



دانشگاه سمنان

Semnan University  
Faculty of Mechanical Engineering

دانشکده مهندسی مکانیک

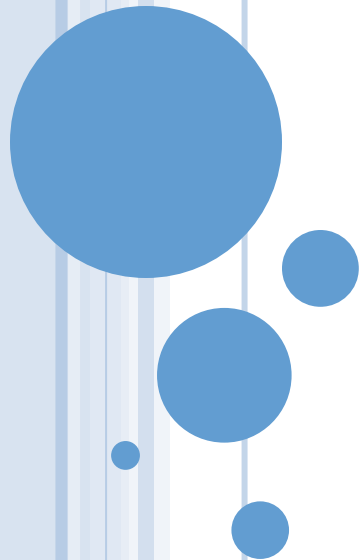


دانشکده مهندسی مکانیک

درس طراحی سیستم های شاسی  
خودرو

**VEHICLE CHASSIS  
SYSTEMS DESIGN**

*Chapter 5 – Ride  
Class Lecture*



□ CONTENTS:

- ❖ Chapter 1: Introduction
- ❖ Chapter 2: Accelerating Performance
- ❖ Chapter 3: Braking Performance
- ❖ Chapter 4: Road Loads
- ❖ Chapter 5: **Ride**
- ❖ Chapter 6: Cornering
- ❖ Chapter 7: Suspension
- ❖ Chapter 8: Steering System
- ❖ Chapter 9: Roll-over



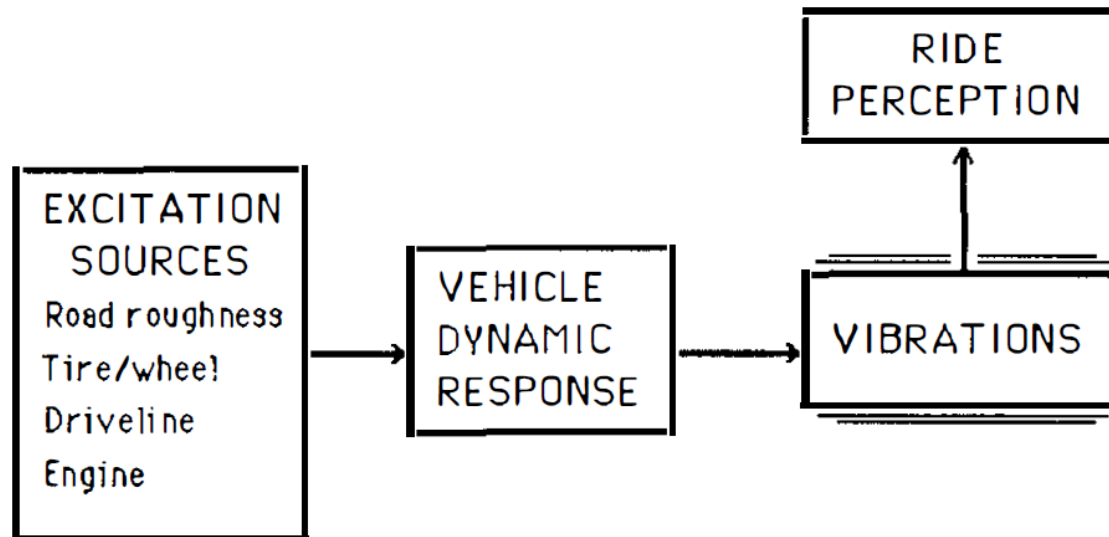
## INTRODUCTION

- ❑ Automobiles travel at high speed, and as a consequence experience a broad spectrum of vibrations.
- ❑ These are transmitted to the passengers either by tactile, visual, or aural paths.
- ❑ The term "ride" is commonly used in reference to tactile and visual vibrations, while the aural vibrations are categorized as "noise."
- ❑ Alternatively, the spectrum of vibrations may be divided up according to frequency and classified as ride (0-25 Hz) and noise (25-20,000Hz).
- ❑ The vibration environment is one of the more important criteria by which people judge the design and construction "quality" of a car.



## INTRODUCTION

- Understanding ride involves the study of three main topics:
  - ❖ Ride excitation sources
  - ❖ Basic mechanics of vehicle vibration response
  - ❖ Human perception and tolerance of vibrations



## EXCITATION SOURCES

- ❑ There are multiple sources from which vehicle ride vibrations may be excited.
  
- ❑ These generally fall into two classes:
  - ❖ Road roughness
  - ❖ On-board sources (arise from rotating components)
    - ✓ Tire/wheel assemblies
    - ✓ The driveline
    - ✓ The engine



## EXCITATION SOURCES

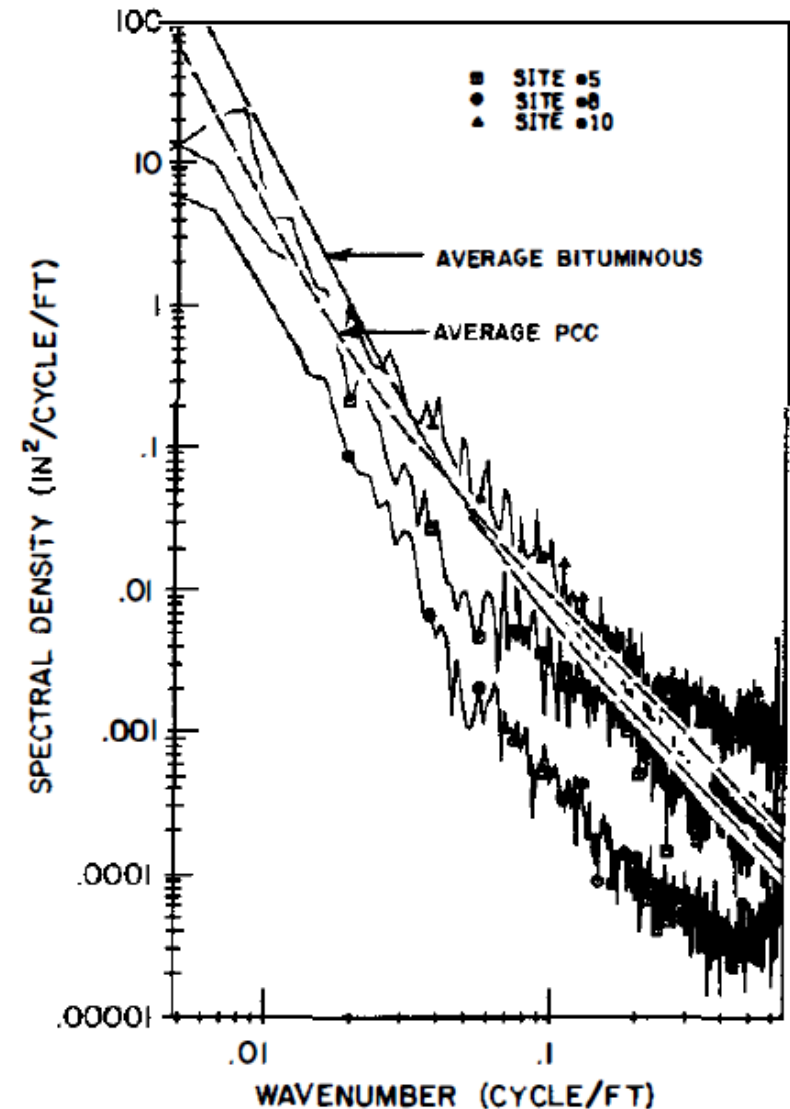
### ❑ Road Roughness

- ❖ Roughness is described by the elevation profile along the wheel tracks over which the vehicle passes.
- ❖ Road profiles fit the general category of "broad-band random signals" and, hence, can be described either by the profile itself or its statistical properties.
- ❖ One of the most useful representations is the Power Spectral Density (PSD) function.
- ❖ Like any random signal, the elevation profile measured over a length of road can be decomposed by the Fourier Transform process.
- ❖ A plot of the amplitudes versus spatial frequency is the PSD.



## EXCITATION SOURCES

- ❖ Road elevation profiles measurement:
  - ✓ Performing close interval rod and level surveys
  - ✓ High-speed profilometers
  
- ❖ Although many ride problems are peculiar to a specific road, or road type, the notion of "average" road properties can often be helpful in understanding the response of a vehicle to road roughness.



## EXCITATION SOURCES

- The PSD for average road properties:

$$G_z(v) = G_0 [1 + (v_0/v)^2] / (2\pi v)^2$$

$G_z(v)$  = PSD amplitude (feet<sup>2</sup>/cycle/foot)

$v$  = Wavenumber (cycles/ft)

$G_0$  = Roughness magnitude parameter (roughness level)

=  $1.25 \times 10^5$  for rough roads

=  $1.25 \times 10^6$  for smooth roads

$v_0$  = Cutoff wavenumber

= .05 cycle/foot for bituminous roads

= .02 cycle/foot for PCC roads

- ❖ The above equation in combination with a random number sequence provides a very useful method to generate road profiles with random roughness





## EXCITATION SOURCES

- ❑ The roughness in a road is the deviation in elevation seen by a vehicle as it moves along the road. That is, the roughness acts as a vertical displacement input to the wheels, thus exciting ride vibrations.
- ❑ Yet the most common and meaningful measure of ride vibration is the acceleration produced.
- ❑ Therefore, for the purpose of understanding the dynamics of ride, the roughness should be viewed as an acceleration input at the wheels.



## EXCITATION SOURCES

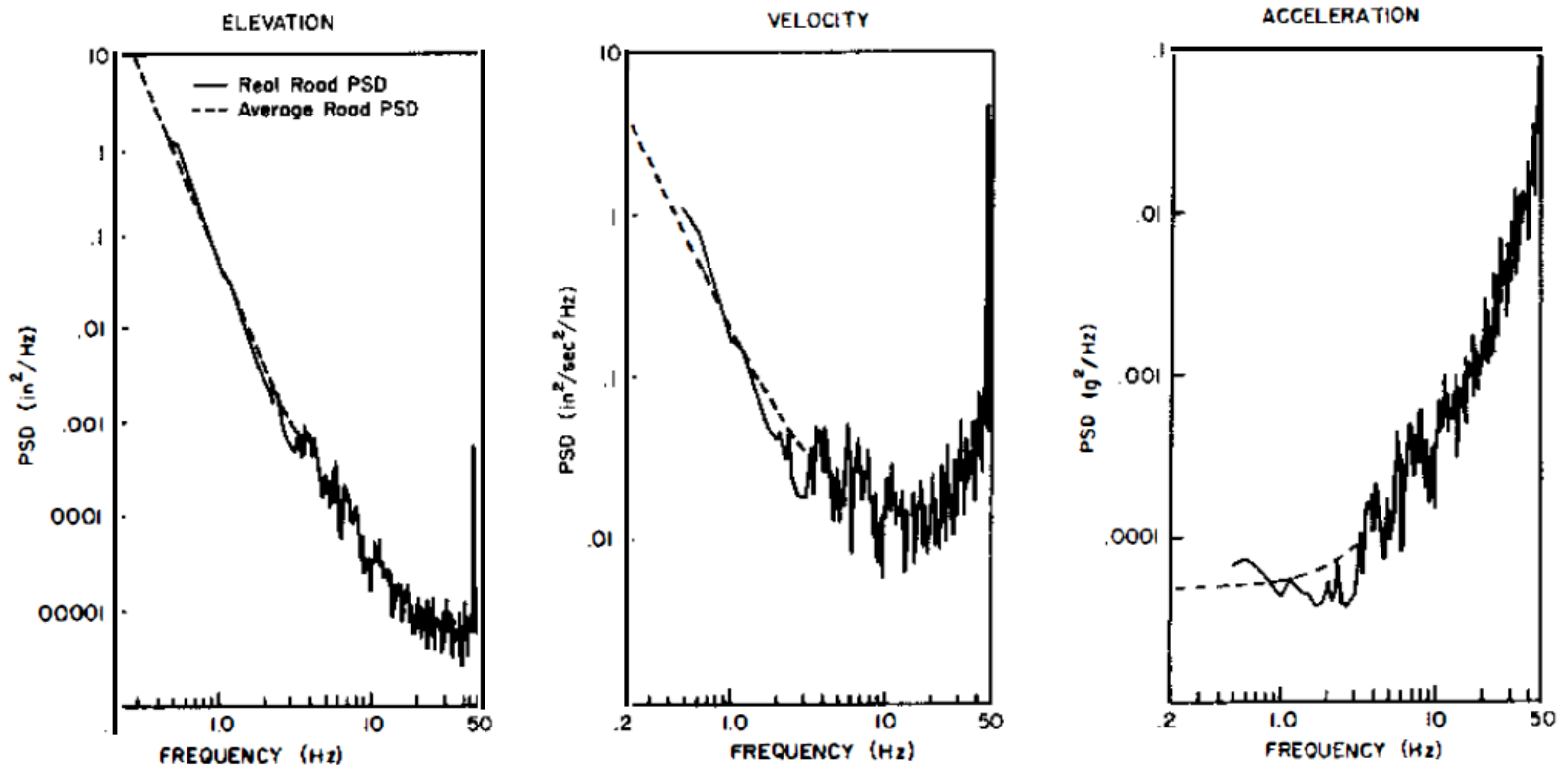
- Two steps are involved:

- ❖ First a speed of travel must be assumed such that the elevation profile is transformed to displacement as a function of time.
- ❖ Then, it may be differentiated once to obtain the velocity of the input at the wheels, and a second time to obtain an acceleration.



## EXCITATION SOURCES

- ❖ Elevation, velocity and acceleration PSDs of road roughness input to a vehicle traveling at 50 mph on a real and average road



## EXCITATION SOURCES

- ❖ Note that the acceleration spectrum has a relatively constant amplitude at low frequency, but begins increasing rapidly above 1 Hz such that it is an order of magnitude greater at 10 Hz.
- ❖ Viewed as an acceleration input, road roughness presents its largest inputs to the vehicle at high frequency, and thus has the greatest potential to excite high-frequency ride vibrations unless attenuated accordingly by the dynamic properties of the vehicle.
- ❖ As will be seen, the vehicle's attenuation of this high-frequency input is an important aspect of the "ride isolation" behavior obtained via the primary suspension commonly used on highway vehicles today



## EXCITATION SOURCES

- Considering a simple sine wave representation of roughness:

$$Z_r = A \sin (2\pi\nu X)$$

$Z_r$  = Profile elevation

$A$  = Sine wave amplitude

$\nu$  = Wavenumber (cycles/foot)

$X$  = Distance along the road

- The distance,  $X$ , equals the velocity,  $V$ , times the time of travel,  $t$ . so:

$$Z_r = A \sin (2\pi\nu Vt)$$

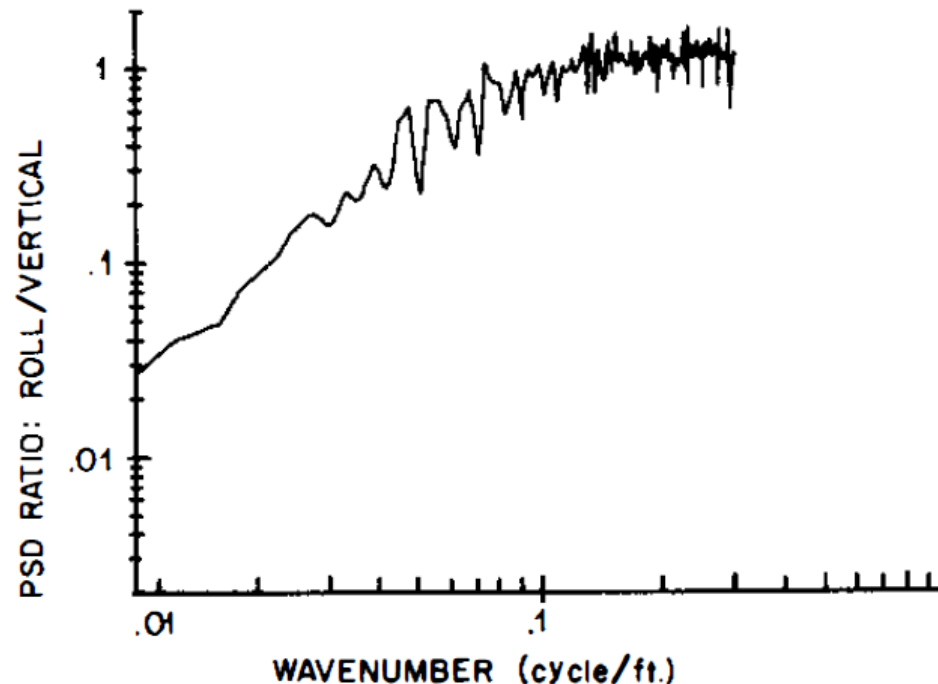
$$\ddot{Z}_r = - (2\pi\nu V)^2 A \sin (2\pi\nu Vt)$$

- ❖ Thus as an acceleration the amplitude coefficient contains the velocity squared.



## EXCITATION SOURCES

- ❑ road profile points in the left and right wheel tracks are usually averaged before processing to obtain the PSD.
- ❑ The difference in elevation between the left and right road profile points represents a roll excitation input to the vehicle.



## EXCITATION SOURCES

### ❑ As an illustration:

- ❖ Consider a vehicle with a roll natural frequency of 1.0 Hz traveling at 60 mph (88 ft/sec).
- ❖ Roll excitation in the road at the 88 ft wavelength (0.011 cycle/ft) will therefore directly excite roll motions.
- ❖ However, the roll amplitude at this wavenumber is only 10 percent of the vertical input, so the vehicle passengers will be more conscious of bounce vibrations rather than roll.



## EXCITATION SOURCES

- ❖ At a low speed, for example at 6 mph, a 1.0 Hz roll resonant frequency would be excited by input from wavenumbers on the order of 0.1 cycle/ft at which the roll and vertical inputs are essentially equal in magnitude.
- ❖ Thus roll and bounce motions would be approximately equal as well.
- ❖ The common case where this is observed is in off-road operation of 4x4 vehicles where the exaggerated ride vibrations are often composed of roll as well as bounce motions.





## TIRE/WHEEL ASSEMBLY

- ❑ Ideally, the tire/wheel assembly is soft and compliant in order to absorb road bumps as part of the ride isolation system. At the same time, it ideally runs true without contributing any excitation to the vehicle.
  
- ❑ In practice, the imperfections in the manufacture of tires, wheels, hubs, brakes and other parts of the rotating assembly may result in nonuniformities of three major types:
  - ❖ Mass imbalance
  - ❖ Dimensional variations
  - ❖ Stiffness variations



## TIRE/WHEEL ASSEMBLY

- ❑ These nonuniformities all combine in a tire/wheel assembly causing it to experience variations in the forces and moments at the ground as it rolls.
- ❑ These in turn are transmitted to the axle of the vehicle and act as excitation sources for ride vibrations.
- ❑ The force variations may be in the vertical (radial) direction, longitudinal (tractive) direction, or the lateral direction.
- ❑ The moment variations in the directions of the overturning moment, aligning torque, and rolling resistance moment generally are not significant as sources of ride excitation, although they can contribute to steering system vibrations.



## TIRE/WHEEL ASSEMBLY

- The imbalance force:

$$F_i = (m r) \omega^2$$

$F_i$  = Imbalance force

$m r$  = The imbalance magnitude (mass times radius)

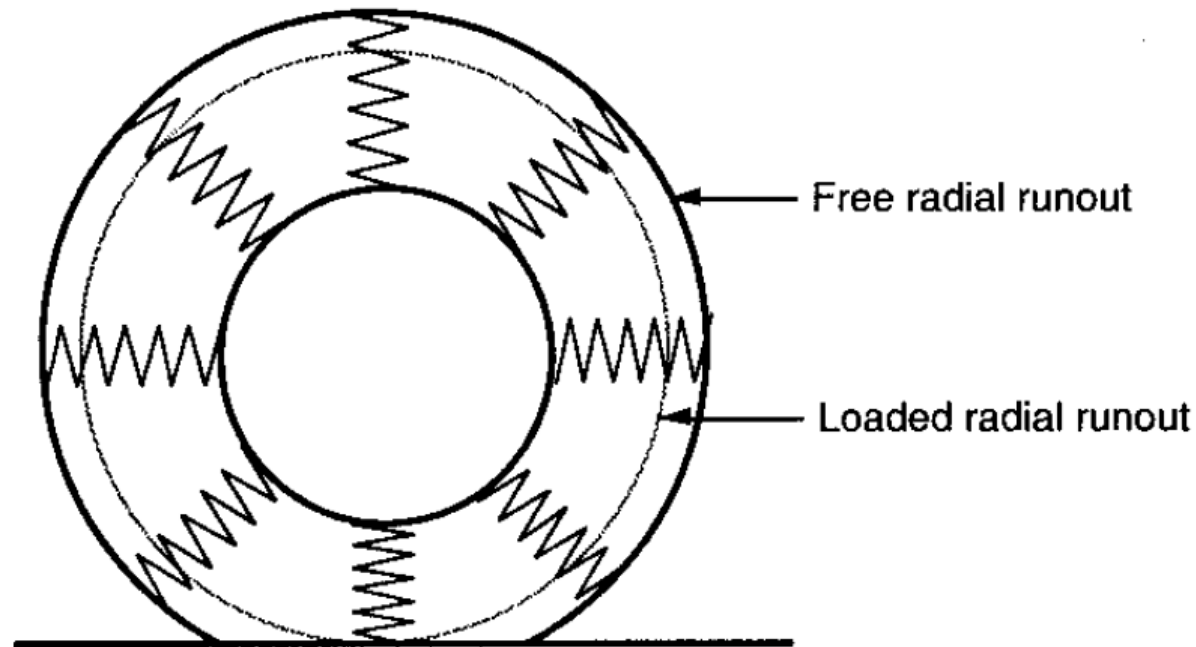
$\omega$  = the rotational speed (radians/second)

- ❖ A nonuniform and asymmetric mass distribution along the axis of rotation causes a dynamic imbalance.
- ❖ Dynamic imbalance creates a rotating torque on the wheel, appearing as variations in overturning moment and aligning torque at the wheel rotational frequency.
- ❖ Dynamic imbalance is most important on steered wheels which may experience steering vibrations as a result of the excitation.



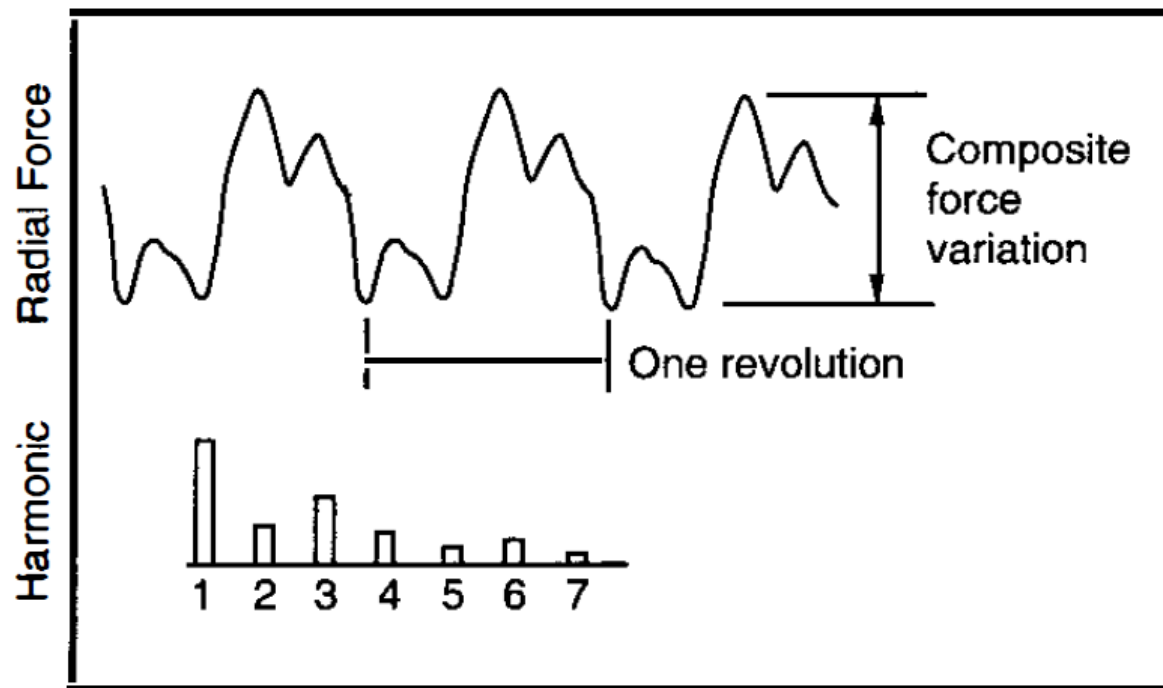
## TIRE/WHEEL ASSEMBLY

- The tire, being an elastic body analogous to an array of radial springs, may exhibit variations in stiffness about its circumference.



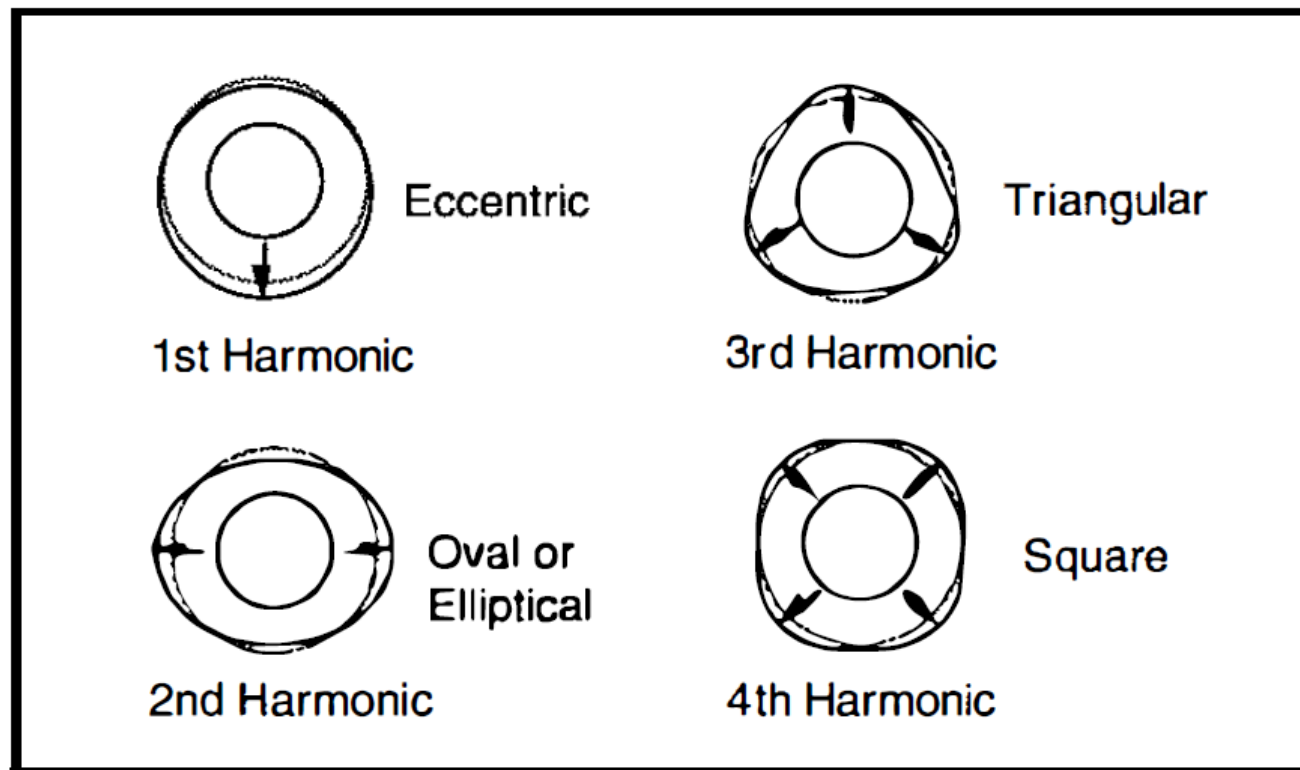
## TIRE/WHEEL ASSEMBLY

- The significant effect of the nonuniformities in a tire/wheel assembly is the generation of excitation forces and displacements at the axle of the vehicle as the wheel rotates.



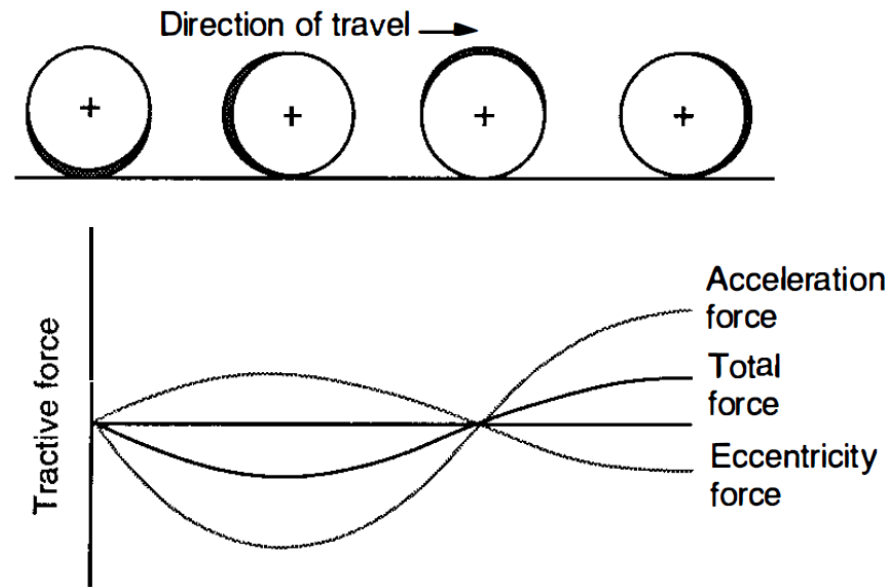
## TIRE/WHEEL ASSEMBLY

- The various harmonics of radial non uniformities in a tire/wheel assembly are functionally equivalent to imperfections in the shape



## TIRE/WHEEL ASSEMBLY

- Tractive force variations arise from dimensional and stiffness nonuniformities.



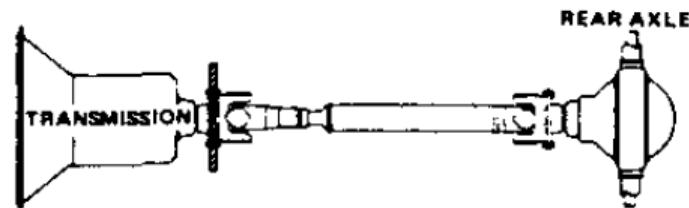
- Lateral force variations may arise from nonuniformities in the tire and will cause wobble.

## DRIVELINE EXCITATION

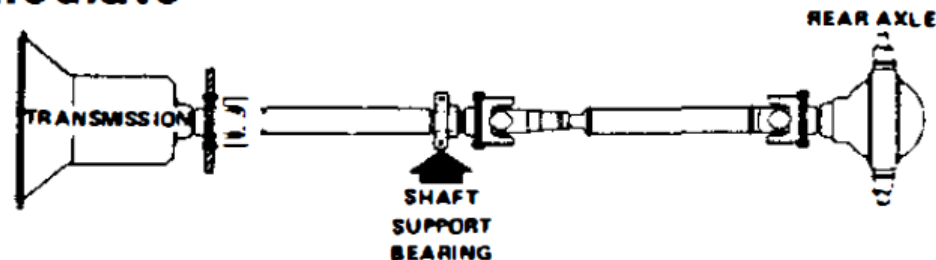
- ❑ The third major source of excitation arises from the rotating driveline.
- ❑ The engine/transmission package will be treated separately.
- ❑ The driveshaft with its spline and universal joints has the most potential for exciting ride vibrations.

### WHEELBASE

Short



Intermediate





## DRIVELINE EXCITATION

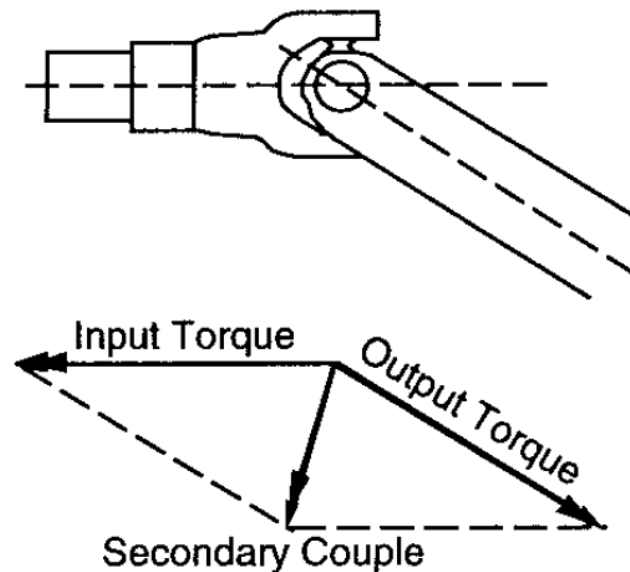
- Excitations to the vehicle arise directly from two sources:
  - ❖ Mass imbalance of the driveshaft hardware
    - ✓ 1) Asymmetry of the rotating parts
    - ✓ 2) The shaft may be off-center on its supporting flange or end yoke
    - ✓ 3) The shaft may not be straight
    - ✓ 4) Running clearances may allow the shaft to run off center
    - ✓ 5) The shaft is an elastic member and may deflect
  - ❖ Secondary couples, or moments, imposed on the driveshaft due to angulation of the cross-type universal joints
- Because the shaft is elastic it may bend in response to the imbalance force allowing additional asymmetry, and an increase in the "dynamic" imbalance.



## DRIVELINE EXCITATION

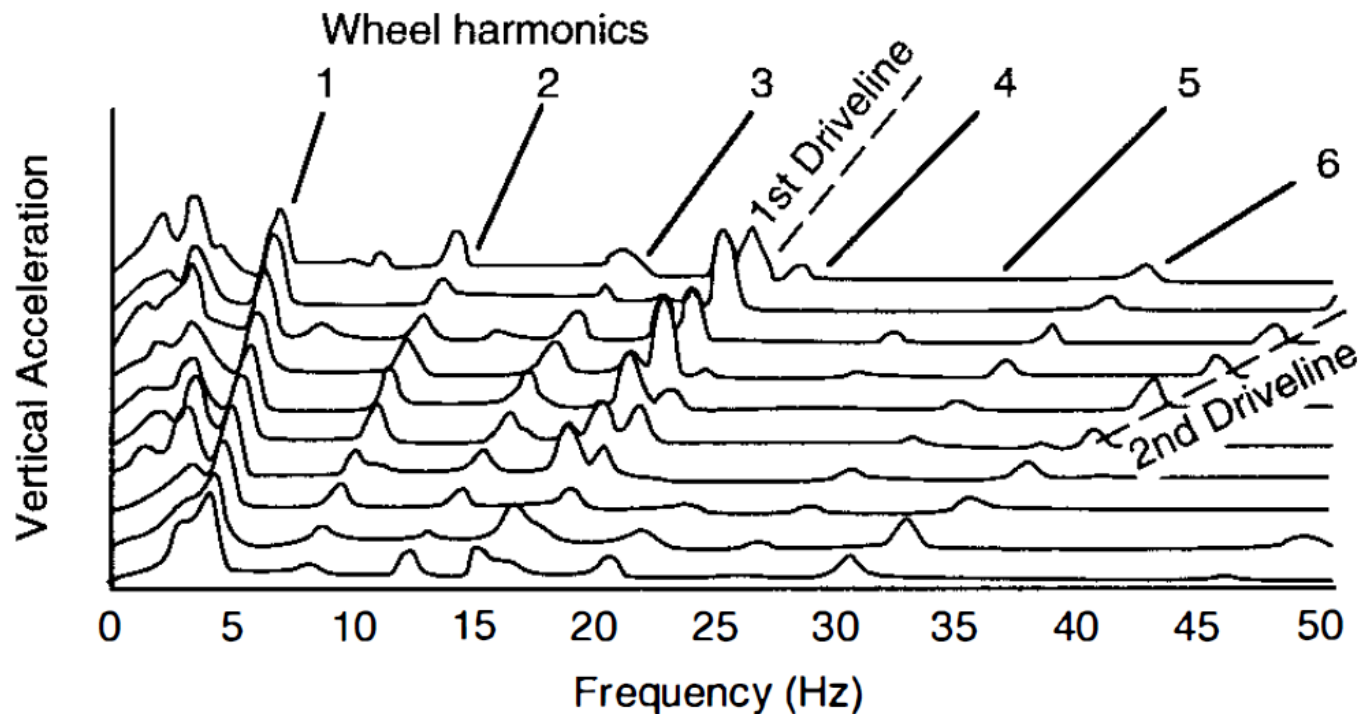
### ❖ Secondary couples

- ✓ The use of universal joints in a driveline opens the way for generation of ride excitation forces when they are operated at an angle, due to the secondary couple that is produced.
- ✓ The magnitude and direction of the secondary couple can be determined by a simple vector summation of torques on the universal joint.



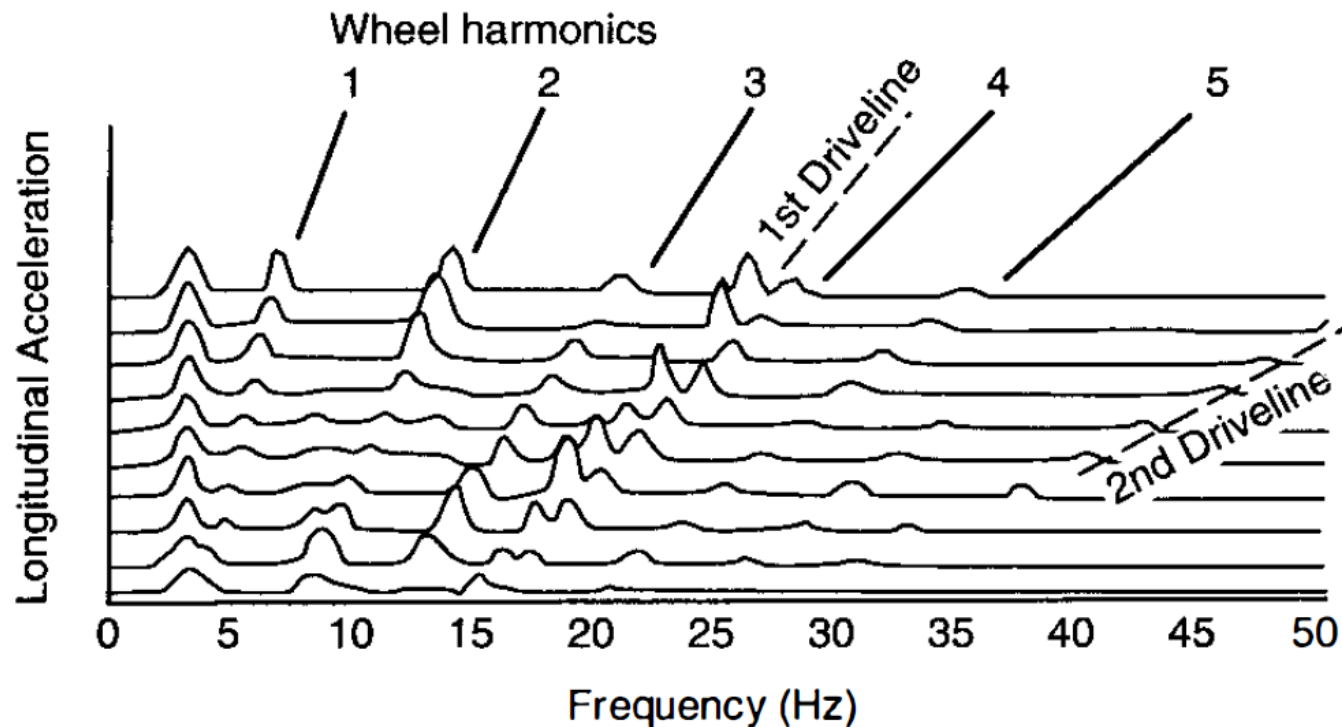
## DRIVELINE EXCITATION

- Spectral map of vibrations arising from drive line and tire/wheel nonuniformities



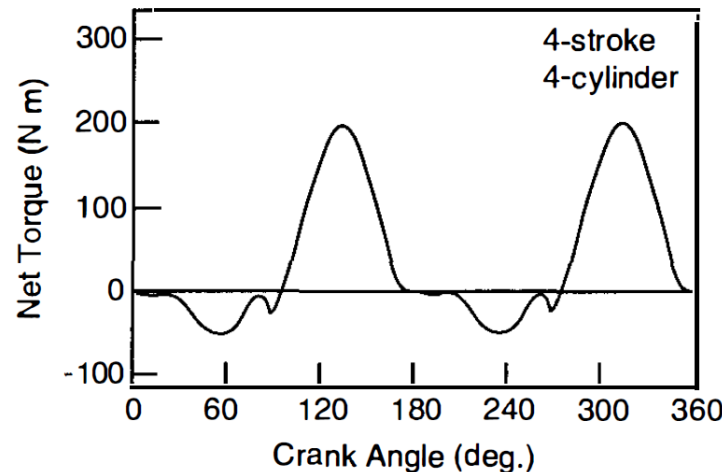
## DRIVELINE EXCITATION

- Spectral map of vibrations arising from drive line and tire/wheel nonuniformities



## ENGINE/TRANSMISSION

- ❖ The fact that it rotates and delivers torque to the driveline opens the possibility that it may be a source for vibration excitation on the vehicle.
- ❖ Further, the mass of the engine in combination with that of the transmission is a substantial part in the chassis, and, if used correctly, can act as a vibration absorber.
- ❖ Piston engines deliver power by a cyclic process; thus the torque delivered by the engine is not constant in magnitude

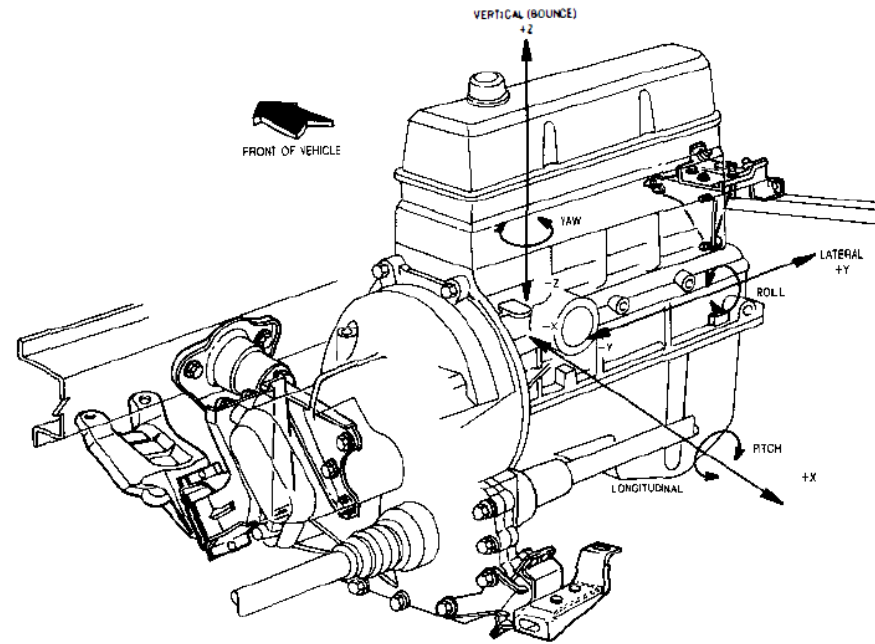


## ENGINE/TRANSMISSION

- ❑ The flywheel acts as an inertial damper along with the inertias and compliances in the transmission.
- ❑ Thus the torque output to the driveshaft consists of a steady-state component plus superimposed torque variations.
- ❑ Those torque variations acting through the drive line may result in excitation forces on the vehicle

## ENGINE/TRANSMISSION

- Because of compliance in the engine/transmission mounts, the system will vibrate in six directions.
- Of all the directions of motion, the most important to vibration is the engine roll direction.



## ENGINE/TRANSMISSION

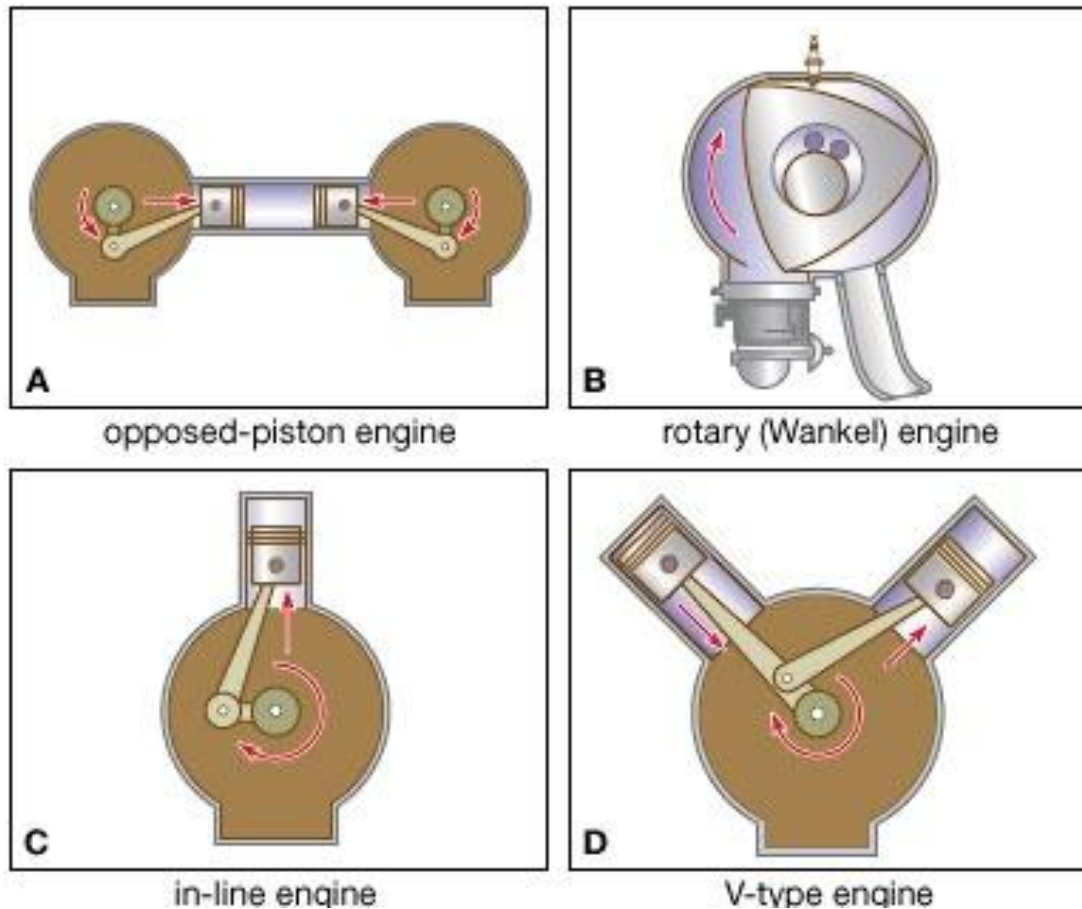
- ❑ A key to isolating these excitations from the vehicle body is to design a mounting system with a roll axis that aligns with the engine inertial roll axis, and provide a resonance about this axis at a frequency that is below the lowest firing frequency.
- ❑ The worst-case problem is isolation of idle speed torque variations for a four-cylinder engine with the transmission in drive, which may have a firing frequency of 20 Hz or below.
- ❑ Therefore, successful isolation requires a roll axis resonance of 10 Hz or below.





## ENGINE/TRANSMISSION

- Balance conditions for more commonly used engine configurations:



## ENGINE/TRANSMISSION

- Balance conditions for more commonly used engine configurations:
  - ❖ 1 ) Four-cylinder, inline
    - ✓ Vertical force at twice engine rotational frequency; can be balanced with counter-rotating shafts.
  - ❖ 2) Four-cylinder, opposed flat
    - ✓ Various forces and moments at rotational frequency and twice rotational frequency depending on crankshaft arrangement.
  - ❖ 3) Six-cylinder, inline
    - ✓ Inherently balanced in all directions.



## ENGINE/TRANSMISSION

- Balance conditions for more commonly used engine configurations:
  - ❖ 4) Six-cylinder, inline two-cycle
    - ✓ Vertical couple generating yaw and pitch moments at the engine rotational frequency; can be balanced.
  - ❖ 5) Six-cylinder, 60-degree V
    - ✓ Generates a counter-rotating couple at rotational frequency that can be balanced with counter-rotating shaft.
  - ❖ 6) Six-cylinder, 90-degree V (uneven firing)
    - ✓ Generates yaw moment of twice rotational frequency; can be balanced with counter-rotating shaft.



## ENGINE/TRANSMISSION

- Balance conditions for more commonly used engine configurations:
  - ❖ 7) Six-cylinder, 90-degree V (even firing)
    - ✓ Generates yaw and pitching moments at crankshaft speed, which can be balanced. Also generates complex yaw and pitching moments at twice rotational speed which are difficult to balance.
  - ❖ 8) Eight-cylinder, inline
    - ✓ Inherently balanced in all directions.
  - ❖ 9) Eight-cylinder, 90-degree V
    - ✓ Couple at primary rotational speed; can be counter-balanced.



## ENGINE/TRANSMISSION

- ❖ With proper design of the mounting system the mass of the engine transmission combination can be utilized as a vibration absorber attenuating other vibrations to which the vehicle is prone.
- ❖ Most often it is used to control vertical shake vibrations arising from the wheel excitations.
- ❖ For this purpose the mounting system is designed to provide a vertical resonance frequency near that of the front wheel hop frequency (12-15 Hz), so that the engine can act as a vibration damper for this mode of vehicle vibration



## VEHICLE RESPONSE PROPERTIES

- ❑ The system is usually treated as sprung and unsprung mass.
- ❑ The dynamic behavior of a vehicle can be characterized most meaningfully by considering the input-output relationships.
- ❑ The input may be any of the excitations discussed in the preceding section, or combinations thereof.
- ❑ The output most commonly of interest will be the vibrations on the body.



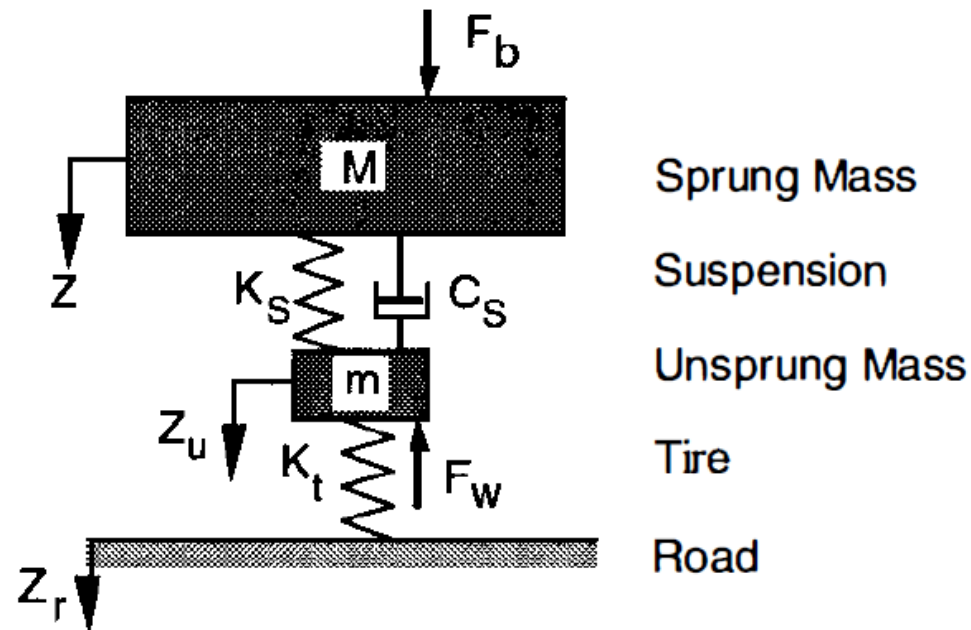
## VEHICLE RESPONSE PROPERTIES

- ❑ The ratio of output and input amplitudes represents a "gain" for the dynamic system.
- ❑ The term "transmissibility" is often used to denote the gain.
- ❑ Transmissibility is the nondimensional ratio of response amplitude to excitation amplitude for a system in steady-state forced vibration.
- ❑ At the most basic level, all highway vehicles share the "ride isolation" properties common to a sprung mass supported by primary suspension systems at each wheel.
- ❑ The dynamic behavior of this system is the first level of isolation from the roughness of the road.



## SUSPENSION ISOLATION

### □ Quarter car model:



*Quarter-car model.*



## SUSPENSION ISOLATION

- The sprung mass resting on the suspension and tire springs is capable of motion in the vertical direction.
- The effective stiffness of the suspension and tire springs in series is called the "ride rate":

$$RR = \frac{K_s K_t}{K_s + K_t}$$

RR = Ride rate

$K_s$  = Suspension stiffness

$K_t$  = Tire stiffness

- Bounce natural frequency:

$$\omega_n = \sqrt{\frac{RR}{M}} \quad (\text{radians/sec})$$

$$f_n = 0.159 \sqrt{\frac{RR}{W/g}} \quad (\text{cycles/sec})$$



## SUSPENSION ISOLATION

- Damped natural frequency:

$$\omega_d = \omega_n \sqrt{1 - \zeta_s^2}$$

$\zeta_s$  = Damping ratio

$$= C_s / \sqrt{4 K_s M}$$

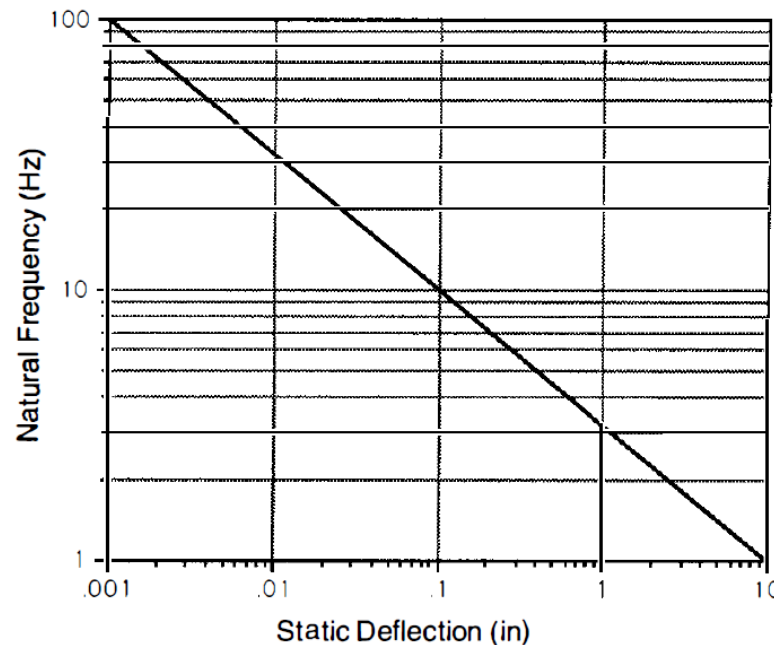
$C_s$  = Suspension damping coefficient

- ❖ For good ride the suspension damping ratio on modern passenger cars usually falls between 0.2 and 0.4.
- ❖ Because there is so little difference the undamped natural frequency, commonly used to characterize the vehicle.



## SUSPENSION ISOLATION

- ❖ The ratio of  $W/K_s$  represents the static deflection of the suspension due to the weight of the vehicle.
- ❖ Because the "static deflection" predominates in determining the natural frequency, it is a straightforward and simple parameter indicative of the lower bound on the isolation of a system.



## SUSPENSION ISOLATION

- ❖ A static deflection of 10 inches (254 mm) is necessary to achieve a 1 Hz natural frequency (considered to be a design optimum for highway vehicles)
- ❖ A 5-inch (127 mm) deflection results in a 1.4Hz frequency, and 1 inch (25 mm) equates to a 3.13 Hz frequency.
- ❖ At least 5 inches of stroke must be available in order to absorb a bump acceleration of one-half "g" without hitting the suspension stops.



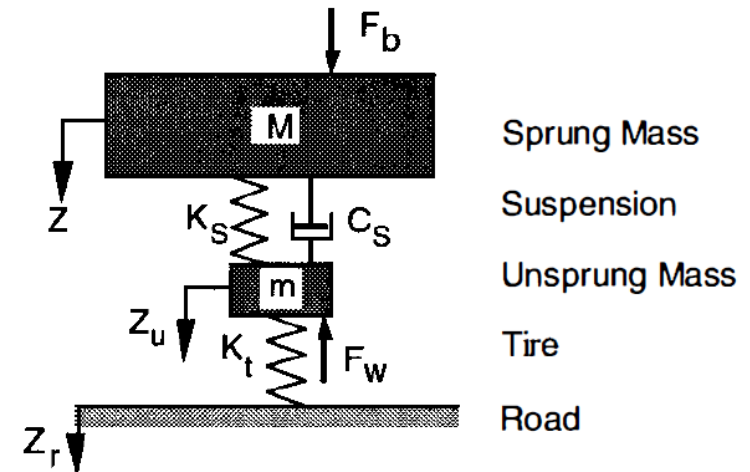
## SUSPENSION ISOLATION

### □ Quarter-car model in steady-state vibration

$$M\ddot{Z} + C_s\dot{Z} + K_s Z = C_s\dot{Z}_u + K_s Z_u + F_b$$

$$m\ddot{Z}_u + C_s\dot{Z}_u + (K_s + K_t)Z_u = C_s\dot{Z} + K_s Z + K_t Z_r + F_w$$

- $Z$  = Sprung mass displacement  
 $Z_u$  = Unsprung mass displacement  
 $Z_r$  = Road displacement  
 $F_b$  = Force on the sprung mass  
 $F_w$  = Force on the unsprung mass



Quarter-car model.

## SUSPENSION ISOLATION

- Closed-form solutions can be obtained for the steady-state harmonic motion by methods

$$\frac{\ddot{z}}{\ddot{z}_r} = \frac{K_1 K_2 + j[K_1 C \omega]}{[\chi \omega^4 - (K_1 + K_2 \chi + K_2) \omega^2 + K_1 K_2] + j[K_1 C \omega - (1 + \chi) C \omega^3]}$$

$$\frac{\ddot{z}}{F_w/M} = \frac{K_2 \omega^2 + j[C \omega^3]}{[\chi \omega^4 - (K_1 + K_2 \chi + K_2) \omega^2 + K_1 K_2] + j K_1 C \omega - (1 + \chi) C \omega^3}$$

$$\frac{\ddot{z}}{F_b/M} = \frac{[\mu \omega^4 - (K_1 + K_2) \omega^2] + j[C \omega^3]}{[\chi \omega^4 - (K_1 + K_2 \chi + K_2) \omega^2 + K_1 K_2] + j[K_1 C \omega - (1 + \chi) C \omega^3]}$$

$\chi = m/M =$  Ratio of unsprung to sprung mass

$C = C_s/M$

$K_1 = K_t/M$

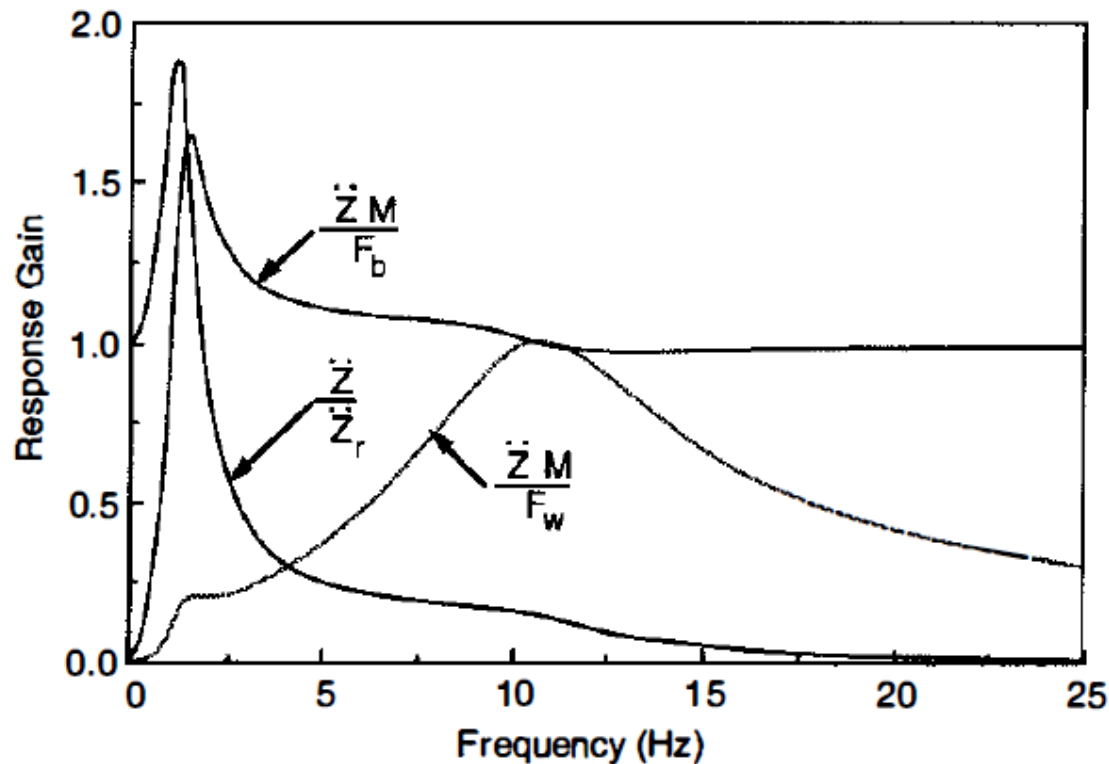
$K_2 = K_s/M$

$j =$  Complex operator



## SUSPENSION ISOLATION

### □ Model Response



*Quarter-car response to road, tire/wheel, and body inputs.*

## SUSPENSION ISOLATION

- The sprung-mass acceleration spectrum can be calculated for a linear model by multiplying the road spectrum by the square of the transfer function.

$$G_{zS}(f) = |H_V(f)|^2 G_{zr}$$

$G_{zS}(f)$  = Acceleration PSD on the sprung mass  
 $H_V(f)$  = Response gain for road input  
 $G_{zr}$  = Acceleration PSD of the road input

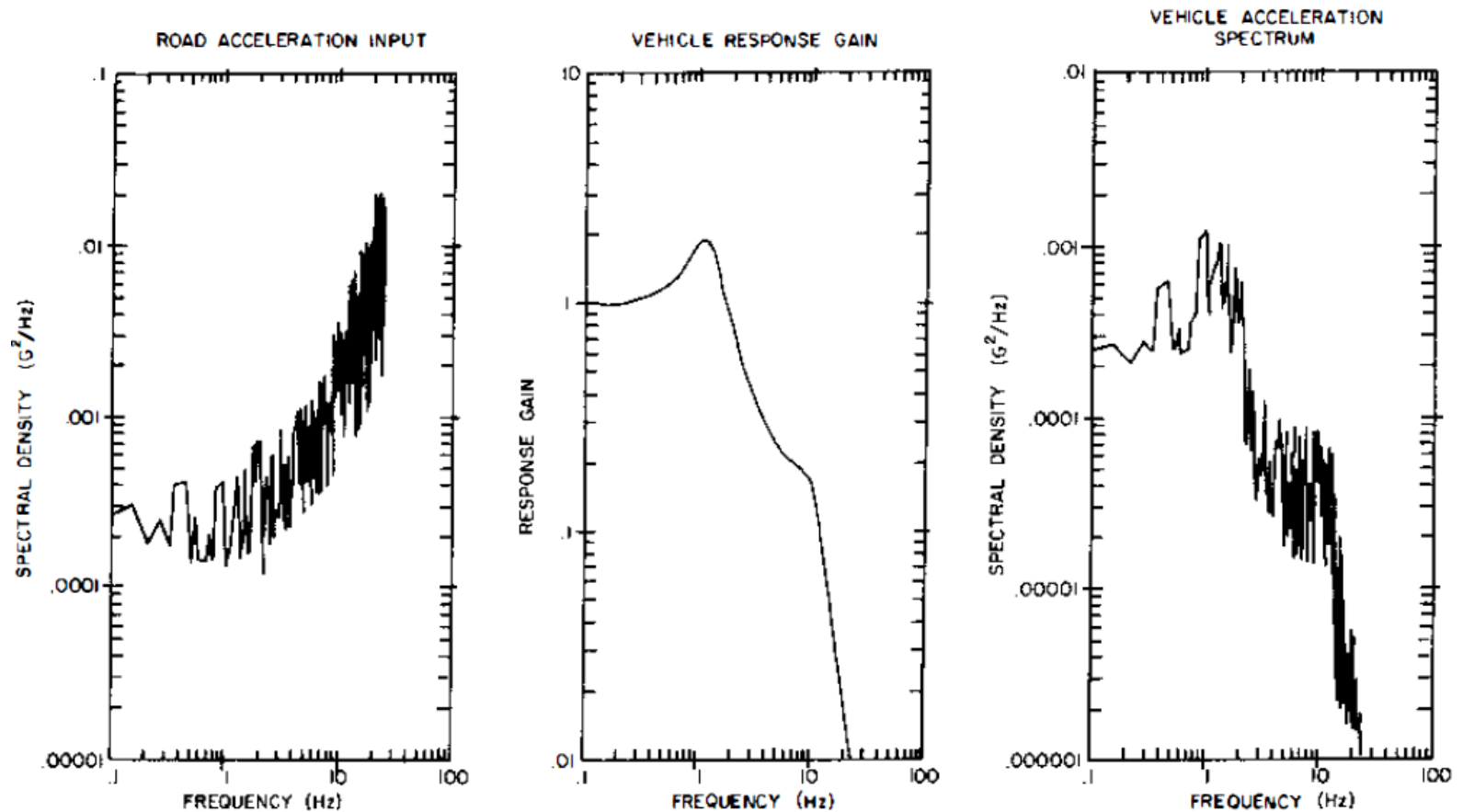
- Even though the road input amplitude increases with frequency, the acceleration response on the vehicle is qualitatively similar to the vehicle's response gain.
- Thus the acceleration spectrum seen on a vehicle does provide some idea of the response gain of the system even when the exact properties of the road are not known.





## SUSPENSION ISOLATION

### □ Vehicle acceleration spectrum



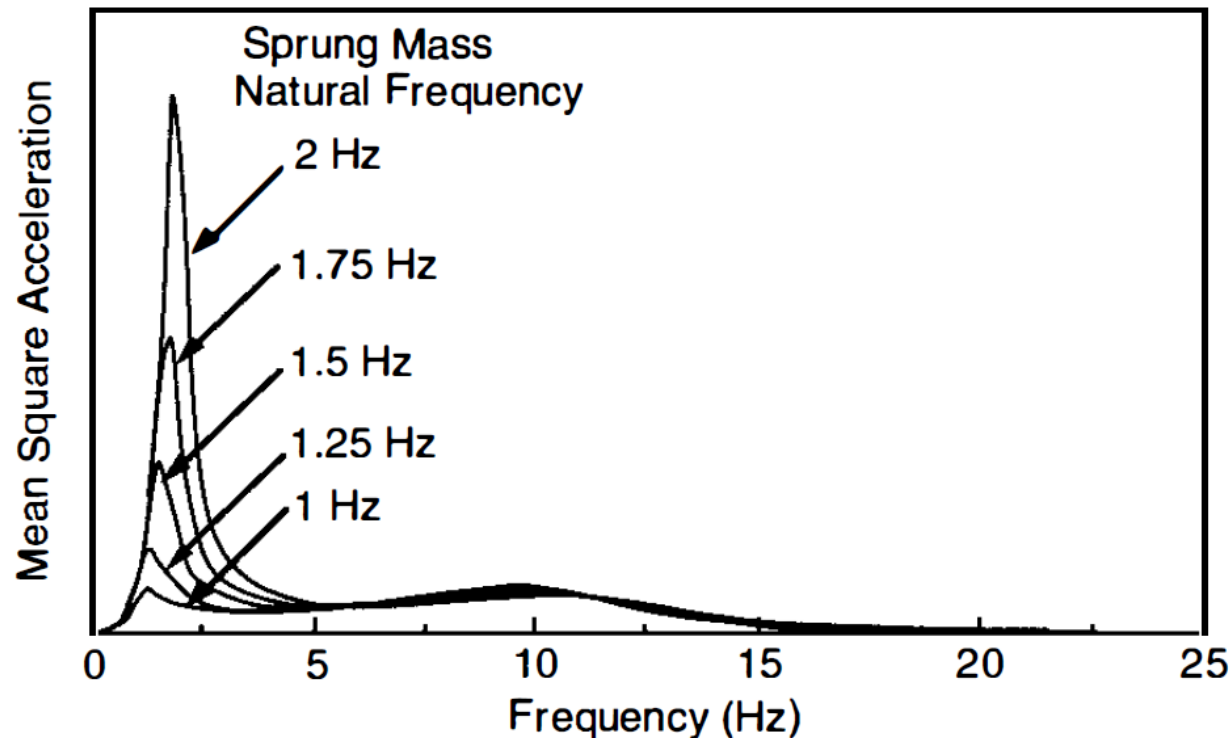
## SUSPENSION STIFFNESS

- ❑ Because the suspension spring is in series with a relatively stiff tire spring, the suspension spring predominates in establishing the ride rate and, hence, the natural frequency of the system in the bounce (vertical) mode.
- ❑ Since road acceleration inputs increase in amplitude at higher frequencies, the best isolation is achieved by keeping the natural frequency as low as possible.



## SUSPENSION STIFFNESS

- The effect on accelerations transmitted to the sprung mass can be estimated analytically by approximating the road acceleration input.
- Acceleration spectra thus calculated for a quarter-car model:



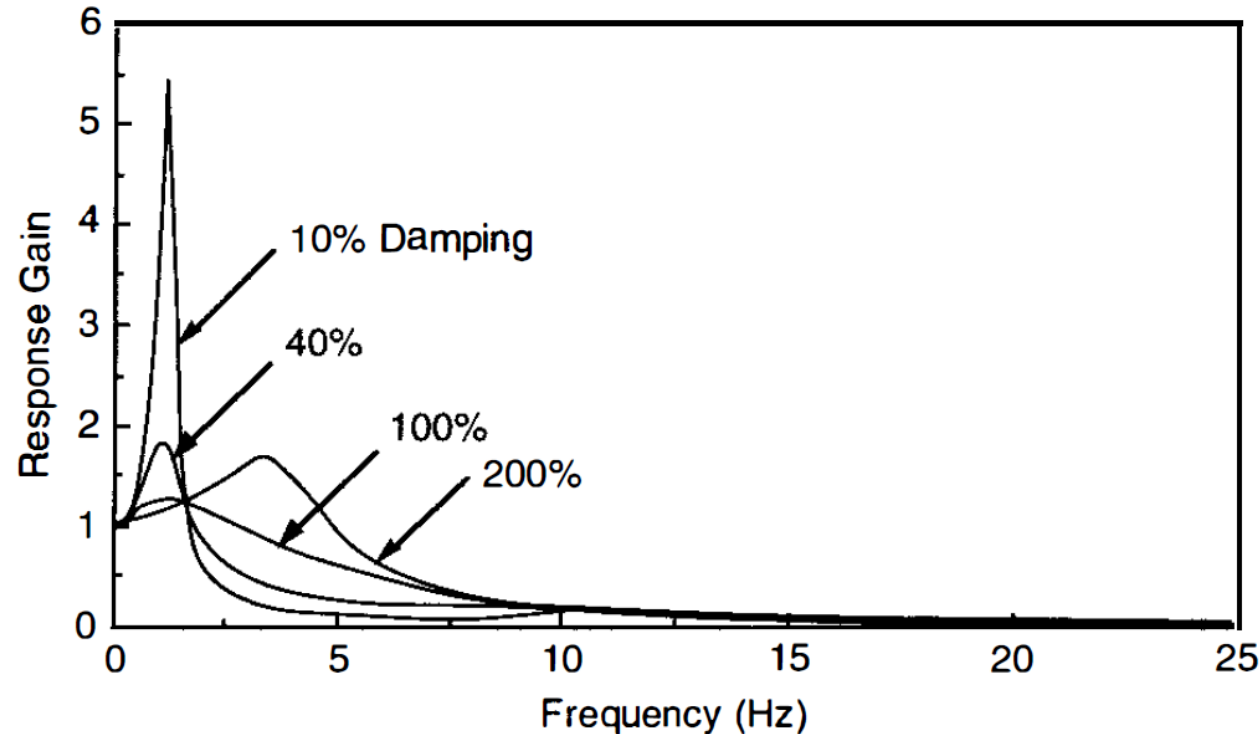
## SUSPENSION STIFFNESS

- ❑ The lowest acceleration occurs at the natural frequency of 1 Hz. At higher values of natural frequency (stiffer suspension springs), the acceleration peak in the 1 to 5 Hz range increases.
- ❑ While this analysis clearly shows the benefits of keeping the suspension soft for ride isolation, the practical limits of stroke that can be accommodated within a given vehicle size and suspension envelope constrain the natural frequency for most cars to a minimum in the 1 to 1.5 Hz range.
- ❑ Performance cars on which ride is sacrificed for the handling benefits of a stiff suspension will have natural frequencies up to 2 or 2.5 Hz.



## SUSPENSION STIFFNESS

- Damping in suspensions comes primarily from the action of hydraulic shock absorbers.
- The nominal effect of damping for the quarter-car model:



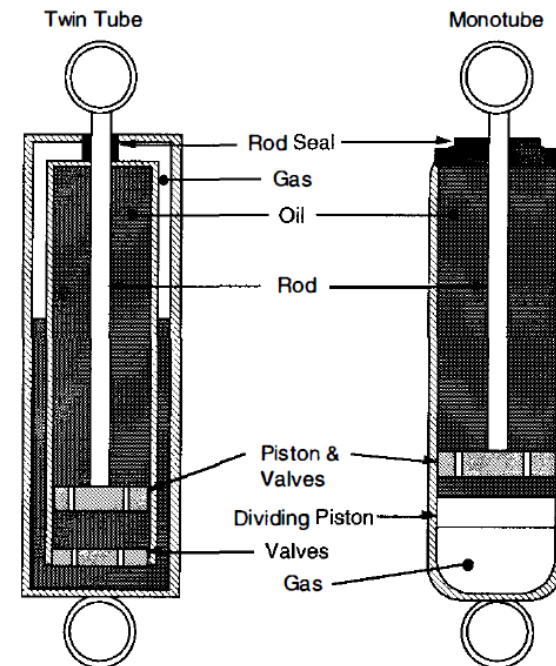
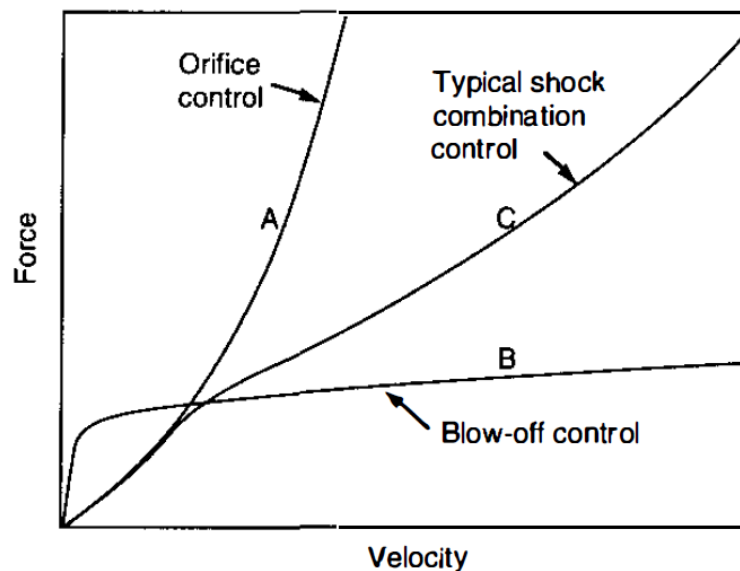
## SUSPENSION DAMPING

- ❑ The 40% damping ratio curve is reasonably representative of most cars, recognizable by the fact that the amplification at the resonant frequency is in the range of 1.5 to 2.0.
- ❑ At 100% (critical) damping, the 1 Hz bounce motions of the sprung mass are well controlled, but with penalties in the isolation at higher frequencies.
- ❑ If damping is pushed beyond the critical, for example to 200%, the damper becomes so stiff that the suspension no longer moves and the vehicle bounces on its tires, resonating in the 3 to 4 Hz range.



## SUSPENSION DAMPING

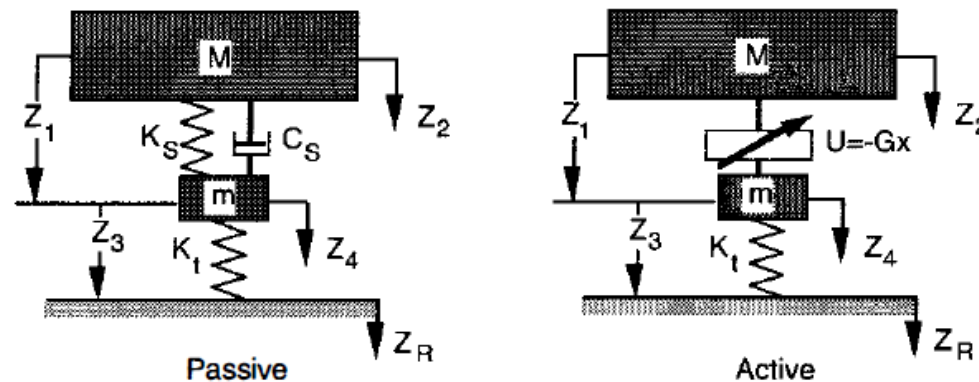
- Shock absorbers must be tailored not only to achieve the desired ride characteristics, but also play a key role in keeping good tire-to-road contact essential for handling and safety.
- Types of absorbers available to produce desired characteristics:



*Twin tube and gas-pressurized monotube shock absorbers.*

## ACTIVE CONTROL

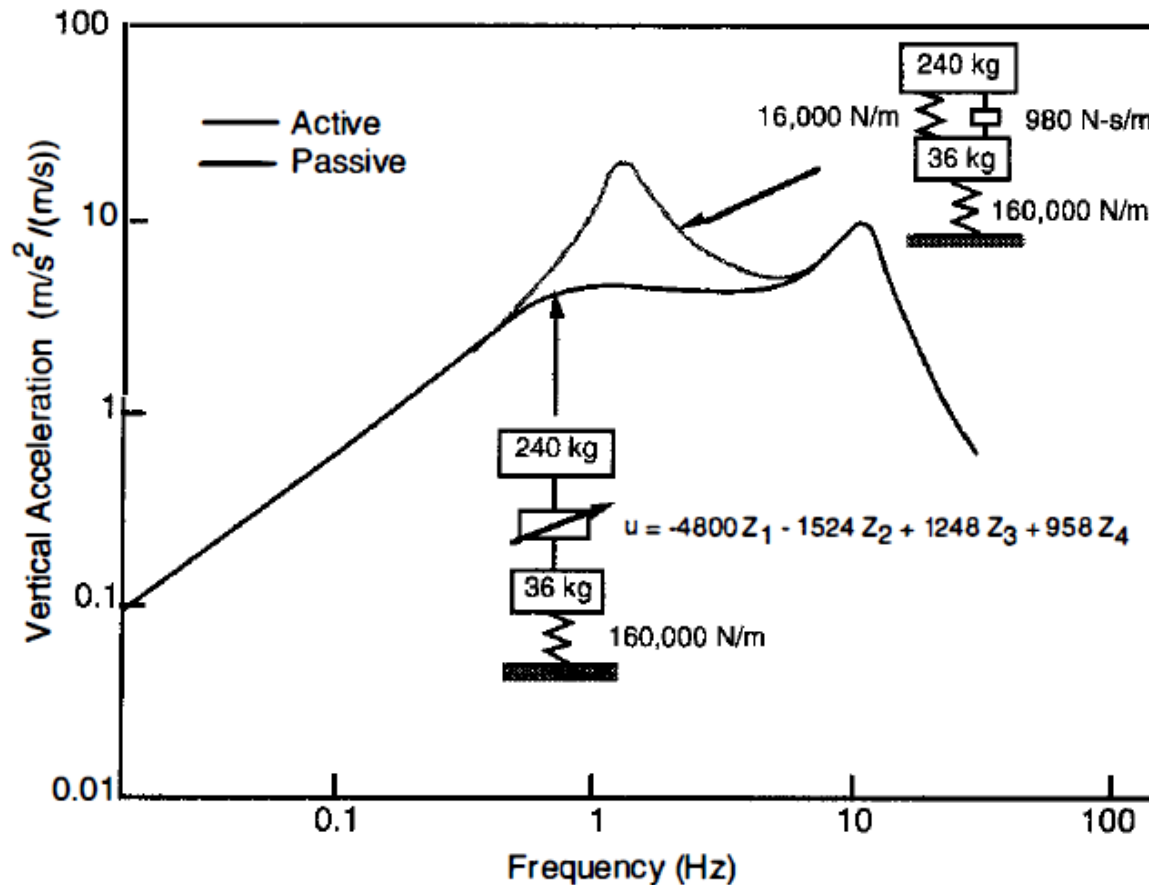
- ❖ In the interest of improving the overall ride performance of automotive vehicles, suspensions incorporating active components have been developed.
- ❖ Most frequently, the active components are hydraulic cylinders that can exert forces in the suspension on command from an electronic controller tailored to produce the desired ride characteristics.
- ❖ The quarter-car model can be used to quantify the comparative ride performance of passive and active systems.





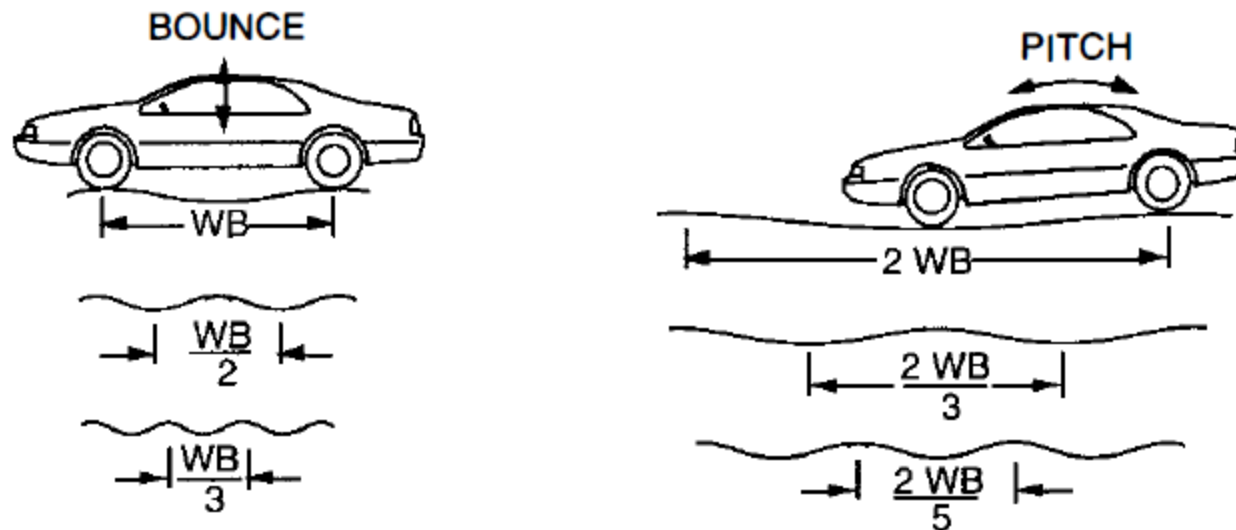
## ACTIVE CONTROL

- Comparison of vertical acceleration response of active and passive systems



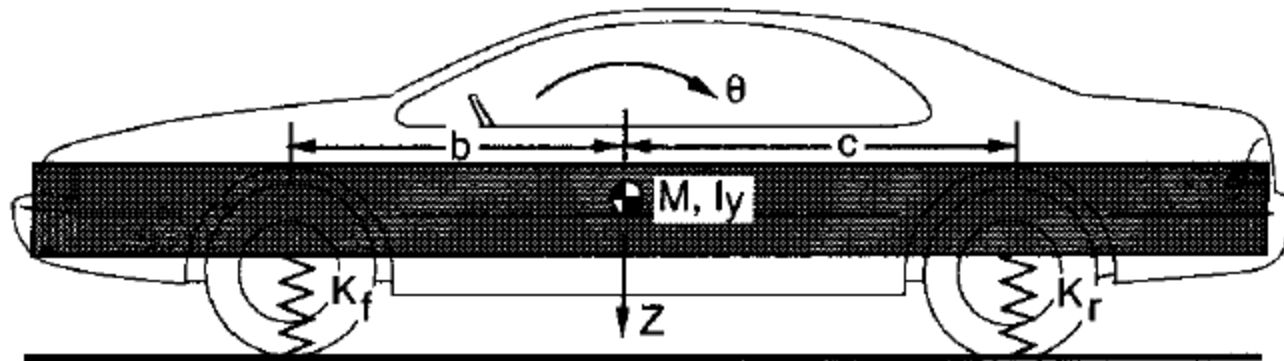
## RIGID BODY BOUNCE/PITCH MOTIONS

- The simple mechanics of the quarter-car model do not fully represent the rigid-body motions that may occur on a motor vehicle.
- Because of the longitudinal distance between the axles, it is a multi-input system that responds with pitch motions as well as vertical bounce.



## BOUNCE/PITCH FREQUENCIES

- On most vehicles there is a coupling of motions in the vertical and pitch directions, such that there are no "pure" bounce and pitch modes.
- For simplicity in analysis, the tire and suspension will be considered as a single stiffness (the ride rate), and damping and unsprung masses will be neglected.



## BOUNCE/PITCH FREQUENCIES

### □ Defining parameters:

$$\alpha = (K_f + K_r)/M$$

$$\beta = (K_r c - K_f b)/M$$

$$\gamma = (K_f b^2 + K_r c^2)/M k^2$$

$K_f$  = Front ride rate

$K_r$  = Rear ride rate

$b$  = Distance from the front axle to the CG

$c$  = Distance from the rear axle to the CG

$I_y$  = Pitch moment of inertia

$k$  = Radius of gyration =  $\sqrt{I_y/M}$

### □ Differential equations for the bounce, $Z$ , and pitch, $\theta$ :

$$\ddot{Z} + \alpha Z + \beta \theta = 0$$

$$\ddot{\theta} + \beta Z k^2 + \gamma \theta = 0$$



## BOUNCE/PITCH FREQUENCIES

- Without damping, the solutions to the differential equations will be sinusoidal

$$Z = Z \sin \omega t$$

$$\theta = \theta \sin \omega t$$

- Substituting  $\theta$  in  $Z$  equation:

$$-Z \omega^2 \sin \omega t + \alpha Z \sin \omega t + \beta \theta \sin \omega t = 0$$

$$(\alpha - \omega^2) Z + \beta \theta = 0$$

$$\longrightarrow Z/\theta = -\beta / (\alpha - \omega^2)$$

- Same for  $\theta$ :

$$\longrightarrow Z/\theta = -k^2 (\gamma - \omega^2) / \beta$$



## BOUNCE/PITCH FREQUENCIES

□ Equating the right sides:

$$(\alpha - \omega^2)(\gamma - \omega^2) = \beta(\beta/k^2) \quad \longrightarrow \quad \omega^4 - (\alpha + \gamma)\omega^2 + \alpha\gamma - \beta^2/k^2 = 0$$

$$(\omega_{1,2})^2 = \frac{(\alpha + \gamma)}{2} \pm \sqrt{\frac{(\alpha + \gamma)^2}{4} - (\alpha\gamma - \beta^2/k^2)}$$

$$= \frac{(\alpha + \gamma)}{2} \pm \sqrt{\frac{(\alpha - \gamma)^2}{4} + \beta^2/k^2}$$

$$\longrightarrow \omega_1 = \sqrt{\frac{(\alpha + \gamma)}{2} + \sqrt{(\alpha - \gamma)^2/4 + \beta^2/k^2}}$$

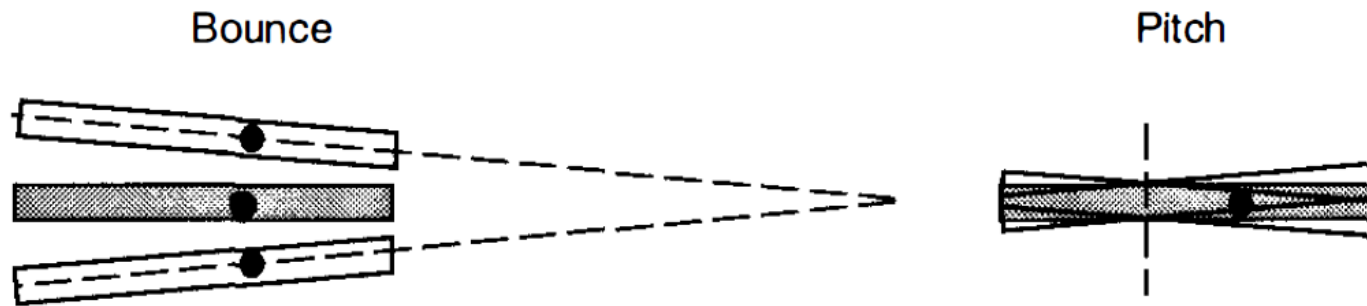
$$\longrightarrow \omega_2 = \sqrt{\frac{(\alpha + \gamma)}{2} - \sqrt{(\alpha - \gamma)^2/4 + \beta^2/k^2}}$$



## BOUNCE/PITCH FREQUENCIES

- The oscillation centers can be found using the amplitude ratios:

### Predominant Modes



- ❖ The motion will be predominantly pitch, and the associated frequency is the pitch frequency

## BOUNCE/PITCH FREQUENCIES

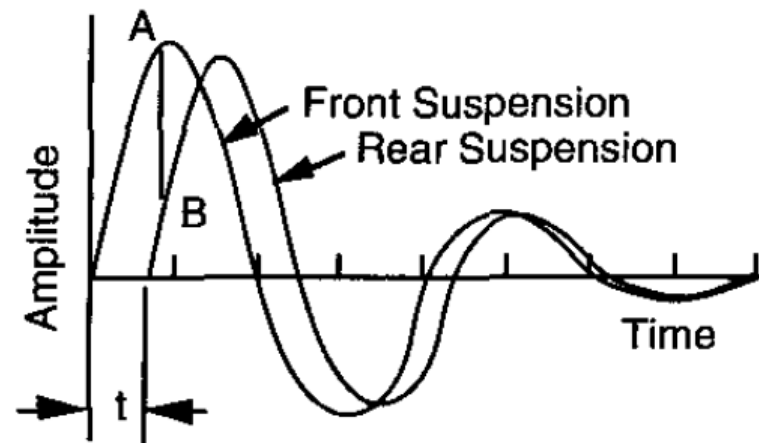
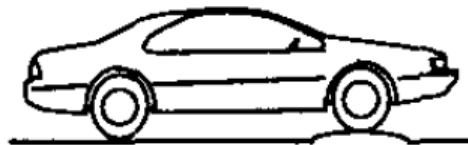
- ❑ The locations of the motion centers are dependent on the relative values of the natural frequencies of the front and rear suspensions.
- ❑ Maurice Olley, one of the founders of modern vehicle dynamics, derived guidelines from experiments with a car modified to allow variation of the pitch moment of inertia.
- ❑ The Olley Criteria:
  - ❖ 1 ) The front suspension should have a 30% lower ride rate than the rear suspension.
  - ❖ 2) The pitch and bounce frequencies should be close together.
  - ❖ 3) Neither frequency should be greater than 1.3 Hz.
  - ❖ 4) The roll frequency should be approximately equal to the pitch and bounce frequencies.





## BOUNCE/PITCH FREQUENCIES

- ❖ The rule that rear suspensions should have a higher spring rate (higher natural frequency) is rationalized by the observation that vehicle bounce is less annoying as a ride motion than pitch.
- ❖ Since excitation inputs from the road to a car affect the front wheels first, the higher rear to front ratio of frequencies will tend to induce bounce.



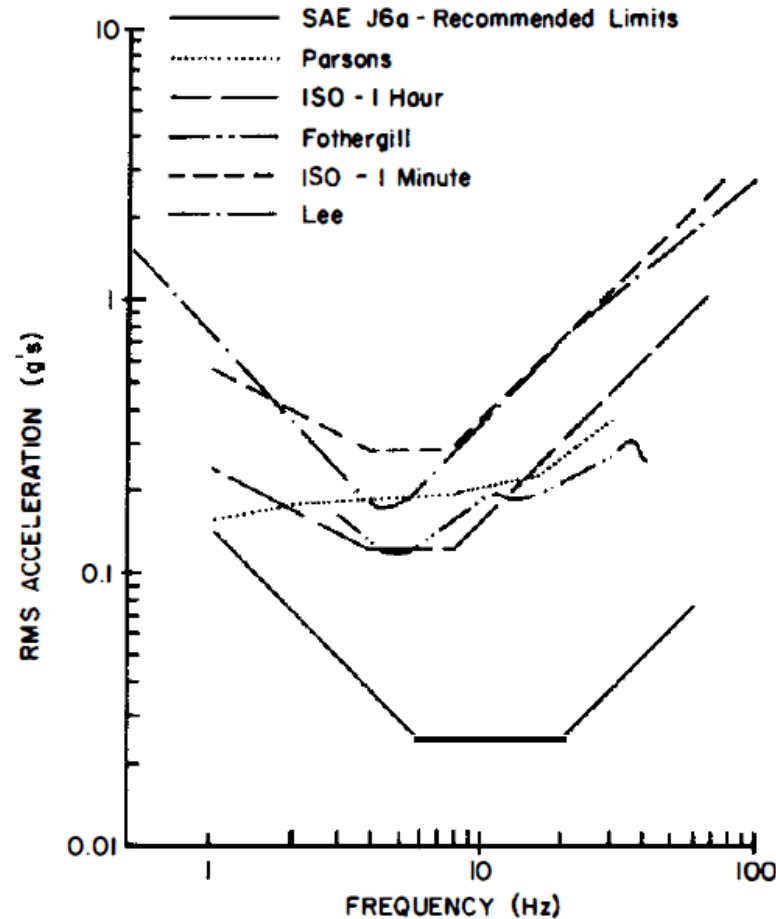
## PERCEPTION OF RIDE

- ❑ Ride is a subjective perception, normally associated with the level of comfort experienced when traveling in a vehicle.
- ❑ The tactile vibrations transmitted to the passenger's body through the seat, and at the hands and feet, are the factors most commonly associated with ride.
- ❑ Yet it is often difficult to separate the influences of acoustic vibrations (noise) in the perception of ride, especially since noise types and levels are usually highly correlated with other vehicle vibrations.
- ❑ Additionally, the general comfort level can be influenced by seat design and its fit to the passenger, temperature, ventilation, interior space, hand holds, and many other factors.



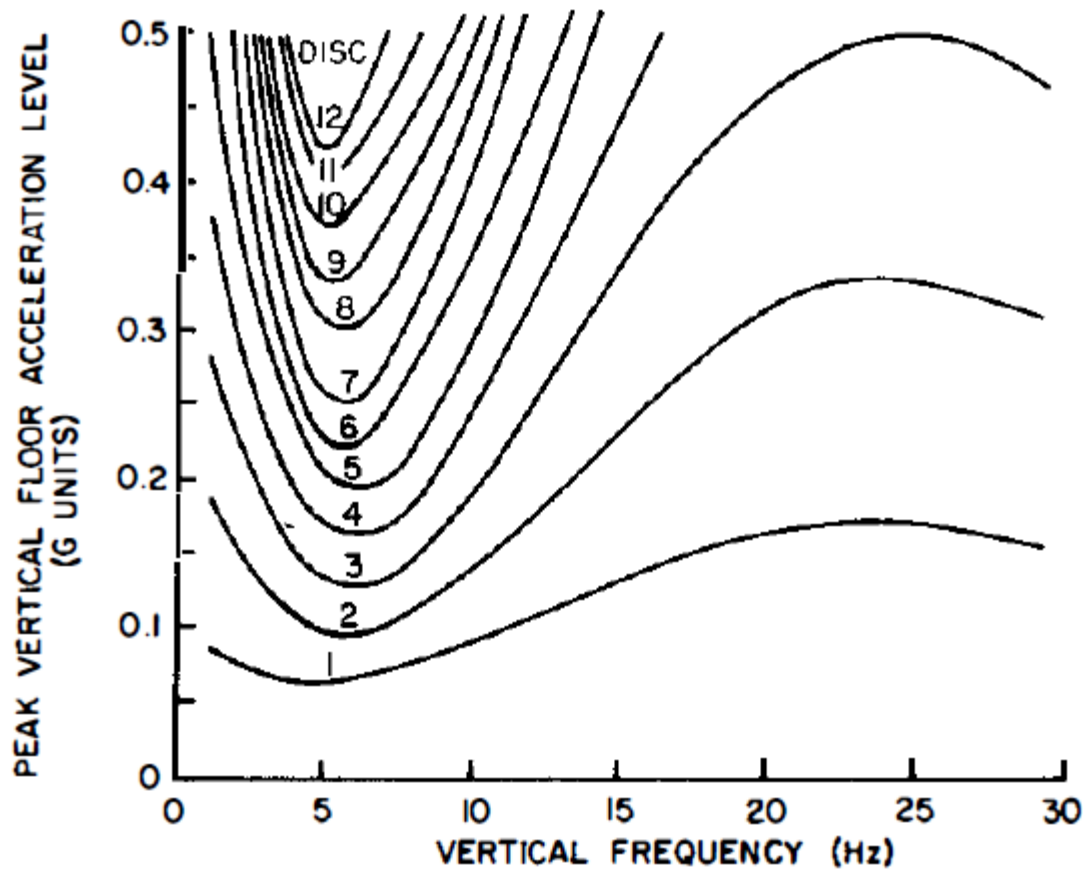
## PERCEPTION OF RIDE

### Human tolerance limits for vertical vibration



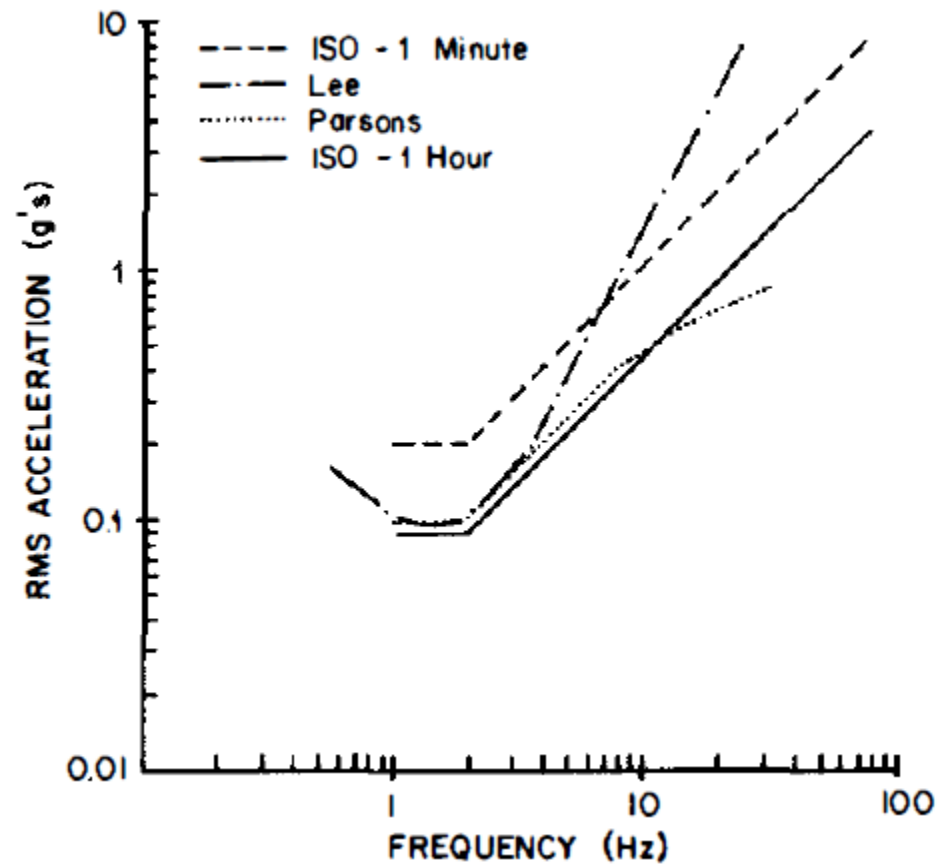
## PERCEPTION OF RIDE

- NASA Discomfort Curves for vibration in transport vehicles



## PERCEPTION OF RIDE

- Human tolerance limits for fore/aft vibrations



## PERCEPTION OF RIDE

- Seat vibrations
  - ❖ Linear-Linear Format
  - ❖ Log-Log Format

