دانشگاه سمنان

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دانشکده مهندسی مکانیک

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نیمسال اول ۹۸-۹۹

ELECTRICAL ENGINEERING

PRINCIPLES AND APPLICATIONS

Allan R. Hambley

5th Edition

- ***CONTENTS:***

1.1 OVERVIEW OF ELECTRICAL ENGINEERING

- Two main objectives of electrical systems:

  1. To gather, store, process, transport, and present information.

  2. To distribute, store, and convert energy between various forms.

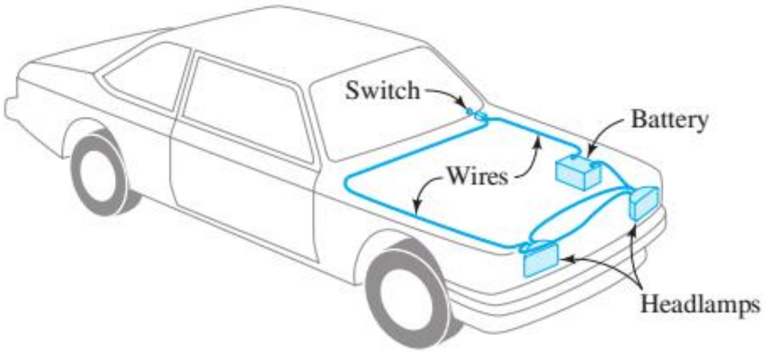
- Subdivisions of Electrical Engineering:

Communication systems, Computer systems, Control systems, Electromagnetics Electronics, Photonics, Power systems, Signal processing

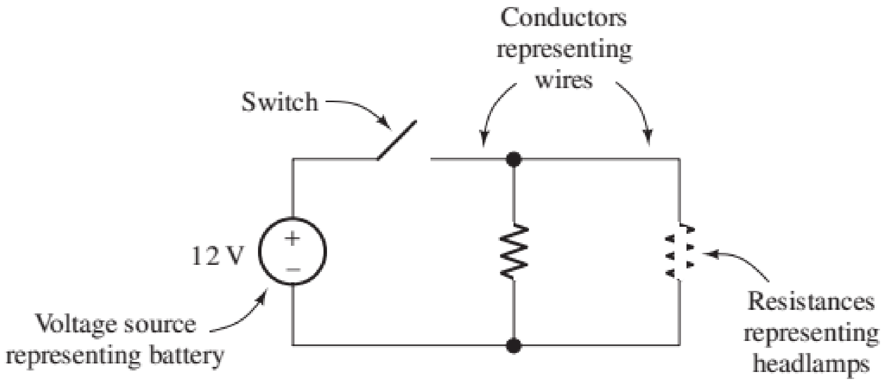
- Electronics integrated with mechanical systems: **Mechatronics**

1.2 CIRCUITS, CURRENTS, AND VOLTAGES

- Example: the headlight circuit of an automobile



(a) Physical configuration



(b) Circuit diagram

1.2 CIRCUITS, CURRENTS, AND VOLTAGES

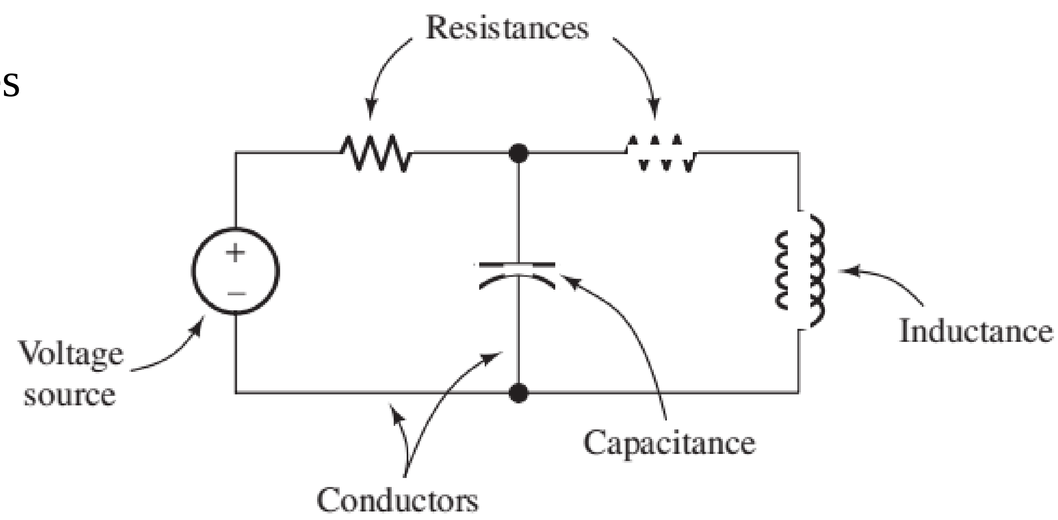
- Fluid-Flow Analogy:
  - Battery: Pump
  - Charge: Fluid
  - Conductors: Pipes
  - Current: Flow Rate
  - Voltage: Pressure difference
  - Switches: Valves
  - Resistance: Constriction

1.2 CIRCUITS, CURRENTS, AND VOLTAGES

- Electrical Circuits:

  Various types of circuit elements connected in closed paths by conductors

  - Resistances
  - Inductances
  - Capacitances
  - Voltage sources



1.2 CIRCUITS, CURRENTS, AND VOLTAGES

- Electrical Current:
- Time rate of flow of electrical charge
- Units: Amperes (A)

Equivalent to: Coulombs per second (C/s)

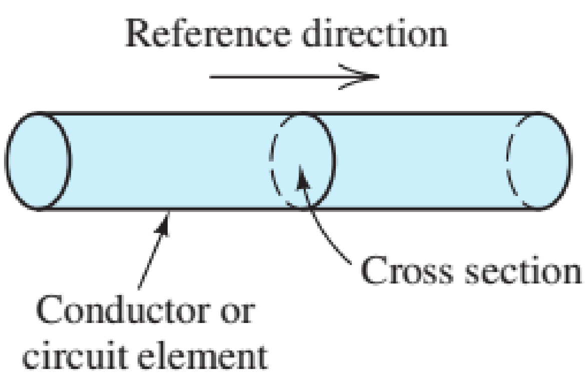
(charge on an electron: -1.602×10^{-19} C)

- To find a current:

Reference direction

$$i(t) = \frac{dq(t)}{dt}$$

$$q(t) = \int_{t_0}^{t} i(t)\,dt + q(t_0)$$



1.2 CIRCUITS, CURRENTS, AND VOLTAGES

- Electrical Current:

Example 1.1 Determining Current Given Charge

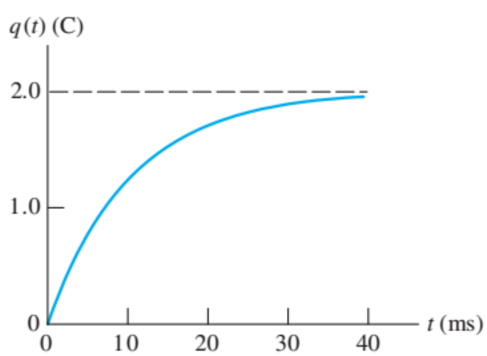
$$q(t) = 0 \qquad \text{for } t < 0$$

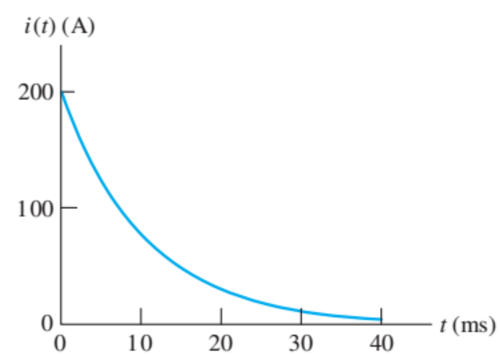
$$q(t) = 2 - 2e^{-100t}\text{C} \qquad \text{for } t > 0$$

$$i(t) = \frac{dq(t)}{dt}$$

$$= 0 \qquad \text{for } t < 0$$

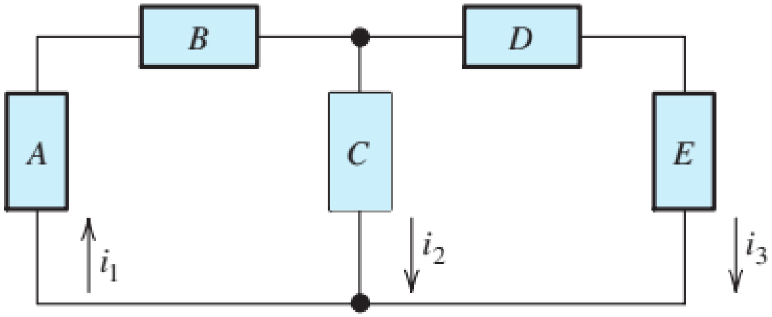
$$= 200e^{-100t}\text{A} \qquad \text{for } t > 0$$





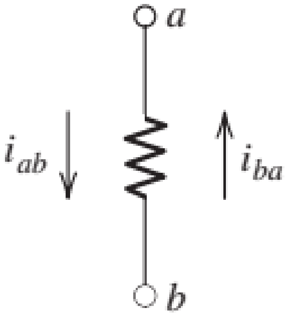
1.2 CIRCUITS, CURRENTS, AND VOLTAGES

- Electrical Current:
  - Assigning current variables
  - Arbitrarily selecting a *reference direction*
  - Solve for the current values
  - Negative value: Opposite direction



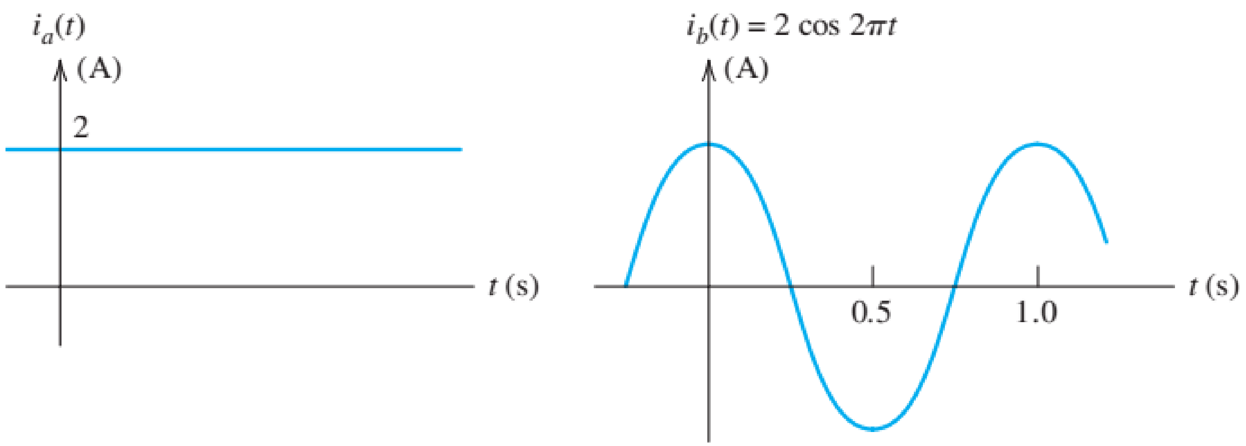
  - Double-Subscript Notation for Currents

$$i_{ab} = -i_{ba}$$



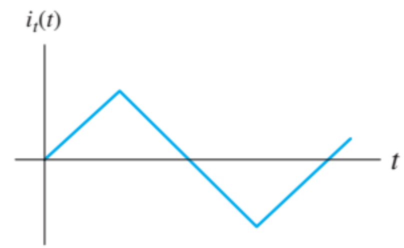
1.2 CIRCUITS, CURRENTS, AND VOLTAGES

- Electrical Current:
  - Direct Current (DC) and Alternating Current (AC)

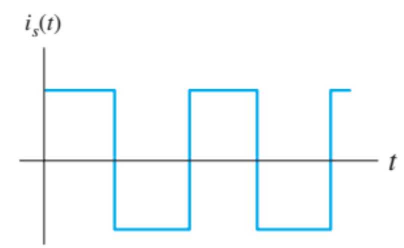


(a) Dc current

(b) Ac current



(a) Triangular waveform

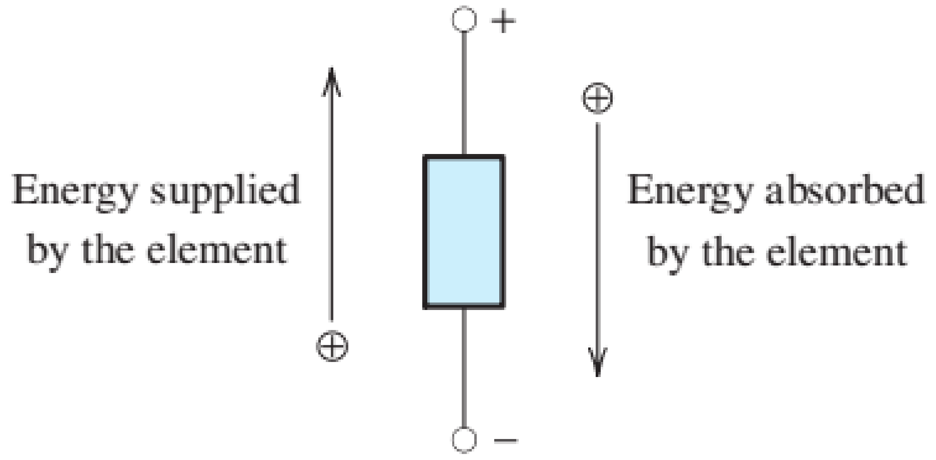


(b) Square waveform

1.2 CIRCUITS, CURRENTS, AND VOLTAGES

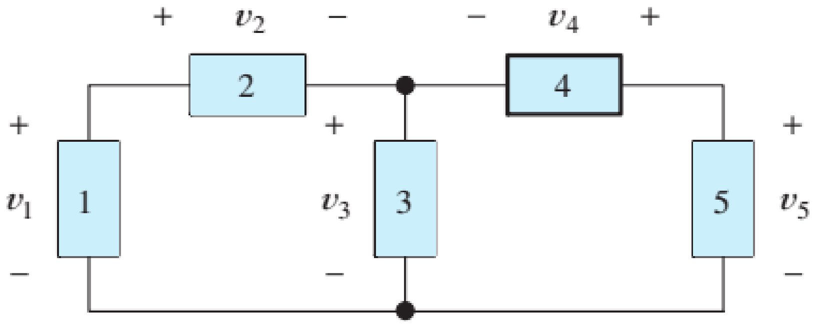
- Voltages:
  - Energy transferred per unit of charge that flows
  - Units: Volts (V)

    Equivalent to: Joules per Coulombs (J/C)
  - Assigned polarities that indicate the direction of energy flow



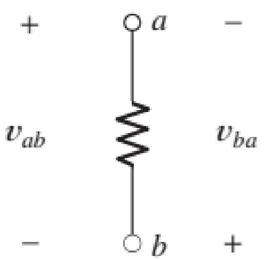
1.2 CIRCUITS, CURRENTS, AND VOLTAGES

- Voltages:
  - Reference Polarities



  - Double-Subscript Notation for Voltages

$$v_{ab} = -v_{ba}$$

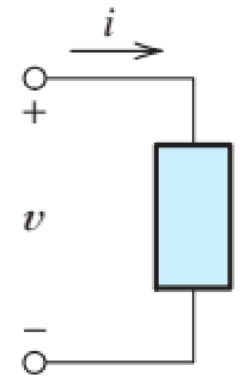


1.3 POWER AND ENERGY

- The product of the current and the voltage is the rate of energy transfer (Power)

$$p = vi$$

$$\begin{aligned} \text{volts} \times \text{amperes} &= \\ (\text{joules/coulomb}) \times (\text{coulombs/second}) &= \\ \text{joules/second} &= \\ \text{watts} & \end{aligned}$$

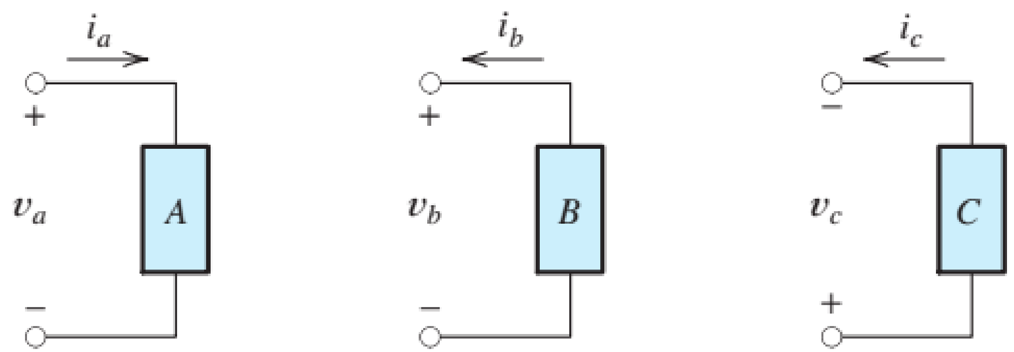


- Passive Reference Configuration
- Functions of time

$$p(t) = v(t)i(t)$$

1.3 POWER AND ENERGY

Example 1.2 Power Calculations



$v_a = 12$ V
$i_a = 2$ A

(a)

$v_b = 12$ V
$i_b = 1$ A

(b)

$v_c = 12$ V
$i_c = -3$ A

(c)

$$p_a = v_a i_a = 12\text{ V} \times 2\text{ A} = 24\text{ W}$$

$$p_c = v_c i_c = 12\text{ V} \times (-3\text{ A}) = -36\text{ W}$$

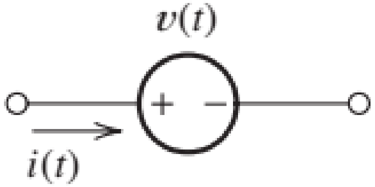
$$p_b = -v_b i_b = -(12\text{ V}) \times 1\text{ A} = -12\text{ W}$$

1.3 POWER AND ENERGY

- Energy Calculations

$$w = \int_{t_1}^{t_2} p(t)\,dt$$

Example 1.3 Energy Calculation



$$v(t) = 12\ \text{V}$$
$$i(t) = 2e^{-t}\ \text{A}$$

$$p(t) = v(t)i(t) = 12 \times 2e^{-t} = 24e^{-t}\ \text{W}$$

$$w = \int_{0}^{\infty} p(t)\,dt = \int_{0}^{\infty} 24e^{-t}\,dt = \left[-24e^{-t}\right]_0^{\infty} = -24e^{-\infty} - (-24e^{0}) = 24\ \text{J}$$

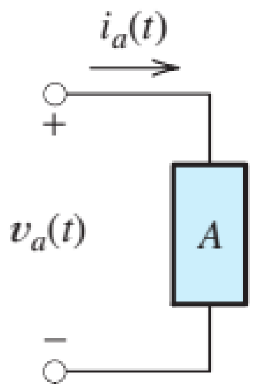
1.3 POWER AND ENERGY

- Prefixes

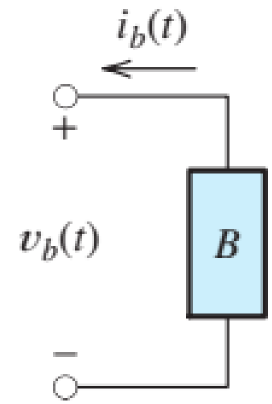
Prefix	Abbreviation	Scale Factor
giga-	G	10^9
meg- or mega-	M	10^6
kilo-	k	10^3
milli-	m	10^{-3}
micro-	μ	10^{-6}
nano-	n	10^{-9}
pico-	p	10^{-12}
femto-	f	10^{-15}

1.3 POWER AND ENERGY

- Exercise:



$i_a(t) = 2t$
$v_a(t) = 10t$



$i_b(t) = 10$
$v_b(t) = 20 - 2t$

$$p_a(t) = v_a(t) i_a(t) = 20t^2$$

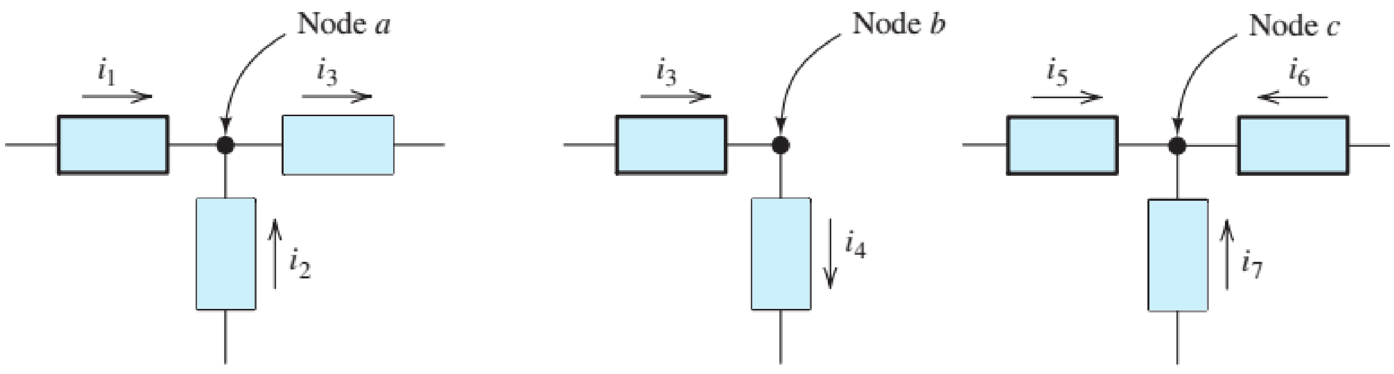
$$w_a = \int_0^{10} p_a(t)dt = \int_0^{10} 20t^2 dt = \left.\frac{20t^3}{3}\right|_0^{10} = \frac{20t^3}{3} = 6667 \text{ J}$$

$$p_b(t) = -v_b(t) i_b(t) = 20t - 200$$

$$w_b = \int_0^{10} p_b(t)dt = \int_0^{10} (20t - 200)dt = 10t^2 - 200t\Big|_0^{10} = -1000 \text{ J}$$

1.4 KIRCHHOFF’S CURRENT LAW (KCL)

- Node: A point at which two or more circuit elements are joined together
- **KCL**: *The net current entering a node is **zero***



(a) (b) (c)

$$\text{Node } a: \quad i_1 + i_2 - i_3 = 0$$

$$\text{Node } b: \quad i_3 - i_4 = 0$$

$$\text{Node } c: \quad i_5 + i_6 + i_7 = 0$$

1.4 KIRCHHOFF'S CURRENT LAW (KCL)

- The net current leaving a node is zero.

Node a: $-i_1 - i_2 + i_3 = 0$

Node b: $-i_3 + i_4 = 0$

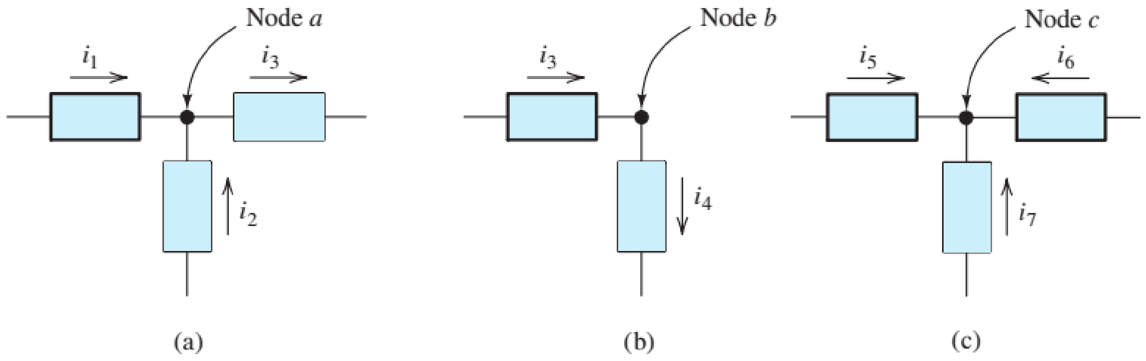
Node c: $-i_5 - i_6 - i_7 = 0$

- The sum of the currents entering a node equals the sum of the currents leaving a node.

Node a: $i_1 + i_2 = i_3$

Node b: $i_3 = i_4$

Node c: $i_5 + i_6 + i_7 = 0$



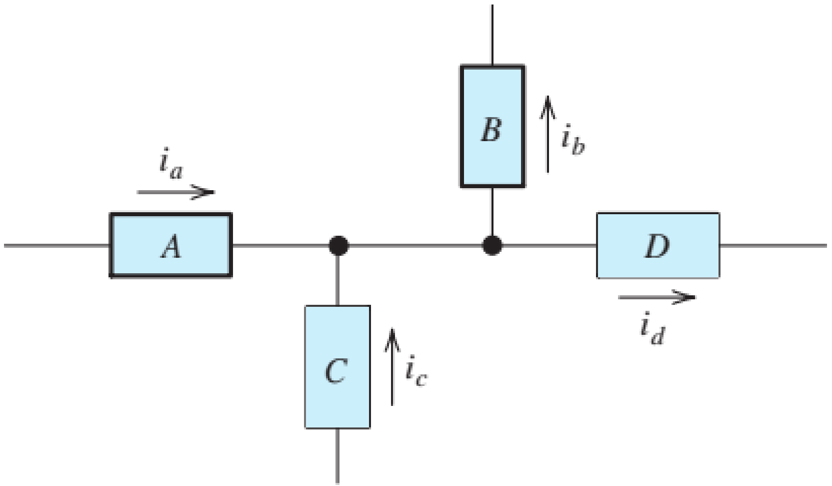
1.4 KIRCHHOFF'S CURRENT LAW (KCL)

- Physical Basis for KCL:

  If violates, The charge would accumulate

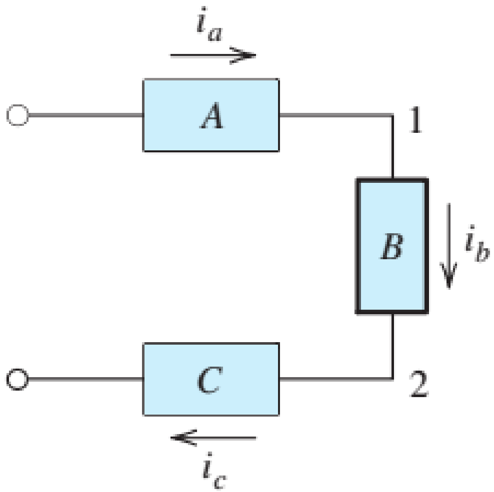
- All points in a circuit that are connected directly by conductors can be considered to be a single node.

$$i_a + i_c = i_b + i_d$$



1.4 KIRCHHOFF'S CURRENT LAW (KCL)

- Series Circuits



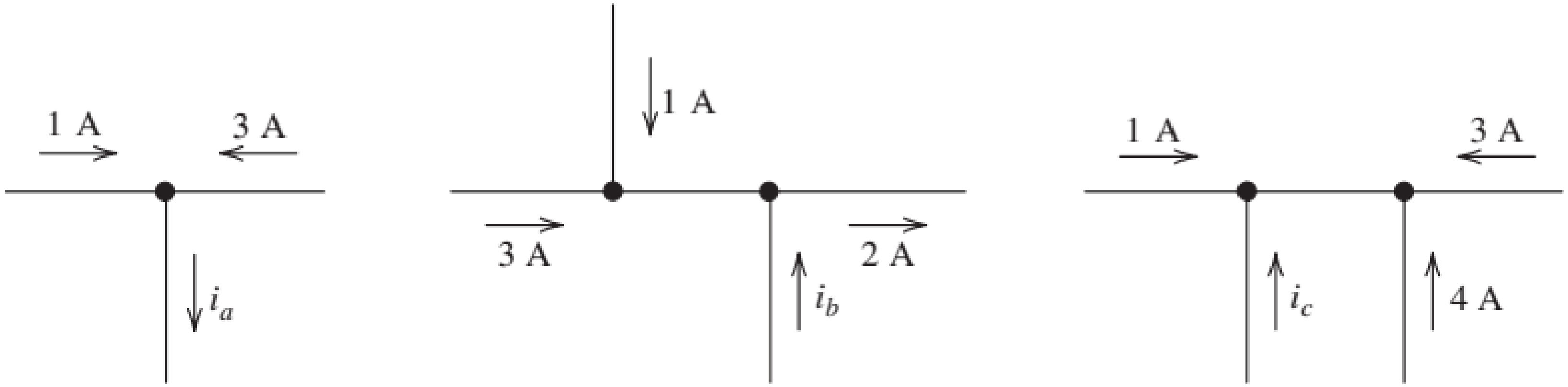
$$i_a = i_b$$

$$i_b = i_c$$

$$\Rightarrow \quad i_a = i_b = i_c$$

1.4 KIRCHHOFF’S CURRENT LAW (KCL)

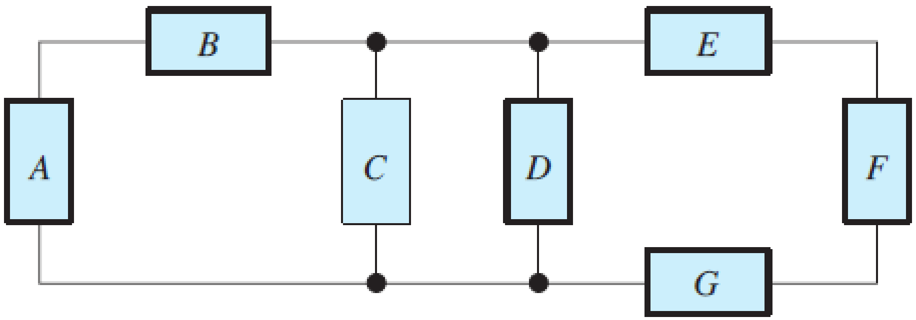
- Exercise:

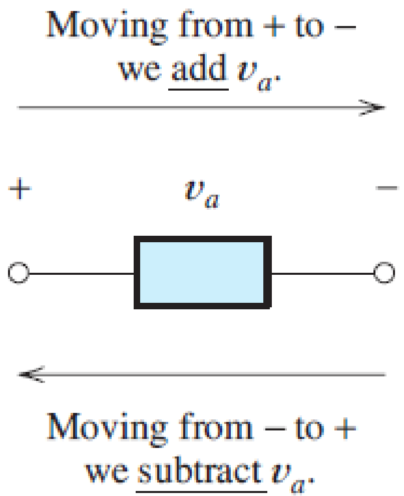


$$i_a = 4\text{ A}, i_b = -2\text{ A}, i_c = -8\text{ A}.$$

1.5 KIRCHHOFF'S VOLTAGE LAW (KVL)

- A **loop** in an electrical circuit is a closed path starting at a node and proceeding through circuit elements, eventually returning to the starting node.





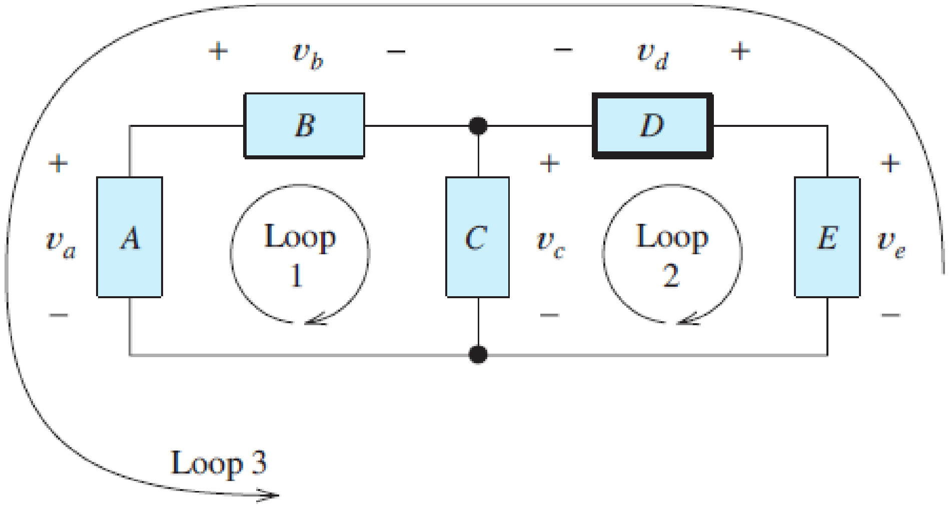
1.5 KIRCHHOFF’S VOLTAGE LAW (KVL)

- **Kirchhoff’s voltage law** (KVL): The algebraic sum of the voltages equals zero for any closed path (loop)

Loop 1: $-v_a + v_b + v_c = 0$

Loop 2: $-v_c - v_d + v_e = 0$

Loop 3: $v_a - v_b + v_d - v_e = 0$



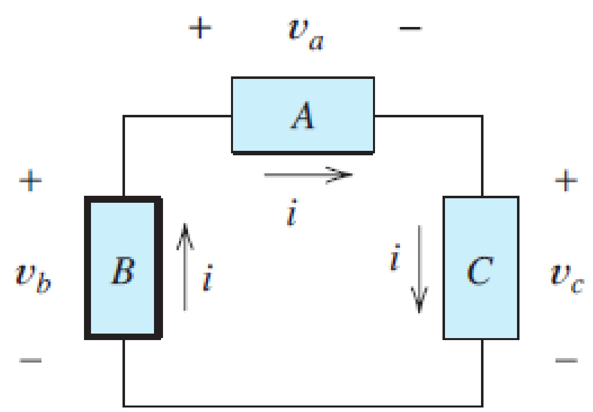
1.5 KIRCHHOFF'S VOLTAGE LAW (KVL)

- **Kirchhoff's Voltage Law Related to Conservation of Energy**

Element A: $p_a = v_a i$

Element B: $p_b = -v_b i$

Element C: $p_c = v_c i$



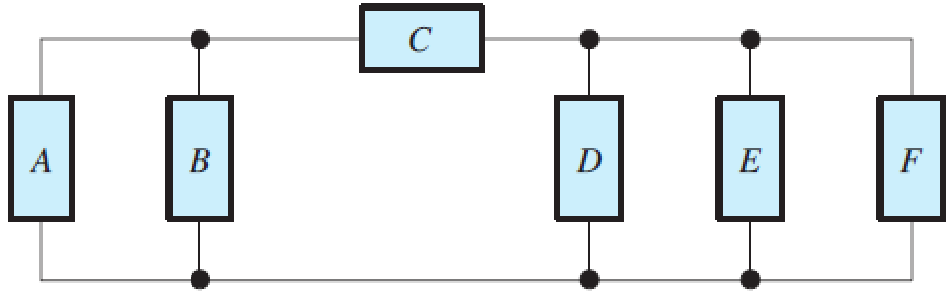
$$p_a + p_b + p_c = 0$$

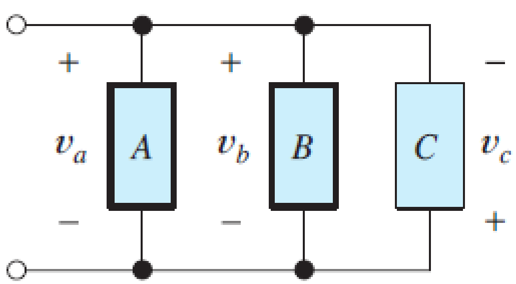
$$\Rightarrow \quad v_a i - v_b i + v_c i = 0$$

$$\Rightarrow \quad v_a - v_b + v_c = 0$$

1.5 KIRCHHOFF’S VOLTAGE LAW (KVL)

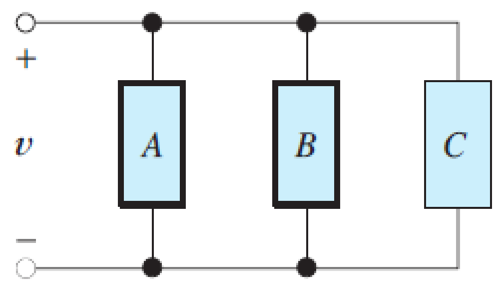
- **Parallel Circuits**





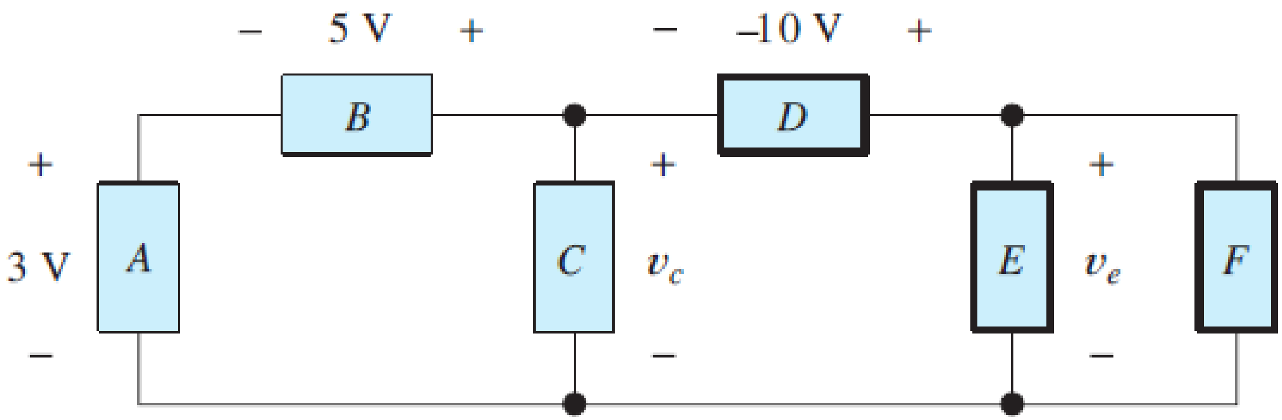
$$-v_a + v_b = 0$$

$$-v_a - v_c = 0$$



1.5 KIRCHHOFF'S VOLTAGE LAW (KVL)

- **Exercise**



$$v_c = 8\text{ V}, v_e = -2\text{ V}.$$

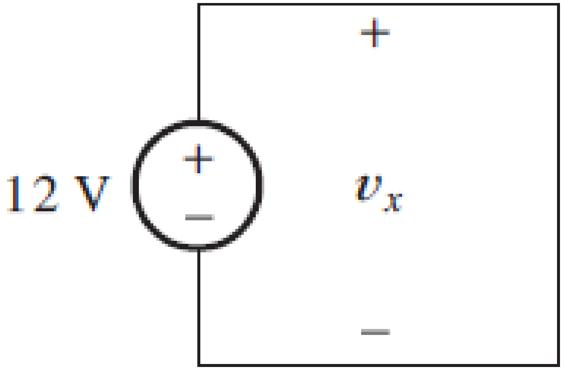
1.6 INTRODUCTION TO CIRCUIT ELEMENTS

Ideal Circuit Elements:

- Conductors
  - Short Circuit – Open Circuit
- Voltage sources
  - Dependent - Independent
- Current sources
  - Dependent - Independent
- Resistors

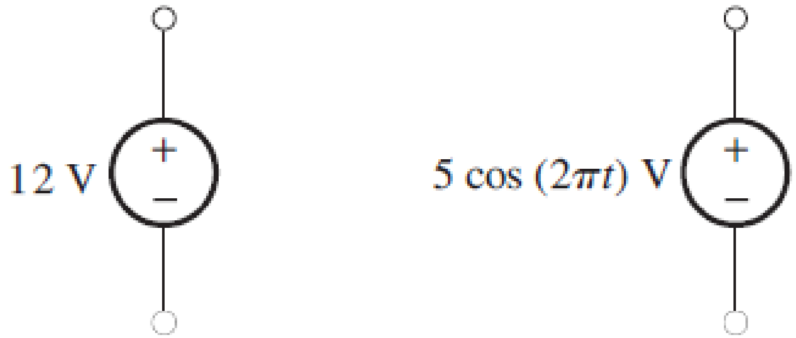
1.6 INTRODUCTION TO CIRCUIT ELEMENTS

- Ideal Circuit Elements versus Reality
  - In our study of ideal circuits, we avoid such conflicts.
  - In the real world, a very large current flows and heat up the wire at high rate.
  - Undesirable situation (such as a fire or destroyed components)
  - Real models:
    - ✓ Source: is an ideal voltage source in series with a resistance
    - ✓ Wire: A small resistance



1.6 INTRODUCTION TO CIRCUIT ELEMENTS

- Independent Voltage Sources
  - Maintains a specified voltage across its terminals.
  - Independent of all other voltages and currents in the circuit.

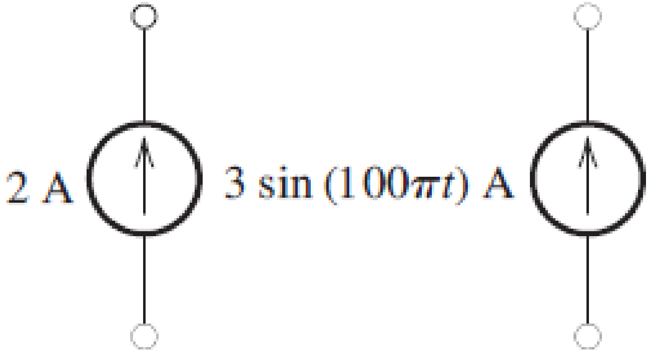


(a) Constant or dc voltage source

(b) Ac voltage source

1.6 INTRODUCTION TO CIRCUIT ELEMENTS

- Independent Current Sources
  - Maintains a specified current through its terminals.
  - Independent of all other voltages and currents in the circuit.
  - A battery is a good example of a voltage source, but an equally familiar example does not exist for a current source.
  - useful in constructing theoretical models



(a) Dc current source

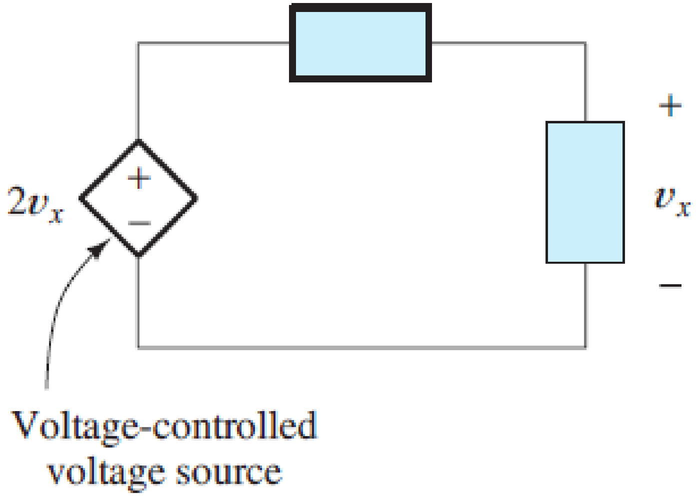
(b) Ac current source

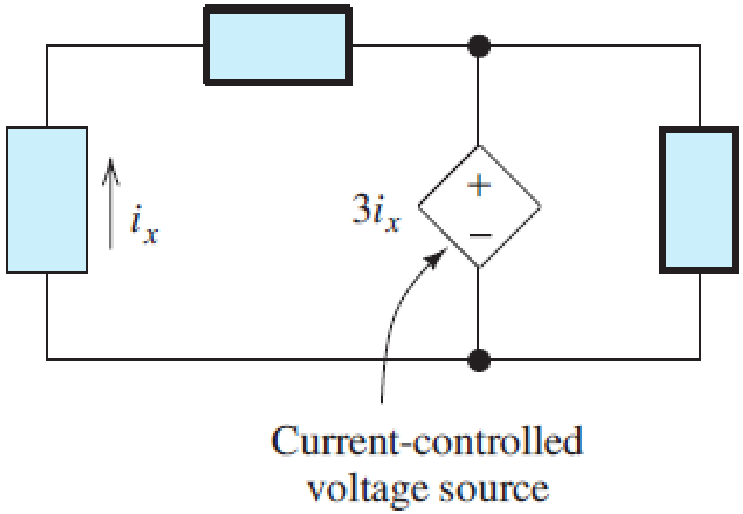
1.6 INTRODUCTION TO CIRCUIT ELEMENTS

- Dependent Voltage Sources
  - Voltage - Current: a function of other voltages or currents
  - four kinds of controlled sources:
    - ✓ **1.** Voltage-controlled voltage sources (VCVS)
    - ✓ **2.** Current-controlled voltage sources (CCVS)
    - ✓ **3.** Voltage-controlled current sources (VCCS)
    - ✓ **4.** Current-controlled current sources (CCCS)

1.6 INTRODUCTION TO CIRCUIT ELEMENTS

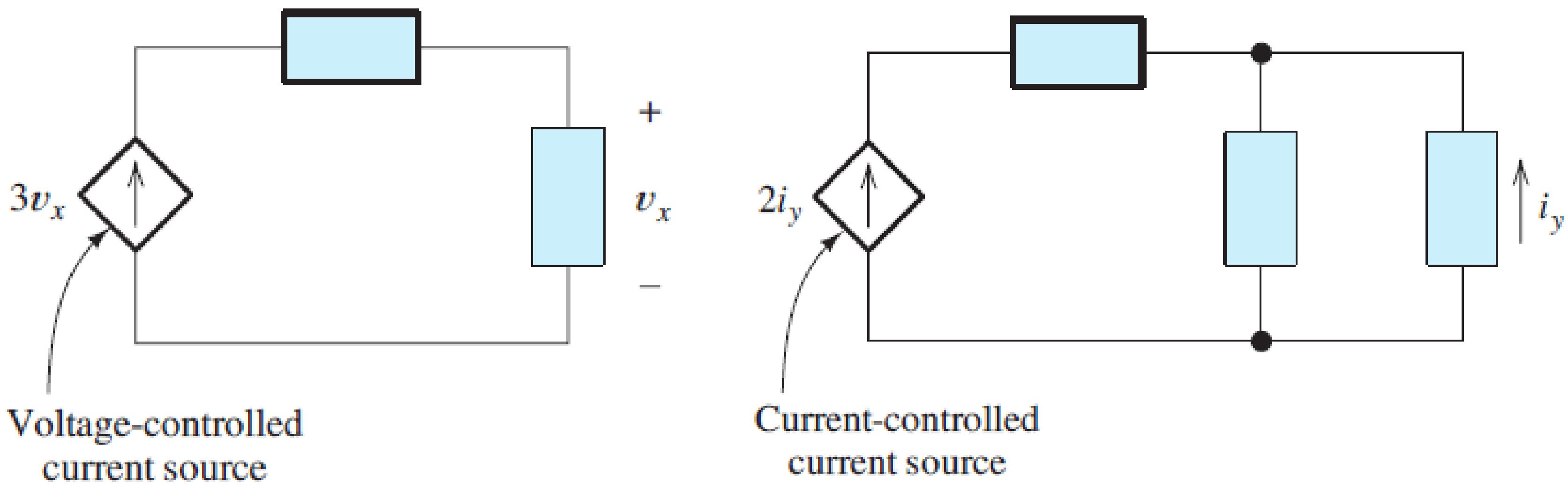
- Dependent Voltage Sources
  - ✓ Voltage-controlled voltage sources (VCVS)
  - ✓ Current-controlled voltage sources (CCVS)





1.6 INTRODUCTION TO CIRCUIT ELEMENTS

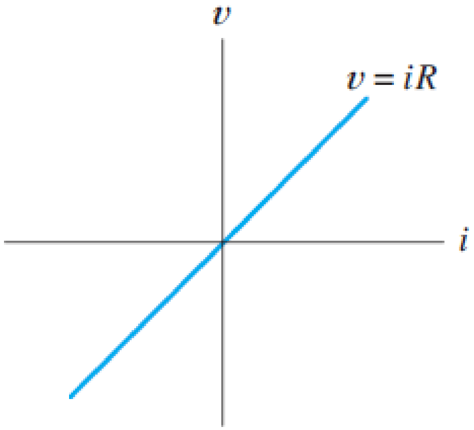
- Dependent Current Sources
  - ✓ Voltage-controlled current sources (VCCS)
  - ✓ Current-controlled current sources (CCCS)



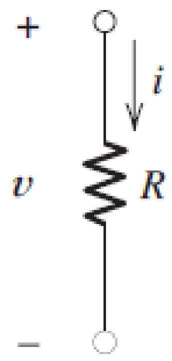
1.6 INTRODUCTION TO CIRCUIT ELEMENTS

- Resistors
  - The **voltage** across an ideal **resistor** is proportional to the **current** through the resistor
  - Ohm's Law
    - ✓ Unit: letter omega (Ω) represents ohms
    - ✓ Ranging from milliohms (mΩ) to megaohms (MΩ).
    - ✓ for now we assume that R is positive.

$$v = iR$$



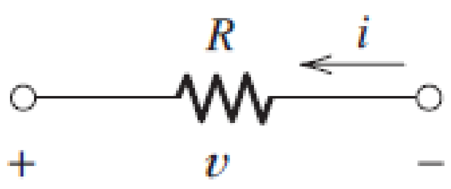
(b) Ohm's law



(a) Resistance symbol

1.6 INTRODUCTION TO CIRCUIT ELEMENTS

- Resistors
  - Reference direction



$$v = -iR$$

$$v_{ab} = i_{ab}R$$

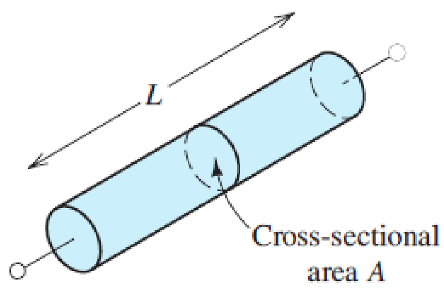
$$v_{ab} = -i_{ba}R$$

  - Conductance (G)
    - ✓ units of inverse ohms (Ω^{-1}), which are called Siemens (abbreviated S).

$$i = \frac{1}{R}v \implies G = \frac{1}{R} \qquad i = Gv$$

1.6 INTRODUCTION TO CIRCUIT ELEMENTS

- Resistors
  - Resistance Related to Physical Parameters



$$R = \frac{\rho L}{A}$$

Table 1.3. Resistivity Values (Ωm) for Selected Materials at 300 K

Conductors	
Aluminum	2.73×10^{-8}
Carbon (amorphous)	3.5×10^{-5}
Copper	1.72×10^{-8}
Gold	2.27×10^{-8}
Nichrome	1.12×10^{-6}
Silver	1.63×10^{-8}
Tungsten	5.44×10^{-8}
Semiconductors	
Silicon (device grade) depends on impurity concentration	10^{-5} to 1
Insulators	
Fused quartz	$> 10^{21}$
Glass (typical)	1×10^{12}
Teflon	1×10^{19}

1.6 INTRODUCTION TO CIRCUIT ELEMENTS

- Resistors

Example 1.4 Resistance Calculation

Compute the resistance of a copper wire having a diameter of 2.05 mm and a length of 10 m.

$$A = \frac{\pi d^2}{4} = \frac{\pi(2.05\times 10^{-3})^2}{4} = 3.3\times 10^{-6}\ \text{m}^2$$

$$R = \frac{\rho L}{A} = \frac{1.72\times 10^{-8}\times 10}{3.3\times 10^{-6}} = 0.052\ \Omega$$

1.6 INTRODUCTION TO CIRCUIT ELEMENTS

- Resistors
  - Power Calculations for Resistances

$$p = vi \quad \Longrightarrow \quad p = Ri^2 \qquad p = \frac{v^2}{R}$$

Example 1.5 Determining Resistance for Given Power and Voltage Ratings

- An Electrical Heater: 1500W, 120V

$$R = \frac{v^2}{p} = \frac{120^2}{1500} = 9.6\ \Omega \qquad i = \frac{v}{R} = \frac{120}{9.6} = 12.5\ \text{A}$$

- Diameter of 1.6 mm: L=17.2 m

1.6 INTRODUCTION TO CIRCUIT ELEMENTS

PRACTICAL APPLICATION 1.1

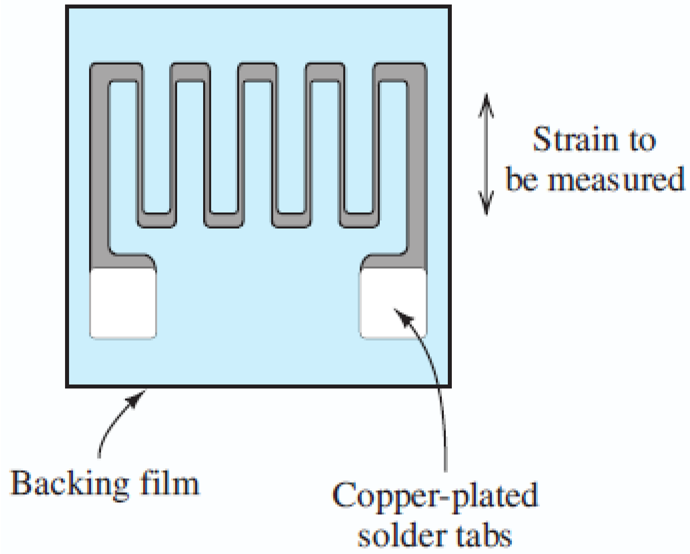
Using Resistance to Measure Strain

$$R = \frac{\rho L}{A}$$

As strain is applied, the length and area change, resulting in changes in resistance. The strain and the change in resistance are related by the gauge factor:

$$G = \frac{\Delta R / R_0}{\epsilon}$$

$$\epsilon = \Delta L / L.$$



1.6 INTRODUCTION TO CIRCUIT ELEMENTS

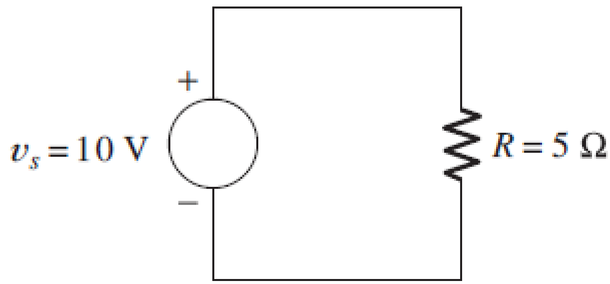
- Exercises:

Exercise 1.12 An electric light bulb is rated for 100 W and 120 V. Find its resistance (at operating temperature) and operating current.
Answer $R = 144\ \Omega$, $i = 0.833$ A. □

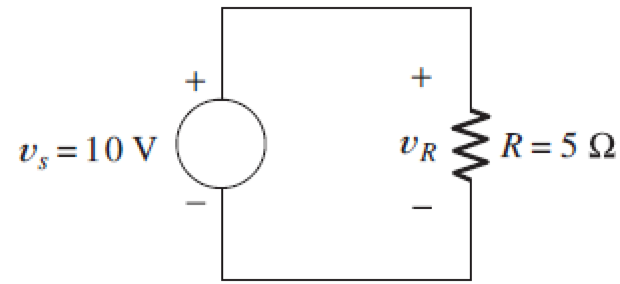
Exercise 1.13 A 1-kΩ resistor used in a television receiver is rated for a maximum power of 1/4 W. Find the current and voltage when the resistor is operated at maximum power.
Answer $v_{max} = 15.8$ V, $i_{max} = 15.8$ mA. □

1.7 INTRODUCTION TO CIRCUITS

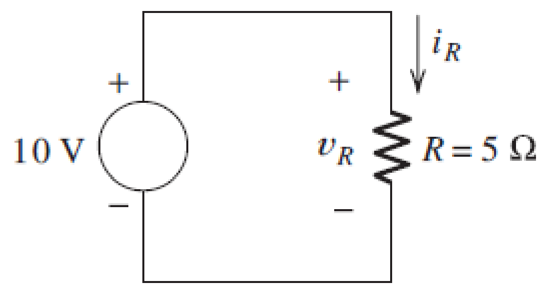
- Basic circuit



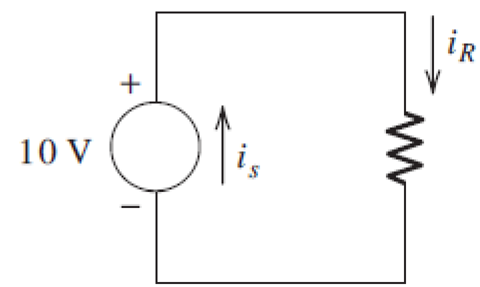
(a) Circuit diagram



(b) KVL requires that $v_R = 10$ V



(c) Ohm's law yields $i_R = v_R/R = 2$ A



(d) KCL requires that $i_s = i_R$

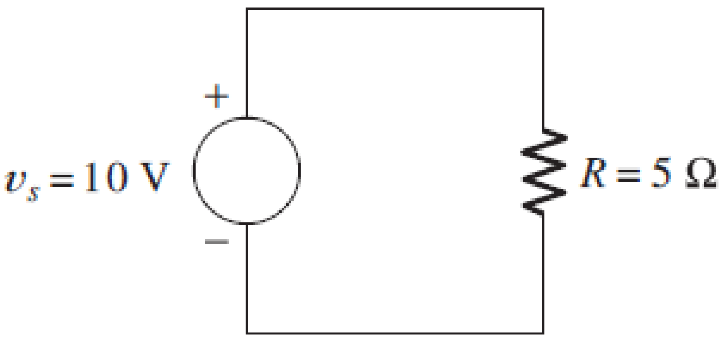
1.7 INTRODUCTION TO CIRCUITS

- Basic circuit

$$p_R = v_R i_R = 10 \times 2 = 20 \text{ W}$$

$$p_R = i_R^2 R = 2^2 \times 5 = 20 \text{ W}$$

$$p_R = \frac{v_R^2}{R} = \frac{10^2}{5} = 20 \text{ W}$$



$$p_s = -v_s i_s = -10 \times 2 = -20 \text{ W}$$

$$\Longrightarrow \quad p_s + p_R = -20 + 20 = 0$$

1.7 INTRODUCTION TO CIRCUITS

- Basic circuit: Using Arbitrary References

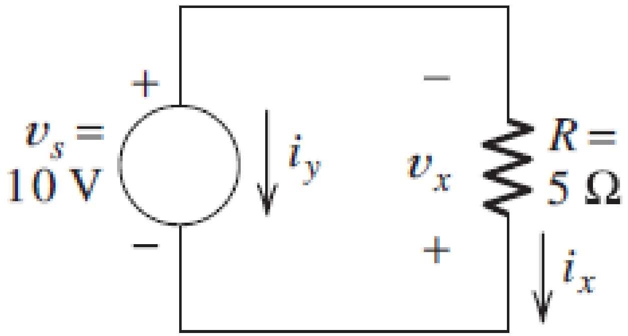
$-v_s - v_x = 0 \Rightarrow v_x = -v_s = -10\text{ V}.$

$i_x = -\frac{v_x}{R} \Rightarrow i_x = -\frac{-10}{5} = 2\text{ A}$

$i_y + i_x = 0 \Rightarrow i_y = -i_x = -2\text{ A}$

$p_s = v_s i_y = 10 \times (-2) = -20\text{ W}$

$p_R = -v_x i_x = -(-10) \times (2) = 20\text{ W}.$



1.7 INTRODUCTION TO CIRCUITS

Example 1.7 Using KVL, KCL, and Ohm's Law to Solve a Circuit

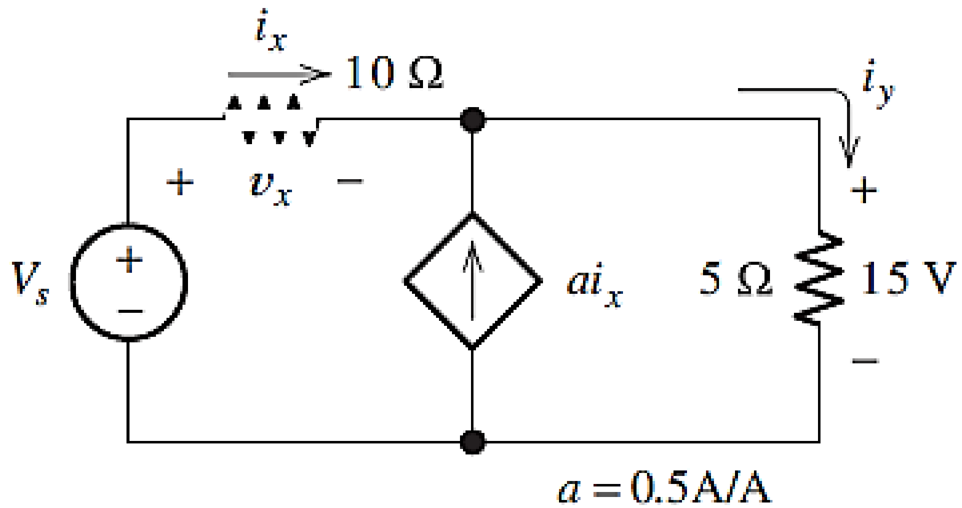
$$i_y = \frac{15\ \text{V}}{5\ \Omega} = 3\ \text{A}$$

$$i_x + 0.5 i_x = i_y$$

$$i_x = 2\ \text{A}.$$

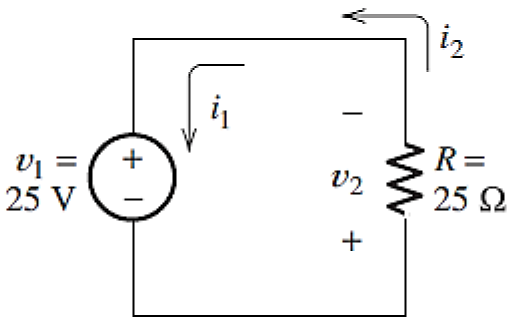
$$v_x = 10 i_x = 20\ \text{V}.$$

$$V_s = v_x + 15 = 35\ \text{V}.$$

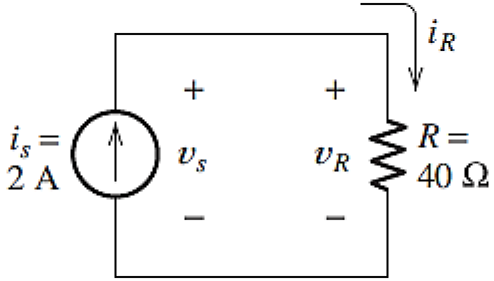


1.7 INTRODUCTION TO CIRCUITS

- Exercises



$i_1 = i_2 = -1$ A, $v_2 = -25$ V, $p_R = 25$ W, $p_s = -25$ W.



$i_R = 2$ A, $v_s = v_R = 80$ V, $p_s = -160$ W, $p_R = 160$ W.

PROBLEMS

***P1.36.** Use KCL to find the values of i_a, i_c, and i_d for the circuit of Figure P1.36. Which elements are connected in series in this circuit?

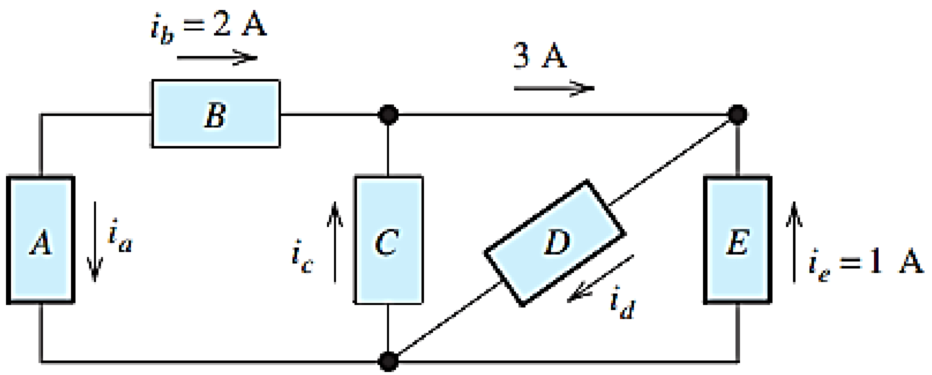


Figure P1.36

$i_a = -2$ A. $i_c = 1$ A. $i_d = 4$ A. Elements A and B are in series.

PROBLEMS

***P1.37.** Given that $i_a = 2$ A, $i_b = 3$ A, $i_d = -5$ A, and $i_h = 4$ A, determine the values of the other currents in Figure P1.37.

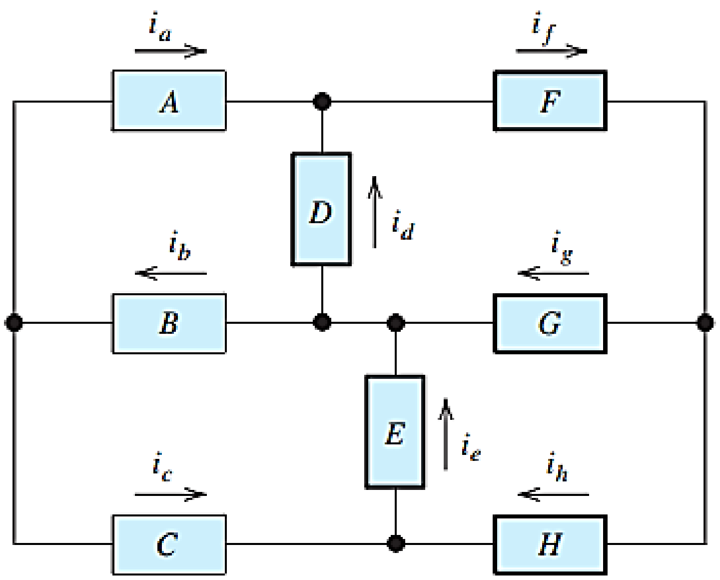


Figure P1.37

$i_c = 1\ A$ $\quad$ $i_e = 5\ A$

$i_f = -3\ A$ $\quad$ $i_g = -7\ A$

PROBLEMS

***P1.41.** Use KVL to solve for the voltages v_a, v_b, and v_c in Figure P1.41.

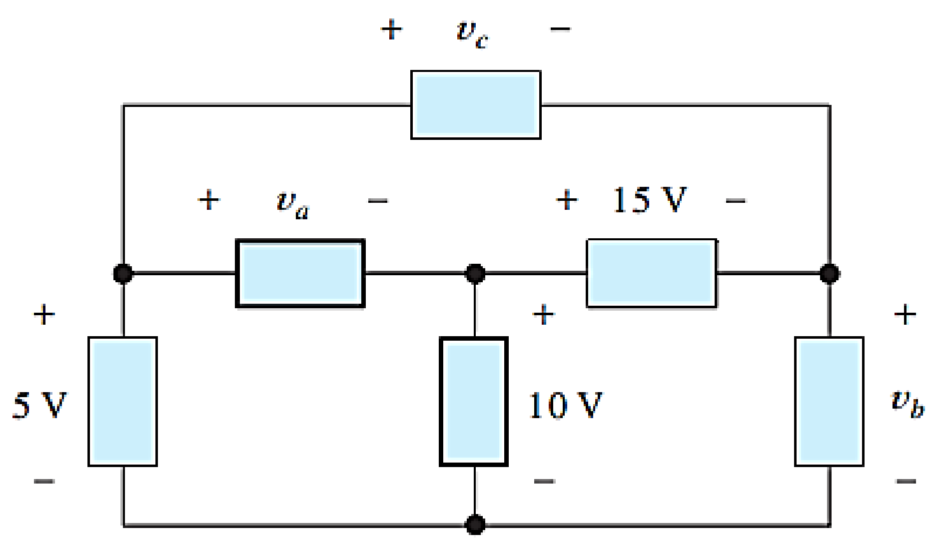


Figure P1.41

$v_a = -5\text{ V}.\quad v_c = 10\text{ V}.\quad v_b = -5\text{ V}.$

PROBLEMS

***P1.42.** Use KVL and KCL to solve for the labeled currents and voltages in Figure P1.42. Compute the power for each element and show that power is conserved (i.e., the algebraic sum of the powers is zero).

$i_c = 1\ A$ $\quad$ $i_b = -2\ A$

$v_b = -6\ V$ $\quad$ $v_c = 4\ V$

$P_A = -20\ W$ $\quad$ $P_B = 12\ W$

$P_C = 4\ W$ $\quad$ $P_D = 4\ W$

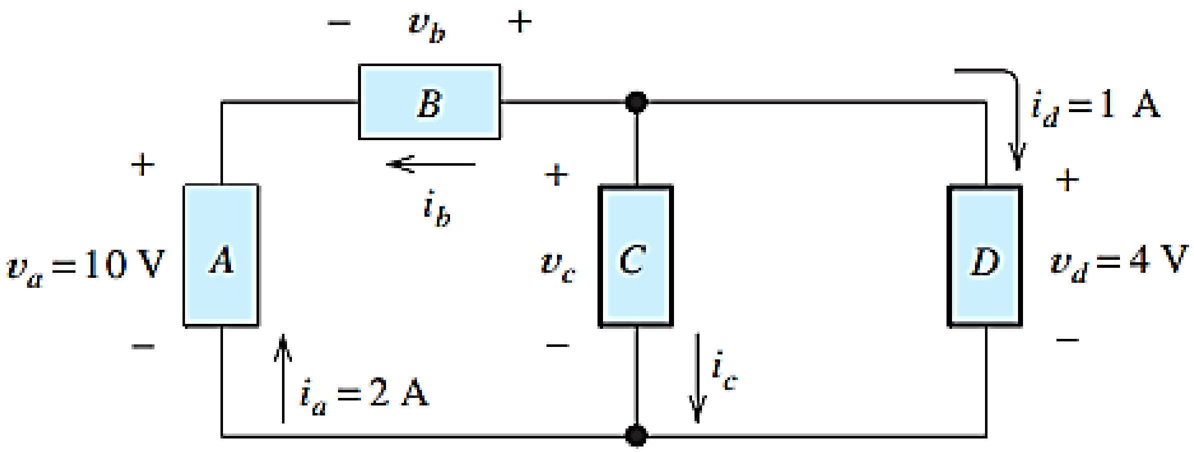


Figure P1.42

PROBLEMS

***P1.63.** Consider the circuit shown in Figure P1.63. Find the current i_R flowing through the resistor. Find the power for each element in the circuit. Which elements are absorbing power?

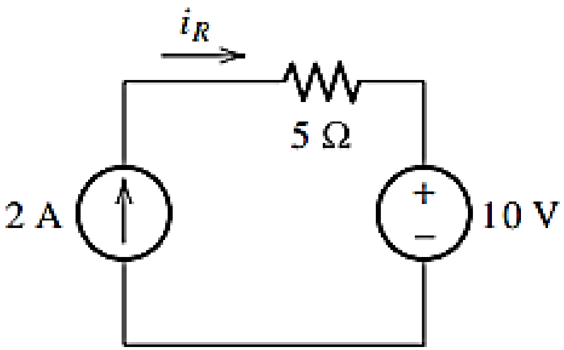


Figure P1.63

$i_R = 2A$

$P_{current-source} = -40$ W. Thus, the current source delivers power.

$P_R = 20$ W. The resistor absorbs power.

$P_{voltage-source} = 20$ W. The voltage source absorbs power.

PROBLEMS

***P1.64.** Consider the circuit shown in Figure P1.64. Use repeated applications of Ohm's law, KVL, and KCL to eventually find V_x.

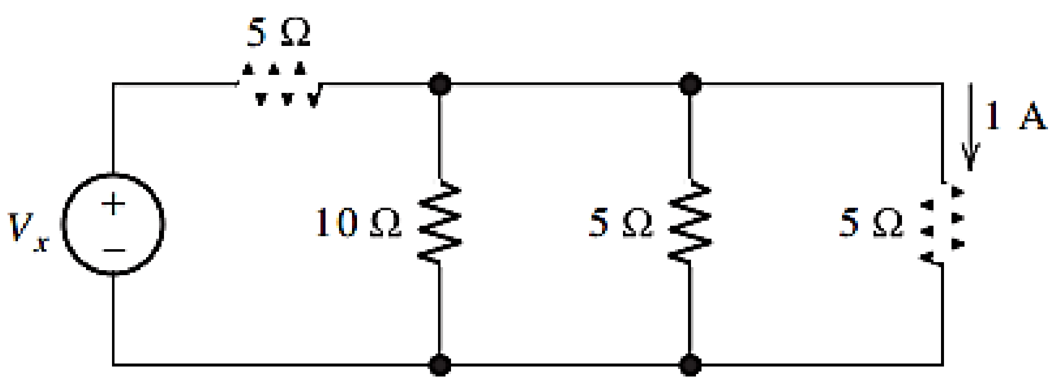


Figure P1.64

$$V_x = 17.5\text{ V}$$

PROBLEMS

***P1.69.** The circuit shown in Figure P1.69 contains a voltage-controlled voltage source. **a.** Use KVL to write an equation relating the voltages and solve for v_x. **b.** Use Ohm's law to find the current i_x. **c.** Find the power for each element in the circuit and verify that power is conserved.

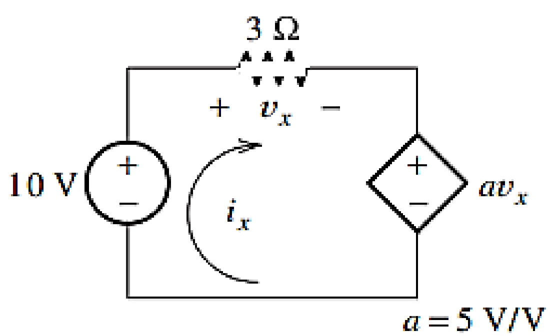


Figure P1.69

(a) $v_x = 10/6 = 1.667\ \text{V}$

(b) $i_x = 0.5556\ \text{A}$

(c) $P_{voltage-source} = -10 i_x = -5.556\ \text{W}.$

the voltage source.)

$P_R = 3(i_x)^2 = 0.926\ \text{W}$ (absorbed)

$P_{controlled-source} = 5 v_x i_x = 4.63\ \text{W}$ (absorbed)

PROBLEMS

***P1.70.** What type of controlled source is shown in the circuit of Figure P1.70? Solve for v_s.

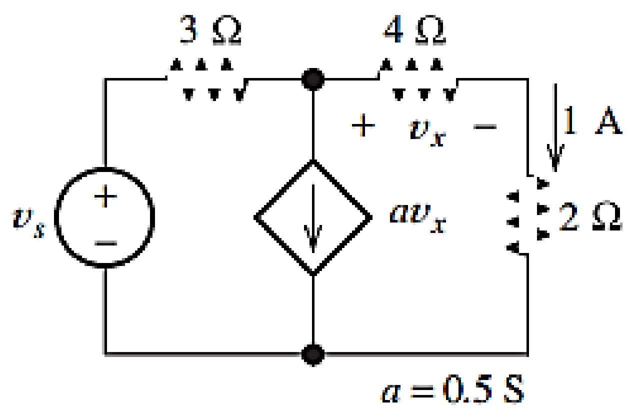


Figure P1.70

The circuit contains a voltage-controlled current source. $v_s = 15$ V