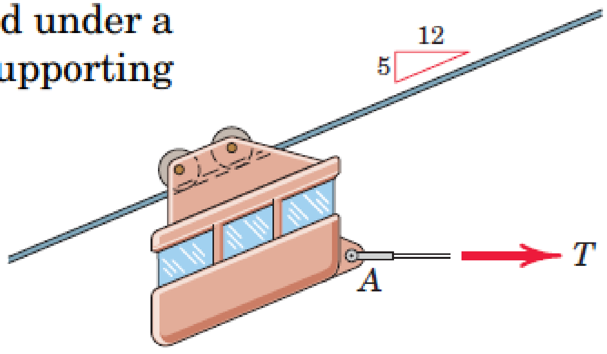
$[\Sigma F_y = ma_y]$ $\quad R - 736 = 75(1.257) \quad R = 830 \text{ N}$

$[\Delta v = \int a\, dt]$ $\quad v - 0 = \int_0^3 1.257\, dt \quad v = 3.77 \text{ m/s}$



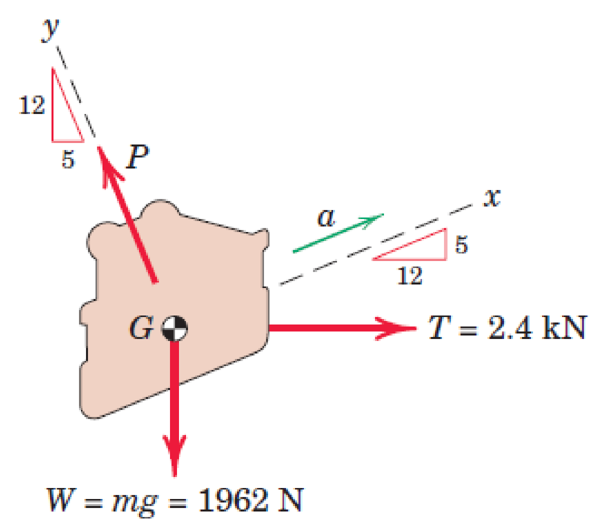
SAMPLE PROBLEM 3/2

A small inspection car with a mass of 200 kg runs along the fixed overhead cable and is controlled by the attached cable at A. Determine the acceleration of the car when the control cable is horizontal and under a tension $T = 2.4$ kN. Also find the total force P exerted by the supporting cable on the wheels.



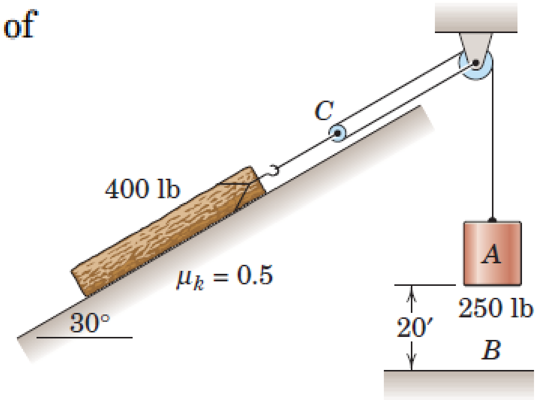
$$[\Sigma F_y = 0] \qquad P - 2.4\left(\frac{5}{13}\right) - 1.962\left(\frac{12}{13}\right) = 0 \qquad P = 2.73\ \text{kN}$$

$$[\Sigma F_x = ma_x] \qquad 2400\left(\frac{12}{13}\right) - 1962\left(\frac{5}{13}\right) = 200a \qquad a = 7.30\ \text{m/s}^2$$



SAMPLE PROBLEM 3/3

The 250-lb concrete block A is released from rest in the position shown and pulls the 400-lb log up the 30° ramp. If the coefficient of kinetic friction between the log and the ramp is 0.5, determine the velocity of the block as it hits the ground at B.



$L = 2s_C + s_A + \text{constant}$ $\qquad$ $0 = 2a_C + a_A$

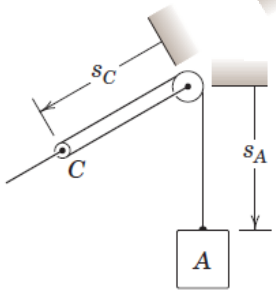
$[\Sigma F_y = 0]$ $\qquad N - 400 \cos 30° = 0 \qquad N = 346 \text{ lb}$

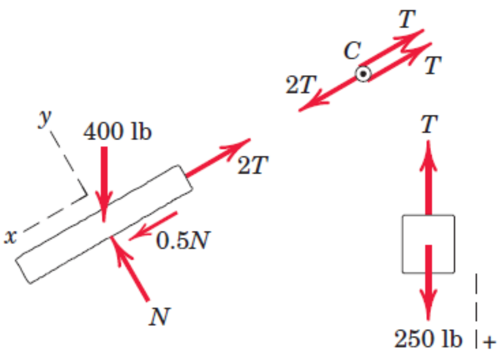
$[\Sigma F_x = ma_x]$ $\qquad 0.5(346) - 2T + 400 \sin 30° = \dfrac{400}{32.2} a_C$

$[+\downarrow \Sigma F = ma]$ $\qquad 250 - T = \dfrac{250}{32.2} a_A$

$a_A = 5.83 \text{ ft/sec}^2 \qquad a_C = -2.92 \text{ ft/sec}^2 \qquad T = 205 \text{ lb}$

$[v^2 = 2ax]$ $\qquad v_A = \sqrt{2(5.83)(20)} = 15.27 \text{ ft/sec}$





3/5 Curvilinear Motion

- Rectangular coordinates:

$$\Sigma F_x = ma_x \qquad \Sigma F_y = ma_y$$

$$a_x = \ddot{x} \quad \text{and} \quad a_y = \ddot{y}$$

- Normal and tangential coordinates:

$$\Sigma F_n = ma_n \qquad \Sigma F_t = ma_t$$

$$a_n = \rho\dot{\beta}^2 = v^2/\rho = v\dot{\beta}, \quad a_t = \dot{v}, \quad \text{and} \quad v = \rho\dot{\beta}$$

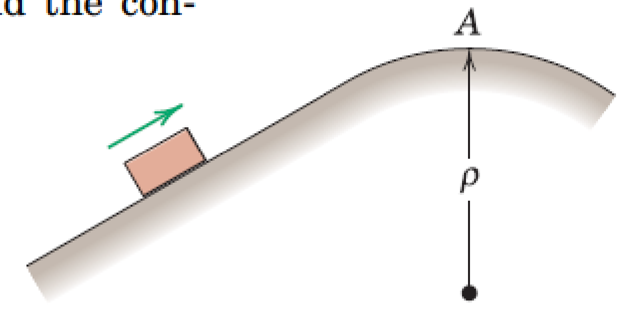
- Polar coordinates:

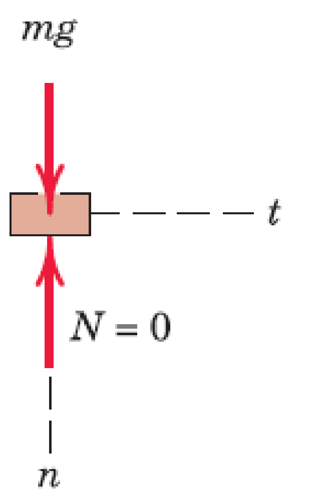
$$\Sigma F_r = ma_r \qquad \Sigma F_\theta = ma_\theta$$

$$a_r = \ddot{r} - r\dot{\theta}^2 \quad \text{and} \quad a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$$

SAMPLE PROBLEM 3/6

Determine the maximum speed v which the sliding block may have as it passes the topmost point A without losing contact with the lower surface. Assume a slightly loose fit between the slider and the constraint surfaces.

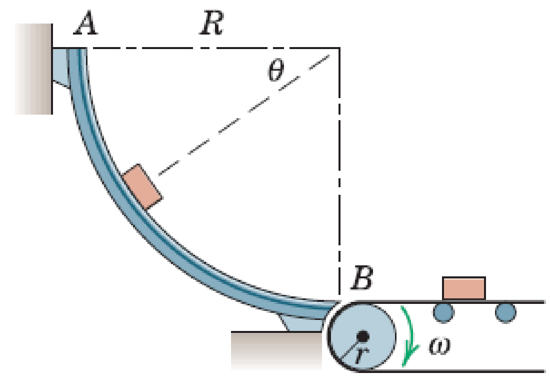




$[\Sigma F_n = ma_n]$ $\qquad mg = m\dfrac{v^2}{\rho} \qquad v = \sqrt{g\rho}$

SAMPLE PROBLEM 3/7

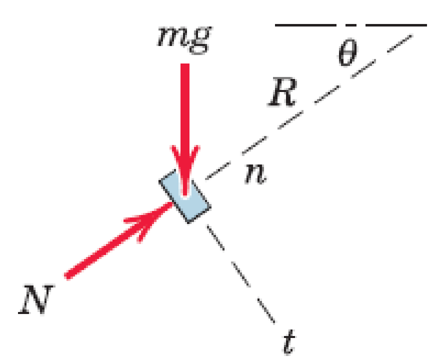
Small objects are released from rest at A and slide down the smooth circular surface of radius R to a conveyor B. Determine the expression for the normal contact force N between the guide and each object in terms of θ and specify the correct angular velocity ω of the conveyor pulley of radius r to prevent any sliding on the belt as the objects transfer to the conveyor.



$[\Sigma F_t = ma_t] \qquad mg\cos\theta = ma_t \qquad a_t = g\cos\theta$

$[v\,dv = a_t\,ds] \qquad \int_0^v v\,dv = \int_0^\theta g\cos\theta\,d(R\theta) \qquad v^2 = 2gR\sin\theta$

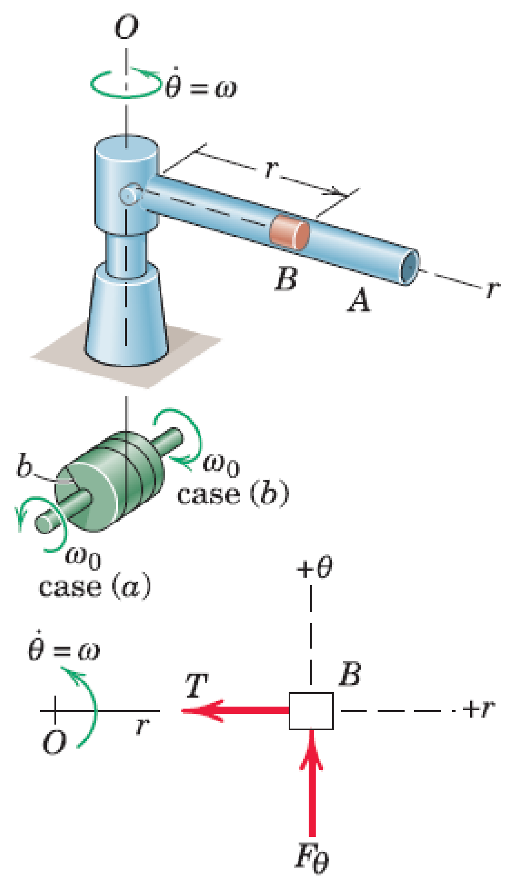
$[\Sigma F_n = ma_n] \qquad N - mg\sin\theta = m\dfrac{v^2}{R} \qquad N = 3mg\sin\theta$



$\omega = \sqrt{2gR}/r$

SAMPLE PROBLEM 3/10

Tube A rotates about the vertical O-axis with a constant angular rate $\dot{\theta} = \omega$ and contains a small cylindrical plug B of mass m whose radial position is controlled by the cord which passes freely through the tube and shaft and is wound around the drum of radius b. Determine the tension T in the cord and the horizontal component F_θ of force exerted by the tube on the plug if the constant angular rate of rotation of the drum is ω_0 first in the direction for case (a) and second in the direction for case (b). Neglect friction.



$[\Sigma F_r = ma_r]$	$-T = m(\ddot{r} - r\dot{\theta}^2)$
$[\Sigma F_\theta = ma_\theta]$	$F_\theta = m(r\ddot{\theta} + 2\dot{r}\dot{\theta})$

Case (a).

$\dot{r} = +b\omega_0$, $\ddot{r} = 0$, and $\ddot{\theta} = 0$,

$T = mr\omega^2 \qquad F_\theta = 2mb\omega_0\omega$

Case (b).

$\dot{r} = -b\omega_0$, $\ddot{r} = 0$, and $\ddot{\theta} = 0$,

$T = mr\omega^2 \qquad F_\theta = -2mb\omega_0\omega$

SECTION B Work and Energy

3/6 Work and Kinetic Energy

- There are two general classes of problems:
  - (1) Integration of the forces with respect to the displacement of the particle
  - (2) Integration of the forces with respect to the time they are applied.

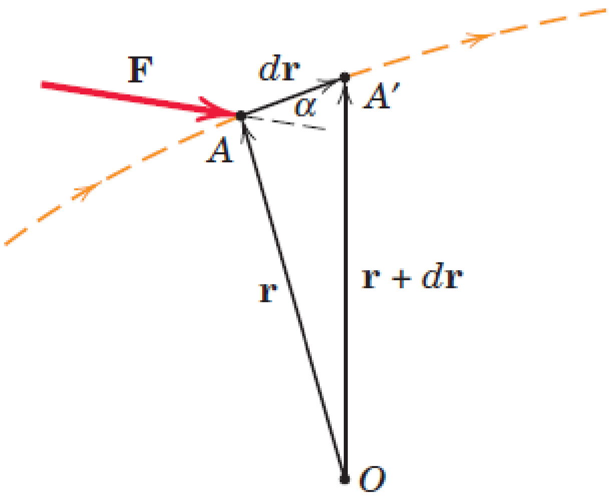
- Integration with respect to displacement leads to the equations of work and energy.

3/6 Work and Kinetic Energy

Definition of Work

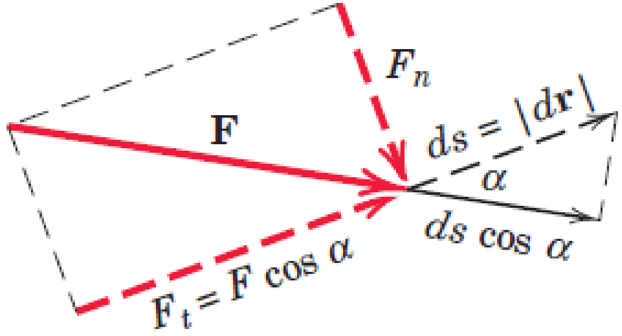
- The work done by the force ***F*** during the displacement ***dr***:

$$dU = \mathbf{F} \cdot d\mathbf{r}$$



→ $dU = F\,ds\cos\alpha$

$F_t = F\cos\alpha$ → $dU = F_t\,ds$



3/6 Work and Kinetic Energy

Units of Work

- The SI units of work are those of force (N) times displacement (m) or N·m.

- This unit is given the special name joule (J), which is defined as the work done by a force of 1 N acting through a distance of 1 m in the direction of the force.

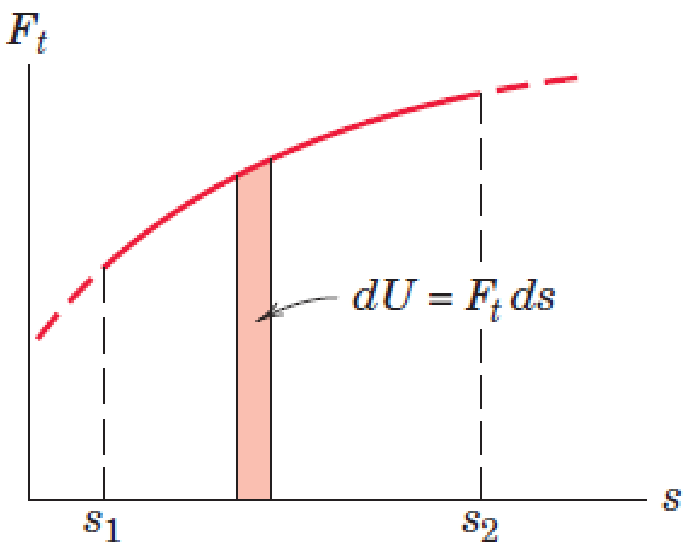
- Consistent use of the joule for work (and energy) rather than the units N·m will avoid possible ambiguity with the units of moment of a force or torque.

3/6 Work and Kinetic Energy

Calculation of Work

$$U = \int_1^2 \mathbf{F}\cdot d\mathbf{r} = \int_1^2 (F_x\, dx + F_y\, dy + F_z\, dz)$$

$$U = \int_{s_1}^{s_2} F_t\, ds$$



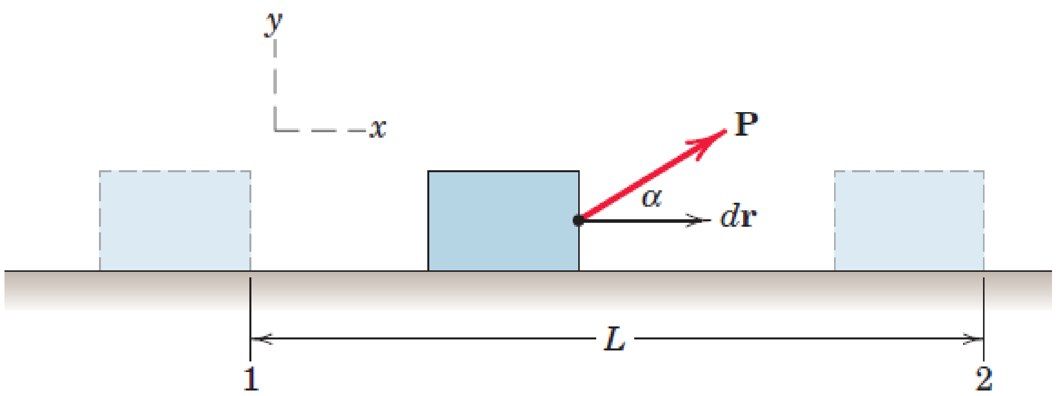
3/6 Work and Kinetic Energy

Examples of Work

- Work Associated with a Constant External Force

$$U_{1\text{-}2} = \int_1^2 \mathbf{F}\cdot d\mathbf{r} = \int_1^2 [(P\cos\alpha)\mathbf{i} + (P\sin\alpha)\mathbf{j}]\cdot dx\,\mathbf{i}$$

$$= \int_{x_1}^{x_2} P\cos\alpha\, dx = P\cos\alpha(x_2 - x_1) = PL\cos\alpha$$

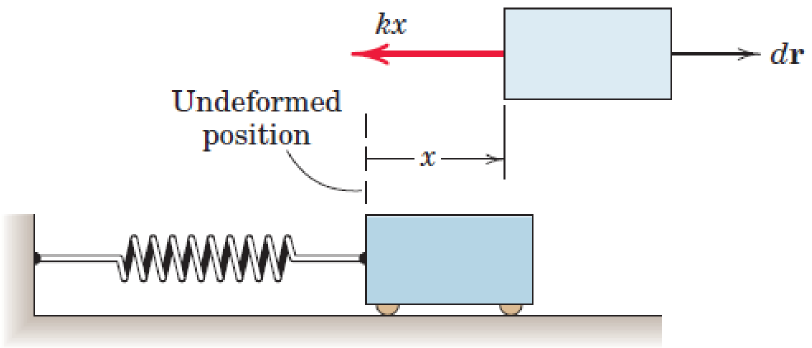


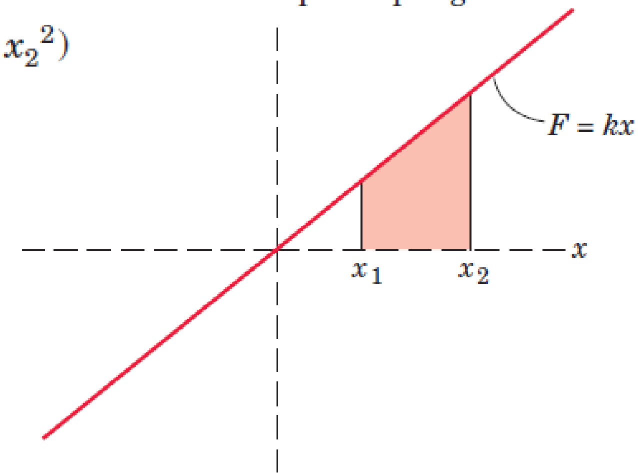
3/6 Work and Kinetic Energy

Examples of Work

- Work Associated with a Spring Force

$$U_{1\text{-}2} = \int_1^2 \mathbf{F}\cdot d\mathbf{r} = \int_1^2 (-kx\mathbf{i})\cdot dx\,\mathbf{i} = -\int_{x_1}^{x_2} kx\,dx = \frac{1}{2}k({x_1}^2 - {x_2}^2)$$





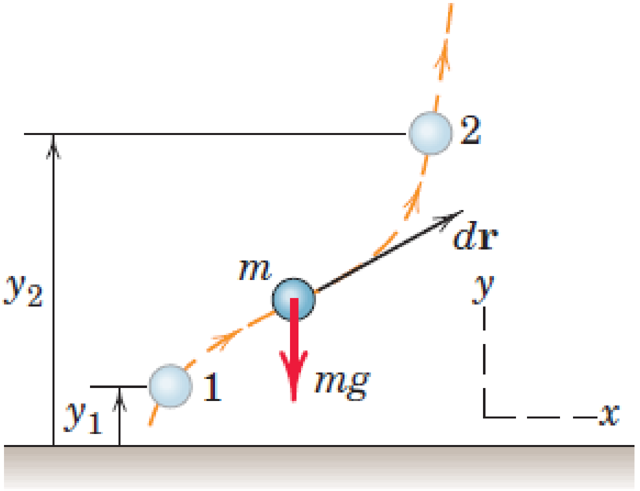
3/6 Work and Kinetic Energy

Examples of Work

- Work Associated with Weight

$$U_{1\text{-}2} = \int_1^2 \mathbf{F}\cdot d\mathbf{r} = \int_1^2 (-mg\mathbf{j})\cdot(dx\mathbf{i} + dy\mathbf{j})$$

$$= -mg\int_{y_1}^{y_2} dy = -mg(y_2 - y_1)$$



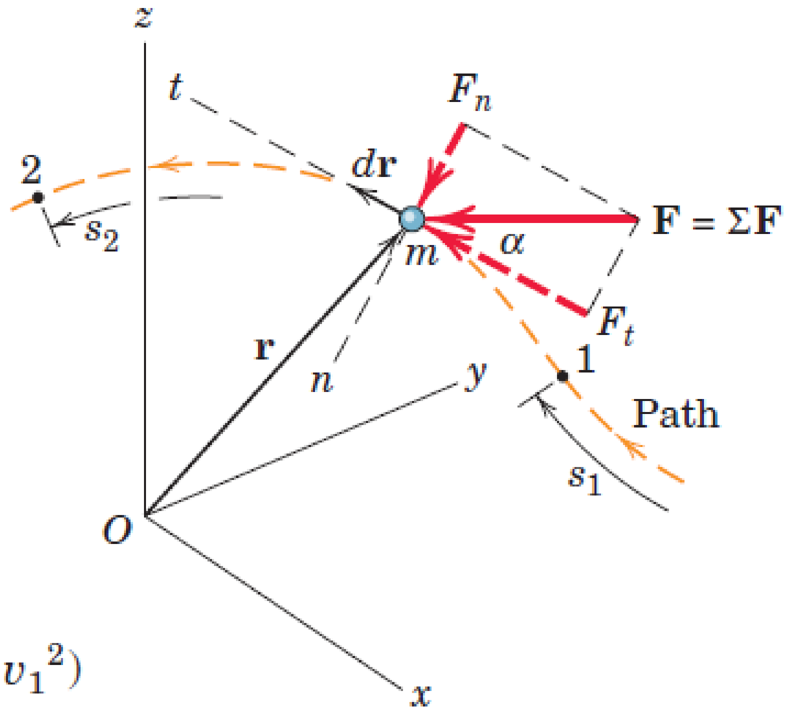
3/6 Work and Kinetic Energy

Work and Curvilinear Motion

$$U_{1\text{-}2} = \int_1^2 \mathbf{F} \cdot d\mathbf{r} = \int_{s_1}^{s_2} F_t \, ds$$

$$U_{1\text{-}2} = \int_1^2 \mathbf{F} \cdot d\mathbf{r} = \int_1^2 m\mathbf{a} \cdot d\mathbf{r}$$

$$U_{1\text{-}2} = \int_1^2 \mathbf{F} \cdot d\mathbf{r} = \int_{v_1}^{v_2} mv \, dv = \frac{1}{2} m (v_2{}^2 - v_1{}^2)$$



3/6 Work and Kinetic Energy

Principle of Work and Kinetic Energy

- The *kinetic energy T* of the particle:

$$T = \frac{1}{2}mv^2$$

- The *work-energy equation* for a particle:

$$T_1 + U_{1\text{-}2} = T_2$$

  - The equation states that the total work done by all forces acting on a particle as it moves from point 1 to point 2 equals the corresponding change in kinetic energy of the particle.

3/6 Work and Kinetic Energy

Power

- The capacity of a machine is rated by its power, which is defined as the time rate of doing work

$$P = dU/dt = \mathbf{F}\cdot d\mathbf{r}/dt \quad \rightarrow \quad \boxed{P = \mathbf{F}\cdot\mathbf{v}}$$

$$1\ \text{W} = 1\ \text{J/s}$$

$$1\ \text{hp} = 550\ \text{ft-lb/sec} = 33{,}000\ \text{ft-lb/min}$$

$$1\ \text{hp} = 746\ \text{W} = 0.746\ \text{kW}$$

3/6 Work and Kinetic Energy

Efficiency

- The ratio of the work done by a machine to the work done on the machine during the same time interval is called the mechanical efficiency e_m of the machine.

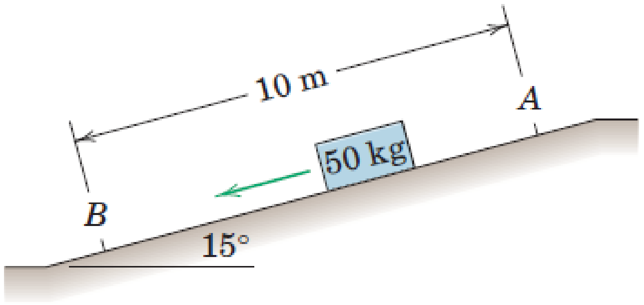
$$e_m = \frac{P_{\text{output}}}{P_{\text{input}}}$$

- In addition to energy loss by mechanical friction, there may also be electrical and thermal energy loss, in which case, the electrical efficiency e_e and thermal efficiency e_t are also involved. The overall efficiency e in such instances is:

$$e = e_m e_e e_t$$

SAMPLE PROBLEM 3/11

Calculate the velocity v of the 50-kg crate when it reaches the bottom of the chute at B if it is given an initial velocity of 4 m/s down the chute at A. The coefficient of kinetic friction is 0.30.



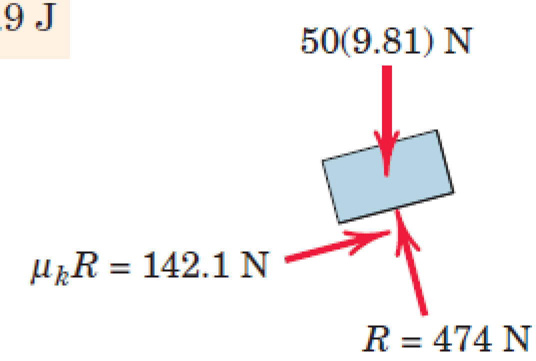
$[U = Fs]$ $\quad U_{1\text{-}2} = 50(9.81)(10 \sin 15°) - 142.1(10) = -151.9 \text{ J}$

$[T_1 + U_{1\text{-}2} = T_2]$

$$\frac{1}{2}mv_1^2 + U_{1\text{-}2} = \frac{1}{2}mv_2^2$$

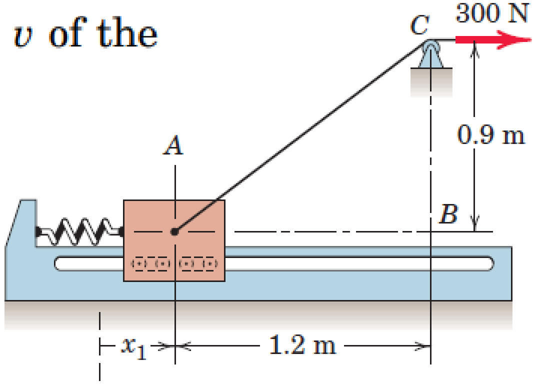
$$\frac{1}{2}(50)(4)^2 - 151.9 = \frac{1}{2}(50)v_2^2$$

$$v_2 = 3.15 \text{ m/s}$$



SAMPLE PROBLEM 3/13

The 50-kg block at A is mounted on rollers so that it moves along the fixed horizontal rail with negligible friction under the action of the constant 300-N force in the cable. The block is released from rest at A, with the spring to which it is attached extended an initial amount $x_1 = 0.233$ m. The spring has a stiffness $k = 80$ N/m. Calculate the velocity v of the block as it reaches position B.



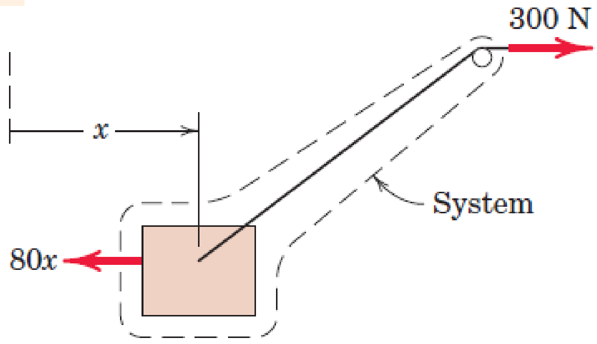
$x_1 = 0.233$ m to $x_2 = 0.233 + 1.2 = 1.433$ m.

$[U_{1\text{-}2} = \frac{1}{2}k(x_1{}^2 - x_2{}^2)]$ $\qquad U_{1\text{-}2} = \frac{1}{2}\,80[0.233^2 - (0.233 + 1.2)^2]$

$$= -80.0 \text{ J}$$

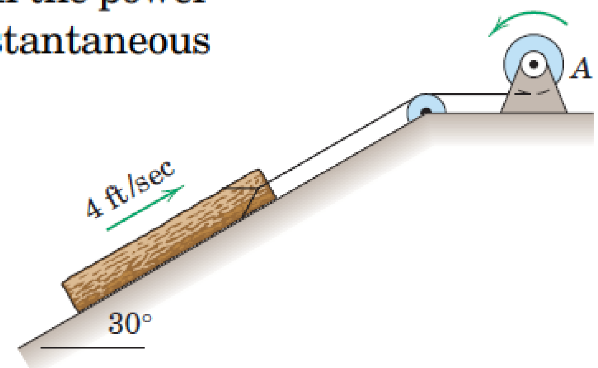
300-N force $\qquad \sqrt{(1.2)^2 + (0.9)^2} - 0.9 = 0.6$ m $\qquad 300(0.6) = 180$ J.

$[T_1 + U_{1\text{-}2} = T_2]$ $\qquad 0 - 80.0 + 180 = \frac{1}{2}(50)v^2 \qquad v = 2.00$ m/s



SAMPLE PROBLEM 3/14

The power winch A hoists the 800-lb log up the 30° incline at a constant speed of 4 ft/sec. If the power output of the winch is 6 hp, compute the coefficient of kinetic friction μ_k between the log and the incline. If the power is suddenly increased to 8 hp, what is the corresponding instantaneous acceleration a of the log?



$$N = 800 \cos 30° = 693 \text{ lb}$$

$$[\Sigma F_x = 0] \qquad T - 693\mu_k - 800 \sin 30° = 0 \qquad T = 693\mu_k + 400$$

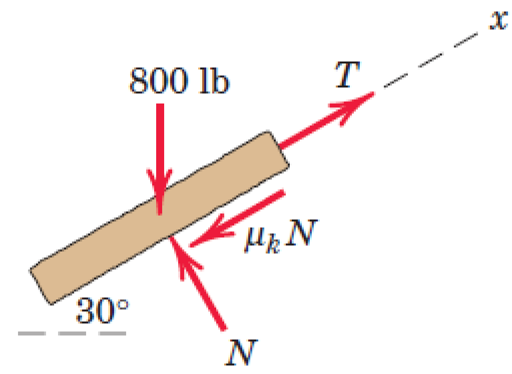
$$[P = Tv] \qquad T = P/v = 6(550)/4 = 825 \text{ lb}$$

$$825 = 693\mu_k + 400 \qquad \mu_k = 0.613$$

$$[P = Tv] \qquad T = P/v = 8(550)/4 = 1100 \text{ lb}$$

$$[\Sigma F_x = ma_x] \qquad 1100 - 693(0.613) - 800 \sin 30° = \frac{800}{32.2} a$$

$$a = 11.07 \text{ ft/sec}^2$$



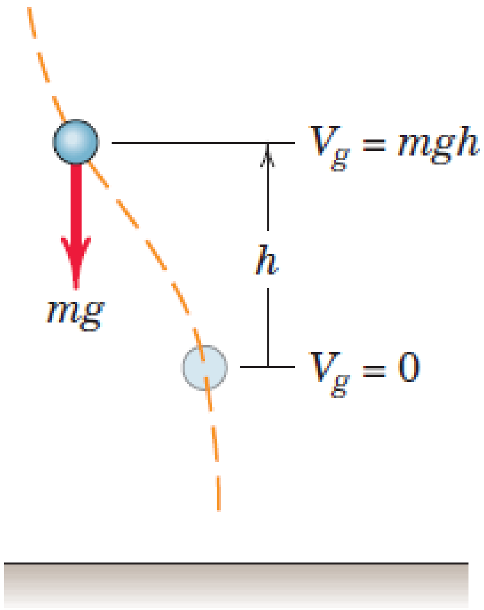
3/7 Potential Energy

Gravitational Potential Energy

- The *gravitational potential energy* V_g of the particle is defined as the work *mgh* done against the gravitational field to elevate the particle a distance h above some arbitrary reference plane.

$$V_g = mgh$$

$$\Delta V_g = mg(h_2 - h_1) = mg\Delta h$$



3/7 Potential Energy

Elastic Potential Energy

- The work which is done on the spring to deform it is stored in the spring and is called its *elastic potential energy* V_e.

$$V_e = \int_0^x kx\,dx = \frac{1}{2}kx^2$$

$$\Delta V_e = \frac{1}{2}k(x_2{}^2 - x_1{}^2)$$

3/7 Potential Energy

Work-Energy Equation

- Work-energy equation modification to account for the potential-energy terms

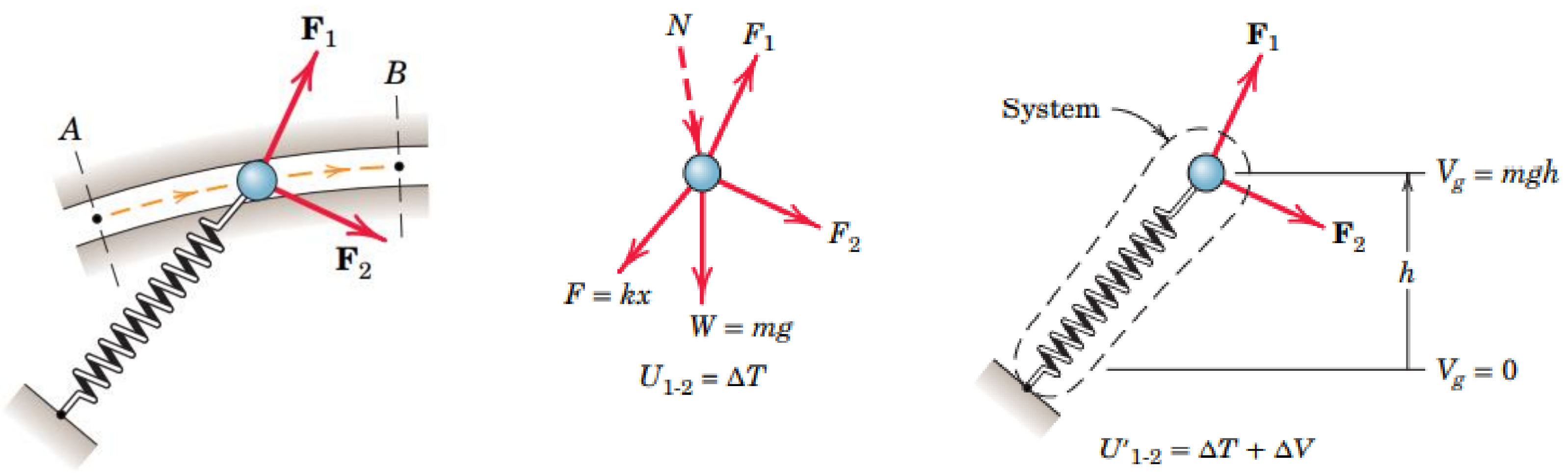
$$U'_{1\text{-}2} + (-\Delta V_g) + (-\Delta V_e) = \Delta T$$

$$U'_{1\text{-}2} = \Delta T + \Delta V$$

$$\boxed{T_1 + V_1 + U'_{1\text{-}2} = T_2 + V_2}$$

3/7 Potential Energy

Work-Energy Equation



3/7 Potential Energy

Work-Energy Equation

- For problems where the only forces are gravitational, elastic, and nonworking constraint forces:

$$T_1 + V_1 = T_2 + V_2 \qquad \text{or} \qquad E_1 = E_2$$

- E=T+V is the total mechanical energy of the particle and its attached spring.

3/7 Potential Energy

Conservative Force Fields

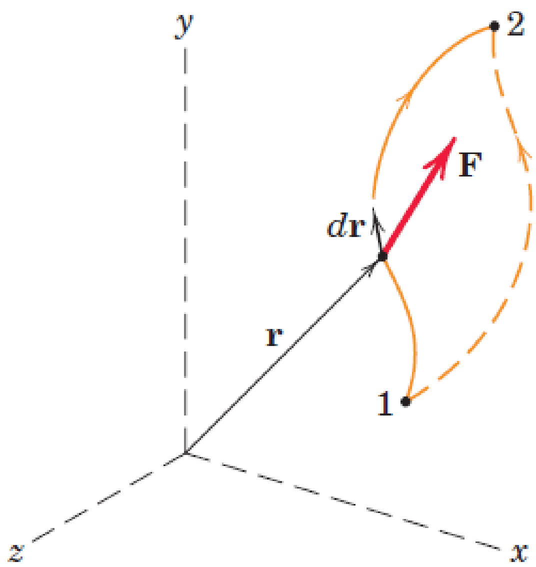
- Conservative force fields: the work done against force depends only on the net change of position and not on the particular path followed in reaching the new position.

$$U = \int \mathbf{F} \cdot d\mathbf{r} = \int (F_x\,dx + F_y\,dy + F_z\,dz)$$

$$\Rightarrow U_{1\text{-}2} = \int_{V_1}^{V_2} -dV = -(V_2 - V_1)$$

$$dV = \frac{\partial V}{\partial x}dx + \frac{\partial V}{\partial y}dy + \frac{\partial V}{\partial z}dz$$

$$F_x = -\frac{\partial V}{\partial x} \qquad F_y = -\frac{\partial V}{\partial y} \qquad F_z = -\frac{\partial V}{\partial z}$$



3/7 Potential Energy

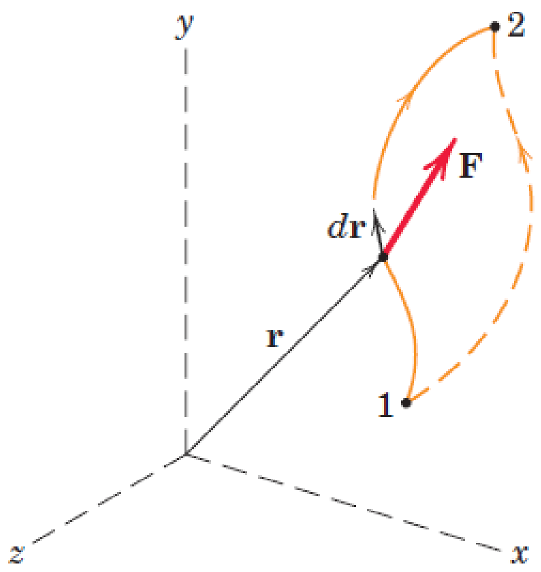
Conservative Force Fields

- The quantity V is known as the potential function.

$$\Rightarrow dV = \frac{\partial V}{\partial x}dx + \frac{\partial V}{\partial y}dy + \frac{\partial V}{\partial z}dz$$

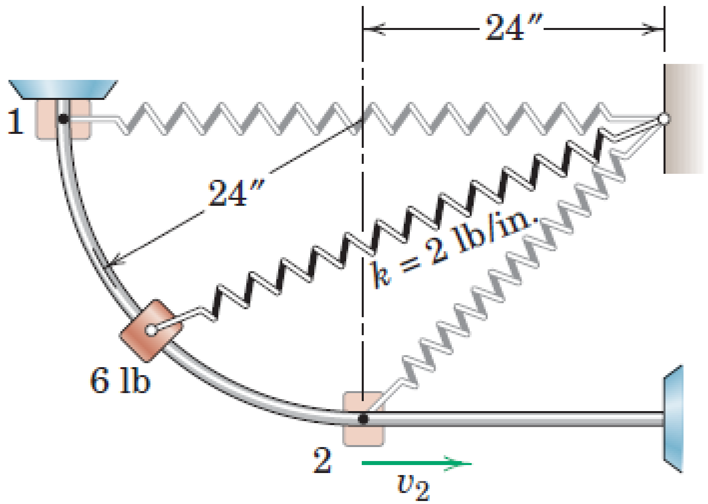
$$F_x = -\frac{\partial V}{\partial x} \qquad F_y = -\frac{\partial V}{\partial y} \qquad F_z = -\frac{\partial V}{\partial z}$$

$$\mathbf{F} = -\nabla V \qquad \nabla = \mathbf{i}\frac{\partial}{\partial x} + \mathbf{j}\frac{\partial}{\partial y} + \mathbf{k}\frac{\partial}{\partial z}$$



SAMPLE PROBLEM 3/16

The 6-lb slider is released from rest at position 1 and slides with negligible friction in a vertical plane along the circular rod. The attached spring has a stiffness of 2 lb/in. and has an unstretched length of 24 in. Determine the velocity of the slider as it passes position 2.



$$V_1 = 0$$

$$V_2 = -mgh = -6\left(\frac{24}{12}\right) = -12 \text{ ft-lb}$$

$$V_1 = \frac{1}{2}kx_1^2 = \frac{1}{2}(2)(12)\left(\frac{24}{12}\right)^2 = 48 \text{ ft-lb}$$

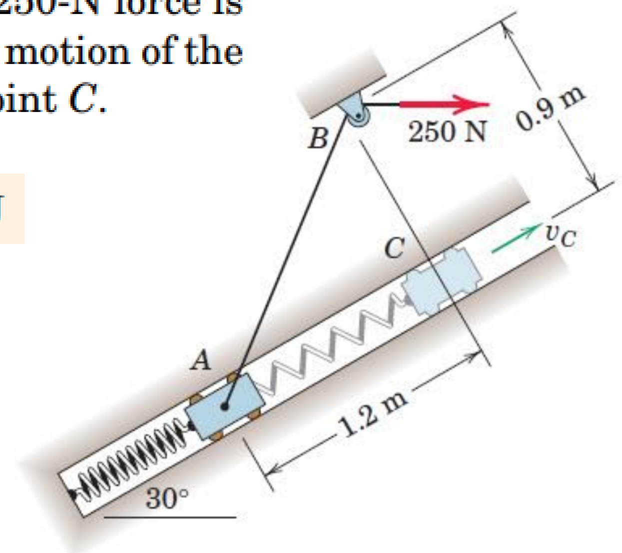
$$V_2 = \frac{1}{2}kx_2^2 = \frac{1}{2}(2)(12)\left(\frac{24\sqrt{2}}{12} - \frac{24}{12}\right)^2 = 8.24 \text{ ft-lb}$$

$$[T_1 + V_1 + U'_{1\text{-}2} = T_2 + V_2] \qquad 0 + 48 + 0 = \frac{1}{2}\left(\frac{6}{32.2}\right)v_2^2 - 12 + 8.24$$

$$v_2 = 23.6 \text{ ft/sec}$$

SAMPLE PROBLEM 3/17

The 10-kg slider moves with negligible friction up the inclined guide. The attached spring has a stiffness of 60 N/m and is stretched 0.6 m in position A, where the slider is released from rest. The 250-N force is constant and the pulley offers negligible resistance to the motion of the cord. Calculate the velocity v_C of the slider as it passes point C.



$\overline{AB} - \overline{BC}$ or $1.5 - 0.9 = 0.6$ m.

$$U'_{A\text{-}C} = 250(0.6) = 150 \text{ J}$$

$$V_A = 0 \qquad V_C = mgh = 10(9.81)(1.2 \sin 30°) = 58.9 \text{ J}$$

$$V_A = \frac{1}{2}k{x_A}^2 = \frac{1}{2}(60)(0.6)^2 = 10.8 \text{ J}$$

$$V_C = \frac{1}{2}k{x_B}^2 = \frac{1}{2}60(0.6 + 1.2)^2 = 97.2 \text{ J}$$

$$[T_A + V_A + U'_{A\text{-}C} = T_C + V_C] \qquad 0 + 0 + 10.8 + 150 = \frac{1}{2}(10){v_C}^2 + 58.9 + 97.2$$

$$v_C = 0.974 \text{ m/s}$$

SECTION C Impulse and Momentum

3/8 Introduction

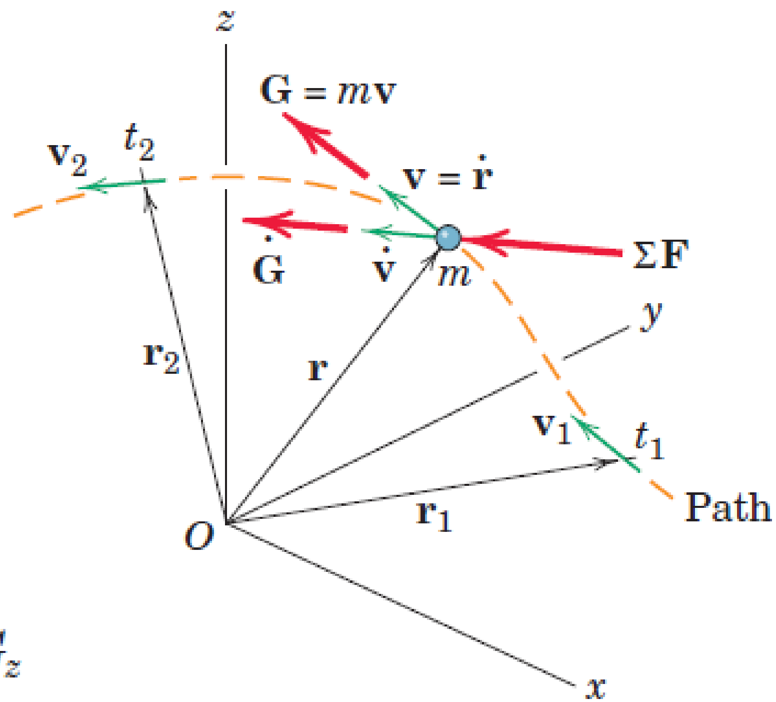
- We can integrate the equation of motion with respect to time rather than displacement.
- This approach leads to the equations of impulse and momentum.
- These equations greatly facilitate the solution of many problems in which the applied forces act during extremely short periods of time (as in impact problems) or over specified intervals of time.

3/9 Linear Impulse and Linear Momentum

- Linear momentum of the particle:
  - **G** = m**v**

$$\Sigma \mathbf{F} = m\dot{\mathbf{v}} = \frac{d}{dt}(m\mathbf{v})$$

$$\Sigma \mathbf{F} = \dot{\mathbf{G}}$$

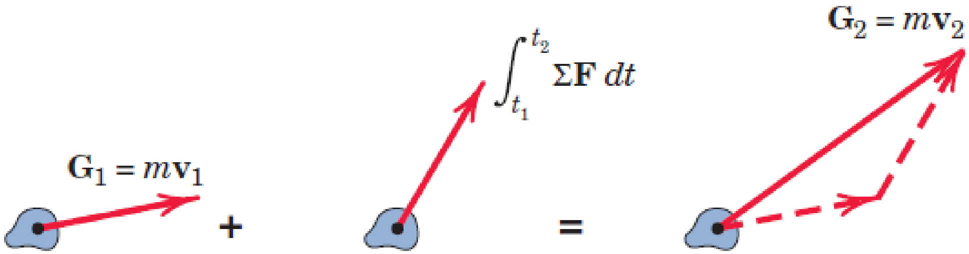


$$\Sigma F_x = \dot{G}_x \qquad \Sigma F_y = \dot{G}_y \qquad \Sigma F_z = \dot{G}_z$$

3/9 Linear Impulse and Linear Momentum

The Linear Impulse-Momentum Principle

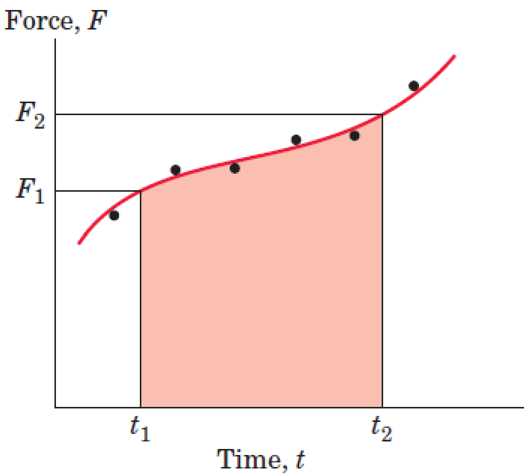
$$\mathbf{G}_1 + \int_{t_1}^{t_2} \Sigma \mathbf{F}\, dt = \mathbf{G}_2$$



$$m(v_1)_x + \int_{t_1}^{t_2} \Sigma F_x\, dt = m(v_2)_x$$

$$m(v_1)_y + \int_{t_1}^{t_2} \Sigma F_y\, dt = m(v_2)_y$$

$$m(v_1)_z + \int_{t_1}^{t_2} \Sigma F_z\, dt = m(v_2)_z$$



3/9 Linear Impulse and Linear Momentum

Conservation of Linear Momentum

- If the resultant force on a particle is zero during an interval of time, its linear momentum **G** remain constant.

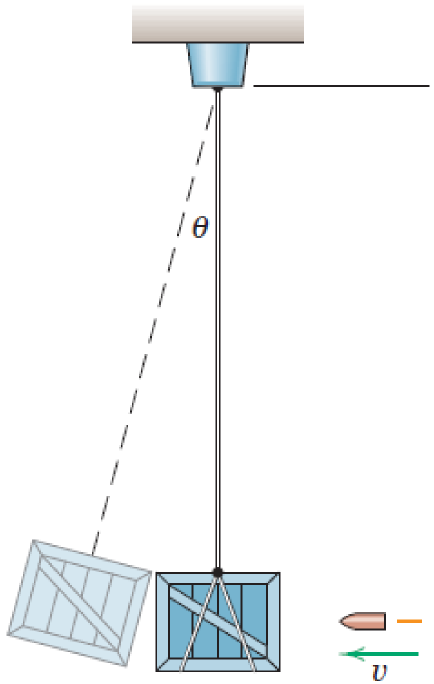
- In this case, the linear momentum of the particle is said to be *conserved*.

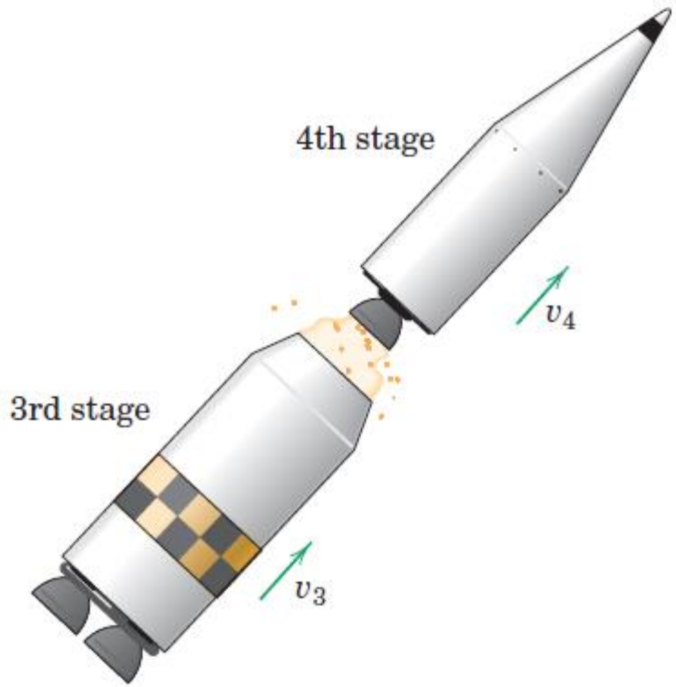
$$\Delta \mathbf{G} = \mathbf{0} \qquad \text{or} \qquad \mathbf{G}_1 = \mathbf{G}_2$$

Principle of conservation of linear momentum

3/9 Linear Impulse and Linear Momentum

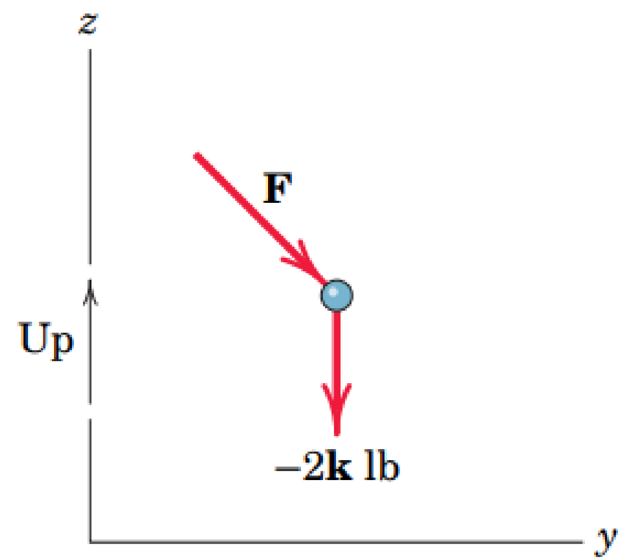
Conservation of Linear Momentum





SAMPLE PROBLEM 3/20

A 2-lb particle moves in the vertical *y*-*z* plane (*z* up, *y* horizontal) under the action of its weight and a force **F** which varies with time. The linear momentum of the particle in pound-seconds is given by the expression $\mathbf{G} = \frac{3}{2}(t^2 + 3)\mathbf{j} - \frac{2}{3}(t^3 - 4)\mathbf{k}$, where t is the time in seconds. Determine **F** and its magnitude for the instant when $t = 2$ sec.



$[\Sigma\mathbf{F} = \dot{\mathbf{G}}]$

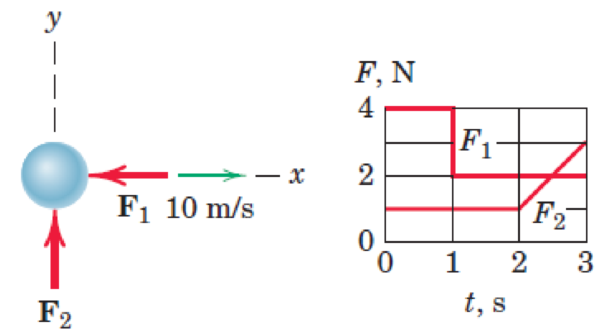
$$\mathbf{F} - 2\mathbf{k} = \frac{d}{dt}\left[\frac{3}{2}(t^2 + 3)\mathbf{j} - \frac{2}{3}(t^3 - 4)\mathbf{k}\right]$$
$$= 3t\mathbf{j} - 2t^2\mathbf{k}$$

$$\mathbf{F} = 2\mathbf{k} + 3(2)\mathbf{j} - 2(2^2)\mathbf{k} = 6\mathbf{j} - 6\mathbf{k} \text{ lb}$$

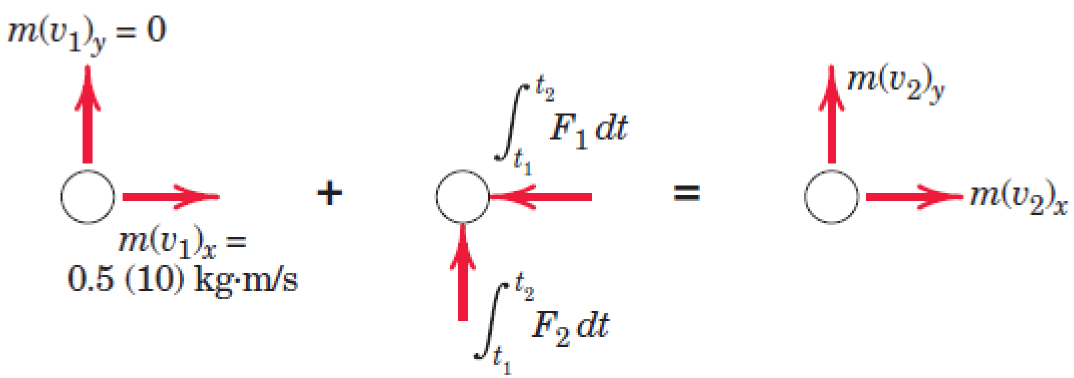
$$F = \sqrt{6^2 + 6^2} = 6\sqrt{2} \text{ lb}$$

SAMPLE PROBLEM 3/21

A particle with a mass of 0.5 kg has a velocity of 10 m/s in the x-direction at time $t = 0$. Forces $\mathbf{F}_1$ and $\mathbf{F}_2$ act on the particle, and their magnitudes change with time according to the graphical schedule shown. Determine the velocity $\mathbf{v}_2$ of the particle at the end of the 3-s interval. The motion occurs in the horizontal x-y plane.



Impulse-momentum diagrams:



$$\left[m(v_1)_x + \int_{t_1}^{t_2} \Sigma F_x \, dt = m(v_2)_x\right] \qquad 0.5(10) - [4(1) + 2(3-1)] = 0.5(v_2)_x$$

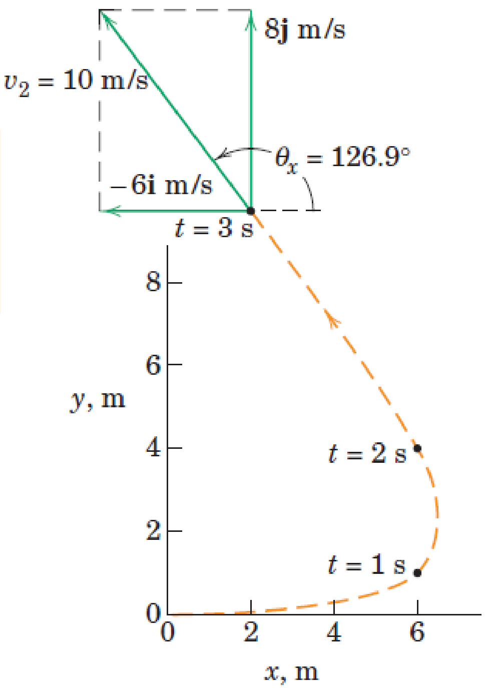
$$(v_2)_x = -6 \text{ m/s}$$

$$\left[m(v_1)_y + \int_{t_1}^{t_2} \Sigma F_y \, dt = m(v_2)_y\right] \qquad 0.5(0) + [1(2) + 2(3-2)] = 0.5(v_2)_y$$

$$(v_2)_y = 8 \text{ m/s}$$

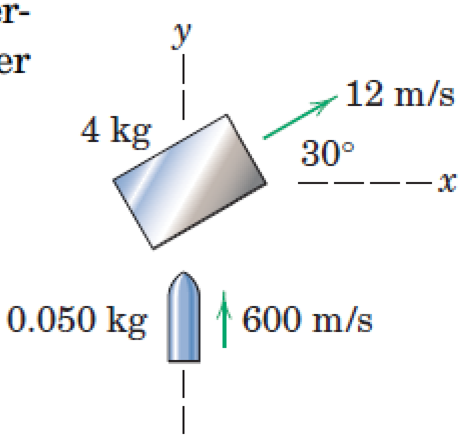
$$\mathbf{v}_2 = -6\mathbf{i} + 8\mathbf{j} \text{ m/s} \qquad \text{and} \qquad v_2 = \sqrt{6^2 + 8^2} = 10 \text{ m/s}$$

$$\theta_x = \tan^{-1}\frac{8}{-6} = 126.9^\circ$$



SAMPLE PROBLEM 3/23

The 50-g bullet traveling at 600 m/s strikes the 4-kg block centrally and is embedded within it. If the block slides on a smooth horizontal plane with a velocity of 12 m/s in the direction shown prior to impact, determine the velocity $\mathbf{v}_2$ of the block and embedded bullet immediately after impact.

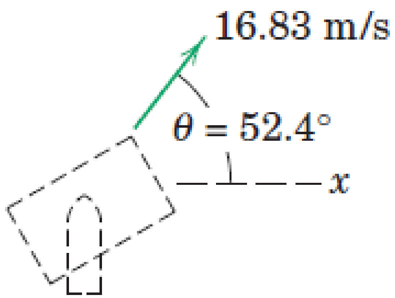


$$[\mathbf{G}_1 = \mathbf{G}_2] \quad 0.050(600\mathbf{j}) + 4(12)(\cos 30°\mathbf{i} + \sin 30°\mathbf{j}) = (4 + 0.050)\mathbf{v}_2$$

$$\mathbf{v}_2 = 10.26\mathbf{i} + 13.33\mathbf{j} \text{ m/s}$$

$$\left[v = \sqrt{v_x{}^2 + v_y{}^2}\right] \qquad v_2 = \sqrt{(10.26)^2 + (13.33)^2} = 16.83 \text{ m/s}$$

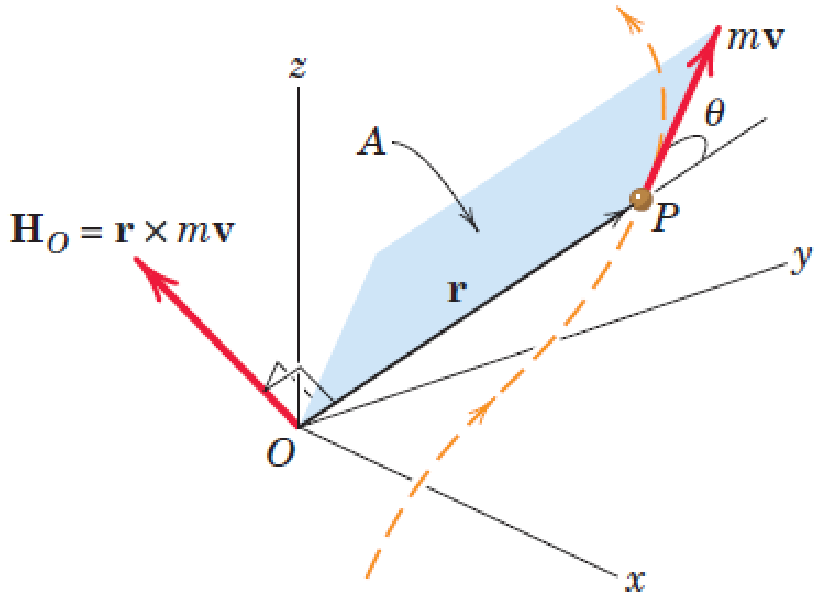
$$[\tan\theta = v_y/v_x] \qquad \tan\theta = \frac{13.33}{10.26} = 1.299 \qquad \theta = 52.4°$$



3/10 Angular Impulse and Angular Momentum

- A particle P of mass m moving along a curve in space.
- The velocity of the particle is v, and its linear momentum is G = mv.
- The moment of the linear momentum vector mv about the origin O is defined as the angular momentum H_O of P about O.

$$\mathbf{H}_O = \mathbf{r} \times m\mathbf{v}$$

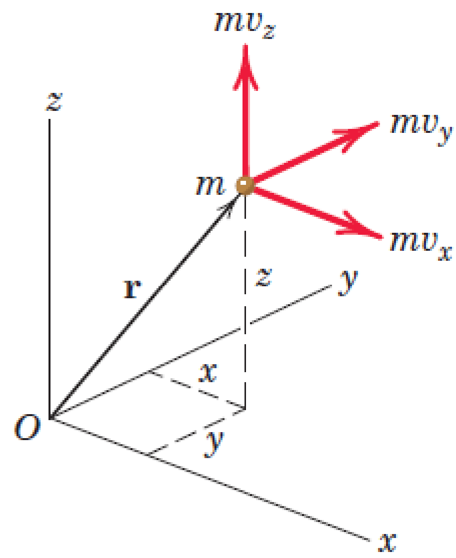


3/10 Angular Impulse and Angular Momentum

- The scalar components of angular momentum:

$$\mathbf{H}_O = \mathbf{r} \times m\mathbf{v} = m(v_z y - v_y z)\mathbf{i} + m(v_x z - v_z x)\mathbf{j} + m(v_y x - v_x y)\mathbf{k}$$

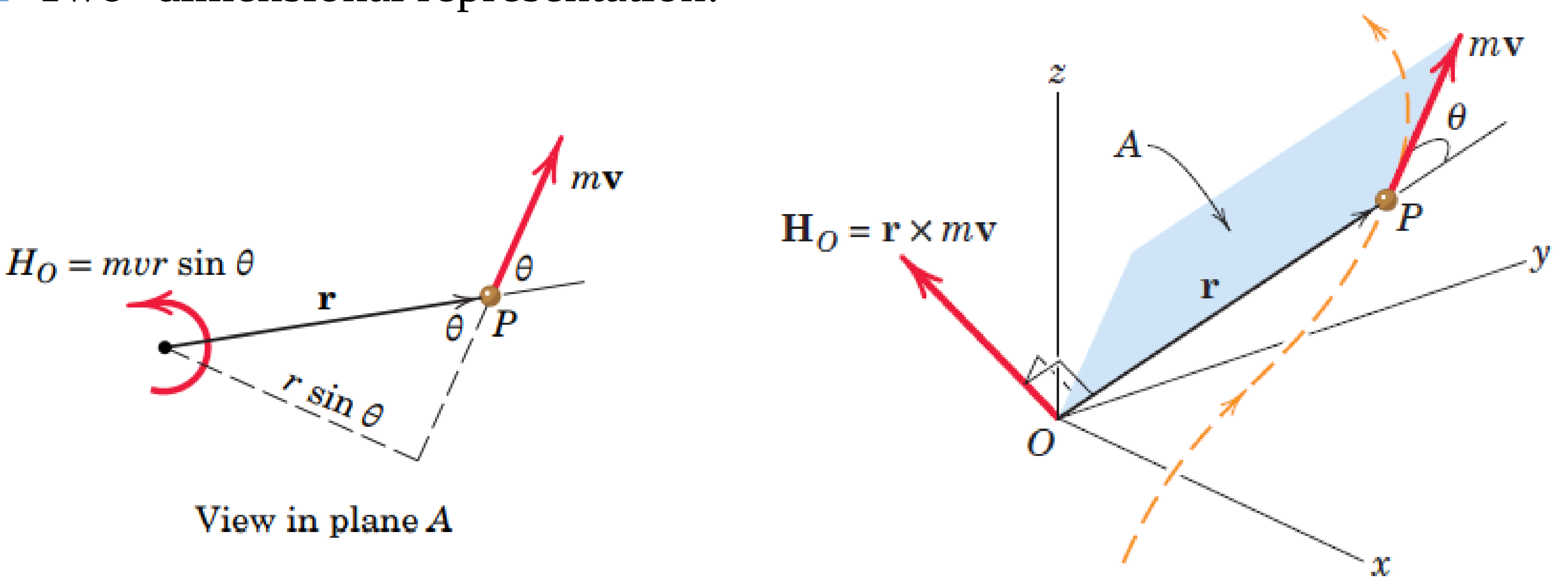
$$\mathbf{H}_O = m\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x & y & z \\ v_x & v_y & v_z \end{vmatrix}$$



$$H_x = m(v_z y - v_y z) \qquad H_y = m(v_x z - v_z x) \qquad H_z = m(v_y x - v_x y)$$

3/10 Angular Impulse and Angular Momentum

- Two- dimensional representation:



3/10 Angular Impulse and Angular Momentum

Rate of Change of Angular Momentum

- The moment of the forces acting on the particle P to its angular momentum relation:

$$\Sigma \mathbf{M}_O = \mathbf{r} \times \Sigma \mathbf{F} = \mathbf{r} \times m\dot{\mathbf{v}}$$

- H_O Differentiation leads to:

$$\dot{\mathbf{H}}_O = \dot{\mathbf{r}} \times m\mathbf{v} + \mathbf{r} \times m\dot{\mathbf{v}} = \mathbf{v} \times m\mathbf{v} + \mathbf{r} \times m\dot{\mathbf{v}}$$

- So:

$$\Sigma \mathbf{M}_O = \dot{\mathbf{H}}_O \qquad \Sigma M_{O_x} = \dot{H}_{O_x} \qquad \Sigma M_{O_y} = \dot{H}_{O_y} \qquad \Sigma M_{O_z} = \dot{H}_{O_z}$$

  - The moment about the fixed point O of all forces acting on m equals the time rate of change of angular momentum of m about O.

3/10 Angular Impulse and Angular Momentum

The Angular Impulse-Momentum Principle

- The total angular impulse on m about the fixed point O equals the corresponding change in angular momentum of m about O.

$$\int_{t_1}^{t_2} \Sigma \mathbf{M}_O \, dt = (\mathbf{H}_O)_2 - (\mathbf{H}_O)_1 = \Delta \mathbf{H}_O$$

$$(\mathbf{H}_O)_1 + \int_{t_1}^{t_2} \Sigma \mathbf{M}_O \, dt = (\mathbf{H}_O)_2$$

$$(H_{O_x})_1 + \int_{t_1}^{t_2} \Sigma M_{O_x} \, dt = (H_{O_x})_2$$

$$m(v_z y - v_y z)_1 + \int_{t_1}^{t_2} \Sigma M_{O_x} \, dt = m(v_z y - v_y z)_2$$

3/10 Angular Impulse and Angular Momentum

Plane-Motion Applications

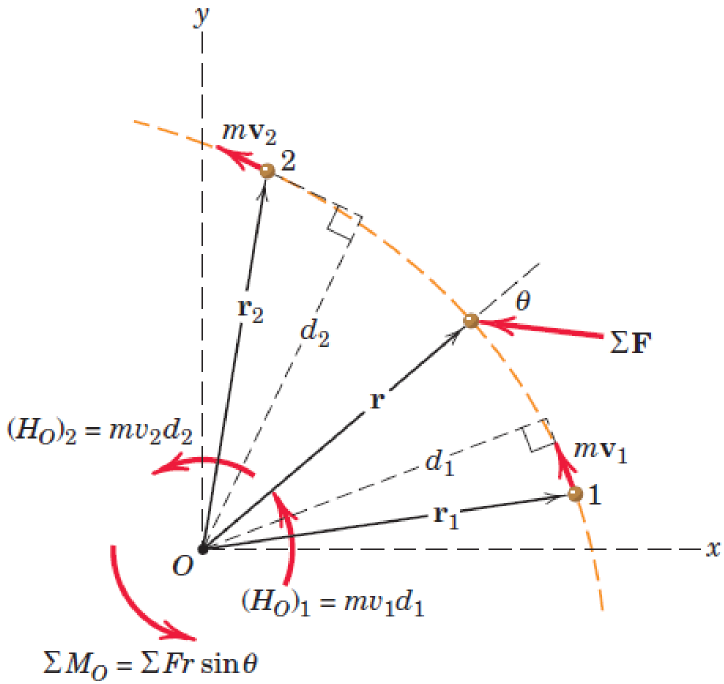
- Most of the applications can be analyzed as plane-motion problems where moments are taken about a single axis normal to the plane of motion.

$$(H_O)_1 = |\mathbf{r}_1 \times m\mathbf{v}_1| = mv_1d_1$$

$$(H_O)_2 = |\mathbf{r}_2 \times m\mathbf{v}_2| = mv_2d_2$$

$$(H_O)_1 + \int_{t_1}^{t_2} \Sigma M_O \, dt = (H_O)_2$$

$$mv_1d_1 + \int_{t_1}^{t_2} \Sigma Fr \sin\theta \, dt = mv_2d_2$$



3/10 Angular Impulse and Angular Momentum

Conservation of Angular Momentum

- If the resultant moment about a fixed point O of all forces acting on a particle is zero during an interval of time, its angular momentum H_O about that point remain constant.
- In this case, the angular momentum of the particle is said to be conserved.

$$\Delta \mathbf{H}_O = \mathbf{0} \quad \text{or} \quad (\mathbf{H}_O)_1 = (\mathbf{H}_O)_2$$

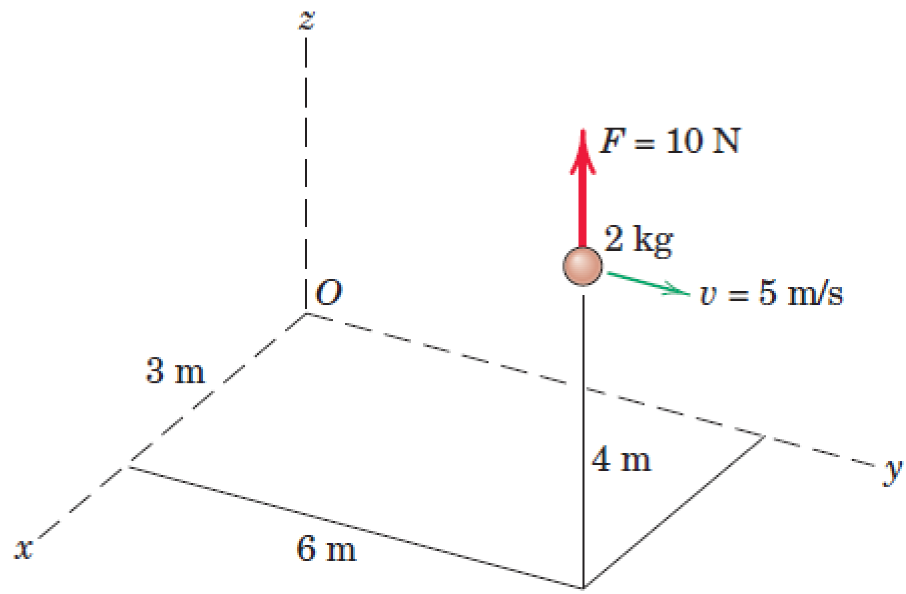
Principle of conservation of angular momentum

SAMPLE PROBLEM 3/24

A small sphere has the position and velocity indicated in the figure and is acted upon by the force F. Determine the angular momentum $\mathbf{H}_O$ about point O and the time derivative $\dot{\mathbf{H}}_O$.

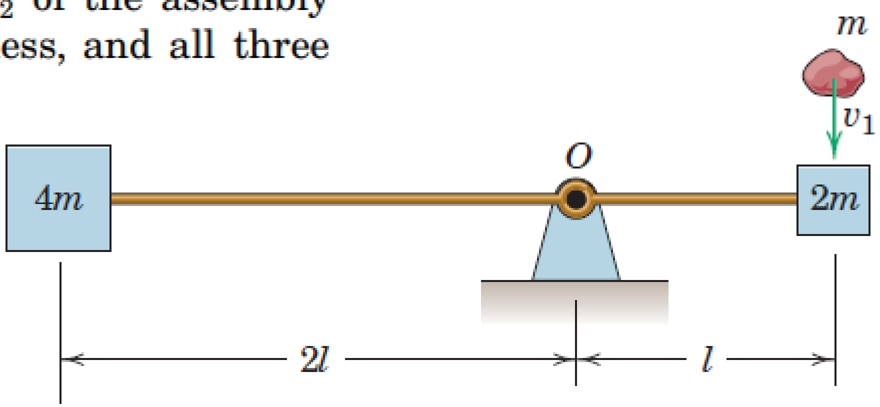
$$\mathbf{H}_O = \mathbf{r} \times m\mathbf{v}$$
$$= (3\mathbf{i} + 6\mathbf{j} + 4\mathbf{k}) \times 2(5\mathbf{j})$$
$$= -40\mathbf{i} + 30\mathbf{k}\ \text{N·m/s}$$

$$\dot{\mathbf{H}}_O = \mathbf{M}_O$$
$$= \mathbf{r} \times \mathbf{F}$$
$$= (3\mathbf{i} + 6\mathbf{j} + 4\mathbf{k}) \times 10\mathbf{k}$$
$$= 60\mathbf{i} - 30\mathbf{j}\ \text{N·m}$$



SAMPLE PROBLEM 3/26

The assembly of the light rod and two end masses is at rest when it is struck by the falling wad of putty traveling with speed v_1 as shown. The putty adheres to and travels with the right-hand end mass. Determine the angular velocity $\dot{\theta}_2$ of the assembly just after impact. The pivot at O is frictionless, and all three masses may be assumed to be particles.



$$(H_O)_1 = (H_O)_2$$

$$mv_1 l = (m + 2m)(l\dot{\theta}_2)l + 4m(2l\dot{\theta}_2)2l$$

$$\dot{\theta}_2 = \frac{v_1}{19l} \text{ CW}$$

SECTION D Special Applications

3/11 Introduction

- Several topics of specialized interest in particle kinetics:
  - 1. Impact
  - 2. Central-force motion
  - 3. Relative motion

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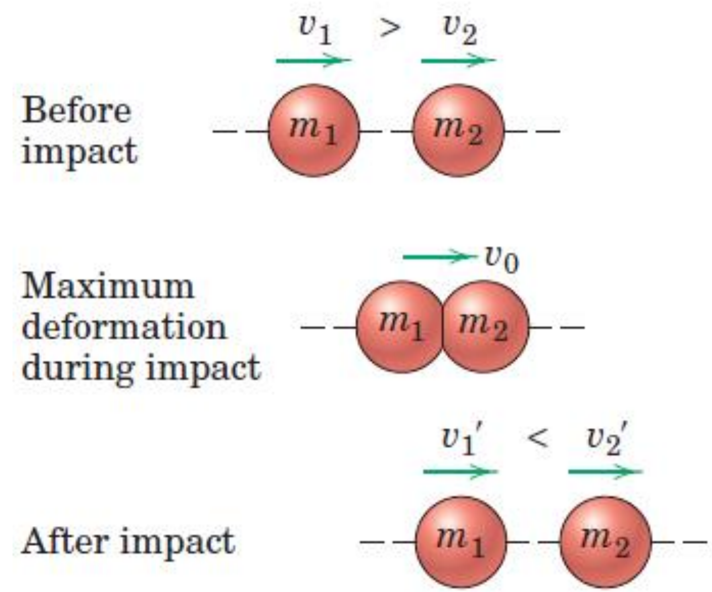
3/12 Impact

Direct Central Impact

- Collision of two spheres with collinear motion
  - Conservation of linear momentum:

$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2'$$



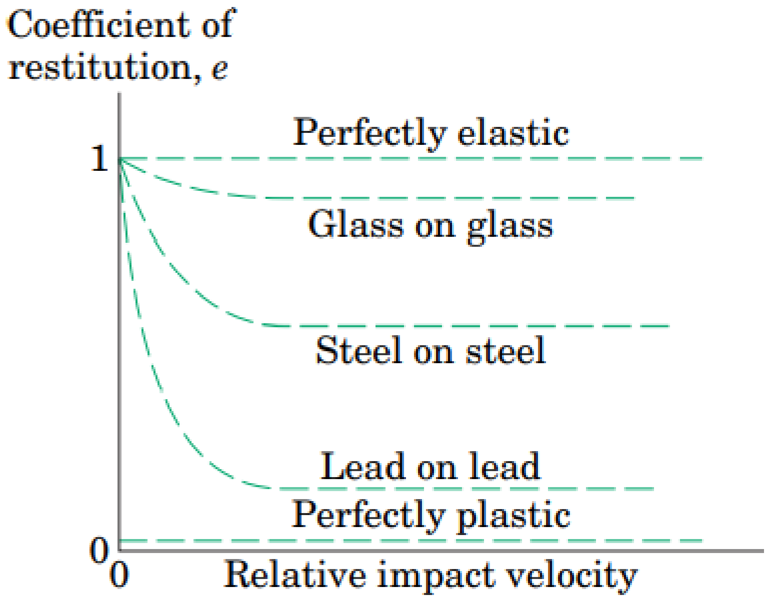
Coefficient of Restitution

$$e = \frac{v_2' - v_1'}{v_1 - v_2}$$

3/12 Impact

Energy Loss During Impact

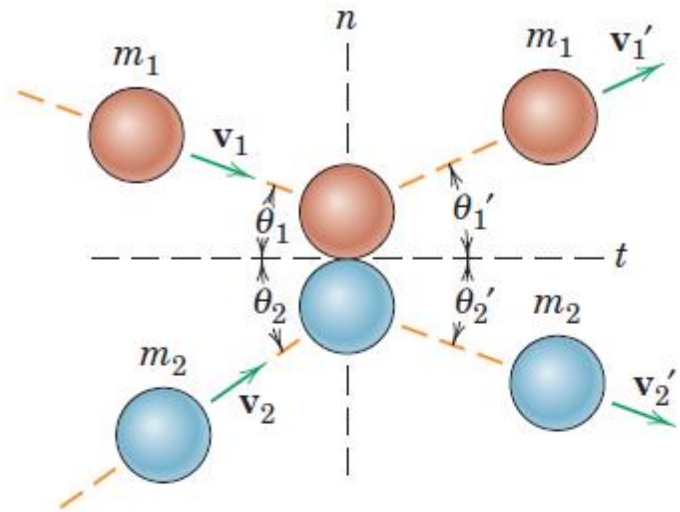
- Impact phenomena are almost always accompanied by energy loss, which may be calculated by subtracting the kinetic energy of the system just after impact from that just before impact.



3/12 Impact

Oblique Central Impact

- Tangent and normal directions



$$m_1(v_1)_t = m_1(v_1{}')_t$$

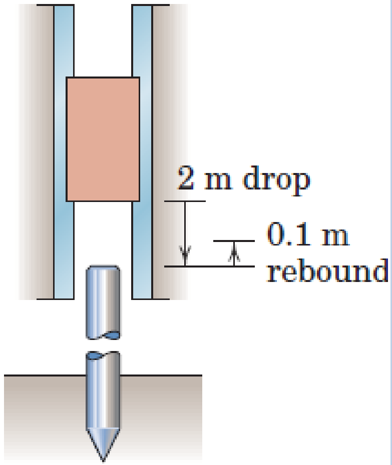
$$m_2(v_2)_t = m_2(v_2{}')_t$$

$$m_1(v_1)_n + m_2(v_2)_n = m_1(v_1{}')_n + m_2(v_2{}')_n$$

$$e = \frac{(v_2{}')_n - (v_1{}')_n}{(v_1)_n - (v_2)_n}$$

SAMPLE PROBLEM 3/28

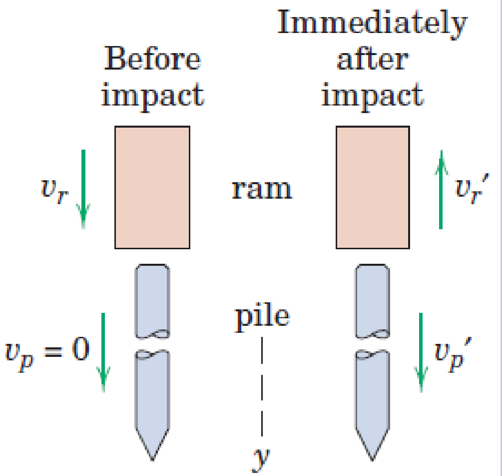
The ram of a pile driver has a mass of 800 kg and is released from rest 2 m above the top of the 2400-kg pile. If the ram rebounds to a height of 0.1 m after impact with the pile, calculate (*a*) the velocity v_p' of the pile immediately after impact, (*b*) the coefficient of restitution e, and (*c*) the percentage loss of energy due to the impact.

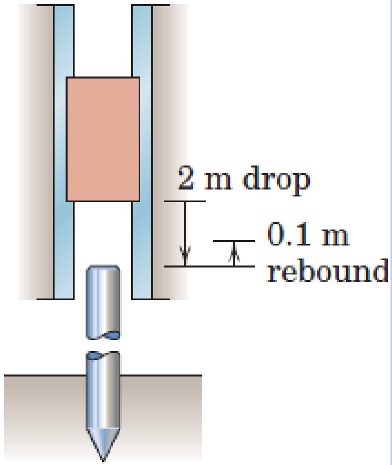


$$v = \sqrt{2gh}$$

$$v_r = \sqrt{2(9.81)(2)} = 6.26 \text{ m/s} \qquad v_r' = \sqrt{2(9.81)(0.1)} = 1.40 \text{ m/s}$$

$$800(6.26) + 0 = 800(-1.401) + 2400v_p' \qquad v_p' = 2.55 \text{ m/s}$$



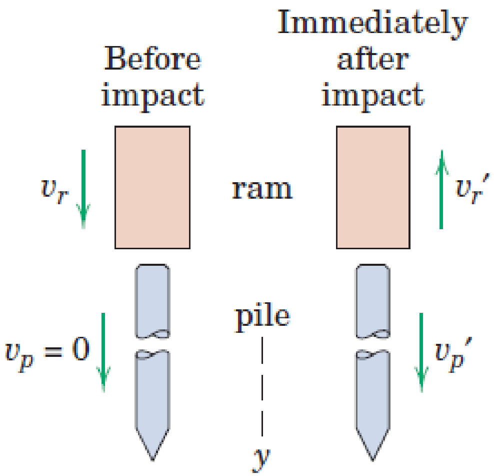


$$e = \frac{|\text{rel. vel. separation}|}{|\text{rel. vel. approach}|} \qquad e = \frac{2.55 + 1.401}{6.26 + 0} = 0.631$$

$$T = V_g = mgh = 800(9.81)(2) = 15\,700 \text{ J}$$

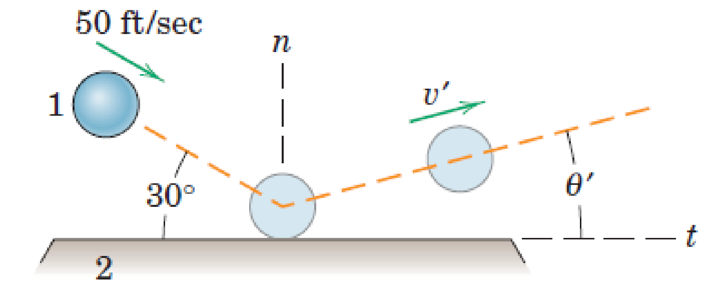
$$T' = \tfrac{1}{2}(800)(1.401)^2 + \tfrac{1}{2}(2400)(2.55)^2 = 8620 \text{ J}$$

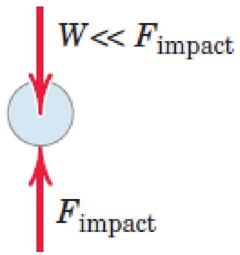
$$\frac{15\,700 - 8620}{15\,700}(100) = 45.1\%$$



SAMPLE PROBLEM 3/29

A ball is projected onto the heavy plate with a velocity of 50 ft/sec at the 30° angle shown. If the effective coefficient of restitution is 0.5, compute the rebound velocity v' and its angle θ'.





$$e = \frac{(v_2')_n - (v_1')_n}{(v_1)_n - (v_2)_n} \qquad 0.5 = \frac{0 - (v_1')_n}{-50 \sin 30° - 0}$$

$$(v_1')_n = 12.5 \text{ ft/sec}$$

$$m(v_1)_t = m(v_1')_t \qquad (v_1')_t = (v_1)_t = 50 \cos 30° = 43.3 \text{ ft/sec}$$

$$v' = \sqrt{(v_1')_n{}^2 + (v_1')_t{}^2} = \sqrt{12.5^2 + 43.3^2} = 45.1 \text{ ft/sec}$$

$$\theta' = \tan^{-1}\left(\frac{(v_1')_n}{(v_1')_t}\right) = \tan^{-1}\left(\frac{12.5}{43.3}\right) = 16.10°$$

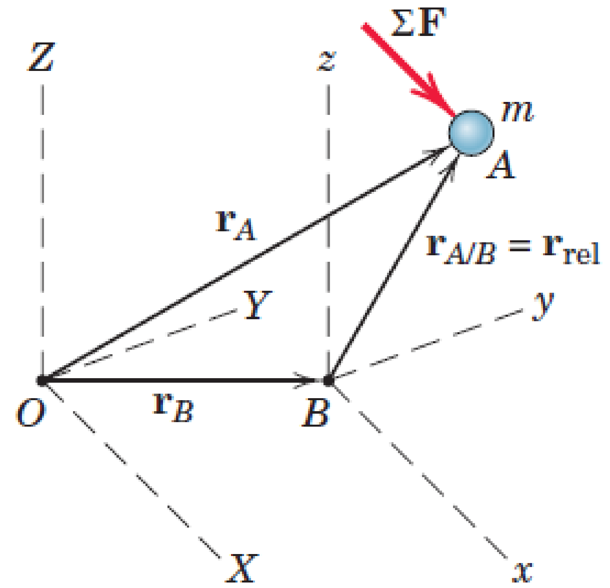
3/14 Relative Motion

Relative-Motion Equation

- A particle **A** of mass m whose motion is observed from a set of axes x-y-z which translate with respect to a fixed reference frame X-Y-Z.

$$\mathbf{a}_A = \mathbf{a}_B + \mathbf{a}_{\text{rel}}$$

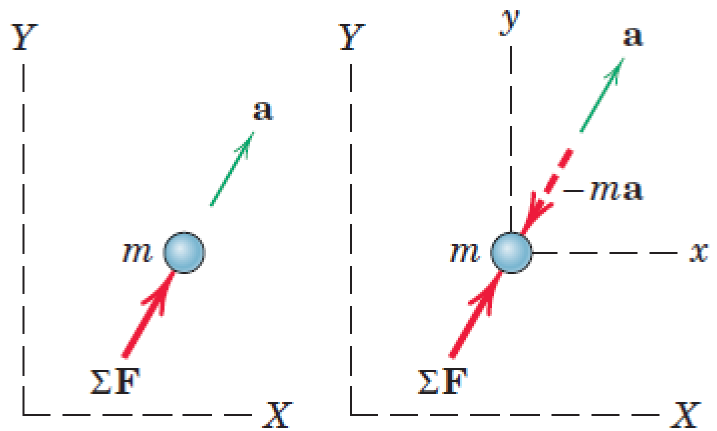
$$\Sigma \mathbf{F} = m(\mathbf{a}_B + \mathbf{a}_{\text{rel}})$$



3/14 Relative Motion

D'Alembert's Principle

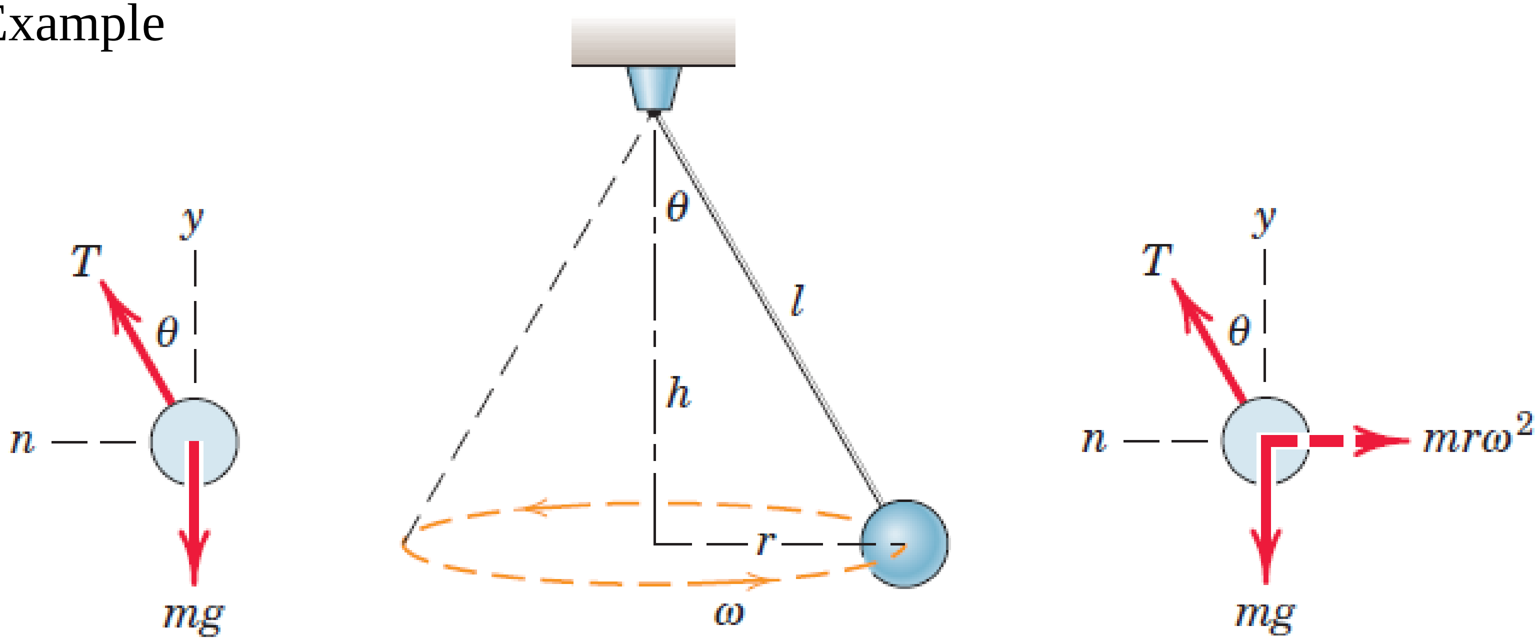
- The particle acceleration we measure from a fixed set of axes X-Y-Z, is its absolute acceleration a. In this case the familiar relation ΣF = ma applies.
- When we observe the particle from a moving system x-y-z attached to the particle, the particle necessarily appears to be at rest or in equilibrium in x-y-z.
- Thus, the observer who is accelerating with x-y-z concludes that a force −ma acts on the particle to balance ΣF.



3/14 Relative Motion

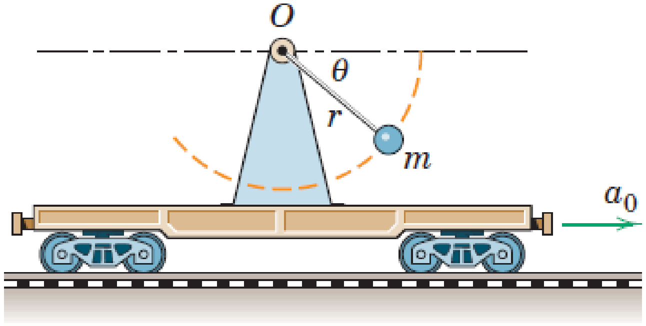
D'Alembert's Principle

- Example



SAMPLE PROBLEM 3/32

A simple pendulum of mass m and length r is mounted on the flatcar, which has a constant horizontal acceleration a_0 as shown. If the pendulum is released from rest relative to the flatcar at the position $\theta = 0$, determine the expression for the tension T in the supporting light rod for any value of θ. Also find T for $\theta = \pi/2$ and $\theta = \pi$.



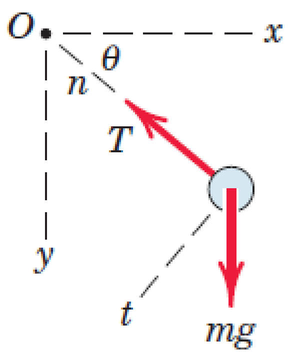
$$\mathbf{a} = \mathbf{a}_0 + \mathbf{a}_{\text{rel}}$$

$[\Sigma F_t = ma_t]$ $$mg\cos\theta = m(r\ddot{\theta} - a_0\sin\theta)$$

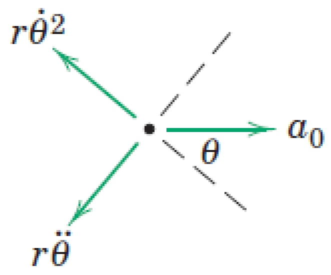
$$r\ddot{\theta} = g\cos\theta + a_0\sin\theta$$

$[\dot{\theta}\, d\dot{\theta} = \ddot{\theta}\, d\theta]$ $$\int_0^{\dot{\theta}} \dot{\theta}\, d\dot{\theta} = \int_0^{\theta} \frac{1}{r}(g\cos\theta + a_0\sin\theta)\, d\theta$$

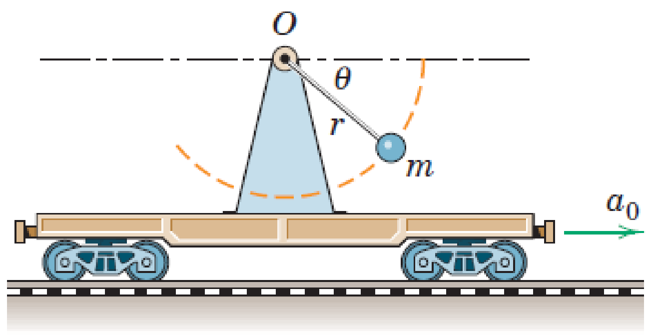
$$\frac{\dot{\theta}^2}{2} = \frac{1}{r}[g\sin\theta + a_0(1 - \cos\theta)]$$



Free-body diagram



Acceleration components



$$[\Sigma F_n = ma_n] \qquad T - mg \sin\theta = m(r\dot{\theta}^2 - a_0 \cos\theta)$$

$$= m[\,2g \sin\theta + 2a_0(1 - \cos\theta) - a_0 \cos\theta\,]$$

$$T = m[\,3g \sin\theta + a_0(2 - 3\cos\theta)]$$

$$T_{\pi/2} = m[3g(1) + a_0\,(2 - 0)] = m(3g + 2a_0)$$

$$T_{\pi} = m[3g(0) + a_0\,(2 - 3[-1])] = 5ma_0$$