دانشگاه سمنان
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دانشکده مهندسی مکانیک

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درس دینامیک

Engineering Mechanics Dynamics

Meriam, Kraige & Bolton

9th Edition

Chapter 2: **Kinematics of Particles**

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CHAPTER 2

Kinematics of Particles

CHAPTER OUTLINE

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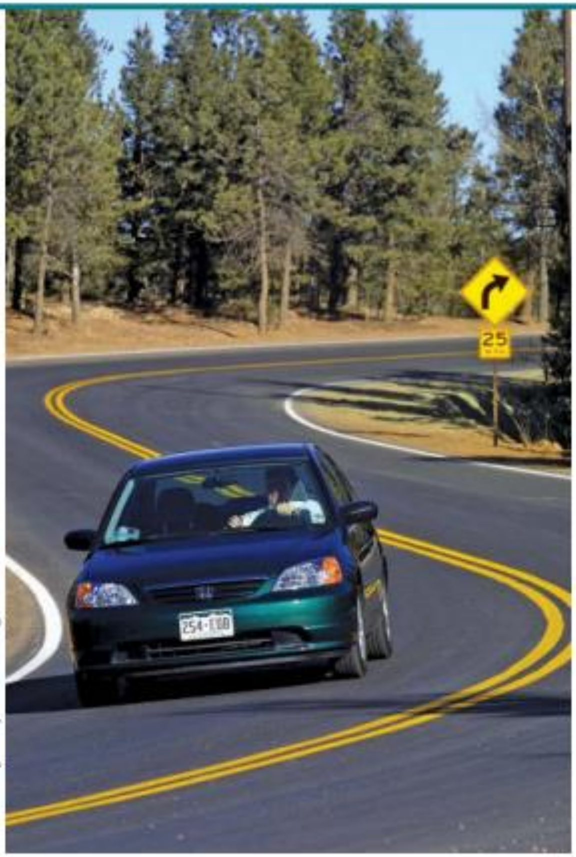


Sean Cayton/The Image Works

Even if this car maintains a constant speed along the winding road, it accelerates laterally, and this acceleration must be considered in the design of the car, its tires, and the roadway itself.

2/1 Introduction

- Kinematics is the branch of dynamics which describes the motion of bodies without reference to the forces which either cause the motion or are generated as a result of the motion.

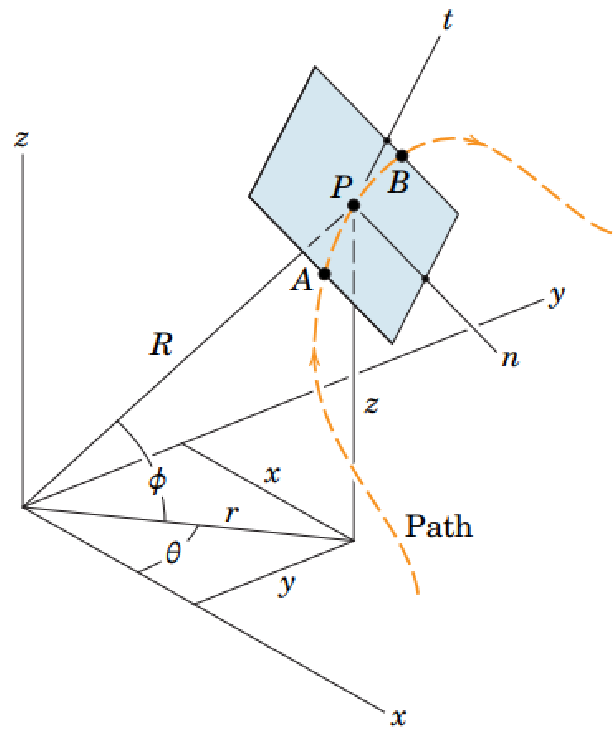
- Kinematics is often described as the “geometry of motion.”

- A thorough working knowledge of kinematics is a prerequisite to kinetics, which is the study of the relationships between motion and the corresponding forces which cause or accompany the motion.

2/1 Introduction

Particle Motion

- A particle is a body whose physical dimensions are so small compared with the radius of curvature of its path that we may treat the motion of the particle as that of a point.

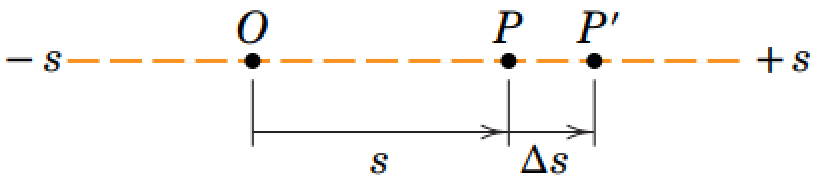


Choice of Coordinates

- Rectangular coordinates x, y, z (Cartesian)
- Cylindrical coordinates r, θ, z
- Spherical coordinates R, θ, ϕ
- Tangent t and normal n to the curve (path variables)

2/2 Rectilinear Motion

- Particle P moving along a straight line



Velocity and Acceleration

- Average velocity of the particle during the interval Δt is the displacement divided by the time interval or $v_{av} = \Delta s / \Delta t$

2/2 Rectilinear Motion

Velocity and Acceleration

- *instantaneous velocity*

$$v = \frac{ds}{dt} = \dot{s}$$

- The average acceleration of the particle during the interval Δt is the change in its velocity divided by the time interval or $a_{av} = \Delta v / \Delta t$.

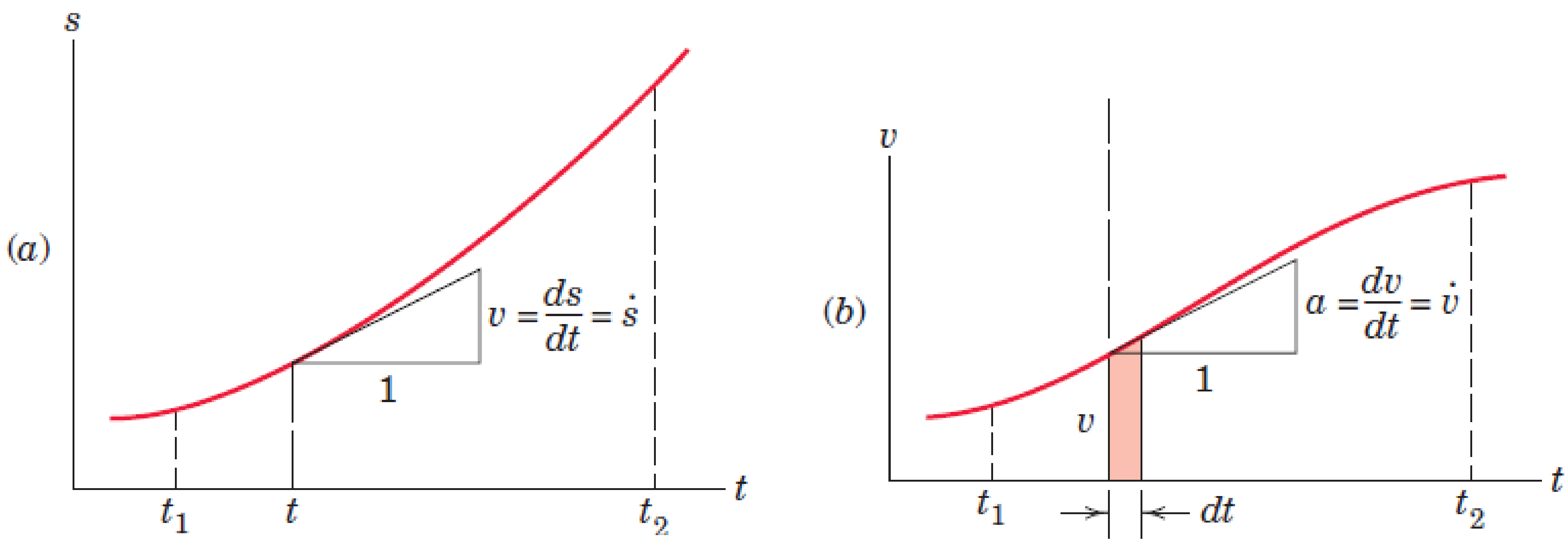
- *instantaneous acceleration*

$$a = \frac{dv}{dt} = \dot{v} \quad \text{or} \quad a = \frac{d^2 s}{dt^2} = \ddot{s}$$

$$v\,dv = a\,ds \quad \text{or} \quad \dot{s}\,d\dot{s} = \ddot{s}\,ds$$

2/2 Rectilinear Motion

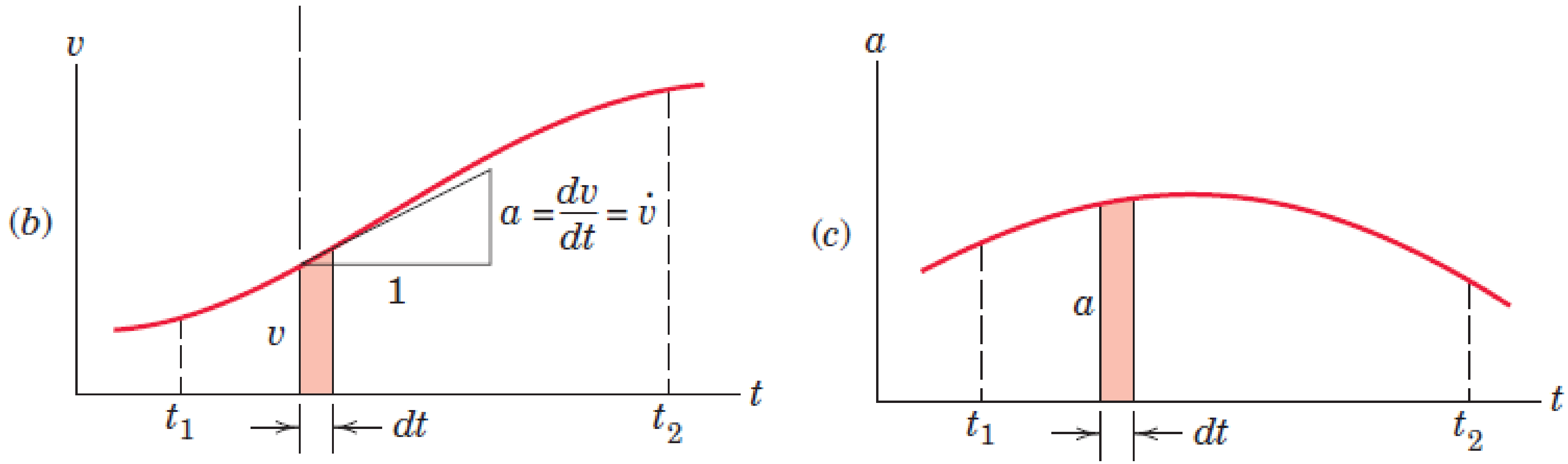
Graphical Interpretations



$$\int_{s_1}^{s_2} ds = \int_{t_1}^{t_2} v\,dt \qquad \text{or} \qquad s_2 - s_1 = (\text{area under } v\text{-}t \text{ curve})$$

2/2 Rectilinear Motion

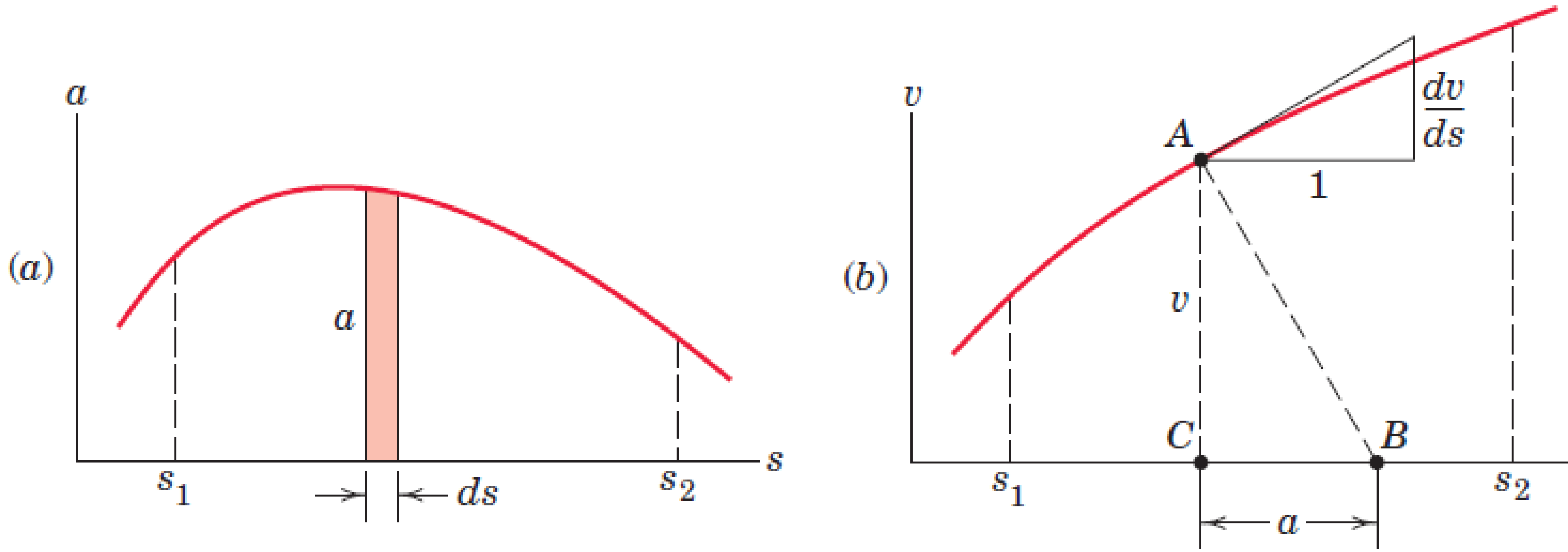
Graphical Interpretations



$$\int_{v_1}^{v_2} dv = \int_{t_1}^{t_2} a\,dt \qquad \text{or} \qquad v_2 - v_1 = (\text{area under } a\text{-}t \text{ curve})$$

2/2 Rectilinear Motion

Graphical Interpretations



$$\int_{v_1}^{v_2} v\,dv = \int_{s_1}^{s_2} a\,ds \qquad \text{or} \qquad \frac{1}{2}(v_2{}^2 - v_1{}^2) = (\text{area under } a\text{-}s \text{ curve})$$

2/2 Rectilinear Motion

Analytical Integration

- *(a) Constant Acceleration*

$$\int_{v_0}^{v} dv = a\int_0^t dt \qquad \text{or} \qquad v = v_0 + at$$

$$\int_{v_0}^{v} v\,dv = a\int_{s_0}^{s} ds \qquad \text{or} \qquad v^2 = {v_0}^2 + 2a(s - s_0)$$

$$\int_{s_0}^{s} ds = \int_0^t (v_0 + at)\,dt \qquad \text{or} \qquad s = s_0 + v_0 t + \frac{1}{2}at^2$$

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2/2 Rectilinear Motion

Analytical Integration

- *(b) Acceleration Given as a Function of Time, a = f(t)*

$$\int_{v_0}^{v} dv = \int_0^t f(t)\,dt \qquad \text{or} \qquad v = v_0 + \int_0^t f(t)\,dt$$

$$\int_{s_0}^{s} ds = \int_0^t v\,dt \qquad \text{or} \qquad s = s_0 + \int_0^t v\,dt$$

2/2 Rectilinear Motion

Analytical Integration

- ***(c) Acceleration Given as a Function of Velocity, a = f(v)***

$$t = \int_0^t dt = \int_{v_0}^{v} \frac{dv}{f(v)}$$

$$\int_{v_0}^{v} \frac{v\,dv}{f(v)} = \int_{s_0}^{s} ds \qquad \text{or} \qquad s = s_0 + \int_{v_0}^{v} \frac{v\,dv}{f(v)}$$

2/2 Rectilinear Motion

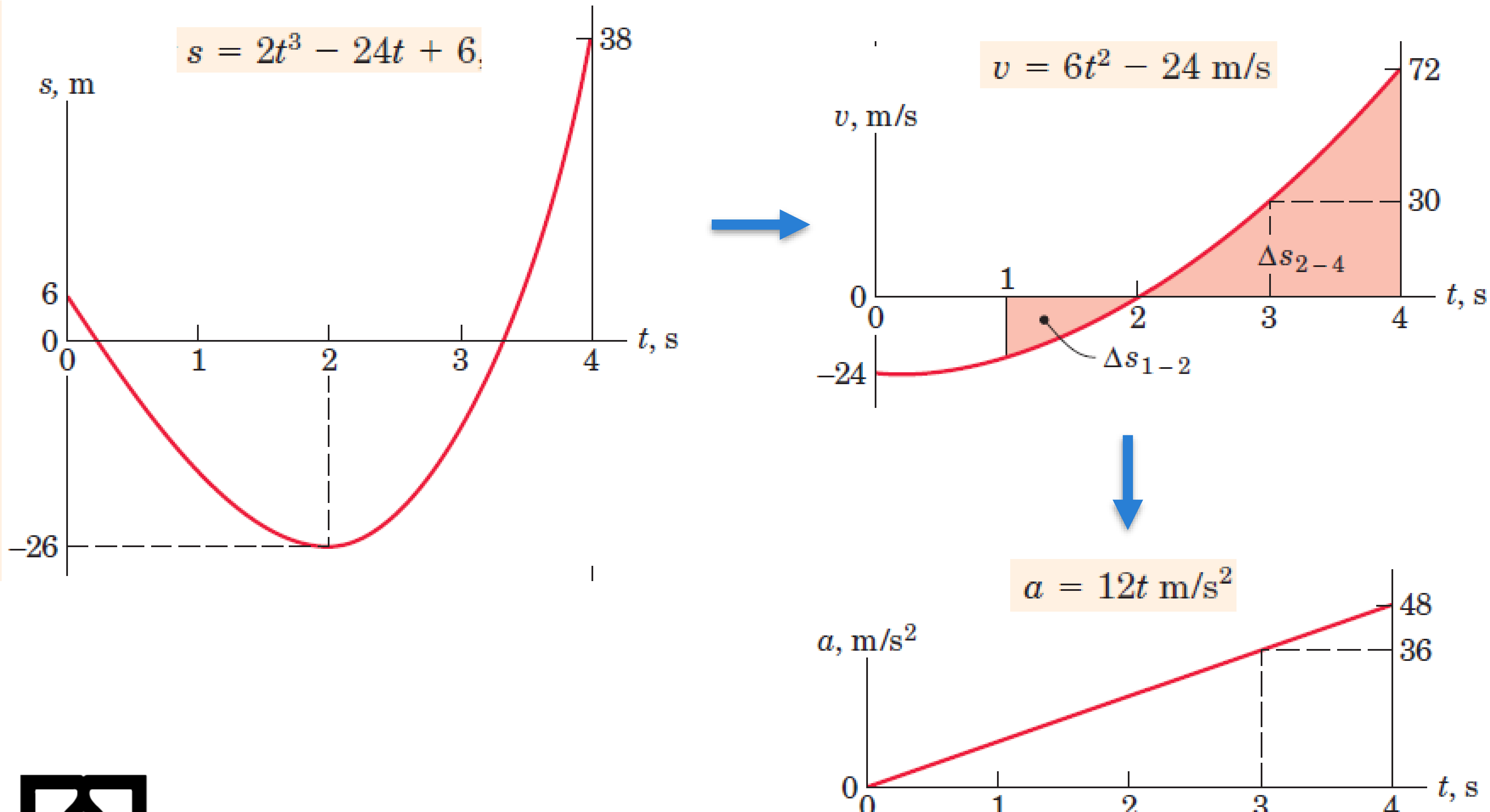
Analytical Integration

- *(d) Acceleration Given as a Function of Displacement, a = f(s)*

$$\int_{v_0}^{v} v\,dv = \int_{s_0}^{s} f(s)\,ds \qquad \text{or} \qquad v^2 = {v_0}^2 + 2\int_{s_0}^{s} f(s)\,ds$$

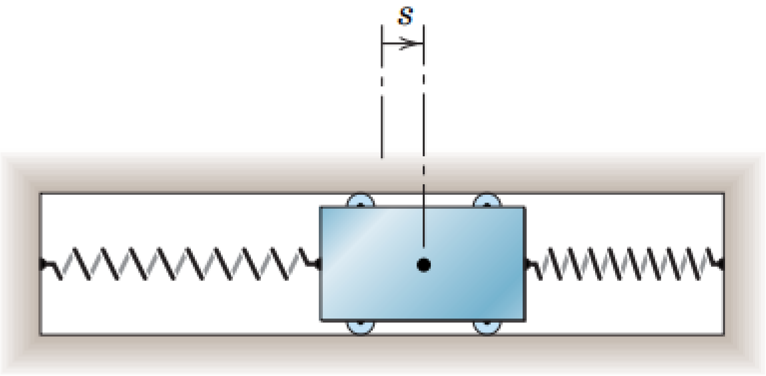
$$\int_{s_0}^{s} \frac{ds}{g(s)} = \int_{0}^{t} dt \qquad \text{or} \qquad t = \int_{s_0}^{s} \frac{ds}{g(s)}$$

SAMPLE PROBLEM 2/1



SAMPLE PROBLEM 2/3

The spring-mounted slider moves in the horizontal guide with negligible friction and has a velocity v_0 in the s-direction as it crosses the mid-position where $s = 0$ and $t = 0$. The two springs together exert a retarding force to the motion of the slider, which gives it an acceleration proportional to the displacement but oppositely directed and equal to $a = -k^2 s$, where k is constant. (The constant is arbitrarily squared for later convenience in the form of the expressions.) Determine the expressions for the displacement s and velocity v as functions of the time t.



SAMPLE PROBLEM 2/3

$$\int v\,dv = \int -k^2 s\,ds + C_1 \text{ a constant,} \quad \text{or} \quad \frac{v^2}{2} = -\frac{k^2 s^2}{2} + C_1$$

When $s = 0, v = v_0$, so that $C_1 = v_0^2/2$

$$v = +\sqrt{v_0^2 - k^2 s^2}$$

$$\int \frac{ds}{\sqrt{v_0^2 - k^2 s^2}} = \int dt + C_2 \text{ a constant,} \quad \text{or} \quad \frac{1}{k}\sin^{-1}\frac{ks}{v_0} = t + C_2$$

$$s = \frac{v_0}{k}\sin kt$$

$$v = v_0 \cos kt$$

Solution II. Since $a = \ddot{s}$, the given relation may be written at once as

$$\ddot{s} + k^2 s = 0$$

$$s = A \sin Kt + B \cos Kt$$

$$v = Ak \cos kt - Bk \sin kt$$

$$s = \frac{v_0}{k}\sin kt \quad \text{and} \quad v = v_0 \cos kt$$

SAMPLE PROBLEM 2/4

A freighter is moving at a speed of 8 knots when its engines are suddenly stopped. ① If it takes 10 minutes for the freighter to reduce its speed to 4 knots, determine and plot the distance s in nautical miles moved by the ship and its speed v in knots as functions of the time t during this interval. The deceleration of the ship is proportional to the square of its speed, so that $a = -kv^2$.

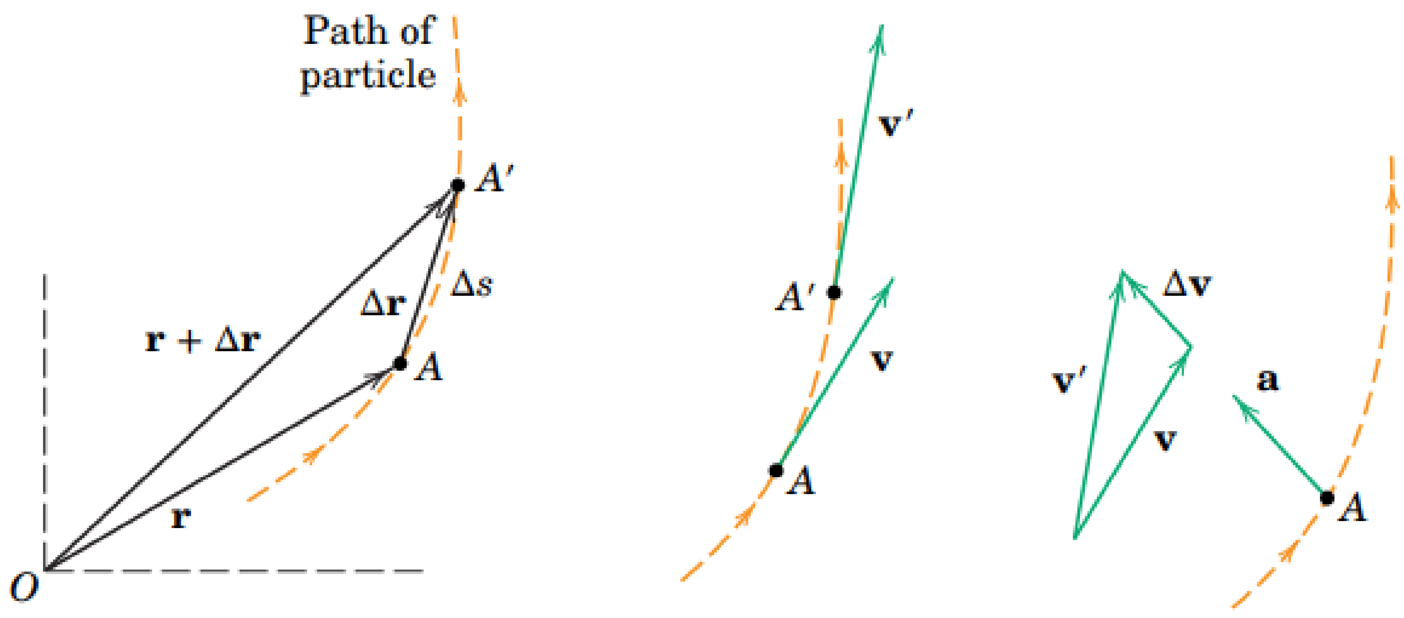
$$-kv^2 = \frac{dv}{dt} \qquad \frac{dv}{v^2} = -k\,dt \qquad \int_8^v \frac{dv}{v^2} = -k\int_0^t dt$$

$$-\frac{1}{v} + \frac{1}{8} = -kt \qquad v = \frac{8}{1 + 8kt}$$

$$\frac{8}{1 + 6t} = \frac{ds}{dt} \qquad \int_0^t \frac{8\,dt}{1 + 6t} = \int_0^s ds \qquad s = \frac{4}{3}\ln(1 + 6t)$$

2/3 Plane Curvilinear Motion

- *Position Vector* **r**



2/3 Plane Curvilinear Motion

Velocity

- *Average velocity* of the particle between *A* and *A′* is defined as $\mathbf{v}_{av} = \Delta\mathbf{r} / \Delta t$

- *instantaneous velocity (*approaches tangent to the path*)*

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = \dot{\mathbf{r}}$$

- The magnitude of **v** is called the *speed* and is the scalar

$$v = |\mathbf{v}| = \frac{ds}{dt} = \dot{s}$$

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2/3 Plane Curvilinear Motion

Acceleration

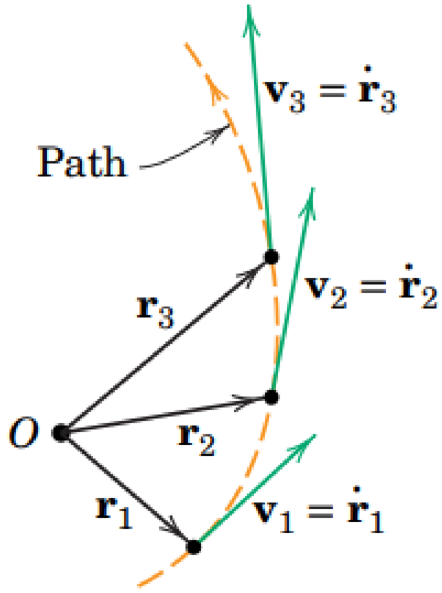
- The *average acceleration* of the particle between A and A' is defined as $\Delta\mathbf{v}/\Delta t$

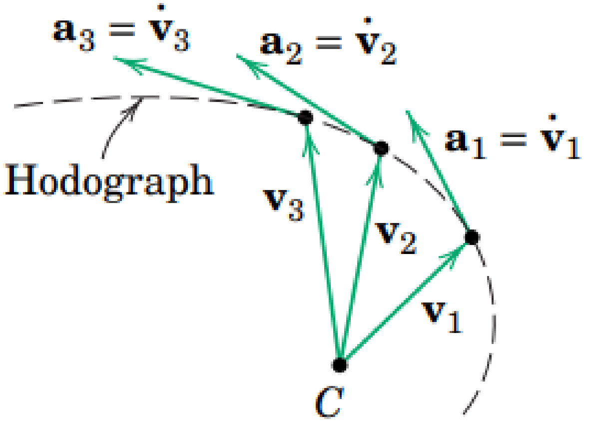
- *instantaneous acceleration*

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = \dot{\mathbf{v}}$$

2/3 Plane Curvilinear Motion

Visualization of Motion





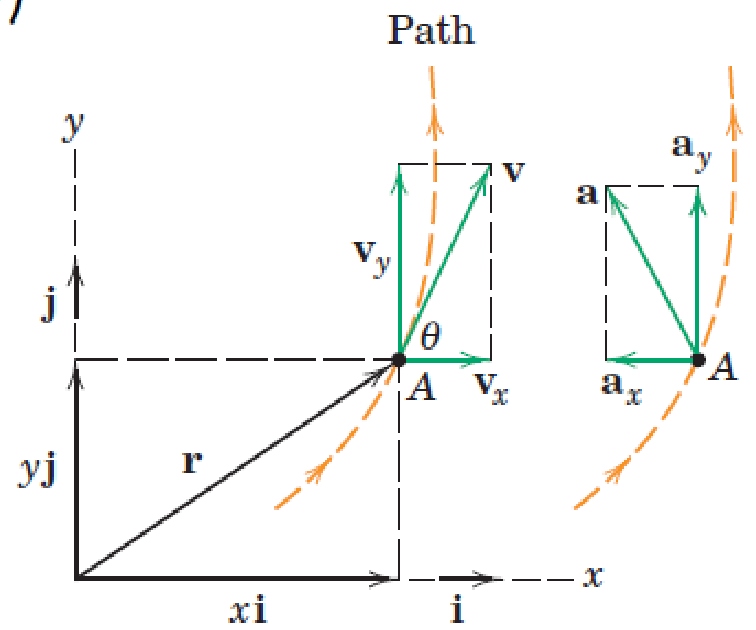
2/4 Rectangular Coordinates (*x-y*)

Vector Representation

$$\mathbf{r} = x\mathbf{i} + y\mathbf{j}$$

$$\mathbf{v} = \dot{\mathbf{r}} = \dot{x}\mathbf{i} + \dot{y}\mathbf{j}$$

$$\mathbf{a} = \dot{\mathbf{v}} = \ddot{\mathbf{r}} = \ddot{x}\mathbf{i} + \ddot{y}\mathbf{j}$$



$$v^2 = v_x{}^2 + v_y{}^2 \qquad v = \sqrt{v_x{}^2 + v_y{}^2} \qquad \tan\theta = \frac{v_y}{v_x}$$

$$a^2 = a_x{}^2 + a_y{}^2 \qquad a = \sqrt{a_x{}^2 + a_y{}^2}$$

2/4 Rectangular Coordinates (x-y)

Projectile Motion

- An important application of two-dimensional kinematic theory is the problem of projectile motion.

- For a first treatment of the subject, we neglect aerodynamic drag and the curvature and rotation of the earth, and we assume that the altitude change is small enough so that the acceleration due to gravity can be considered constant.

$$a_x = 0 \qquad a_y = -g$$

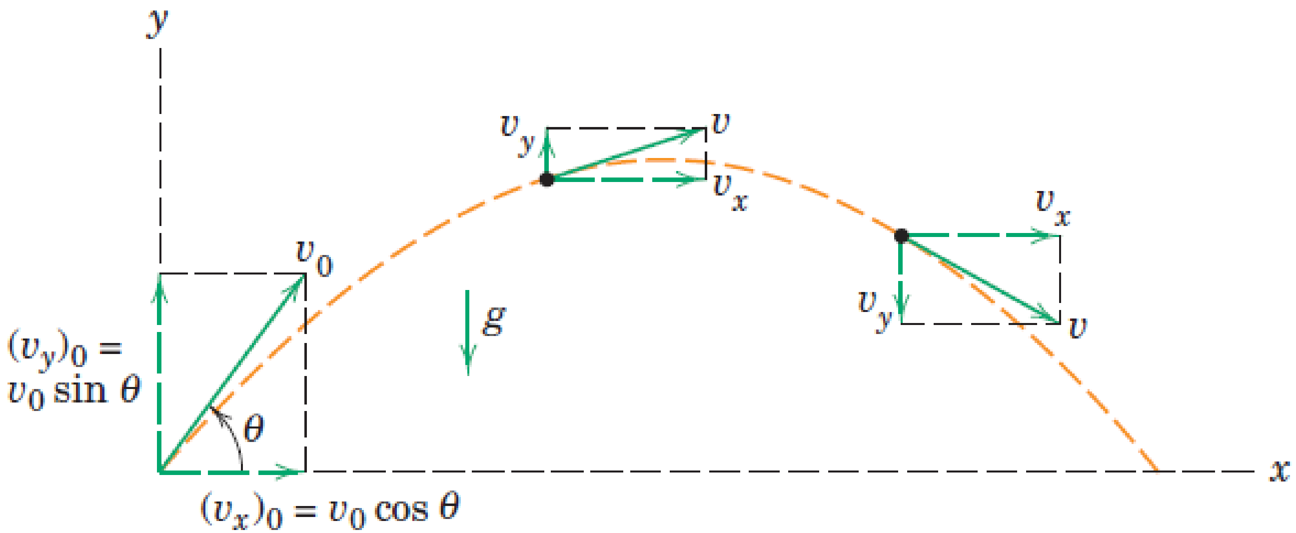
2/4 Rectangular Coordinates (*x*-*y*)

Projectile Motion

$$v_x = (v_x)_0 \qquad v_y = (v_y)_0 - gt$$

$$x = x_0 + (v_x)_0 t \qquad y = y_0 + (v_y)_0 t - \frac{1}{2}gt^2$$

$$v_y{}^2 = (v_y)_0{}^2 - 2g(y - y_0)$$



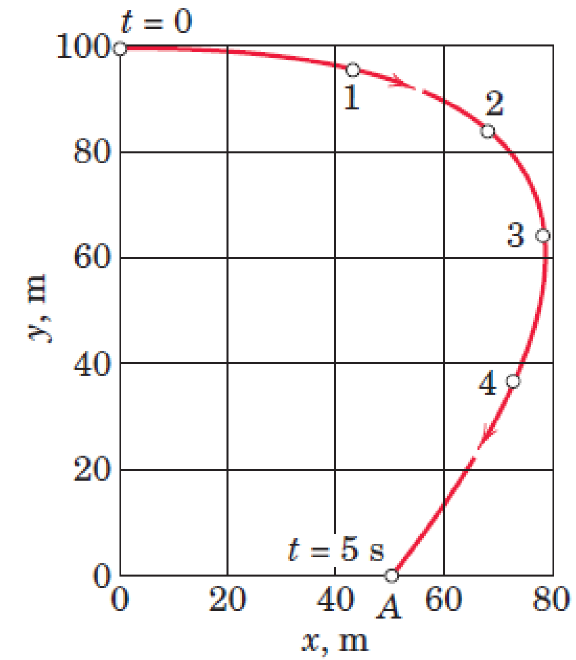
2/4 Rectangular Coordinates (*x*-*y*)

Projectile Motion

This stroboscopic photograph of a bouncing ping-pong ball suggests not only the parabolic nature of the path, but also the fact that the speed is lower near the apex.

SAMPLE PROBLEM 2/5

The curvilinear motion of a particle is defined by $v_x = 50 - 16t$ and $y = 100 - 4t^2$, where v_x is in meters per second, y is in meters, and t is in seconds. It is also known that $x = 0$ when $t = 0$. Plot the path of the particle and determine its velocity and acceleration when the position $y = 0$ is reached.



$$\left[\int dx = \int v_x\, dt\right] \qquad \int_0^x dx = \int_0^t (50 - 16t)\, dt \qquad x = 50t - 8t^2 \text{ m}$$

$$[a_x = \dot{v}_x] \qquad a_x = \frac{d}{dt}(50 - 16t) \qquad a_x = -16 \text{ m/s}^2$$

$$[a_y = \dot{v}_y] \qquad a_y = \frac{d}{dt}(-8t) \qquad a_y = -8 \text{ m/s}^2$$

$$[v_y = \dot{y}] \qquad v_y = \frac{d}{dt}(100 - 4t^2) \qquad v_y = -8t \text{ m/s}$$

SAMPLE PROBLEM 2/5

$$v_x = 50 - 16(5) = -30 \text{ m/s}$$

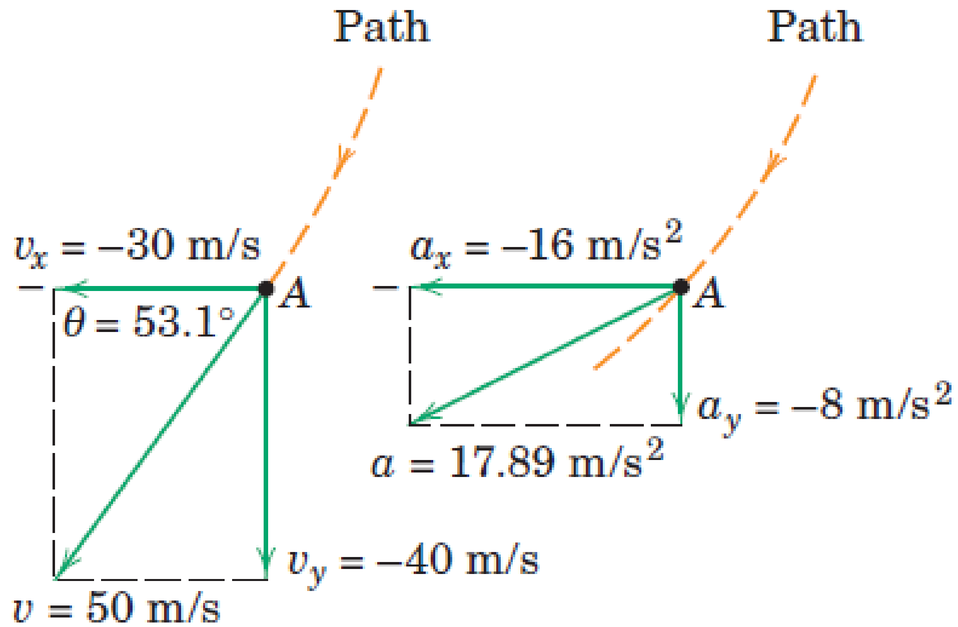
$$v_y = -8(5) = -40 \text{ m/s}$$

$$v = \sqrt{(-30)^2 + (-40)^2} = 50 \text{ m/s}$$

$$a = \sqrt{(-16)^2 + (-8)^2} = 17.89 \text{ m/s}^2$$

$$\mathbf{v} = -30\mathbf{i} - 40\mathbf{j} \text{ m/s}$$

$$\mathbf{a} = -16\mathbf{i} - 8\mathbf{j} \text{ m/s}^2$$



SAMPLE PROBLEM 2/6

A team of engineering students designs a medium-size catapult which launches 8-lb steel spheres. The launch speed is $v_0 = 80$ ft/sec, the launch angle is $\theta = 35°$ above the horizontal, and the launch position is 6 ft above ground level. The students use an athletic field with an adjoining slope topped by an 8-ft fence as shown. Determine:

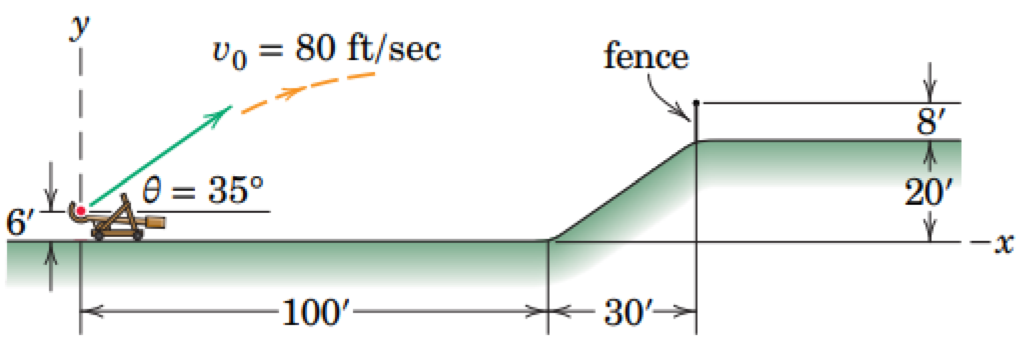
(*a*) the time duration t_f of the flight

(*b*) the *x*-*y* coordinates of the point of first impact

(*c*) the maximum height *h* above the horizontal field attained by the ball

(*d*) the velocity (expressed as a vector) with which the projectile strikes the ground (or the fence)

Repeat part (*b*) for a launch speed of $v_0 = 75$ ft/sec.



SAMPLE PROBLEM 2/6

$[x = x_0 + (v_x)_0 t]$ $\quad 100 + 30 = 0 + (80 \cos 35°)t \quad t = 1.984 \text{ sec}$

$[y = y_0 + (v_y)_0 t - \frac{1}{2}gt^2]$ $\quad y = 6 + 80 \sin 35°(1.984) - \frac{1}{2}(32.2)(1.984)^2 = 33.7 \text{ ft}$

$[y = y_0 + (v_y)_0 t - \frac{1}{2}gt^2]$ $\quad 20 = 6 + 80 \sin 35°(t_f) - \frac{1}{2}(32.2){t_f}^2 \quad t_f = 2.50 \text{ sec}$ *Ans.*

$[x = x_0 + (v_x)_0 t]$ $\quad x = 0 + 80 \cos 35°(2.50) = 164.0 \text{ ft}$

(b) Thus the point of first impact is $(x, y) = (164.0, 20)$ ft. *Ans.*

SAMPLE PROBLEM 2/6

(c) For the maximum height:

$[v_y{}^2 = (v_y)_0{}^2 - 2g(y - y_0)]$ $0^2 = (80 \sin 35°)^2 - 2(32.2)(h - 6)$ $h = 38.7$ ft *Ans.*

(d) For the impact velocity:

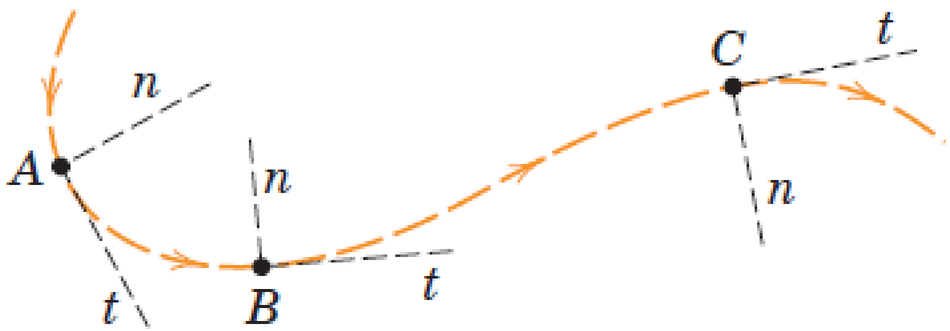
$[v_x = (v_x)_0]$ $v_x = 80 \cos 35° = 65.5$ ft/sec

$[v_y = (v_y)_0 - gt]$ $v_y = 80 \sin 35° - 32.2(2.50) = -34.7$ ft/sec

So the impact velocity is $\mathbf{v} = 65.5\mathbf{i} - 34.7\mathbf{j}$ ft/sec. *Ans.*

2/5 Normal and Tangential Coordinates (*n*-*t*)

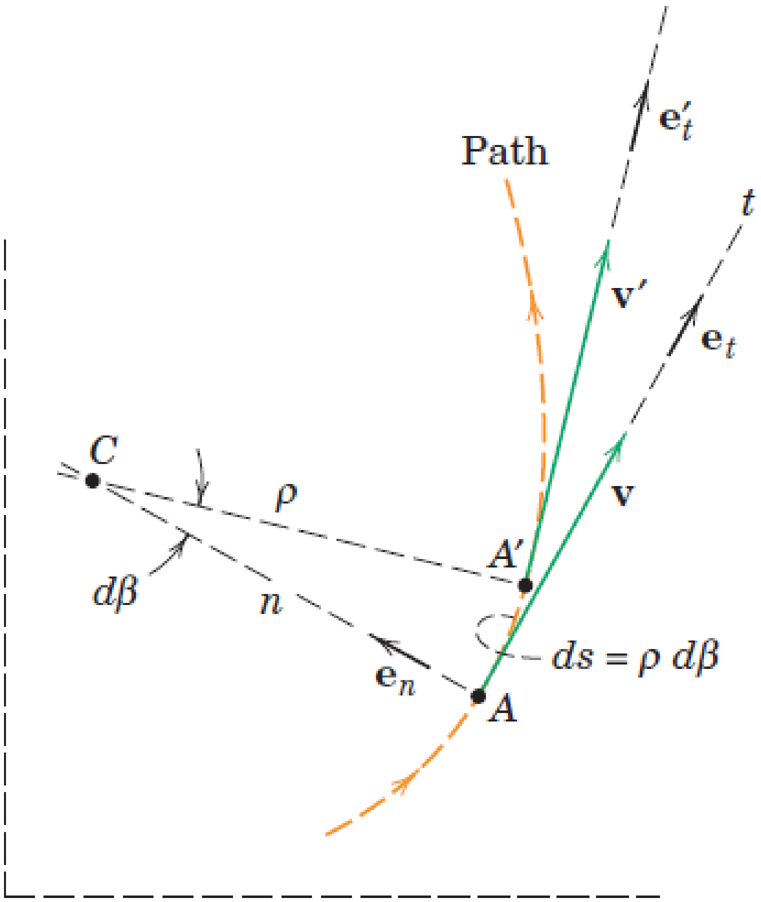
- Measurements made along the tangent *t* and normal *n* to the path of the particle



2/5 Normal and Tangential Coordinates (*n*-*t*)

Velocity and Acceleration

$$\mathbf{v} = v\mathbf{e}_t = \rho\dot{\beta}\mathbf{e}_t$$



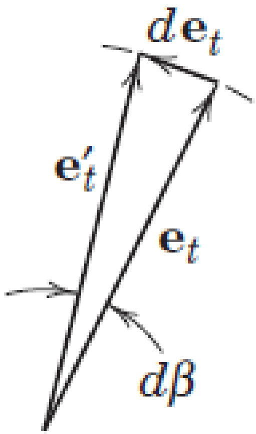
2/5 Normal and Tangential Coordinates (*n*-*t*)

Velocity and Acceleration

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = \frac{d(v\mathbf{e}_t)}{dt} = v\dot{\mathbf{e}}_t + \dot{v}\mathbf{e}_t$$

$$d\mathbf{e}_t = \mathbf{e}_n \, d\beta$$

$$\dot{\mathbf{e}}_t = \dot{\beta}\mathbf{e}_n$$



2/5 Normal and Tangential Coordinates (*n*-*t*)

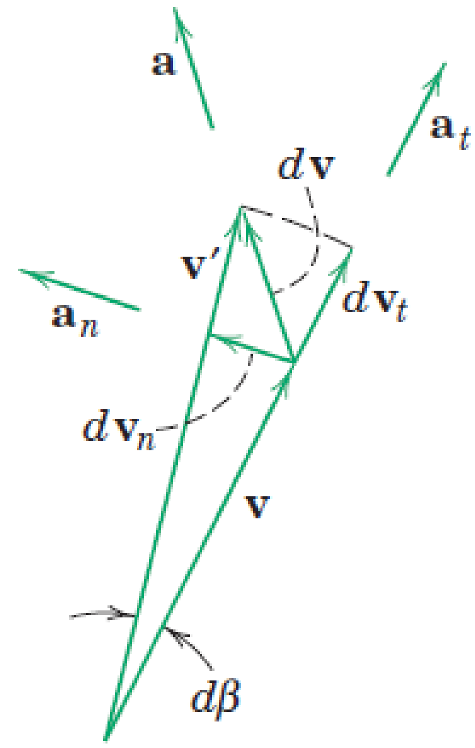
Velocity and Acceleration

$$\mathbf{a} = \frac{v^2}{\rho}\mathbf{e}_n + \dot{v}\mathbf{e}_t$$

$$a_n = \frac{v^2}{\rho} = \rho\dot{\beta}^2 = v\dot{\beta}$$

$$a_t = \dot{v} = \ddot{s} \quad \rightarrow \quad a_t = \dot{v} = d(\rho\dot{\beta})/dt = \rho\ddot{\beta} + \dot{\rho}\dot{\beta}$$

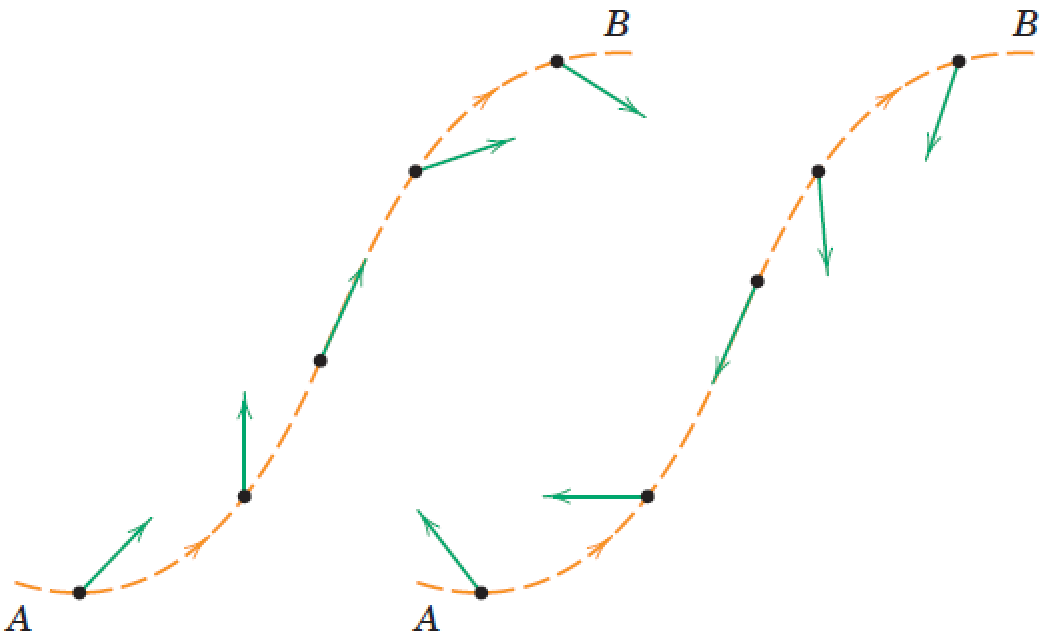
$$a = \sqrt{{a_n}^2 + {a_t}^2}$$



2/5 Normal and Tangential Coordinates (*n*-*t*)

Geometric Interpretation

- a_n is *always directed toward the center of curvature C*



Speed increasing
(*a*)

Speed decreasing
(*b*)

2/5 Normal and Tangential Coordinates (*n-t*)

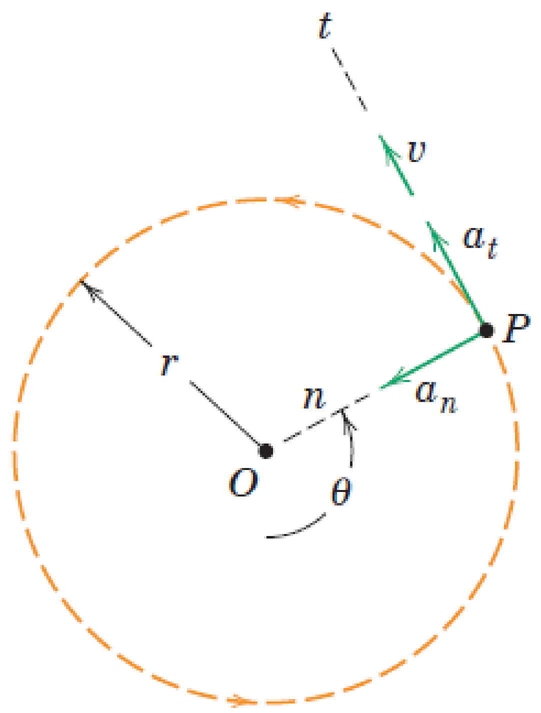
Circular Motion

- Circular motion is an important special case of plane curvilinear motion where the radius of curvature ρ becomes the constant radius r of the circle.

$$v = r\dot{\theta}$$

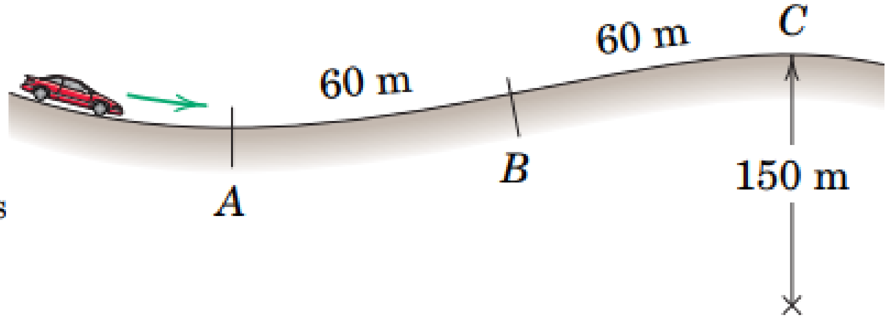
$$a_n = v^2/r = r\dot{\theta}^2 = v\dot{\theta}$$

$$a_t = \dot{v} = r\ddot{\theta}$$



SAMPLE PROBLEM 2/7

To anticipate the dip and hump in the road, the driver of a car applies her brakes to produce a uniform deceleration. Her speed is 100 km/h at the bottom *A* of the dip and 50 km/h at the top *C* of the hump, which is 120 m along the road from *A*. If the passengers experience a total acceleration of 3 m/s² at *A* and if the radius of curvature of the hump at *C* is 150 m, calculate (*a*) the radius of curvature ρ at *A*, (*b*) the acceleration at the inflection point *B*, and (*c*) the total acceleration at *C*.



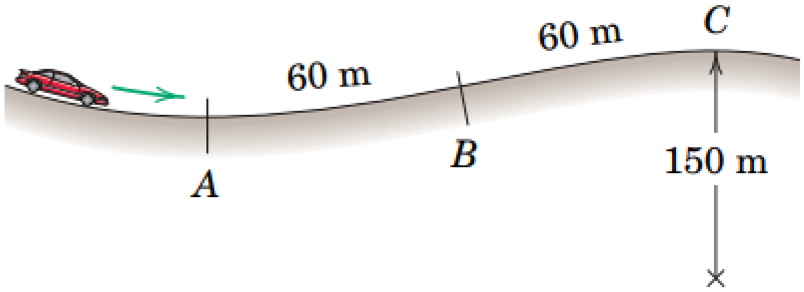
$$v_A = \left(100\,\frac{\text{km}}{\text{h}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right)\left(1000\,\frac{\text{m}}{\text{km}}\right) = 27.8\text{ m/s}$$

$$v_C = 50\,\frac{1000}{3600} = 13.89\text{ m/s}$$

$$\left[\int v\,dv = \int a_t\,ds\right] \qquad \int_{v_A}^{v_C} v\,dv = a_t \int_0^s ds$$

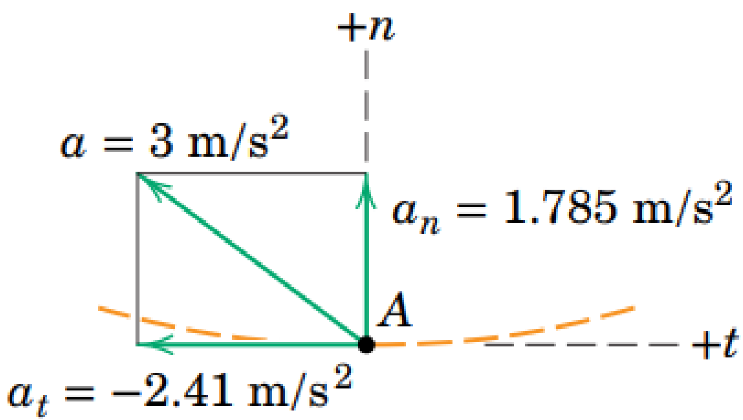
$$a_t = \frac{1}{2s}\left(v_C^{\,2} - v_A^{\,2}\right) = \frac{(13.89)^2 - (27.8)^2}{2(120)} = -2.41\text{ m/s}^2$$

SAMPLE PROBLEM 2/7

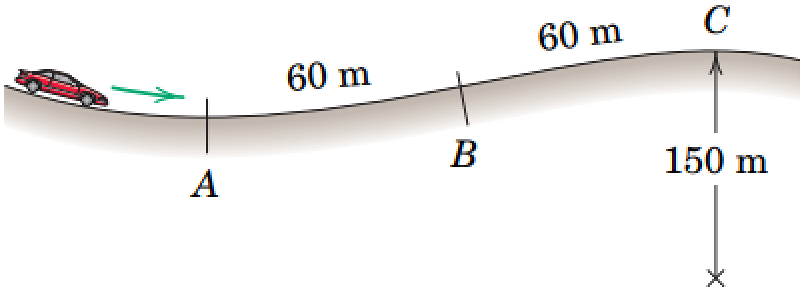


$[a^2 = a_n{}^2 + a_t{}^2]$ $\quad a_n{}^2 = 3^2 - (2.41)^2 = 3.19 \quad a_n = 1.785\ \text{m/s}^2$

$[a_n = v^2/\rho]$ $\quad \rho = v^2/a_n = (27.8)^2/1.785 = 432\ \text{m}$

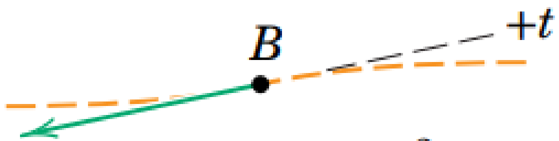


SAMPLE PROBLEM 2/7



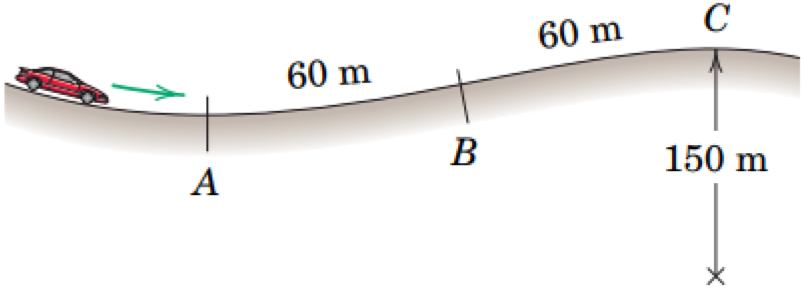
$a_n = 0$ and

$$a = a_t = -2.41 \text{ m/s}^2$$



$$a = a_t = -2.41 \text{ m/s}^2$$

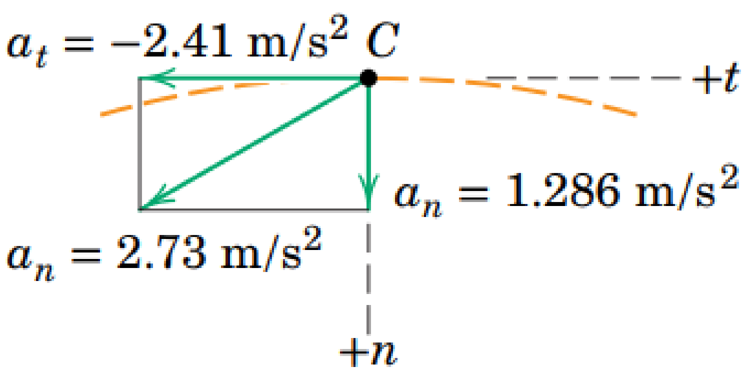
SAMPLE PROBLEM 2/7



$$[a_n = v^2/\rho] \qquad a_n = (13.89)^2/150 = 1.286 \text{ m/s}^2$$

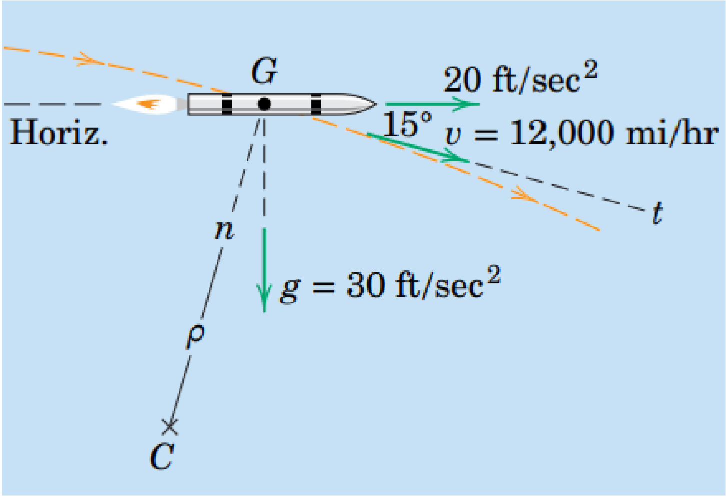
$$\mathbf{a} = 1.286\mathbf{e}_n - 2.41\mathbf{e}_t \text{ m/s}^2$$

$$\left[a = \sqrt{a_n{}^2 + a_t{}^2}\right] \qquad a = \sqrt{(1.286)^2 + (-2.41)^2} = 2.73 \text{ m/s}^2$$



SAMPLE PROBLEM 2/8

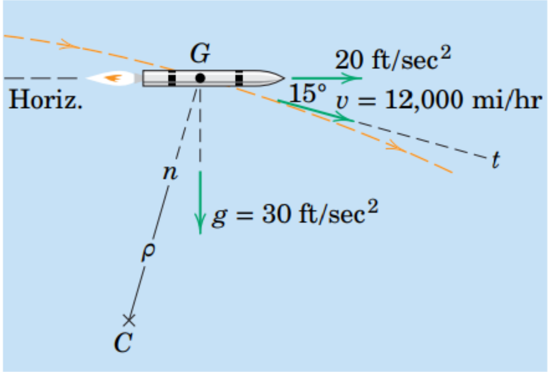
A certain rocket maintains a horizontal attitude of its axis during the powered phase of its flight at high altitude. The thrust imparts a horizontal component of acceleration of 20 ft/sec^2, and the downward acceleration component is the acceleration due to gravity at that altitude, which is g = 30 ft/sec^2. At the instant represented, the velocity of the mass center G of the rocket along the 15° direction of its trajectory is 12,000 mi/hr. For this position determine (a) the radius of curvature of the flight trajectory, (b) the rate at which the speed v is increasing, (c) the angular rate $\dot{\beta}$ of the radial line from G to the center of curvature C, and (d) the vector expression for the total acceleration **a** of the rocket.



SAMPLE PROBLEM 2/8

$$a_n = 30 \cos 15° - 20 \sin 15° = 23.8 \text{ ft/sec}^2$$

$$a_t = 30 \sin 15° + 20 \cos 15° = 27.1 \text{ ft/sec}^2$$

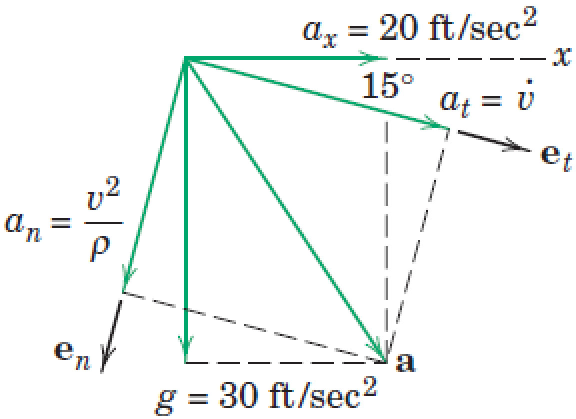


$$[a_n = v^2/\rho] \qquad \rho = \frac{v^2}{a_n} = \frac{[(12{,}000)(44/30)]^2}{23.8} = 13.01(10^6) \text{ ft}$$

$$[\dot{v} = a_t] \qquad \dot{v} = 27.1 \text{ ft/sec}^2$$

$$[v = \rho\dot{\beta}] \qquad \dot{\beta} = v/\rho = \frac{12{,}000(44/30)}{13.01(10^6)} = 13.53(10^{-4}) \text{ rad/sec}$$

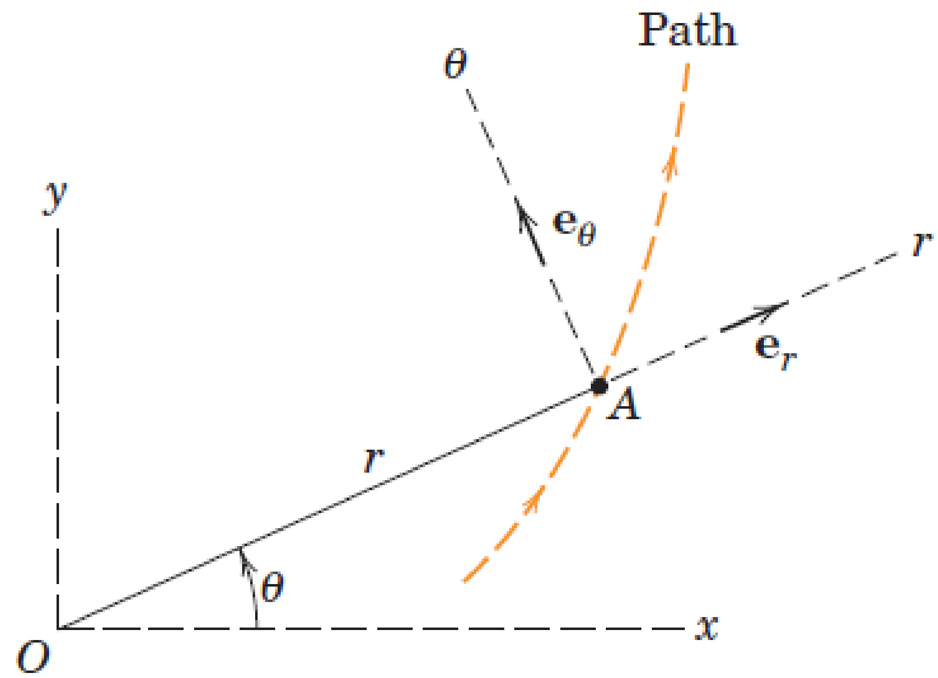
$$\mathbf{a} = 23.8\mathbf{e}_n + 27.1\mathbf{e}_t \text{ ft/sec}^2$$



2/6 Polar Coordinates (r-θ)

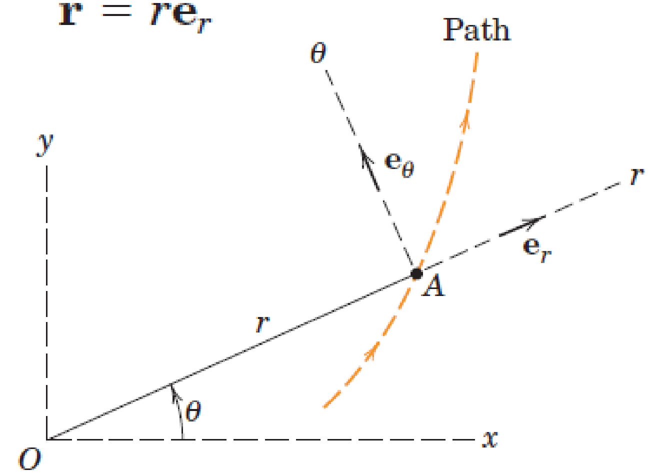
- Particle is located by the radial distance r from a fixed point and by an angular measurement θ to the radial line

$$\mathbf{r} = r\mathbf{e}_r$$



2/6 Polar Coordinates (r-θ)

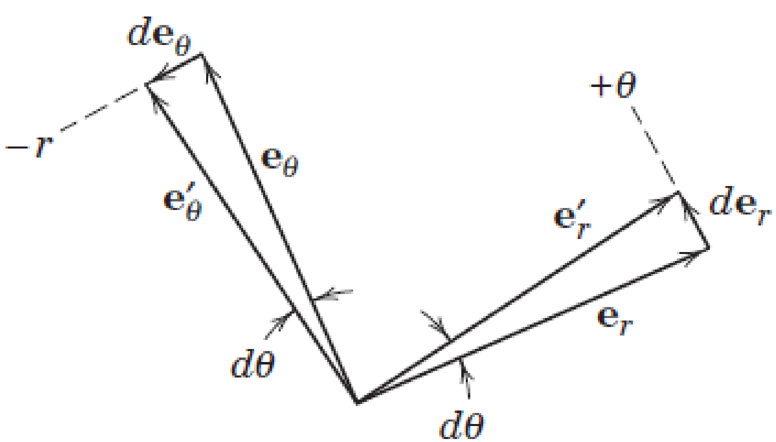
$$\mathbf{r} = r\mathbf{e}_r$$



Time Derivatives of the Unit Vectors

$$\frac{d\mathbf{e}_r}{d\theta} = \mathbf{e}_\theta \quad \text{and} \quad \frac{d\mathbf{e}_\theta}{d\theta} = -\mathbf{e}_r$$

$$\dot{\mathbf{e}}_r = \dot{\theta}\mathbf{e}_\theta \quad \text{and} \quad \dot{\mathbf{e}}_\theta = -\dot{\theta}\mathbf{e}_r$$



2/6 Polar Coordinates (r-θ)

Velocity

$$\mathbf{v} = \dot{\mathbf{r}} = \dot{r}\mathbf{e}_r + r\dot{\mathbf{e}}_r$$

$$\boxed{\mathbf{v} = \dot{r}\mathbf{e}_r + r\dot{\theta}\mathbf{e}_\theta}$$

$$v_r = \dot{r}$$
$$v_\theta = r\dot{\theta}$$
$$v = \sqrt{{v_r}^2 + {v_\theta}^2}$$

Acceleration

$$\mathbf{a} = \dot{\mathbf{v}} = (\ddot{r}\mathbf{e}_r + \dot{r}\dot{\mathbf{e}}_r) + (\dot{r}\dot{\theta}\mathbf{e}_\theta + r\ddot{\theta}\mathbf{e}_\theta + r\dot{\theta}\dot{\mathbf{e}}_\theta)$$

$$\boxed{\mathbf{a} = (\ddot{r} - r\dot{\theta}^2)\mathbf{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\mathbf{e}_\theta}$$

$$a_r = \ddot{r} - r\dot{\theta}^2$$
$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$$
$$a = \sqrt{{a_r}^2 + {a_\theta}^2}$$

$$a_\theta = \frac{1}{r}\frac{d}{dt}(r^2\dot{\theta})$$

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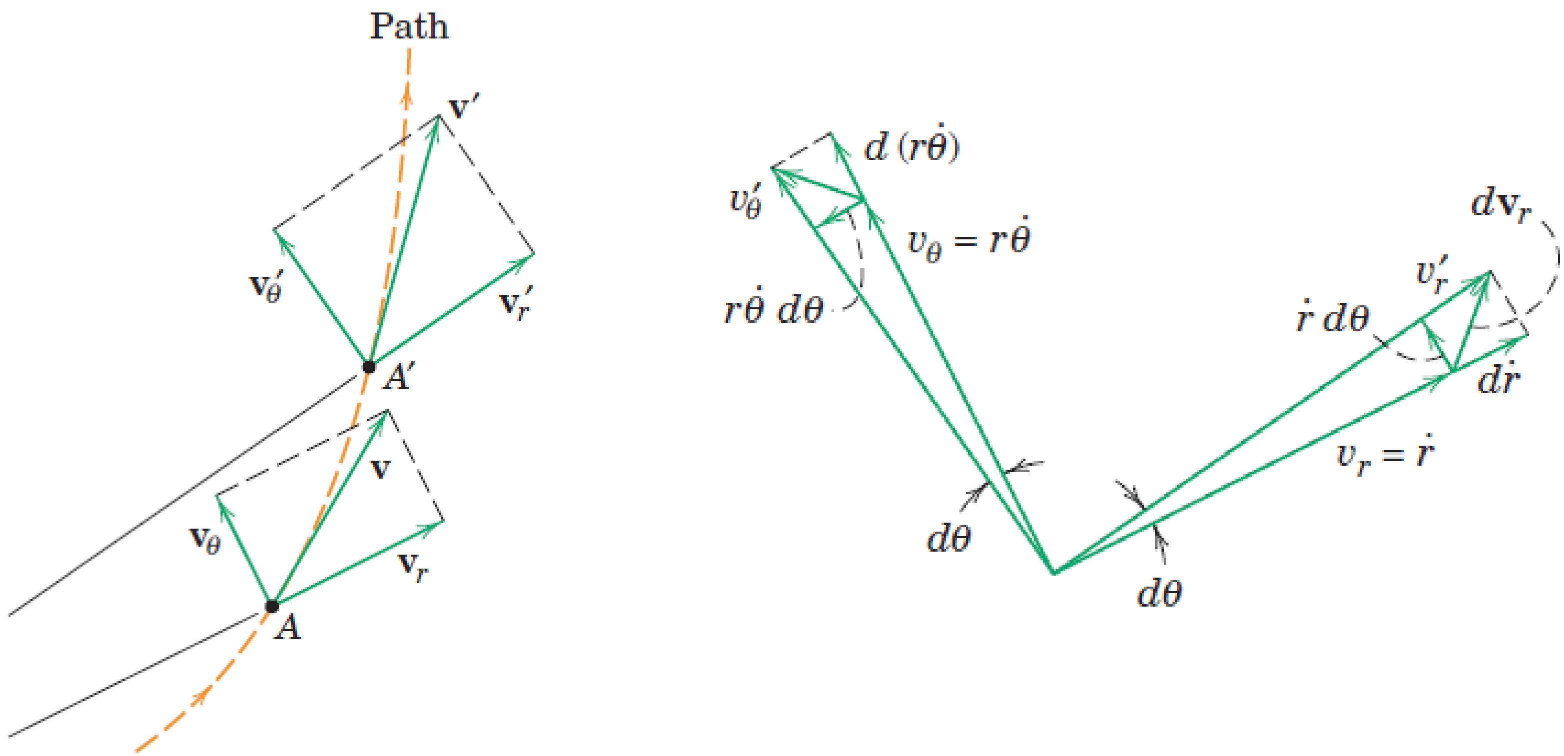
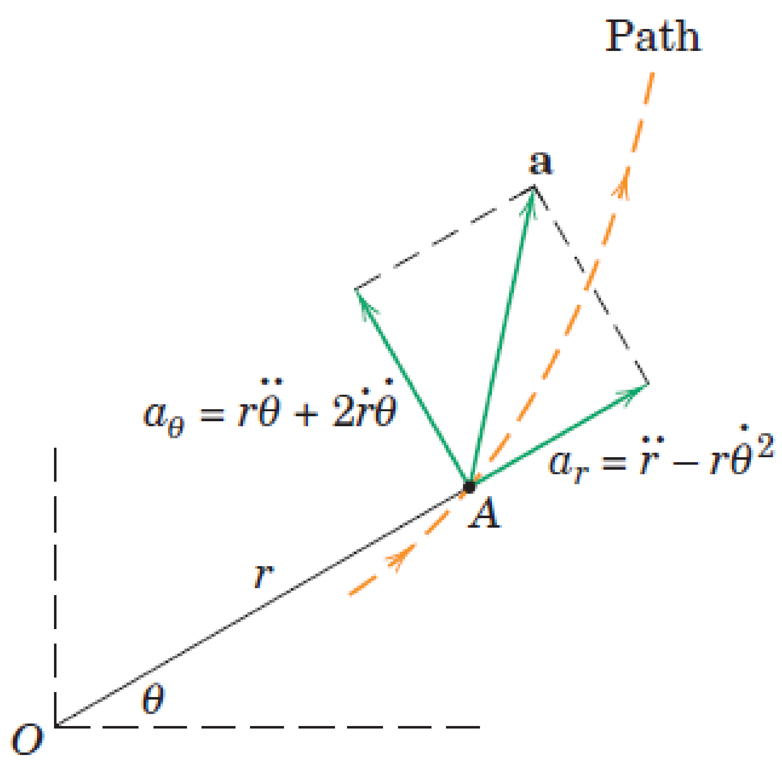
2/6 Polar Coordinates (r-θ)

Geometric Interpretation



2/6 Polar Coordinates (r-θ)

Geometric Interpretation



2/6 Polar Coordinates (r-θ)

Circular Motion

- Same as that obtained with n- and t-components, where the n- and t-directions coincide but the positive r-direction is in the negative n-direction.

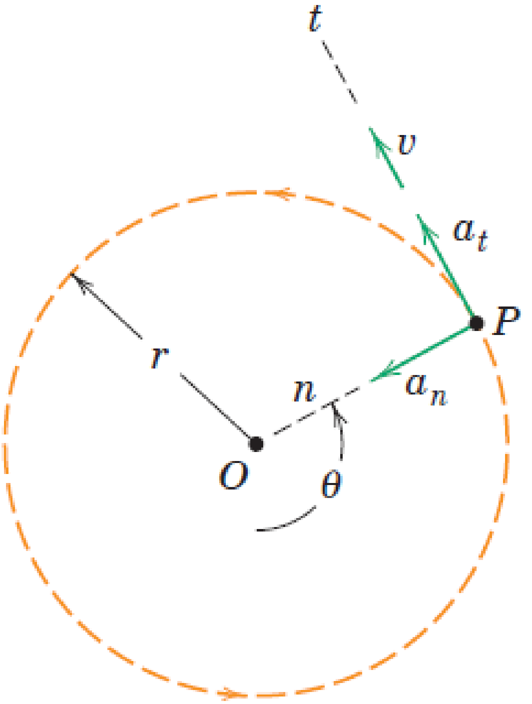
$$v = r\dot{\theta}$$

$$a_n = v^2/r = r\dot{\theta}^2 = v\dot{\theta}$$

$$a_t = \dot{v} = r\ddot{\theta}$$

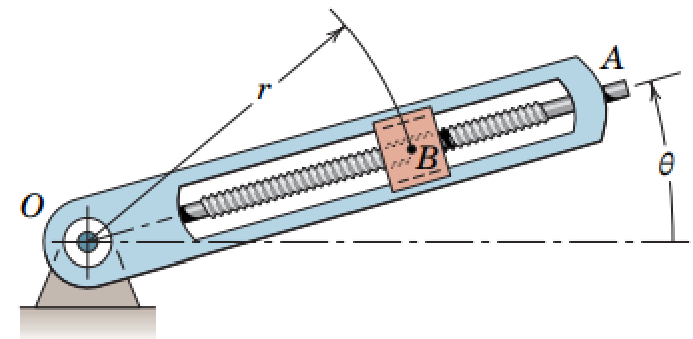
$$v_r = 0 \qquad v_\theta = r\dot{\theta}$$

$$a_r = -r\dot{\theta}^2 \qquad a_\theta = r\ddot{\theta}$$



SAMPLE PROBLEM 2/9

Rotation of the radially slotted arm is governed by $\theta = 0.2t + 0.02t^3$, where θ is in radians and t is in seconds. Simultaneously, the power screw in the arm engages the slider B and controls its distance from O according to $r = 0.2 + 0.04t^2$, where r is in meters and t is in seconds. Calculate the magnitudes of the velocity and acceleration of the slider for the instant when $t = 3$ s.



$$r = 0.2 + 0.04t^2 \qquad r_3 = 0.2 + 0.04(3^2) = 0.56 \text{ m}$$

$$\dot{r} = 0.08t \qquad \dot{r}_3 = 0.08(3) = 0.24 \text{ m/s}$$

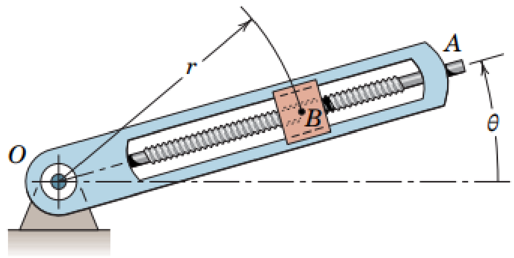
$$\ddot{r} = 0.08 \qquad \ddot{r}_3 = 0.08 \text{ m/s}^2$$

$$[v_\theta = r\dot{\theta}] \qquad v_r = 0.24 \text{ m/s}$$

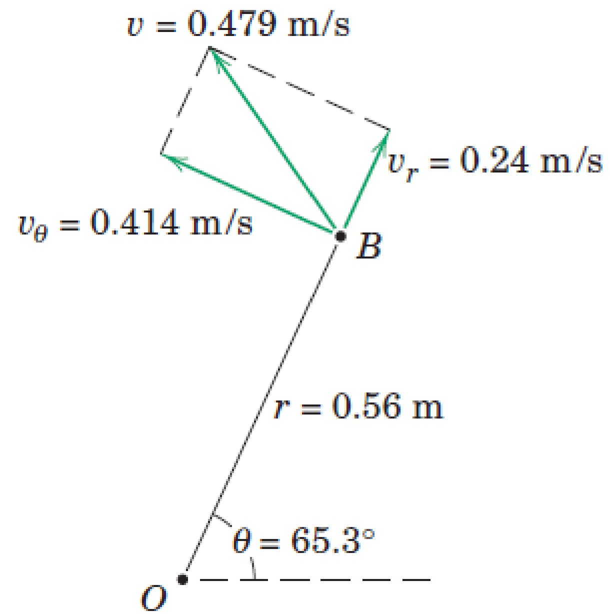
$$[v_\theta = r\dot{\theta}] \qquad v_\theta = 0.56(0.74) = 0.414 \text{ m/s}$$

$$[v = \sqrt{v_r{}^2 + v_\theta{}^2}] \qquad v = \sqrt{(0.24)^2 + (0.414)^2} = 0.479 \text{ m/s}$$

SAMPLE PROBLEM 2/9

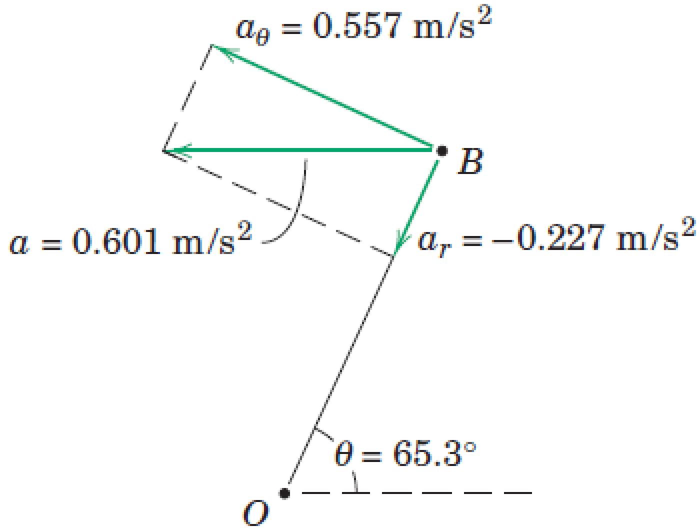


$\theta = 0.2t + 0.02t^3$	$\theta_3 = 0.2(3) + 0.02(3^3) = 1.14 \text{ rad}$
	or $\theta_3 = 1.14(180/\pi) = 65.3°$
$\dot{\theta} = 0.2 + 0.06t^2$	$\dot{\theta}_3 = 0.2 + 0.06(3^2) = 0.74 \text{ rad/s}$
$\ddot{\theta} = 0.12t$	$\ddot{\theta}_3 = 0.12(3) = 0.36 \text{ rad/s}^2$



SAMPLE PROBLEM 2/9

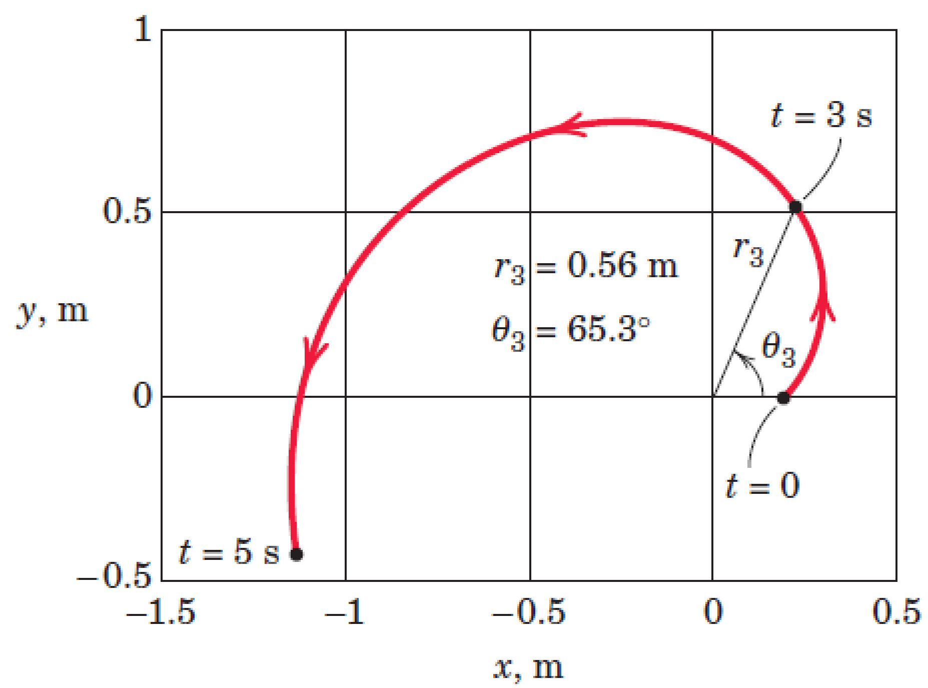
$[a_r = \ddot{r} - r\dot{\theta}^2]$	$a_r = 0.08 - 0.56(0.74)^2 = -0.227 \text{ m/s}^2$
$[a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}]$	$a_\theta = 0.56(0.36) + 2(0.24)(0.74) = 0.557 \text{ m/s}^2$
$[a = \sqrt{a_r{}^2 + a_\theta{}^2}]$	$a = \sqrt{(-0.227)^2 + (0.557)^2} = 0.601 \text{ m/s}^2$



SAMPLE PROBLEM 2/9

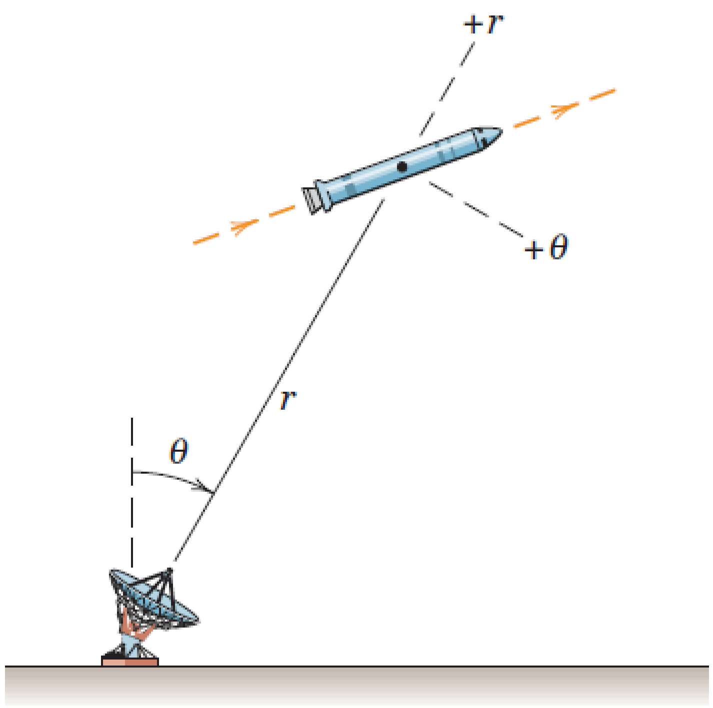
- Conversion from polar to rectangular coordinates

$$x = r\cos\theta \qquad y = r\sin\theta$$



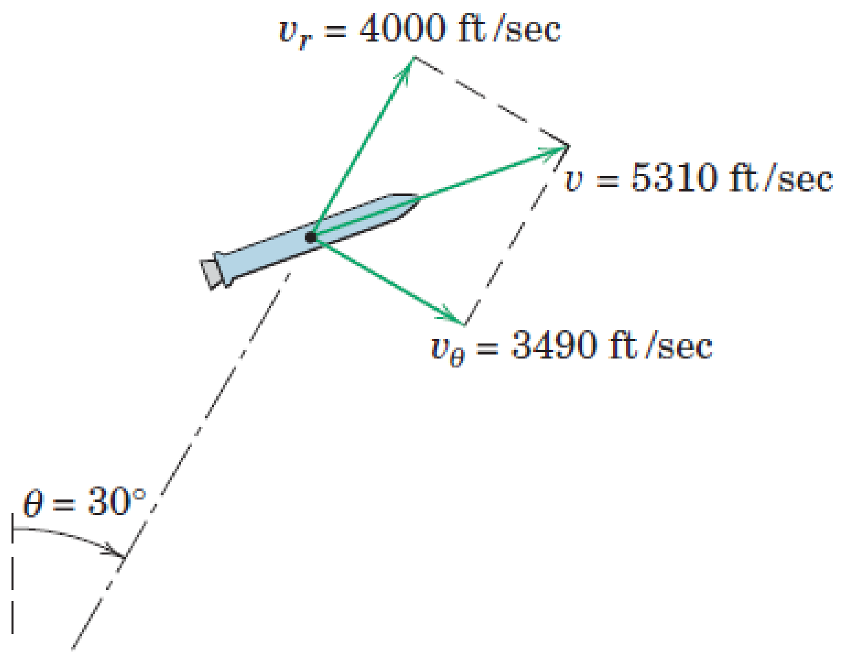
SAMPLE PROBLEM 2/10

A tracking radar lies in the vertical plane of the path of a rocket which is coasting in unpowered flight above the atmosphere. For the instant when $\theta = 30°$, the tracking data give $r = 25(10^4)$ ft, $\dot{r} = 4000$ ft/sec, and $\dot{\theta} = 0.80$ deg/sec. The acceleration of the rocket is due only to gravitational attraction and for its particular altitude is 31.4 ft/sec^2 vertically down. For these conditions determine the velocity v of the rocket and the values of $\ddot{r}$ and $\ddot{\theta}$.

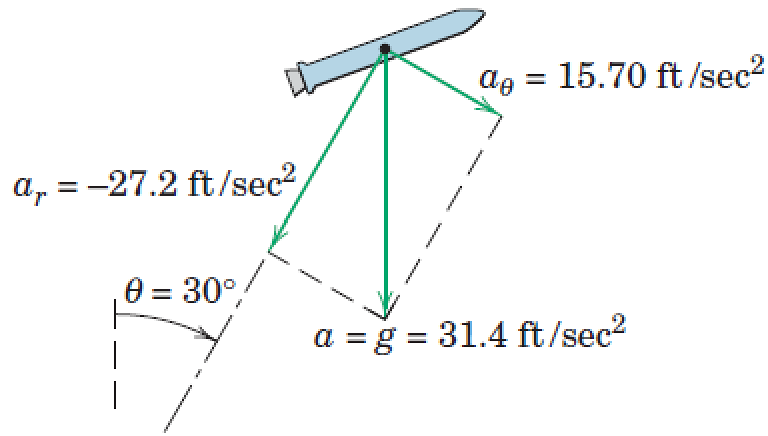


SAMPLE PROBLEM 2/10

$[v_r = \dot{r}]$	$v_r = 4000$ ft/sec
$[v_\theta = r\dot{\theta}]$	$v_\theta = 25(10^4)(0.80)\left(\frac{\pi}{180}\right) = 3490$ ft/sec
$[v = \sqrt{v_r{}^2 + v_\theta{}^2}]$	$v = \sqrt{(4000)^2 + (3490)^2} = 5310$ ft/sec



SAMPLE PROBLEM 2/10



$$a_r = -31.4 \cos 30° = -27.2 \text{ ft/sec}^2$$

$$a_\theta = 31.4 \sin 30° = 15.70 \text{ ft/sec}^2$$

$$[a_r = \ddot{r} - r\dot{\theta}^2] \qquad -27.2 = \ddot{r} - 25(10^4)\left(0.80\frac{\pi}{180}\right)^2$$

$$\ddot{r} = 21.5 \text{ ft/sec}^2$$

$$[a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}] \qquad 15.70 = 25(10^4)\ddot{\theta} + 2(4000)\left(0.80\frac{\pi}{180}\right)$$

$$\ddot{\theta} = -3.84(10^{-4}) \text{ rad/sec}^2$$

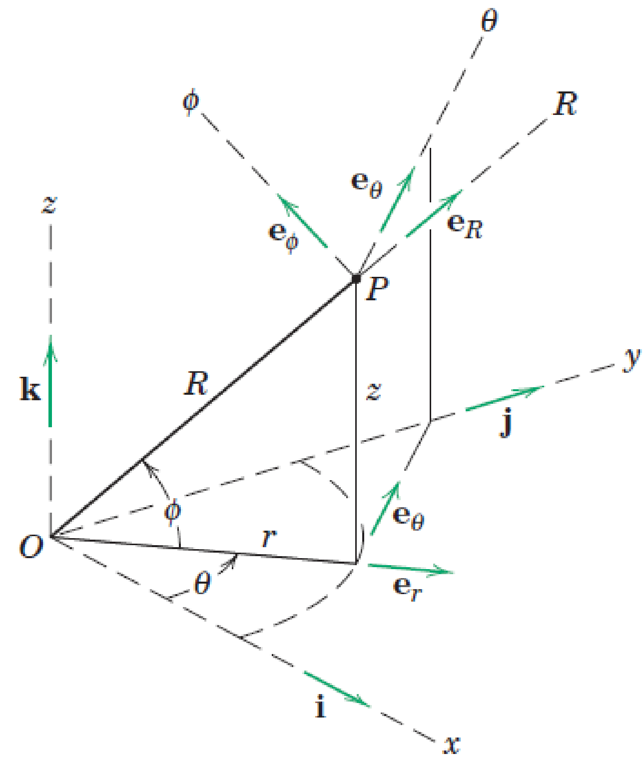
2/7 Space Curvilinear Motion

Rectangular Coordinates (*x*-*y*-*z*)

$$\mathbf{R} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$$

$$\mathbf{v} = \dot{\mathbf{R}} = \dot{x}\mathbf{i} + \dot{y}\mathbf{j} + \dot{z}\mathbf{k}$$

$$\mathbf{a} = \dot{\mathbf{v}} = \ddot{\mathbf{R}} = \ddot{x}\mathbf{i} + \ddot{y}\mathbf{j} + \ddot{z}\mathbf{k}$$



2/7 Space Curvilinear Motion

Cylindrical Coordinates (*r*-*θ*-*z*)

$$\mathbf{R} = r\mathbf{e}_r + z\mathbf{k}$$

$$\mathbf{v} = \dot{r}\mathbf{e}_r + r\dot{\theta}\mathbf{e}_\theta + \dot{z}\mathbf{k}$$

$$v_r = \dot{r}$$

$$v_\theta = r\dot{\theta}$$

$$v_z = \dot{z}$$

$$v = \sqrt{{v_r}^2 + {v_\theta}^2 + {v_z}^2}$$

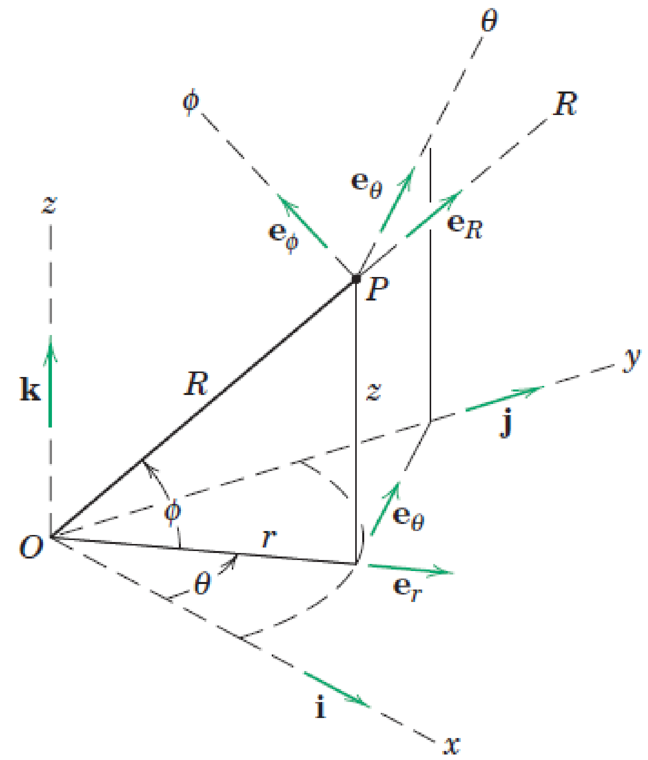
$$\mathbf{a} = (\ddot{r} - r\dot{\theta}^2)\mathbf{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\mathbf{e}_\theta + \ddot{z}\mathbf{k}$$

$$a_r = \ddot{r} - r\dot{\theta}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = \frac{1}{r}\frac{d}{dt}(r^2\dot{\theta})$$

$$a_z = \ddot{z}$$

$$a = \sqrt{{a_r}^2 + {a_\theta}^2 + {a_z}^2}$$



2/7 Space Curvilinear Motion

Spherical Coordinates (R-θ-ϕ)

$$\mathbf{v} = v_R\mathbf{e}_R + v_\theta\mathbf{e}_\theta + v_\phi\mathbf{e}_\phi$$

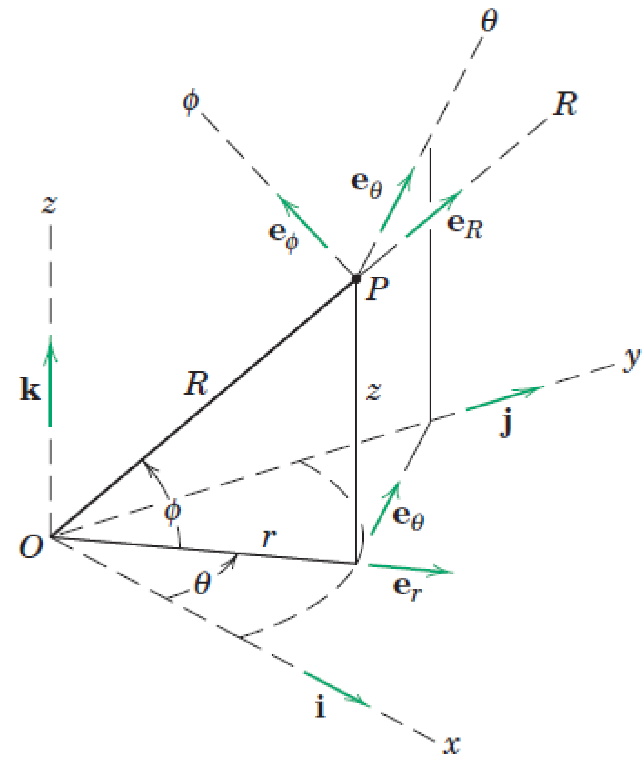
$$v_R = \dot{R}$$
$$v_\theta = R\dot{\theta}\cos\phi$$
$$v_\phi = R\dot{\phi}$$

$$\mathbf{a} = a_R\mathbf{e}_R + a_\theta\mathbf{e}_\theta + a_\phi\mathbf{e}_\phi$$

$$a_R = \ddot{R} - R\dot{\phi}^2 - R\dot{\theta}^2\cos^2\phi$$

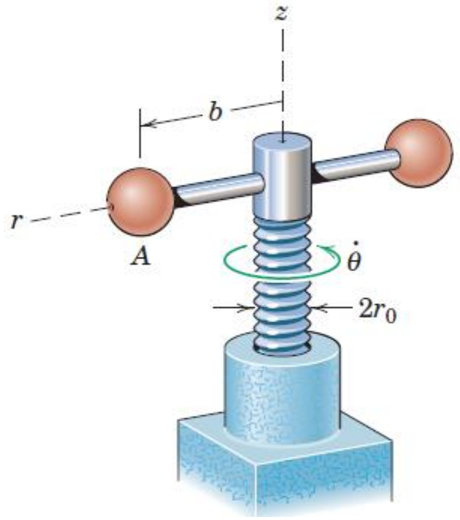
$$a_\theta = \frac{\cos\phi}{R}\frac{d}{dt}(R^2\dot{\theta}) - 2R\dot{\theta}\dot{\phi}\sin\phi$$

$$a_\phi = \frac{1}{R}\frac{d}{dt}(R^2\dot{\phi}) + R\dot{\theta}^2\sin\phi\cos\phi$$



SAMPLE PROBLEM 2/11

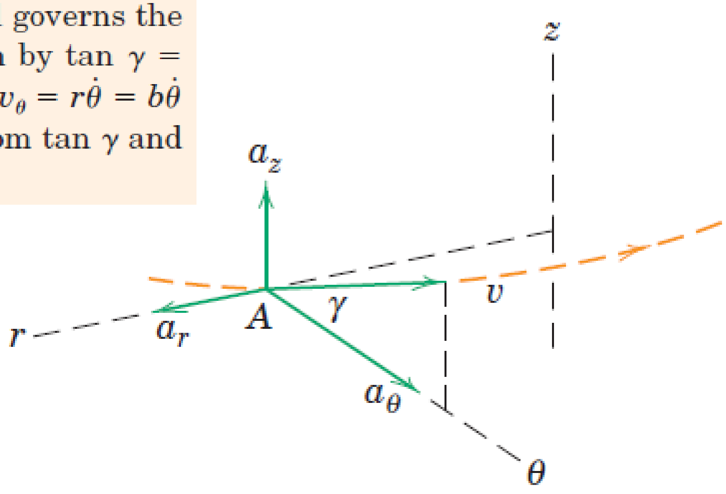
The power screw starts from rest and is given a rotational speed $\dot{\theta}$ which increases uniformly with time t according to $\dot{\theta} = kt$, where k is a constant. Determine the expressions for the velocity v and acceleration a of the center of ball A when the screw has turned through one complete revolution from rest. The lead of the screw (advancement per revolution) is L.



$$\theta = \Delta\theta = \int \dot{\theta}\, dt = \frac{1}{2}kt^2 \qquad 2\pi = \frac{1}{2}kt^2 \qquad t = 2\sqrt{\pi/k}$$

$$\dot{\theta} = kt = k(2\sqrt{\pi/k}) = 2\sqrt{\pi k}$$

The helix angle γ of the path followed by the center of the ball governs the relation between the θ- and z-components of velocity and is given by $\tan \gamma = L/(2\pi b)$. Now from the figure we see that $v_\theta = v \cos \gamma$. Substituting $v_\theta = r\dot{\theta} = b\dot{\theta}$ from Eq. 2/16 gives $v = v_\theta/\cos \gamma = b\dot{\theta}/\cos \gamma$. With $\cos \gamma$ obtained from $\tan \gamma$ and with $\dot{\theta} = 2\sqrt{\pi k}$, we have for the one-revolution position



SAMPLE PROBLEM 2/11

$\tan\ \gamma = L/(2\pi b).$

$v_\theta = v \cos\gamma.$ $v_\theta = r\dot\theta = b\dot\theta$ $v = v_\theta/\cos\gamma = b\dot\theta/\cos\gamma.$

that for $\tan\beta = a/b$ the cosine of β becomes $b/\sqrt{a^2 + b^2}$.

$$v = 2b\sqrt{\pi k}\,\frac{\sqrt{L^2 + 4\pi^2 b^2}}{2\pi b} = \sqrt{\frac{k}{\pi}}\sqrt{L^2 + 4\pi^2 b^2}$$

$[a_r = \ddot r - r\dot\theta^2]$ $\quad a_r = 0 - b(2\sqrt{\pi k})^2 = -4b\pi k$

$[a_\theta = r\ddot\theta + 2\dot r\dot\theta]$ $\quad a_\theta = bk + 2(0)(2\sqrt{\pi k}) = bk$

$[a_z = \ddot z = \dot v_z]$ $\quad a_z = \frac{d}{dt}(v_z) = \frac{d}{dt}(v_\theta \tan\gamma) = \frac{d}{dt}(b\dot\theta\tan\gamma)$

$$= (b\tan\gamma)\ddot\theta = b\,\frac{L}{2\pi b}\,k = \frac{kL}{2\pi}$$

$$a = \sqrt{(-4b\pi k)^2 + (bk)^2 + \left(\frac{kL}{2\pi}\right)^2}$$

$$= bk\sqrt{(1 + 16\pi^2) + L^2/(4\pi^2 b^2)}$$

2/8 Relative Motion (Translating Axes)

- It is not always possible or convenient, however, to use a fi xed set of axes to describe or to measure motion.

- In addition, there are many engineering problems for which the analysis of motion is simplified by using measurements made with respect to a moving reference system.

- These measurements, when combined with the absolute motion of the moving coordinate system, enable us to determine the absolute motion in question.

- This approach is called a relative-motion analysis.

2/8 Relative Motion (Translating Axes)

Choice of Coordinate System

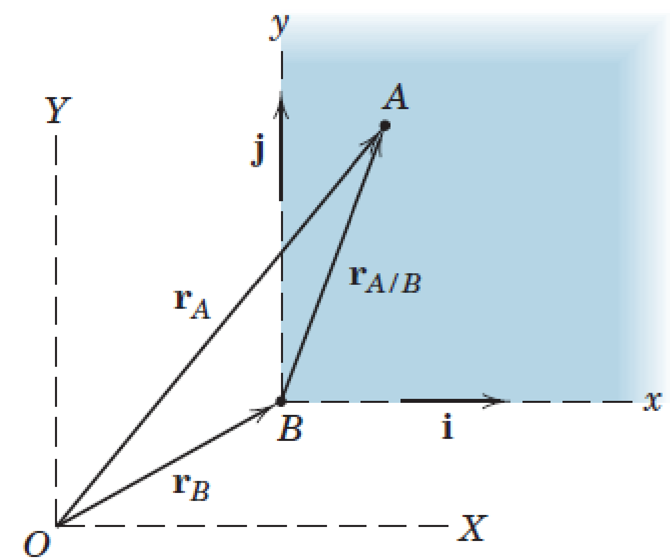
- The motion of the moving coordinate system is specified with respect to a fixed coordinate system.

Vector Representation

$$\mathbf{r}_A = \mathbf{r}_B + \mathbf{r}_{A/B}$$

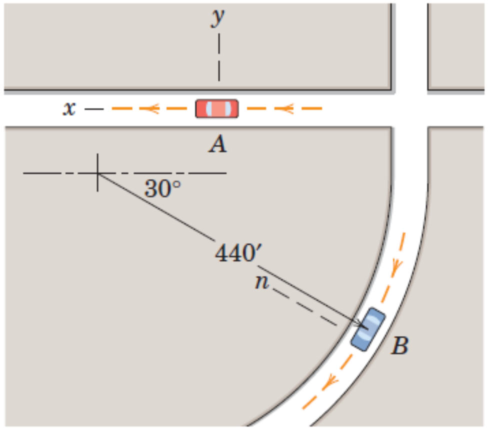
$$\dot{\mathbf{r}}_A = \dot{\mathbf{r}}_B + \dot{\mathbf{r}}_{A/B} \quad \text{or} \quad \boxed{\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}}$$

$$\ddot{\mathbf{r}}_A = \ddot{\mathbf{r}}_B + \ddot{\mathbf{r}}_{A/B} \quad \text{or} \quad \boxed{\mathbf{a}_A = \mathbf{a}_B + \mathbf{a}_{A/B}}$$



SAMPLE PROBLEM 2/14

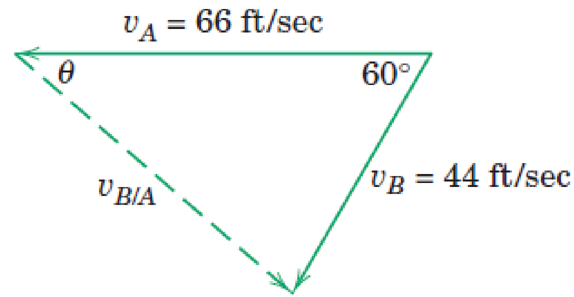
Car A is accelerating in the direction of its motion at the rate of 3 ft/sec^2. Car B is rounding a curve of 440-ft radius at a constant speed of 30 mi/hr. Determine the velocity and acceleration which car B appears to have to an observer in car A if car A has reached a speed of 45 mi/hr for the positions represented.



$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$$

$$v_A = 45\frac{5280}{60^2} = 45\frac{44}{30} = 66 \text{ ft/sec} \qquad v_B = 30\frac{44}{30} = 44 \text{ ft/sec}$$

$$v_{B/A} = 58.2 \text{ ft/sec} \qquad \theta = 40.9°$$



$$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$$

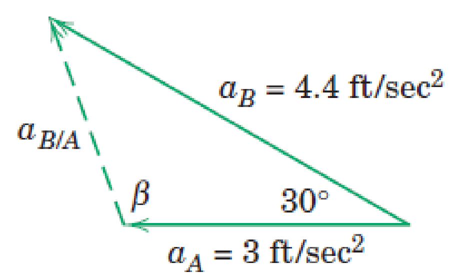
$$[a_n = v^2/\rho] \qquad a_B = (44)^2/440 = 4.4 \text{ ft/sec}^2$$

$$(a_{B/A})_x = 4.4\cos 30° - 3 = 0.810 \text{ ft/sec}^2$$

$$(a_{B/A})_y = 4.4\sin 30° = 2.2 \text{ ft/sec}^2$$

$$a_{B/A} = \sqrt{(0.810)^2 + (2.2)^2} = 2.34 \text{ ft/sec}^2$$

$$\frac{4.4}{\sin\beta} = \frac{2.34}{\sin 30°} \qquad \beta = \sin^{-1}\left(\frac{4.4}{2.34}0.5\right) = 110.2°$$



2/9 Constrained Motion of Connected Particles

- Sometimes the motions of particles are interrelated because of the constraints imposed by interconnecting members.

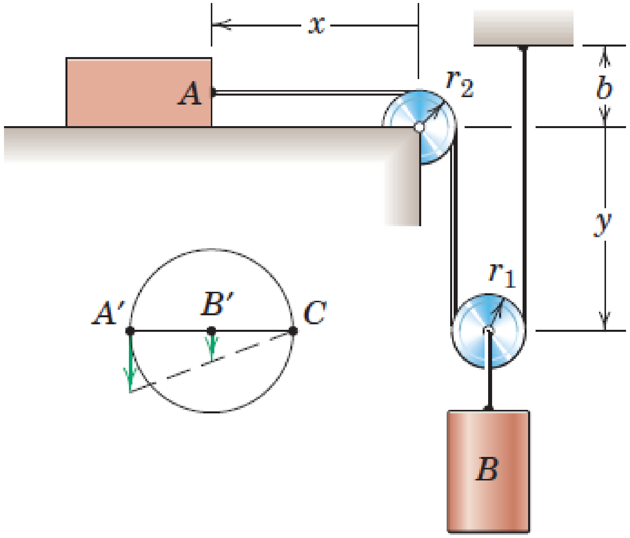
One Degree of Freedom

- One degree of freedom: only one variable, either x or y, is needed to specify the positions of all parts of the system.

$$L = x + \frac{\pi r_2}{2} + 2y + \pi r_1 + b$$

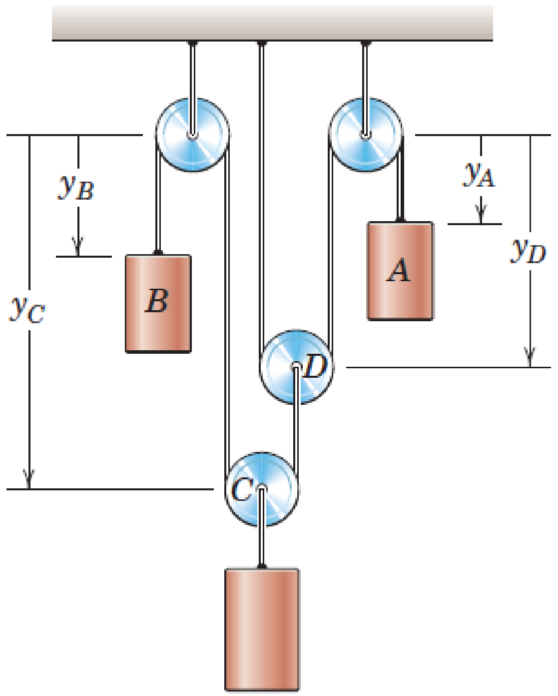
$$0 = \dot{x} + 2\dot{y} \qquad \text{or} \qquad 0 = v_A + 2v_B$$

$$0 = \ddot{x} + 2\ddot{y} \qquad \text{or} \qquad 0 = a_A + 2a_B$$



2/9 Constrained Motion of Connected Particles

Two Degrees of Freedom



$$L_A = y_A + 2y_D + \text{constant}$$

$$L_B = y_B + y_C + (y_C - y_D) + \text{constant}$$

$$0 = \dot{y}_A + 2\dot{y}_D \quad \text{and} \quad 0 = \dot{y}_B + 2\dot{y}_C - \dot{y}_D$$

$$0 = \ddot{y}_A + 2\ddot{y}_D \quad \text{and} \quad 0 = \ddot{y}_B + 2\ddot{y}_C - \ddot{y}_D$$

$$\dot{y}_A + 2\dot{y}_B + 4\dot{y}_C = 0 \quad \text{or} \quad v_A + 2v_B + 4v_C = 0$$

$$\ddot{y}_A + 2\ddot{y}_B + 4\ddot{y}_C = 0 \quad \text{or} \quad a_A + 2a_B + 4a_C = 0$$

SAMPLE PROBLEM 2/16

The tractor A is used to hoist the bale B with the pulley arrangement shown. If A has a forward velocity v_A, determine an expression for the upward velocity v_B of the bale in terms of x.

$$L = 2(h - y) + l = 2(h - y) + \sqrt{h^2 + x^2}$$

$$0 = -2\dot{y} + \frac{x\dot{x}}{\sqrt{h^2 + x^2}}$$

$$v_B = \frac{1}{2}\frac{xv_A}{\sqrt{h^2 + x^2}}$$