دانشگاه سمنان

Semnan University
Faculty of Mechanical Engineering

دانشکده مهندسی مکانیک

دانشکده مهندسی مکانیک

درس مبانی برق ۱

نیمسال اول ۹۸-۹۹

Electrical Engineering

Principles and Applications

Allan R. Hambley

5th Edition

❑ *Contents:*

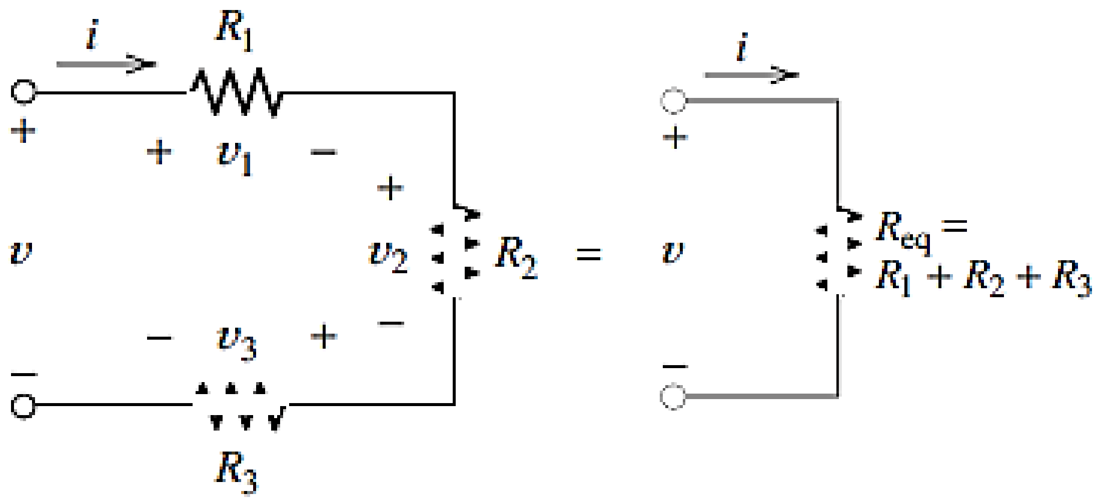
INTRODUCTION

- Network analysis
- Voltage-division and current-division principles
- Node-voltage technique
- Mesh-current technique
- Thévenin and Norton equivalents
- Use MATLAB® to solve circuit equations
- Superposition principle

2.1 RESISTANCES IN SERIES AND PARALLEL

- Series Resistances

$$v_1 = R_1 i$$
$$v_2 = R_2 i$$
$$v_3 = R_3 i$$



- KVL:

$$v = v_1 + v_2 + v_3$$
$$= R_1 i + R_2 i + R_3 i$$
$$= (R_1 + R_2 + R_3) i$$

- Equivalent Resistance:

$$v = R_{eq} i \qquad R_{eq} = R_1 + R_2 + R_3$$

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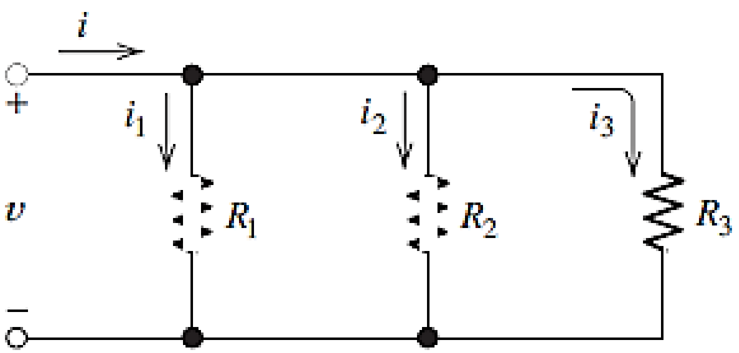
2.1 RESISTANCES IN SERIES AND PARALLEL

- Parallel Resistances

$$i_1 = \frac{v}{R_1}$$

$$i_2 = \frac{v}{R_2}$$

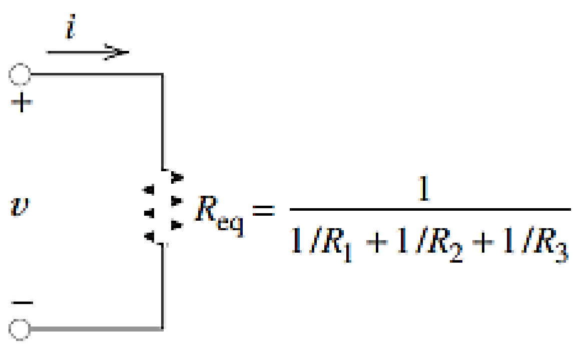
$$i_3 = \frac{v}{R_3}$$



- KCL:

$$i = i_1 + i_2 + i_3 = \frac{v}{R_1} + \frac{v}{R_2} + \frac{v}{R_3}$$

$$= \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) v$$



- Equivalent Resistance

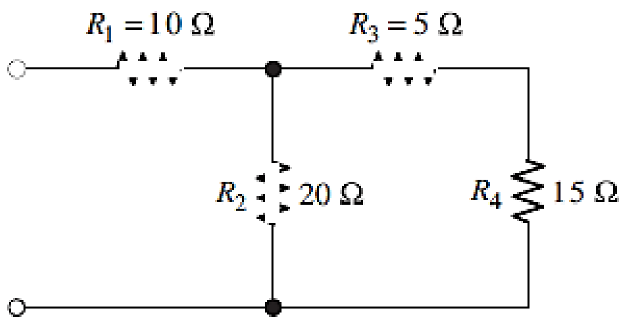
$$i = \frac{1}{R_{eq}} v$$

$$R_{eq} = \frac{1}{1/R_1 + 1/R_2 + 1/R_3}$$

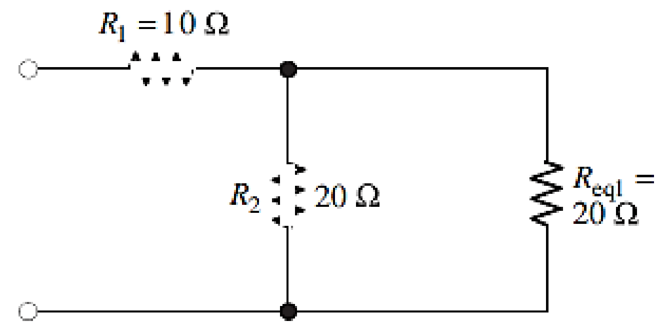
$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

2.1 RESISTANCES IN SERIES AND PARALLEL

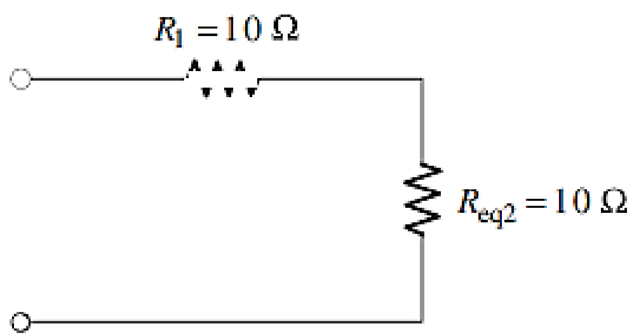
Example 2.1 Combining Resistances in Series and Parallel



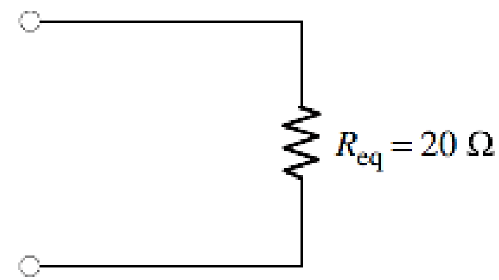
(a) Original network



(b) Network after replacing R_3 and R_4 by their equivalent resistance



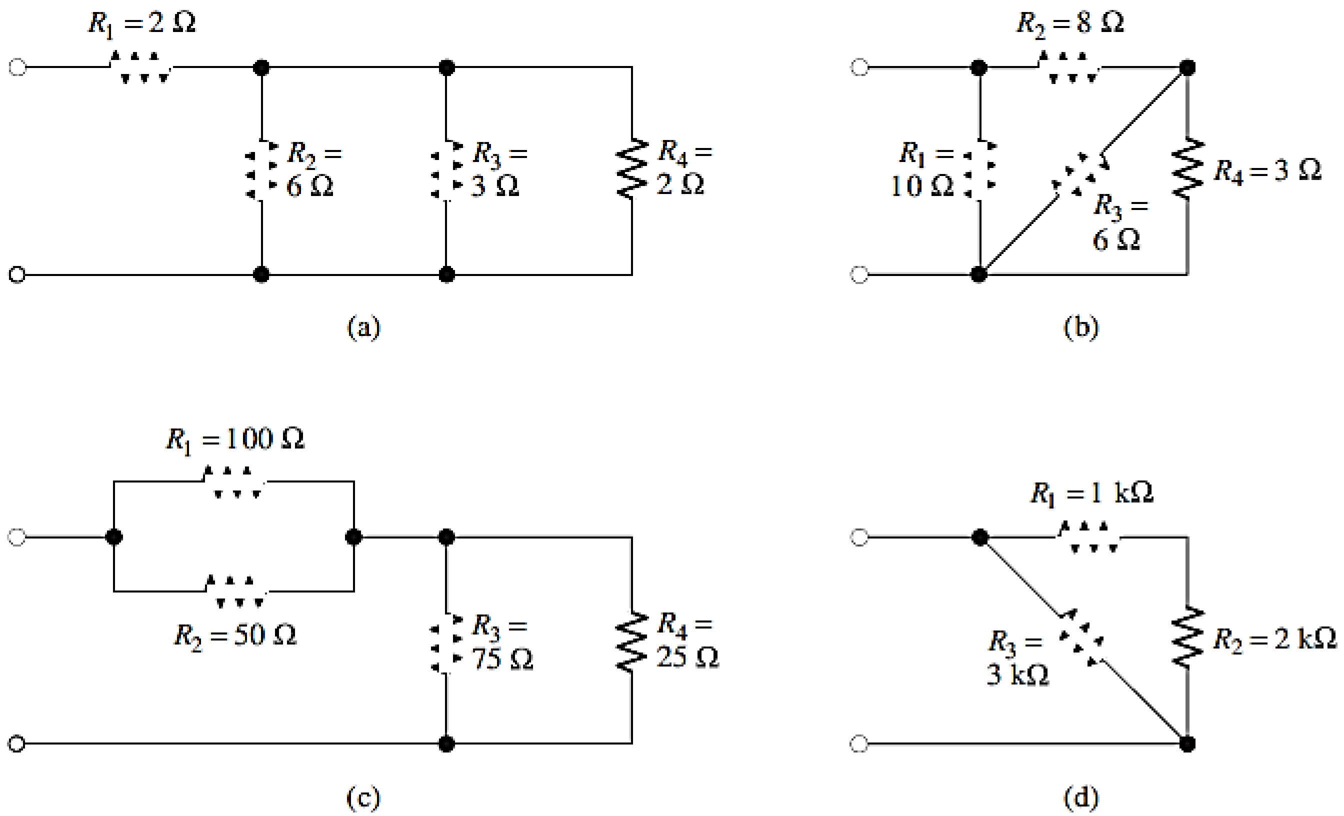
(c) Network after replacing R_2 and R_{eq1} by their equivalent



(d) Combining R_1 and R_{eq2} in series yields the equivalent resistance of the entire network

2.1 RESISTANCES IN SERIES AND PARALLEL

- Exercise



a. 3 Ω; **b.** 5 Ω; **c.** 52.1 Ω; **d.** 1.5 kΩ.

2.1 RESISTANCES IN SERIES AND PARALLEL

- Conductances in Series and Parallel
  - Series

$$G_{eq} = \frac{1}{1/G_1 + 1/G_2 + \cdots + 1/G_n}$$

  - Parallel

$$G_{eq} = G_1 + G_2 + \cdots + G_n$$

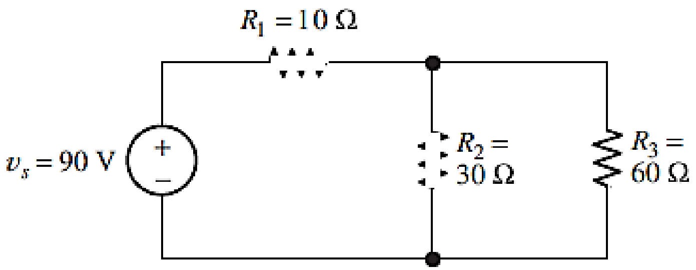
- Series versus Parallel Circuits
  - Parallel: Distribute power from single voltage source
  - Series

2.2 NETWORK ANALYSIS BY USING SERIES AND PARALLEL EQUIVALENTS

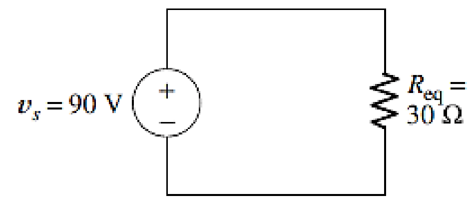
- Circuit Analysis Using Series/Parallel Equivalents
  - **1.** Locating a combination of series or parallel resistances
  - **2.** Redraw the circuit with the equivalent resistances
  - **3.** Repeat steps 1 and 2
  - **4.** Solve final equivalent circuit, transfer results back
  - **5.** Check your results for KCL and KVL

2.2 NETWORK ANALYSIS BY USING SERIES AND PARALLEL EQUIVALENTS

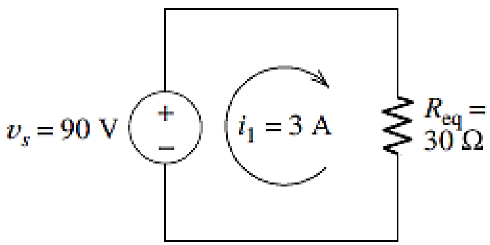
Example 2.2 Circuit Analysis Using Series/Parallel Equivalents



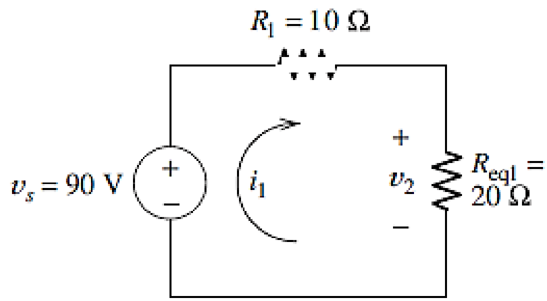
(a) Original circuit



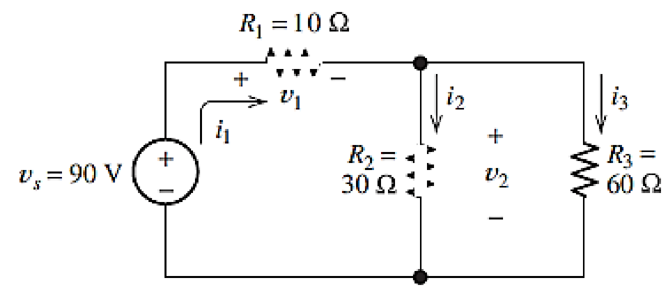
(c) Circuit after replacing R_1 and R_{eq1} by their equivalent



(c) First, we solve for $i_1 = \dfrac{v_s}{R_{eq}} = 3$ A



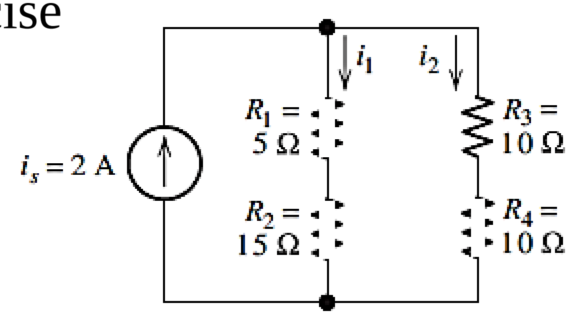
(b) Second, we find $v_2 = R_{eq1}\, i_1 = 60$ V



(a) Third, we use known values of i_1 and v_2 to solve for the remaining currents and voltages

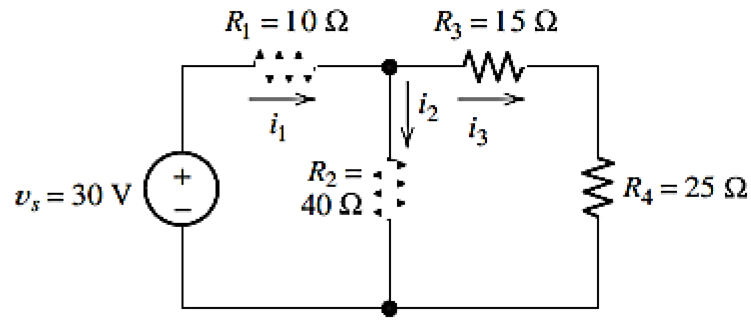
2.2 NETWORK ANALYSIS BY USING SERIES AND PARALLEL EQUIVALENTS

- Exercise



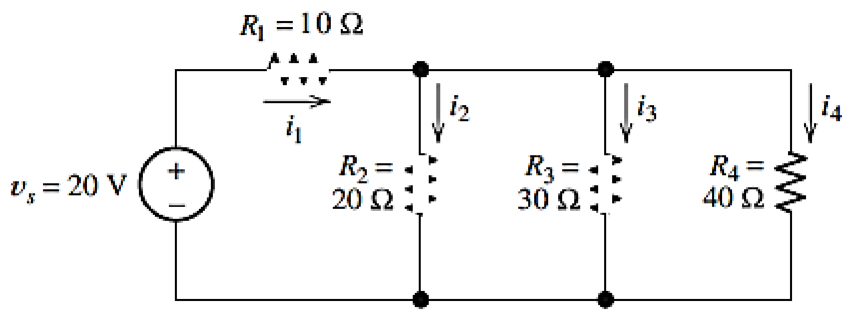
(b)

b. $i_1 = 1$ A, $i_2 = 1$ A



(c)

c. $i_1 = 1$ A, $i_2 = 0.5$ A, $i_3 = 0.5$ A.



(a)

a. $i_1 = 1.04$ A, $i_2 = 0.480$ A, $i_3 = 0.320$ A, $i_4 = 0.240$ A

2.3 VOLTAGE-DIVIDER AND CURRENT-DIVIDER CIRCUITS

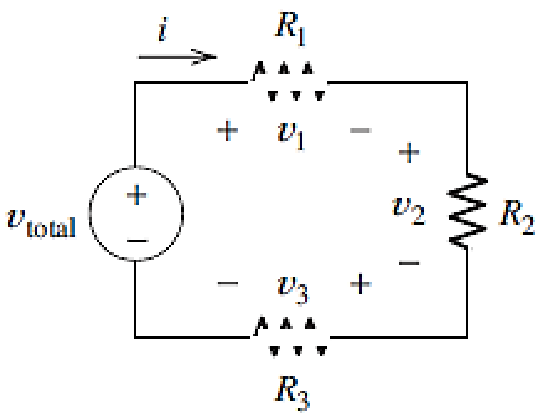
- Voltage Division

$$i = \frac{v_{\text{total}}}{R_{\text{eq}}} = \frac{v_{\text{total}}}{R_1 + R_2 + R_3}$$

$$v_1 = R_1 i = \frac{R_1}{R_1 + R_2 + R_3} v_{\text{total}}$$

$$v_2 = R_2 i = \frac{R_2}{R_1 + R_2 + R_3} v_{\text{total}}$$

$$v_3 = R_3 i = \frac{R_3}{R_1 + R_2 + R_3} v_{\text{total}}$$

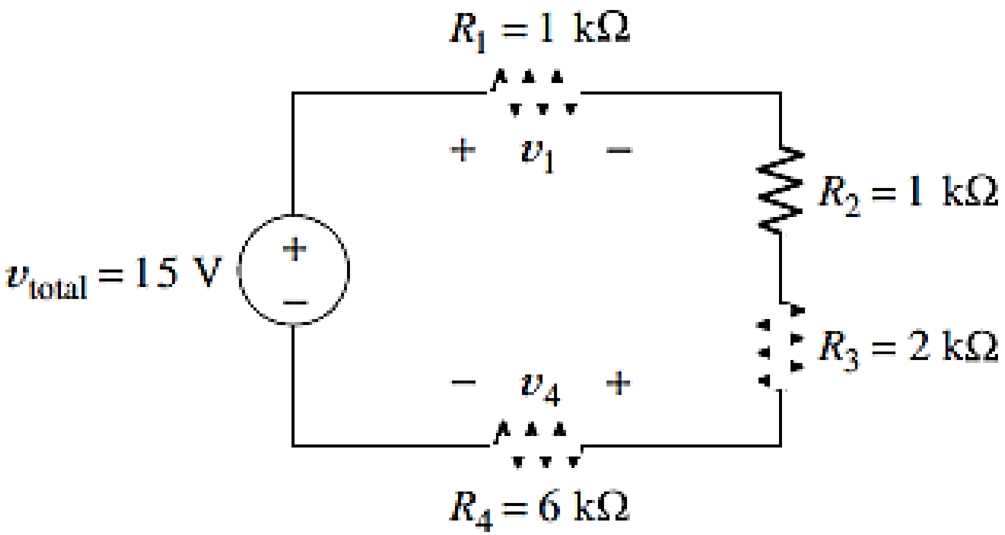


- Voltage-division principle:

  The Voltage fraction that appears across a given resistance in a series circuit is the ratio of the given resistance to the total series resistance.

2.3 VOLTAGE-DIVIDER AND CURRENT-DIVIDER CIRCUITS

Example 2.3 Application of the Voltage-Division Principle



$V_1 = 1.5$ V $\quad V_2 = 1.5$ V $\quad V_1 = 3$ V $\quad V_1 = 9$ V

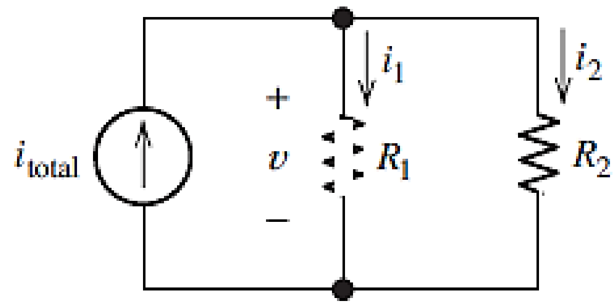
2.3 VOLTAGE-DIVIDER AND CURRENT-DIVIDER CIRCUITS

Current Division

$$v = R_{eq} i_{total} = \frac{R_1 R_2}{R_1 + R_2} i_{total}$$

$$i_1 = \frac{v}{R_1} = \frac{R_2}{R_1 + R_2} i_{total}$$

$$i_2 = \frac{v}{R_2} = \frac{R_1}{R_1 + R_2} i_{total}$$



- Current-division principle:

  The fraction of the total current owing in a resistance is the ratio of the other resistance to the sum of the two resistances.

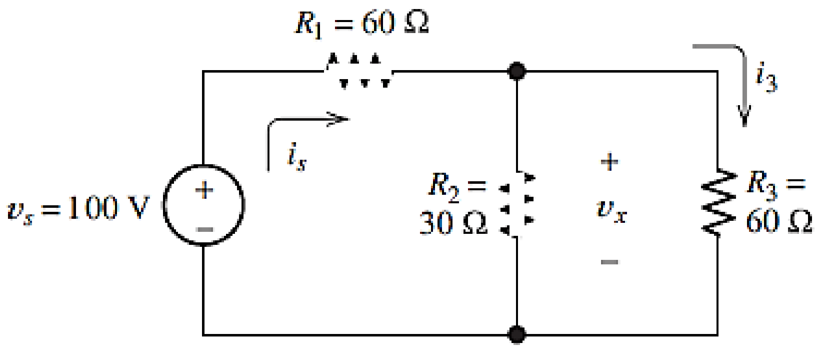
- Using Conductances:

$$i_1 = \frac{G_1}{G_1 + G_2 + \cdots + G_n} i_{total}$$

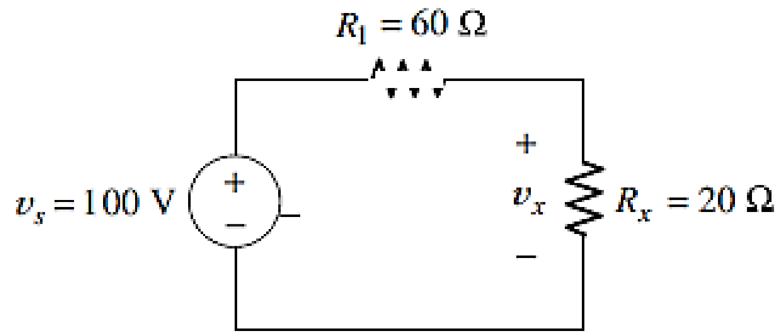
$$i_2 = \frac{G_2}{G_1 + G_2 + \cdots + G_n} i_{total}$$

2.3 VOLTAGE-DIVIDER AND CURRENT-DIVIDER CIRCUITS

Example 2.4 Applying the Current- and Voltage-Division Principles



(a) Original circuit

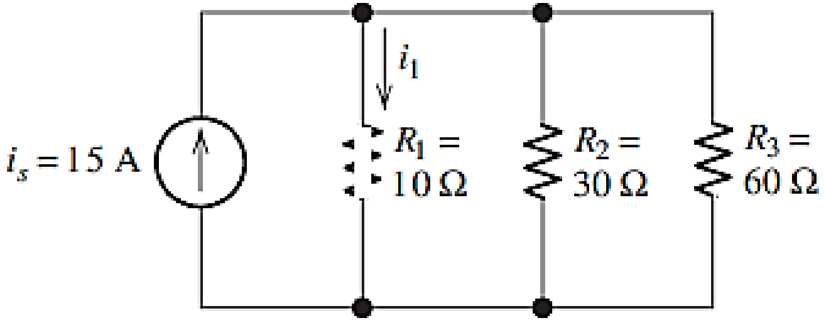


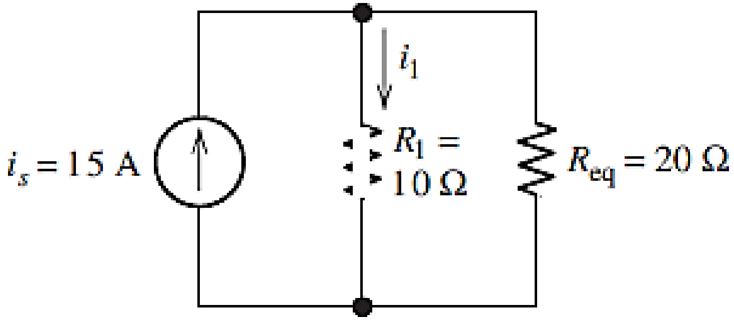
$$R_x = \frac{R_2 R_3}{R_2 + R_3} = \frac{30 \times 60}{30 + 60} = 20\ \Omega \quad \Longrightarrow \quad v_x = \frac{R_x}{R_1 + R_x} v_s = \frac{20}{60 + 20} \times 100 = 25\ \text{V}$$

$$i_s = \frac{v_s}{R_1 + R_x} = \frac{100}{60 + 20} = 1.25\ \text{A} \quad \Longrightarrow \quad i_3 = \frac{R_2}{R_2 + R_3} i_s = \frac{30}{30 + 60} \times 1.25 = 0.417\ \text{A}$$

2.3 VOLTAGE-DIVIDER AND CURRENT-DIVIDER CIRCUITS

Example 2.5 Application of the Current-Division Principle





$$R_{eq} = \frac{R_2 R_3}{R_2 + R_3} = \frac{30 \times 60}{30 + 60} = 20\ \Omega \quad \Rightarrow \quad i_1 = \frac{R_{eq}}{R_1 + R_{eq}} i_s = \frac{20}{10 + 20} 15 = 10\ \text{A}$$

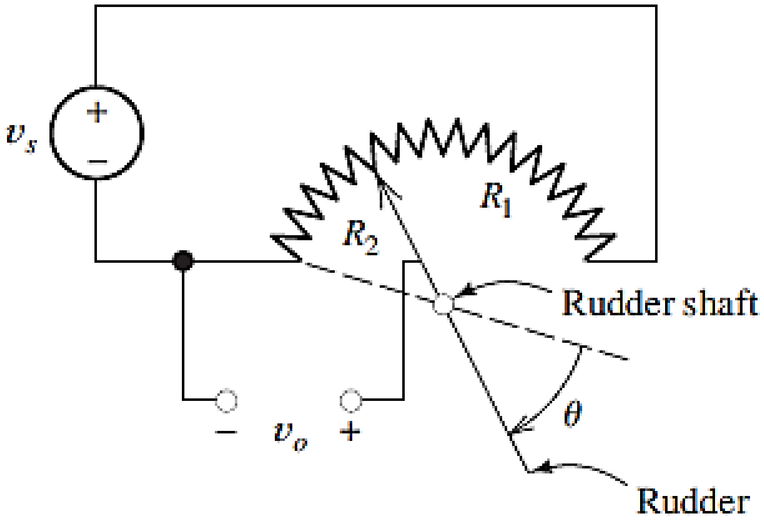
$$G_1 = \frac{1}{R_1} = 100\,\text{mS}, \quad G_2 = \frac{1}{R_2} = 33.33\,\text{mS}, \quad \text{and} \quad G_3 = \frac{1}{R_3} = 16.67\,\text{mS}$$

$$\Rightarrow \quad i_1 = \frac{G_1}{G_1 + G_2 + G_3} i_s = \frac{100}{100 + 33.33 + 16.67} 15 = 10\ \text{A}$$

2.3 VOLTAGE-DIVIDER AND CURRENT-DIVIDER CIRCUITS

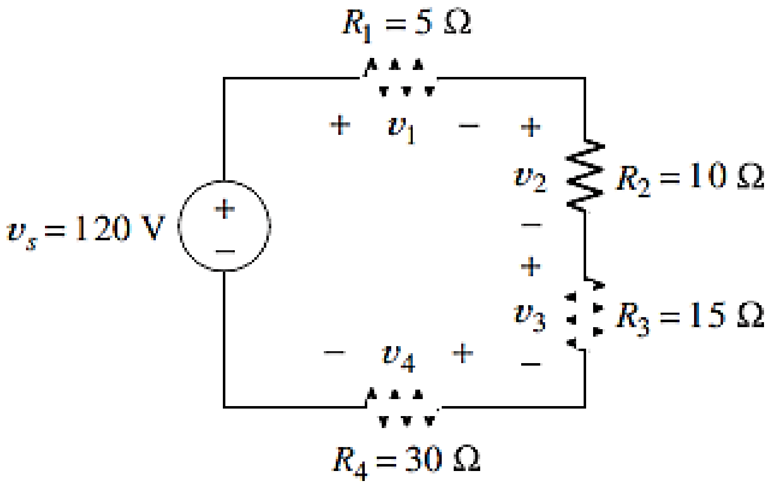
- Position Transducers Based on the Voltage-Division Principle

$$v_o = v_s \frac{R_2}{R_1 + R_2} = K\theta$$

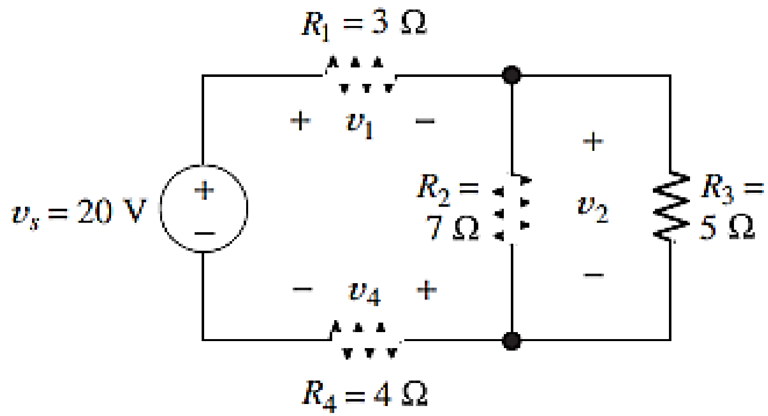


2.3 VOLTAGE-DIVIDER AND CURRENT-DIVIDER CIRCUITS

- Exercise



(a)



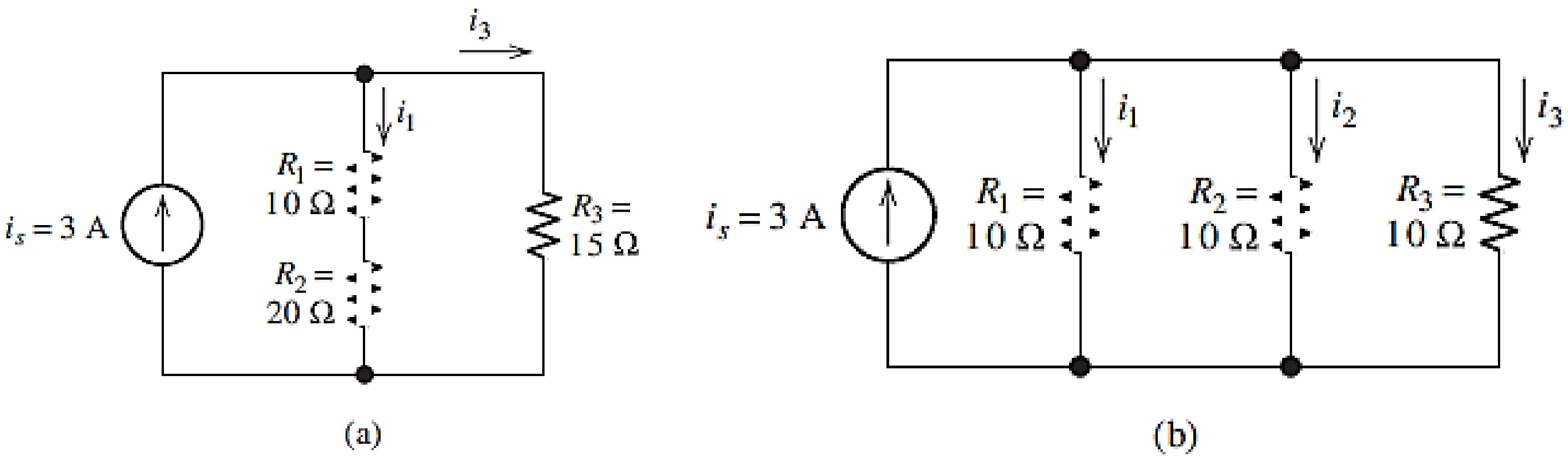
(b)

a. $v_1 = 10$ V, $v_2 = 20$ V, $v_3 = 30$ V, $v_4 = 60$ V;

b. $v_1 = 6.05$ V, $v_2 = 5.88$ V, $v_4 = 8.07$ V.

2.3 VOLTAGE-DIVIDER AND CURRENT-DIVIDER CIRCUITS

- Exercise

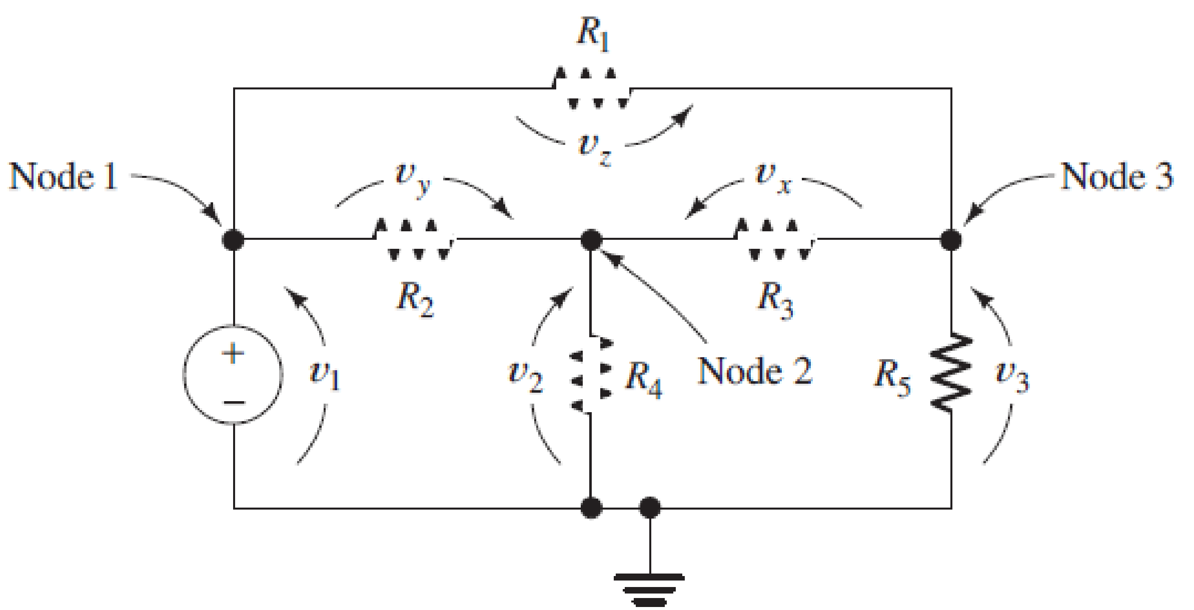


a. $i_1 = 1$ A, $i_3 = 2$ A

b. $i_1 = i_2 = i_3 = 1$ A.

2.4 NODE-VOLTAGE ANALYSIS

- Selecting the Reference Node
- Assigning Node Voltages
- Finding Element Voltages in Terms of the Node Voltages $v_x = v_2 - v_3$
- Writing KCL Equations in Terms of the Node Voltages $\frac{v_n - v_k}{R}$



2.4 NODE-VOLTAGE ANALYSIS

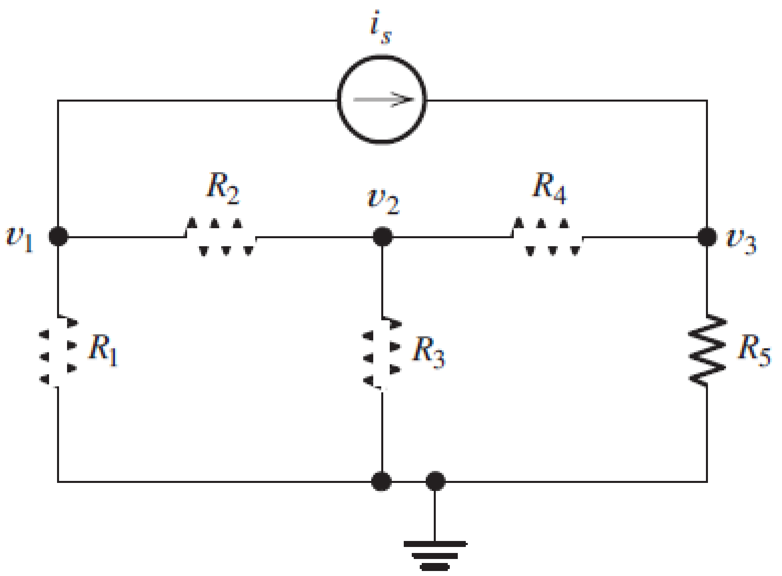
Example 2.6 Node-Voltage Analysis

- KCL for nodes 1,2 and 3:

$$\frac{v_1}{R_1} + \frac{v_1 - v_2}{R_2} + i_s = 0$$

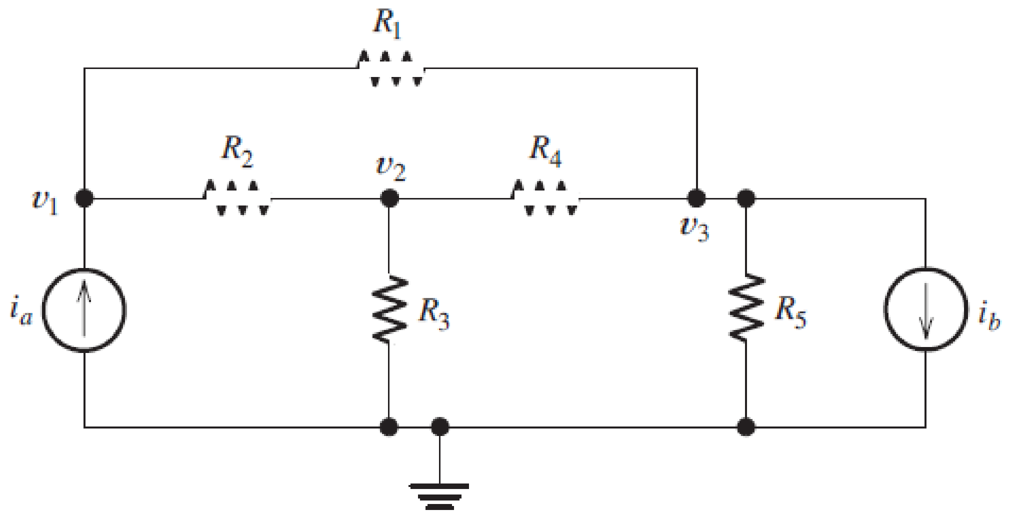
$$\frac{v_2 - v_1}{R_2} + \frac{v_2}{R_3} + \frac{v_2 - v_3}{R_4} = 0$$

$$\frac{v_3}{R_5} + \frac{v_3 - v_2}{R_4} = i_s$$



2.4 NODE-VOLTAGE ANALYSIS

- Exercise



Node 1: $\dfrac{v_1 - v_3}{R_1} + \dfrac{v_1 - v_2}{R_2} = i_a$

Node 2: $\dfrac{v_2 - v_1}{R_2} + \dfrac{v_2}{R_3} + \dfrac{v_2 - v_3}{R_4} = 0$

Node 3: $\dfrac{v_3}{R_5} + \dfrac{v_3 - v_2}{R_4} + \dfrac{v_3 - v_1}{R_1} + i_b = 0$

2.4 NODE-VOLTAGE ANALYSIS

- Circuit Equations in Standard Form
  - For two node voltages:

$$g_{11}v_1 + g_{12}v_2 = i_1$$
$$g_{21}v_1 + g_{22}v_2 = i_2$$

  - For three node voltages:

$$g_{11}v_1 + g_{12}v_2 + g_{13}v_3 = i_1$$
$$g_{21}v_1 + g_{22}v_2 + g_{23}v_3 = i_2$$
$$g_{31}v_1 + g_{32}v_2 + g_{33}v_3 = i_3$$

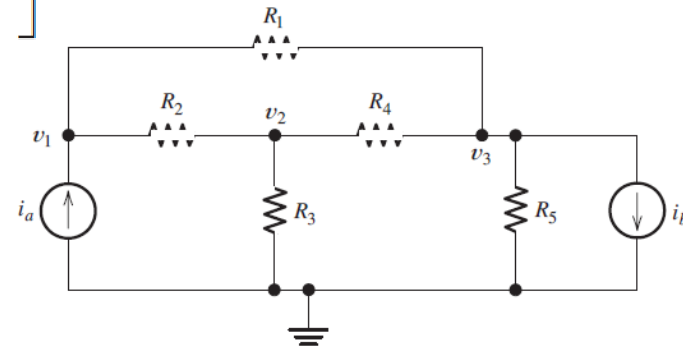
  - Matrix form:

$$\mathbf{GV} = \mathbf{I} \qquad \mathbf{V} = \mathbf{G}^{-1}\mathbf{I}$$

2.4 NODE-VOLTAGE ANALYSIS

- A Shortcut to Writing the Matrix Equations

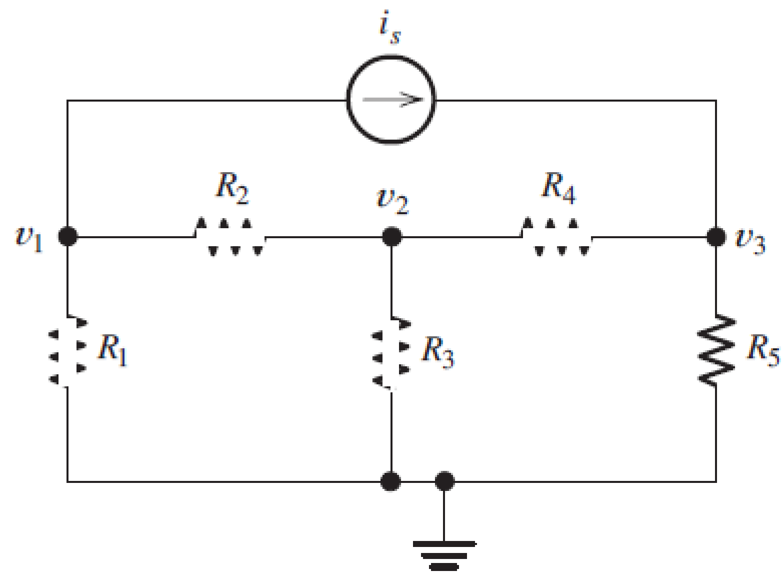
$$\begin{bmatrix} \frac{1}{R_1}+\frac{1}{R_2} & -\frac{1}{R_2} & -\frac{1}{R_1} \\ -\frac{1}{R_2} & \frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_4} & -\frac{1}{R_4} \\ -\frac{1}{R_1} & -\frac{1}{R_4} & \frac{1}{R_1}+\frac{1}{R_4}+\frac{1}{R_5} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} i_a \\ 0 \\ -i_b \end{bmatrix}$$



- Diagonal elements: sum of the conductances connected to node
- Off-diagonal terms: negative of the conductance connected between node *j* and *k*
- Terms in **I** matrix: currents pushed into corresponding nodes by current sources

2.4 NODE-VOLTAGE ANALYSIS

- Exercise



$$\begin{bmatrix} \frac{1}{R_1}+\frac{1}{R_2} & -\frac{1}{R_2} & 0 \\ -\frac{1}{R_2} & \frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_4} & -\frac{1}{R_4} \\ 0 & -\frac{1}{R_4} & \frac{1}{R_4}+\frac{1}{R_5} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} -i_s \\ 0 \\ i_s \end{bmatrix}$$

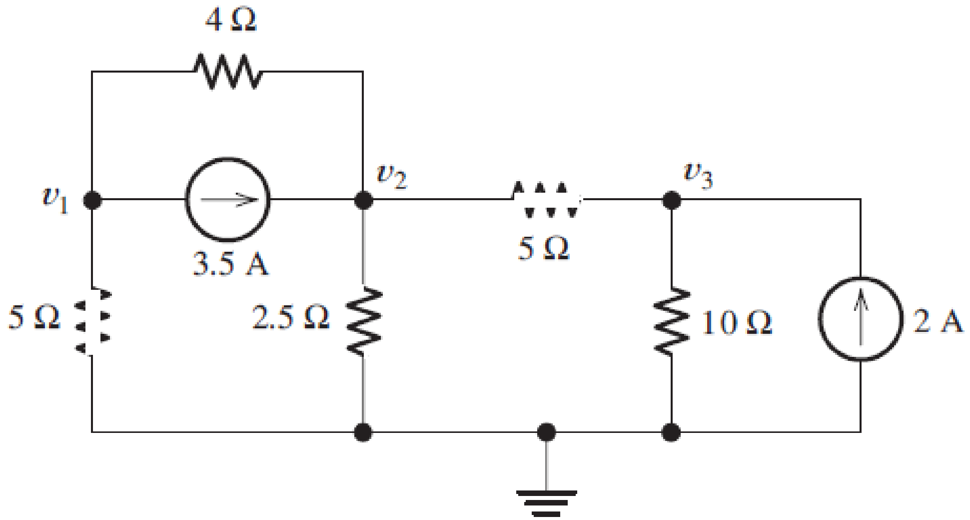
2.4 NODE-VOLTAGE ANALYSIS

Example 2.7 Node-Voltage Analysis

$$\frac{v_1}{5} + \frac{v_1 - v_2}{4} + 3.5 = 0$$

$$\frac{v_2 - v_1}{4} + \frac{v_2}{2.5} + \frac{v_2 - v_3}{5} = 3.5$$

$$\frac{v_3 - v_2}{5} + \frac{v_3}{10} = 2$$



$$0.45v_1 - 0.25v_2 = -3.5$$

$$-0.25v_1 + 0.85v_2 - 0.2v_3 = 3.5$$

$$-0.2v_2 + 0.35v_3 = 2$$

$$\Rightarrow \begin{bmatrix} 0.45 & -0.25 & 0 \\ -0.25 & 0.85 & -0.20 \\ 0 & -0.20 & 0.30 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} -3.5 \\ 3.5 \\ 2 \end{bmatrix}$$

2.4 NODE-VOLTAGE ANALYSIS

- Using MATLAB to Solve Network Equations

$$\begin{bmatrix} 0.45 & -0.25 & 0 \\ -0.25 & 0.85 & -0.20 \\ 0 & -0.20 & 0.30 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} -3.5 \\ 3.5 \\ 2 \end{bmatrix}$$

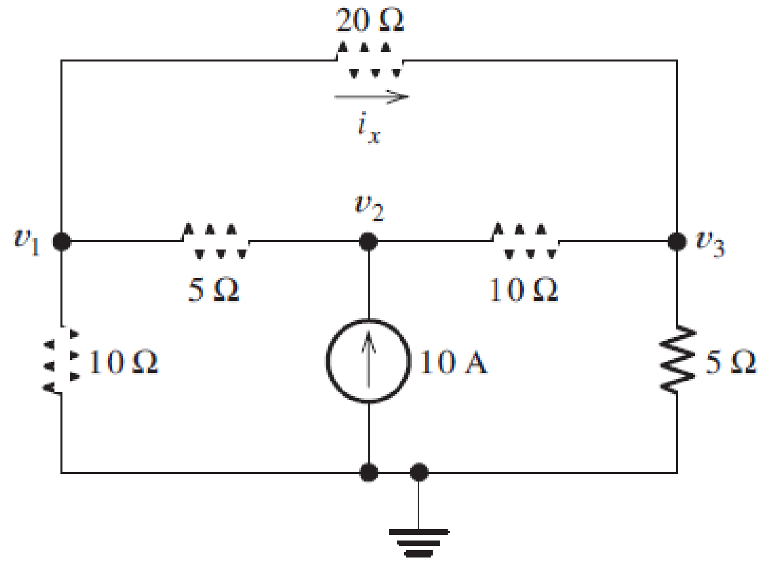
```
>> I = [-3.5; 3.5; 2]
I =
   -3.5000
    3.5000
    2.0000

>> G = [0.45 -0.25 0; -0.25 0.85 -0.2; 0 -0.2 0.30]
G =
    0.4500   -0.2500         0
   -0.2500    0.8500   -0.2000
         0   -0.2000    0.3000

>> V = G\I
V =
   -5.0000
    5.0000
   10.0000
```

2.4 NODE-VOLTAGE ANALYSIS

Example 2.8 Node-Voltage Analysis



Node 1: $$\frac{v_1}{10} + \frac{v_1 - v_2}{5} + \frac{v_1 - v_3}{20} = 0$$

Node 2: $$\frac{v_2 - v_1}{5} + \frac{v_2 - v_3}{10} = 10$$

Node 3: $$\frac{v_3}{5} + \frac{v_3 - v_2}{10} + \frac{v_3 - v_1}{20} = 0$$

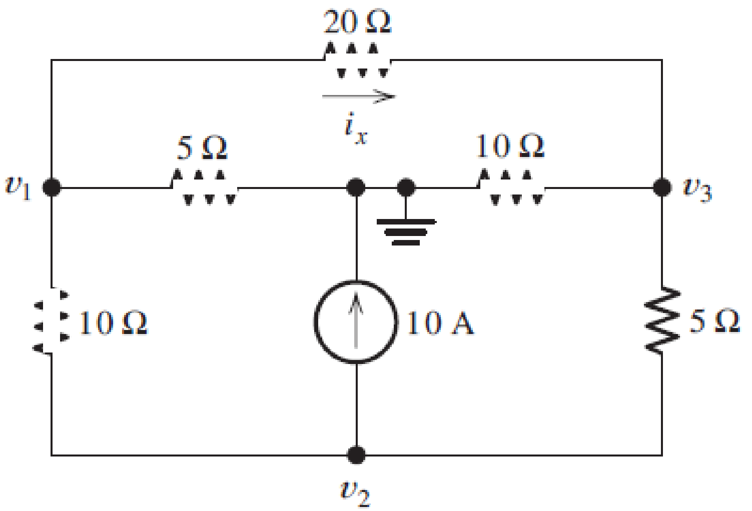
$$0.35v_1 - 0.2v_2 - 0.05v_3 = 0$$
$$\Rightarrow \quad -0.2v_1 + 0.3v_2 - 0.10v_3 = 10$$
$$-0.05v_1 - 0.10v_2 + 0.35v_3 = 0$$

```
>> V = G\I
V =
    45.4545
    72.7273
    27.2727
```

```
>> Ix = (V(1) - V(3))/20
Ix =
    0.9091
```

2.4 NODE-VOLTAGE ANALYSIS

- Exercise



$$\frac{v_1 - v_3}{20} + \frac{v_1}{5} + \frac{v_1 - v_2}{10} = 0$$
$$\frac{v_2 - v_1}{10} + 10 + \frac{v_2 - v_3}{5} = 0$$
$$\frac{v_3 - v_1}{20} + \frac{v_3}{10} + \frac{v_3 - v_2}{5} = 0$$

$$\Rightarrow \begin{aligned} 0.35v_1 - 0.10v_2 - 0.05v_3 &= 0 \\ -0.10v_1 + 0.30v_2 - 0.20v_3 &= -10 \\ -0.05v_1 - 0.20v_2 + 0.35v_3 &= 0 \end{aligned}$$

$\Rightarrow$ $v_1 = -27.27; v_2 = -72.73; v_3 = -45.45$ $\quad$ $i_x = 0.909$ A

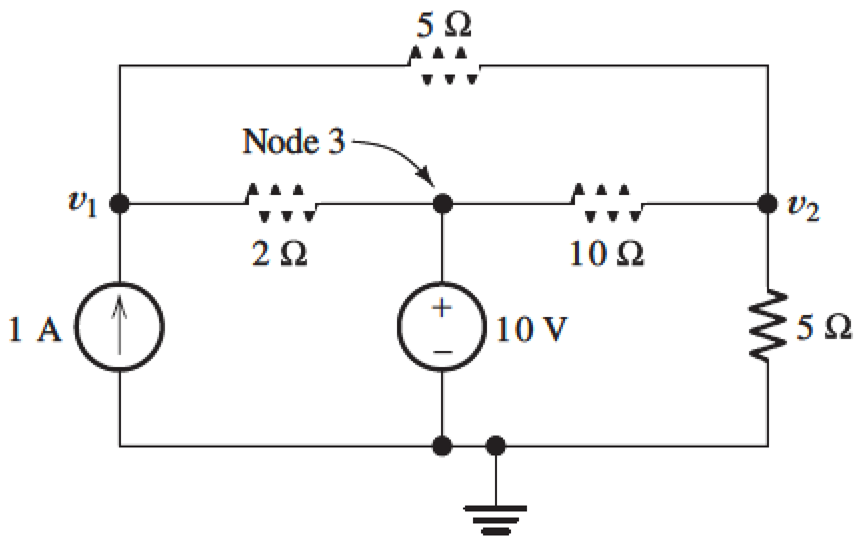
2.4 NODE-VOLTAGE ANALYSIS

- Circuits with Voltage Sources
  - Pick reference node at one end of source, one less unknown node voltage

Example 2.9 Node-Voltage Analysis

$$\frac{v_1 - v_2}{5} + \frac{v_1 - 10}{2} = 1$$

$$\frac{v_2}{5} + \frac{v_2 - 10}{10} + \frac{v_2 - v_1}{5} = 0$$

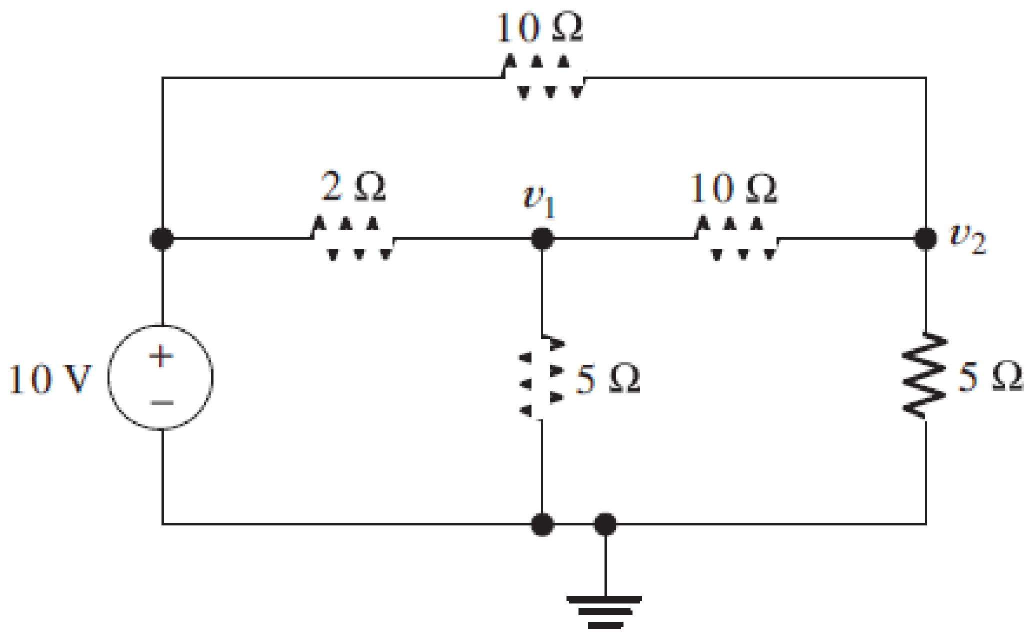


$$\Rightarrow \quad 0.7v_1 - 0.2v_2 = 6$$
$$-0.2v_1 + 0.5v_2 = 1$$

$$\Rightarrow \quad v_1 = 10.32 \text{ V}; v_2 = 6.129 \text{ V}.$$

2.4 NODE-VOLTAGE ANALYSIS

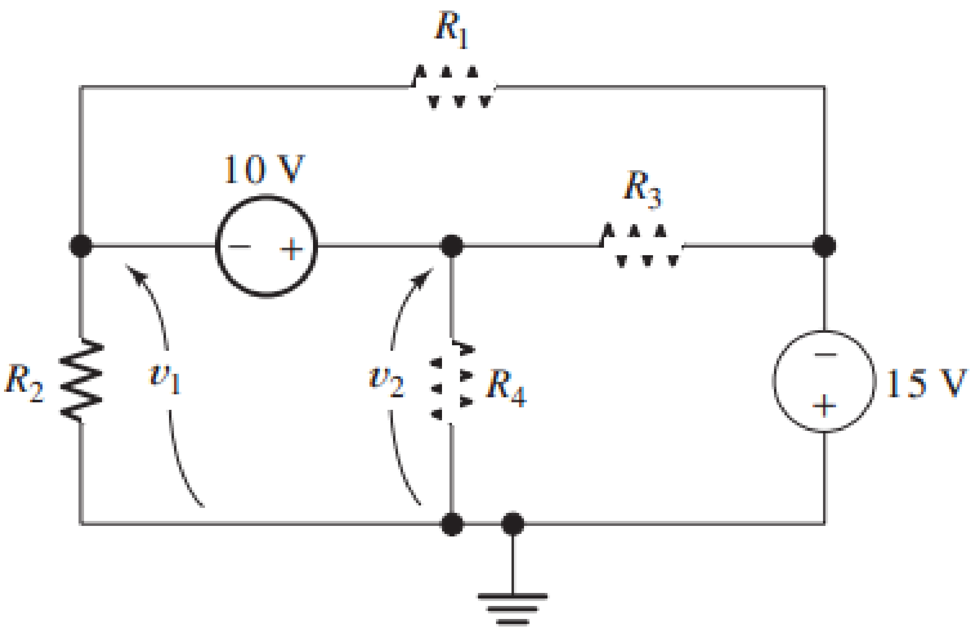
- Exercise



$$v_1 = 6.77\ \text{V},\ v_2 = 4.19\ \text{V}.$$

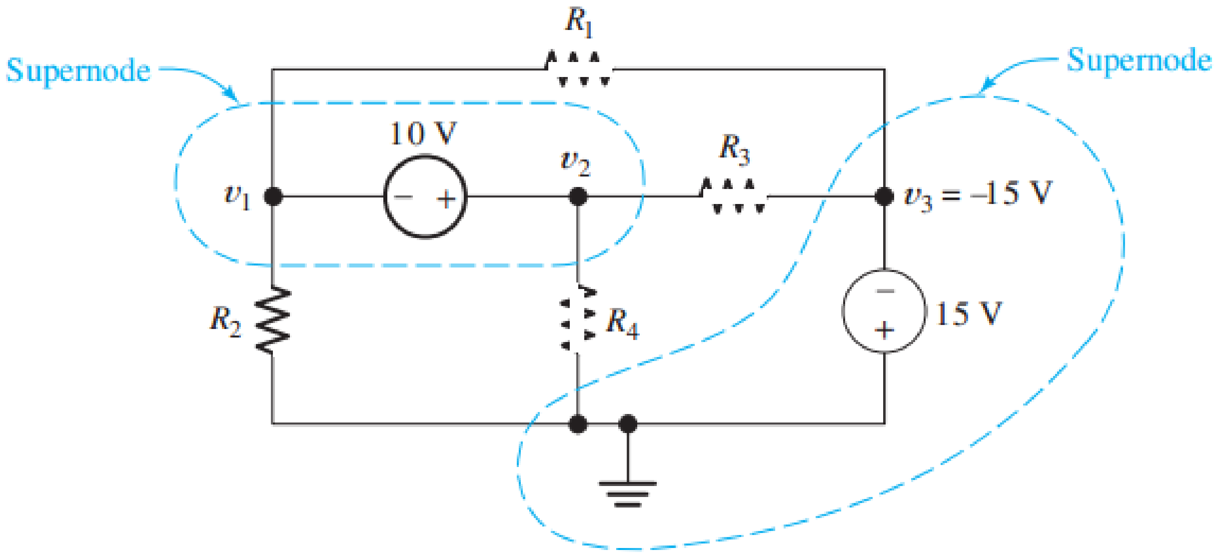
2.4 NODE-VOLTAGE ANALYSIS

- Supernode
  - Decrease equations order (ignore source current)
  - Forming closed surface (Supernode)
  - Writing KCL for Supernode



2.4 NODE-VOLTAGE ANALYSIS

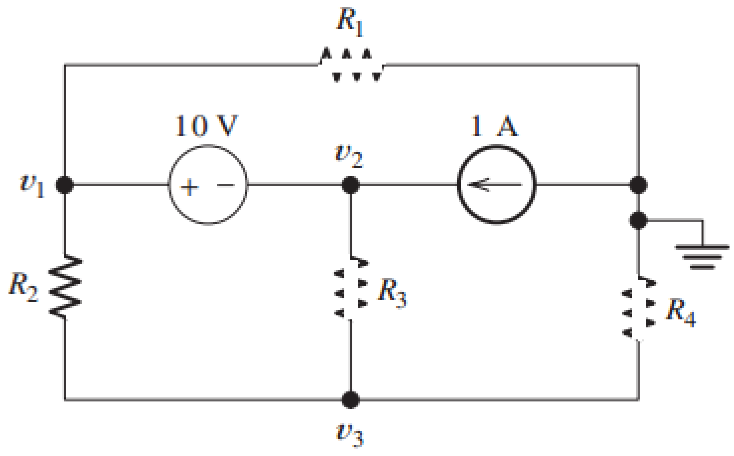
- Supernode



- KCL for Supernde: $$\frac{v_1}{R_2} + \frac{v_1 - (-15)}{R_1} + \frac{v_2}{R_4} + \frac{v_2 - (-15)}{R_3} = 0$$
- KVL for 10V source: $$-v_1 - 10 + v_2 = 0$$

2.4 NODE-VOLTAGE ANALYSIS

- Exercise:



KCL for the supernode enclosing the 10-V source:

$$\frac{v_1}{R_1} + \frac{v_1 - v_3}{R_2} + \frac{v_2 - v_3}{R_3} = 1$$

KCL for node 3:

$$\frac{v_3 - v_1}{R_2} + \frac{v_3 - v_2}{R_3} + \frac{v_3}{R_4} = 0$$

KCL at the reference node:

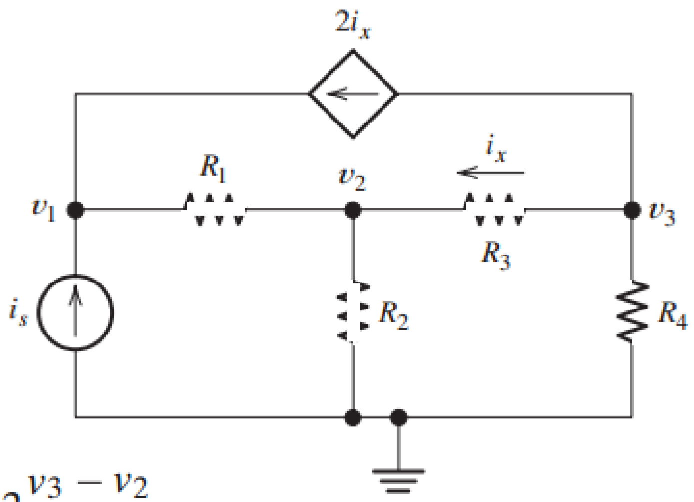
$$\frac{v_1}{R_1} + \frac{v_3}{R_4} = 1$$

$$-v_1 + 10 + v_2 = 0$$

2.4 NODE-VOLTAGE ANALYSIS

- Circuits with controlled sources

Example 2.10 Node-Voltage Analysis with a Dependent Source



- KCLs:

$$\frac{v_1 - v_2}{R_1} = i_s + 2i_x$$

$$\frac{v_2 - v_1}{R_1} + \frac{v_2}{R_2} + \frac{v_2 - v_3}{R_3} = 0$$

$$\frac{v_3 - v_2}{R_3} + \frac{v_3}{R_4} + 2i_x = 0$$

- Substituting:

$$i_x = \frac{v_3 - v_2}{R_3} \Longrightarrow$$

$$\frac{v_1 - v_2}{R_1} = i_s + 2\frac{v_3 - v_2}{R_3}$$

$$\frac{v_2 - v_1}{R_1} + \frac{v_2}{R_2} + \frac{v_2 - v_3}{R_3} = 0$$

$$\frac{v_3 - v_2}{R_3} + \frac{v_3}{R_4} + 2\frac{v_3 - v_2}{R_3} = 0$$

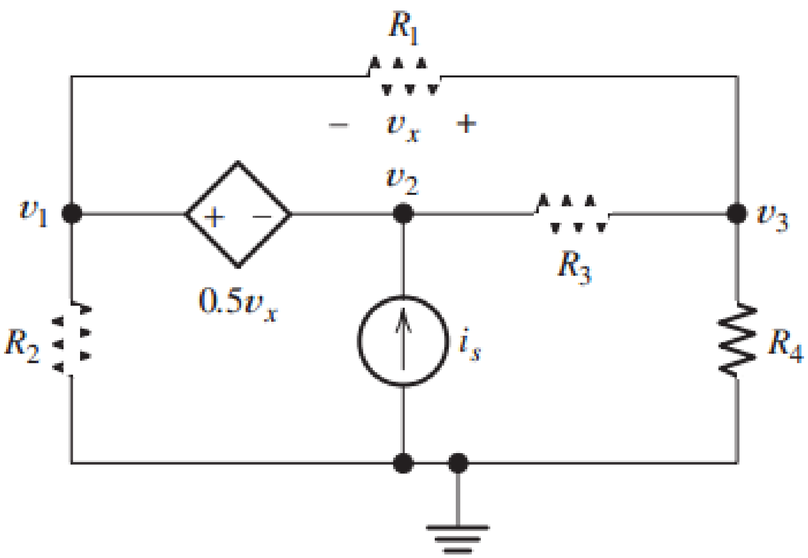
2.4 NODE-VOLTAGE ANALYSIS

Example 2.11 Node-Voltage Analysis with a Dependent Source

$$\frac{v_1}{R_2} + \frac{v_1 - v_3}{R_1} + \frac{v_2 - v_3}{R_3} = i_s$$

$$\frac{v_3}{R_4} + \frac{v_3 - v_2}{R_3} + \frac{v_3 - v_1}{R_1} = 0$$

$$\frac{v_1}{R_2} + \frac{v_3}{R_4} = i_s$$

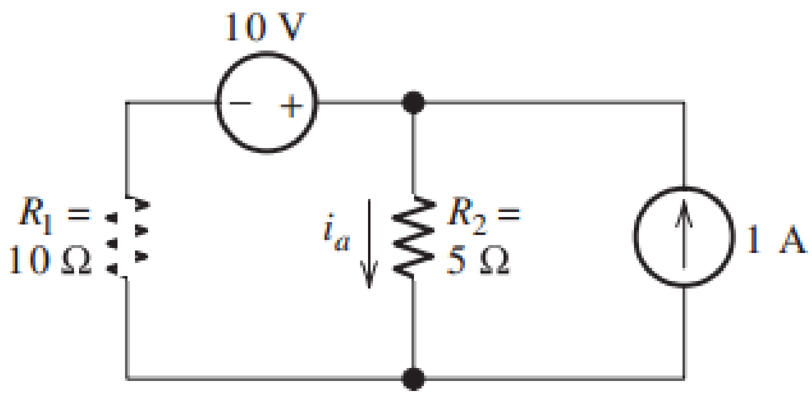


$$-v_1 + 0.5v_x + v_2 = 0$$

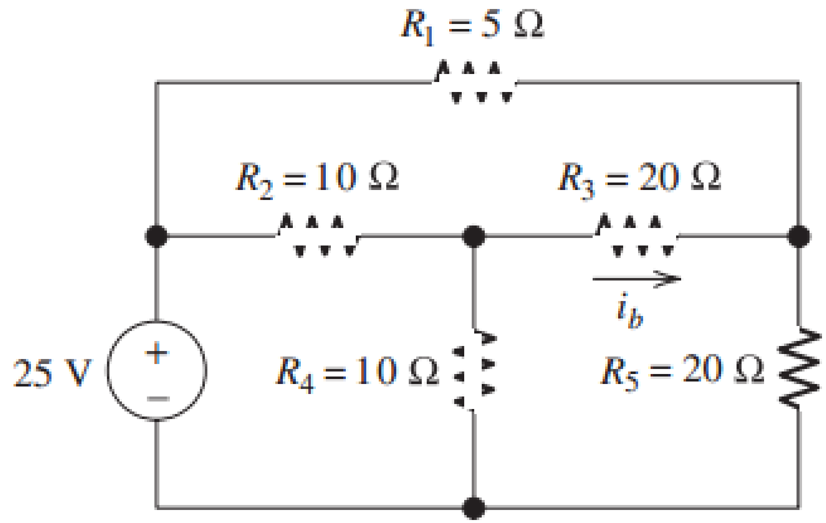
$$v_x = v_3 - v_1$$

2.4 NODE-VOLTAGE ANALYSIS

- Exercise



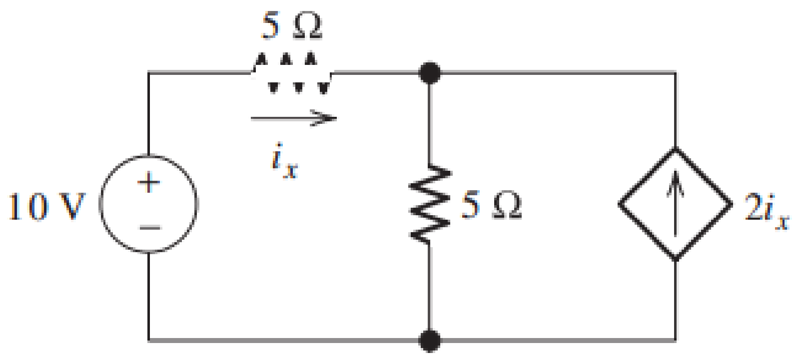
$i_a = 1.33$ A



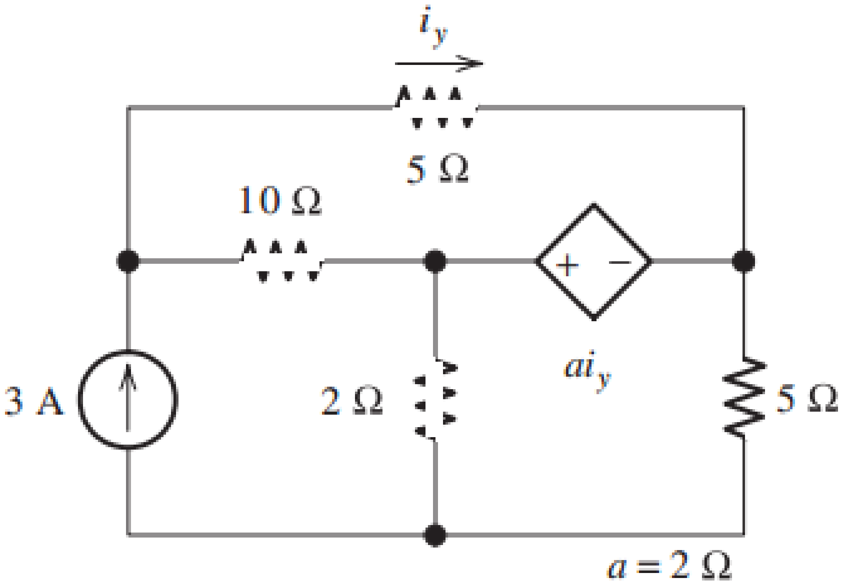
$i_b = -0.259$ A

2.4 NODE-VOLTAGE ANALYSIS

- Exercise



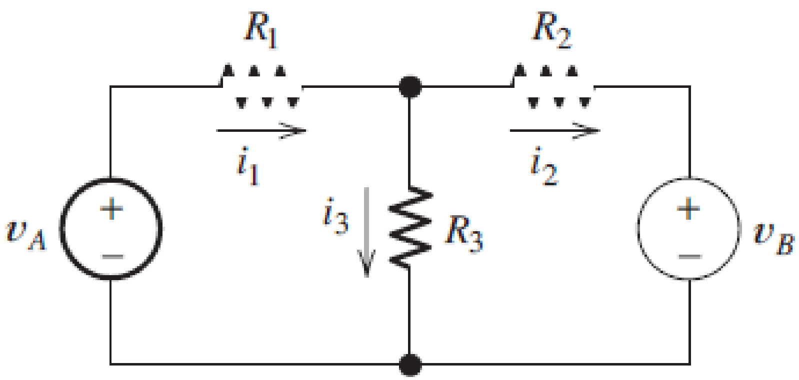
$$i_x = 0.5\ \text{A}$$



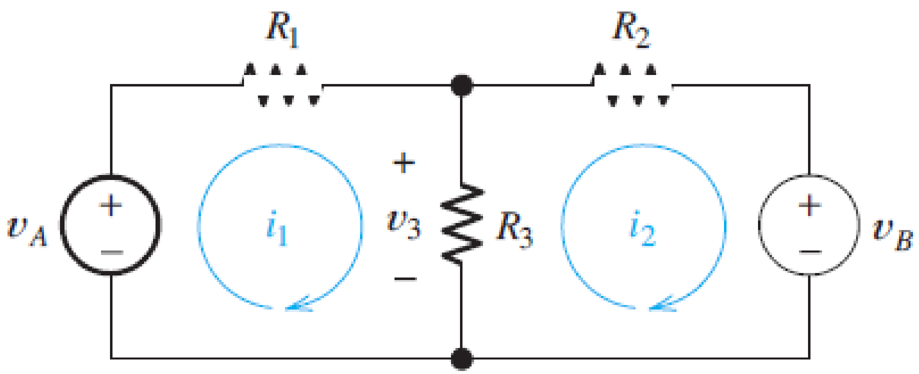
$$i_y = 2.31\ \text{A}.$$

2.5 MESH-CURRENT ANALYSIS

- For solving currents
  - Branch currents
  - Mesh currents



(a) Circuit with branch currents



(b) Circuit with mesh currents

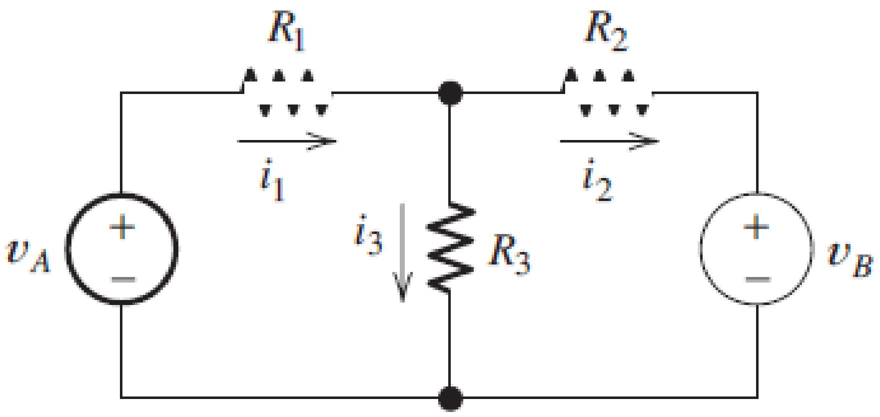
2.5 MESH-CURRENT ANALYSIS

- Normal KVL-KCL:

$$R_1 i_1 + R_3 i_3 = v_A$$

$$-R_3 i_3 + R_2 i_2 = -v_B$$

$$i_1 = i_2 + i_3$$

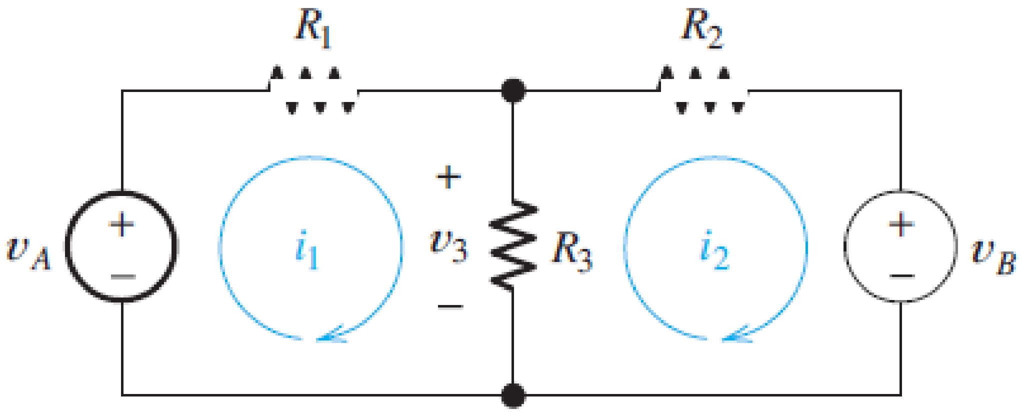


$$\Rightarrow \quad R_1 i_1 + R_3 (i_1 - i_2) = v_A$$

$$-R_3 (i_1 - i_2) + R_2 i_2 = -v_B$$

2.5 MESH-CURRENT ANALYSIS

- Mesh-Current Method:

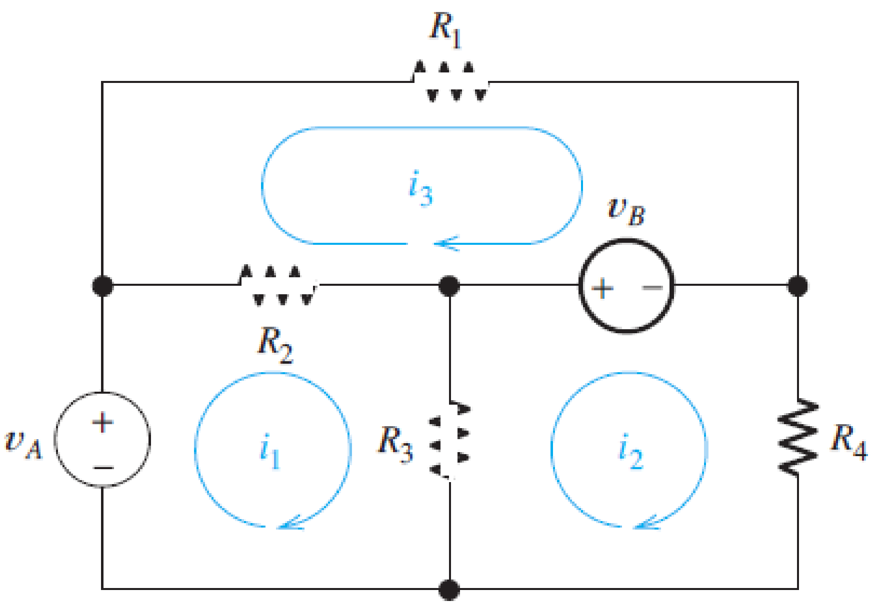


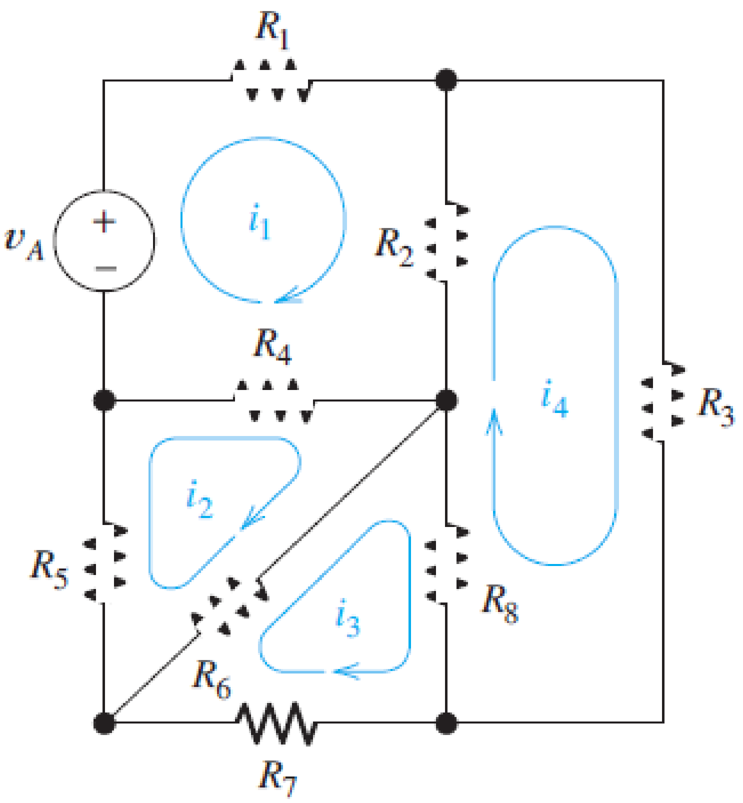
$$R_1 i_1 + R_3(i_1 - i_2) = v_A$$

$$-R_3(i_1 - i_2) + R_2 i_2 = -v_B$$

2.5 MESH-CURRENT ANALYSIS

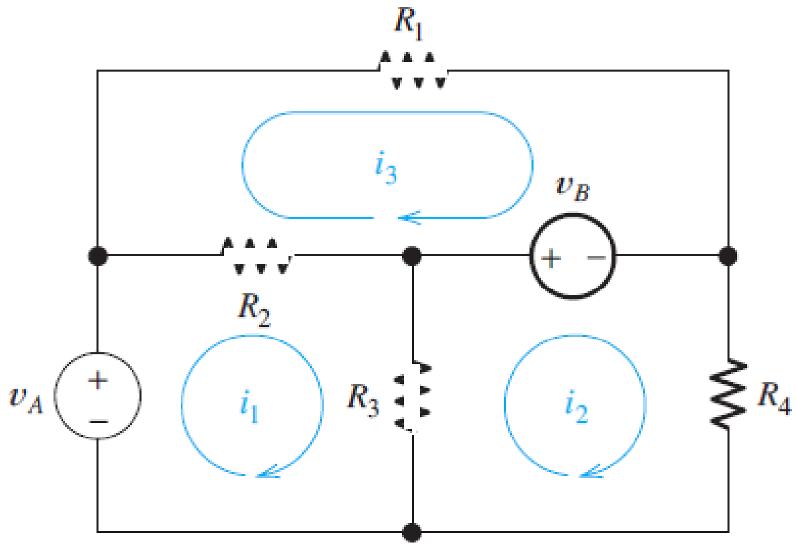
- Example





2.5 MESH-CURRENT ANALYSIS

Example 2.12 Mesh-Current Analysis



$$R_2(i_1 - i_3) + R_3(i_1 - i_2) - v_A = 0$$

$$R_3(i_2 - i_1) + R_4 i_2 + v_B = 0$$

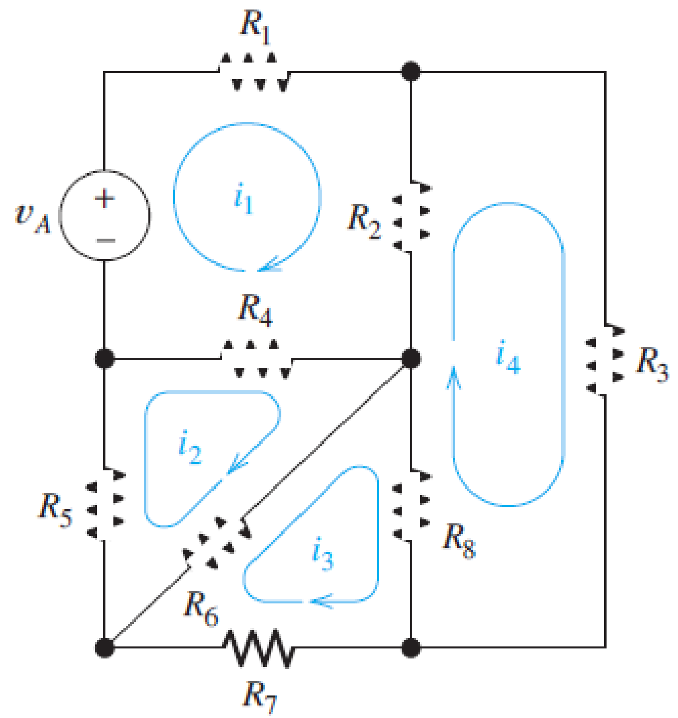
$$R_2(i_3 - i_1) + R_1 i_3 - v_B = 0$$

$$\Rightarrow \begin{bmatrix} (R_2 + R_3) & -R_3 & -R_2 \\ -R_3 & (R_3 + R_4) & 0 \\ -R_2 & 0 & (R_1 + R_2) \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} v_A \\ -v_B \\ v_B \end{bmatrix}$$

2.5 MESH-CURRENT ANALYSIS

- Exercise

$$R_1 i_1 + R_2(i_1 - i_4) + R_4(i_1 - i_2) - v_A = 0$$
$$R_5 i_2 + R_4(i_2 - i_1) + R_6(i_2 - i_3) = 0$$
$$R_7 i_3 + R_6(i_3 - i_2) + R_8(i_3 - i_4) = 0$$
$$R_3 i_4 + R_2(i_4 - i_1) + R_8(i_4 - i_3) = 0$$



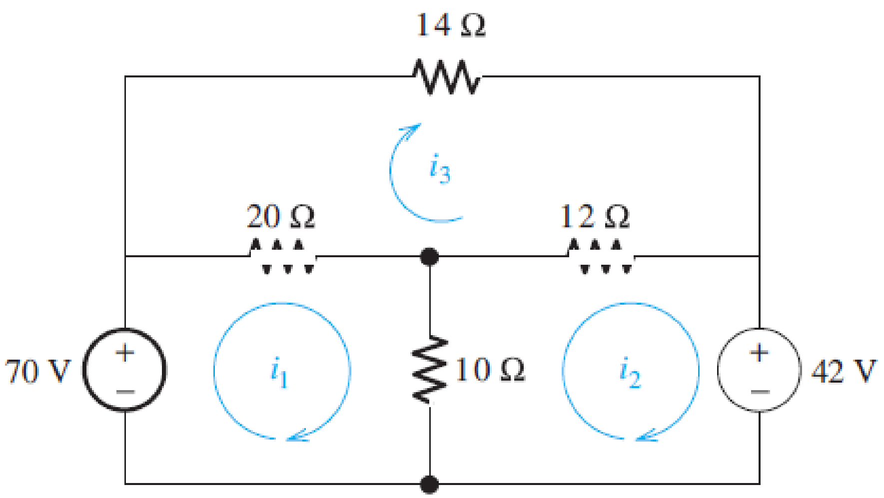
2.5 MESH-CURRENT ANALYSIS

Example 2.13 Mesh-Current Analysis

$$20(i_1 - i_3) + 10(i_1 - i_2) - 70 = 0$$

$$10(i_2 - i_1) + 12(i_2 - i_3) + 42 = 0$$

$$20(i_3 - i_1) + 14i_3 + 12(i_3 - i_2) = 0$$

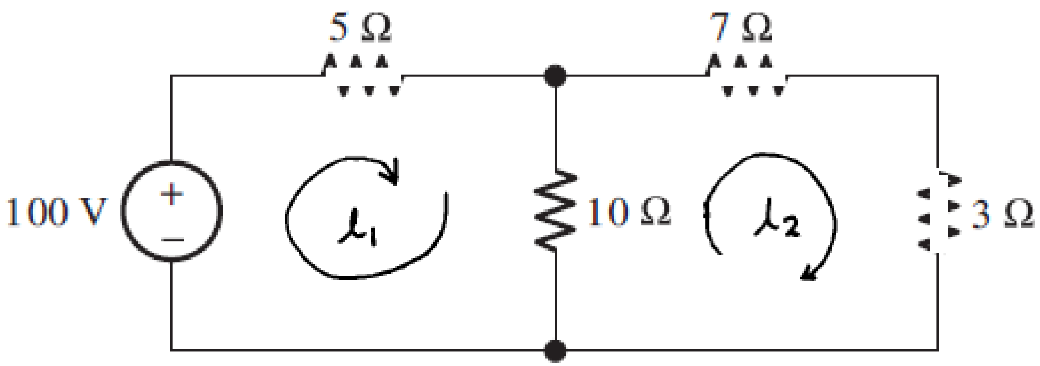


$$\begin{bmatrix} 30 & -10 & -20 \\ -10 & 22 & -12 \\ -20 & -12 & 46 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 70 \\ -42 \\ 0 \end{bmatrix}$$

```
>> R = [30 -10 -20; -10 22 -12; -20 -12 46];
>> V = [70; -42; 0];
>> I = R\V % Try to avoid using i, which represents the square root of
>> % -1 in MATLAB.
I =
    4.0000
    1.0000
    2.0000
```

2.5 MESH-CURRENT ANALYSIS

- Exercise
  - Current flowing through the 10 ohm



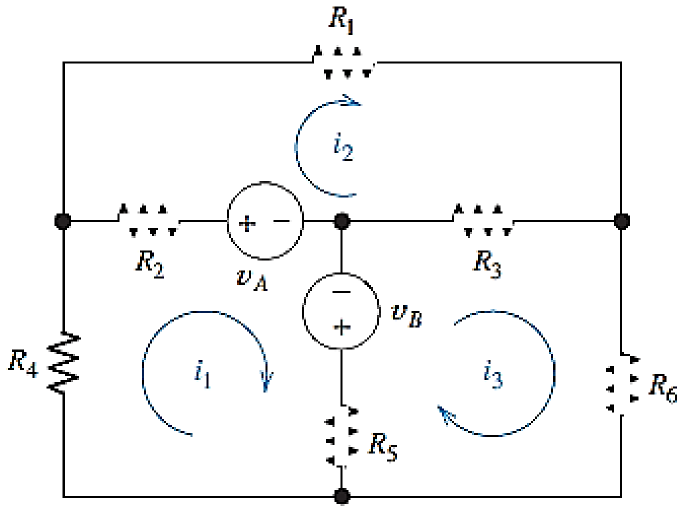
$$5i_1 + 10(i_1 - i_2) = 100 \quad \text{and} \quad 10(i_2 - i_1) + 7i_2 + 3i_2 = 0$$

$$\Longrightarrow i_1 - i_2 = 5\ A.$$

2.5 MESH-CURRENT ANALYSIS

- A Shortcut to Writing the Matrix Equations: R.I=V

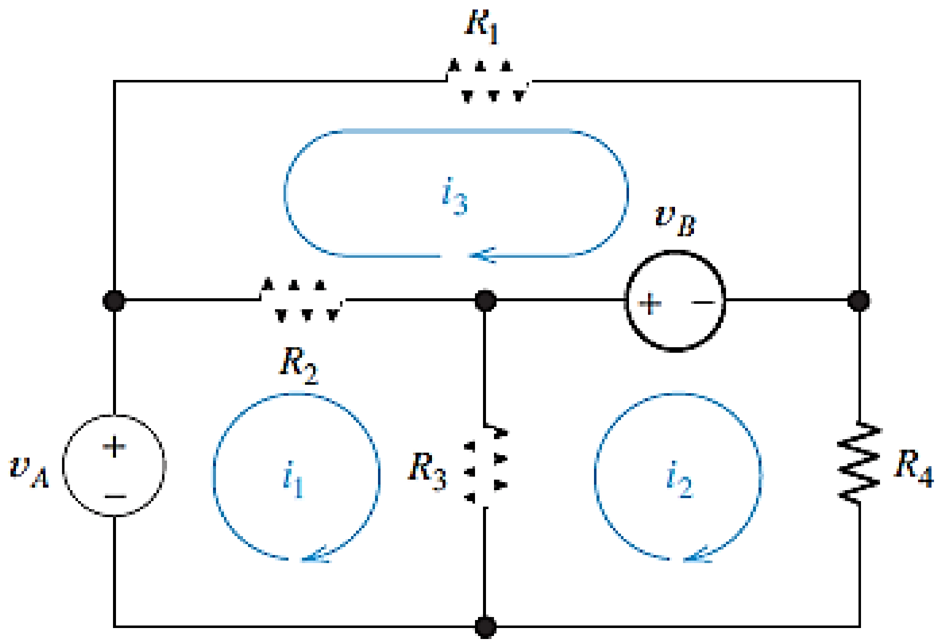
$$\begin{bmatrix} (R_2+R_4+R_5) & -R_2 & -R_5 \\ -R_2 & (R_1+R_2+R_3) & -R_3 \\ -R_5 & -R_3 & (R_3+R_5+R_6) \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} -v_A+v_B \\ v_A \\ -v_B \end{bmatrix}$$



- Diagonal elements: sum of the resistances in mesh
- Off-diagonal terms: negative of the resistance shared between mesh *j* and *k*
- Terms in **V** matrix: mesh voltage sources (+ or -)

2.5 MESH-CURRENT ANALYSIS

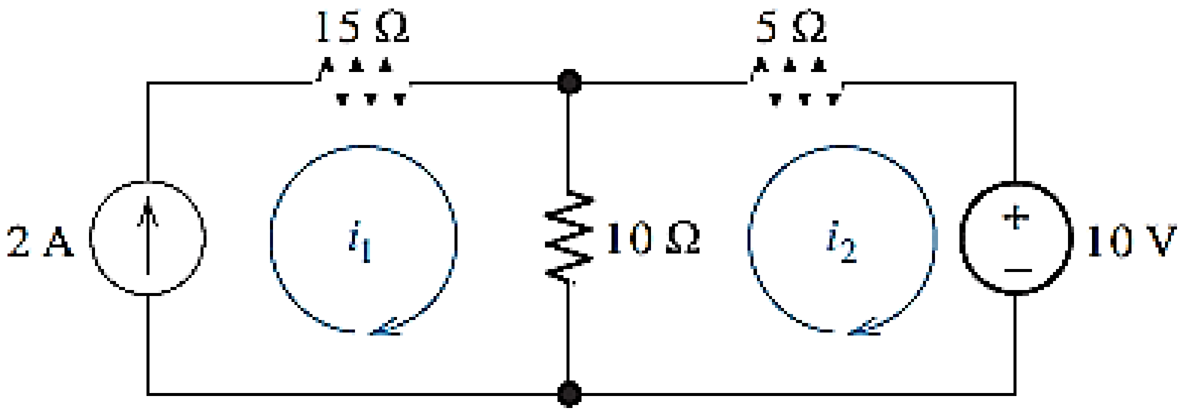
- Exercise



$$\begin{bmatrix} (R_2+R_3) & -R_3 & -R_2 \\ -R_3 & (R_3+R_4) & 0 \\ -R_2 & 0 & (R_1+R_2) \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} v_A \\ -v_B \\ v_B \end{bmatrix}$$

2.5 MESH-CURRENT ANALYSIS

- Mesh Currents in Circuits Containing Current Sources



$$i_1 = 2\text{ A}$$

$$10(i_2 - i_1) + 5i_2 + 10 = 0$$

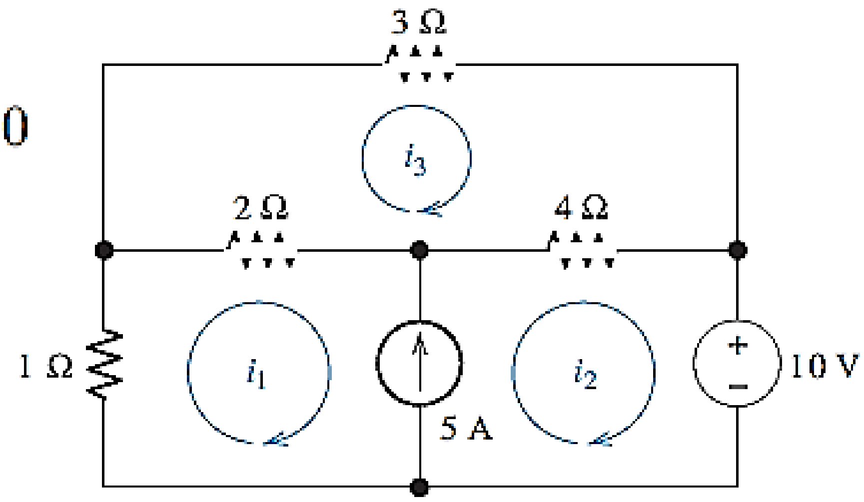
2.5 MESH-CURRENT ANALYSIS

- Supermesh
  - Write a KVL equation around the periphery of meshes 1 and 2 combined

$$i_1 + 2(i_1 - i_3) + 4(i_2 - i_3) + 10 = 0$$

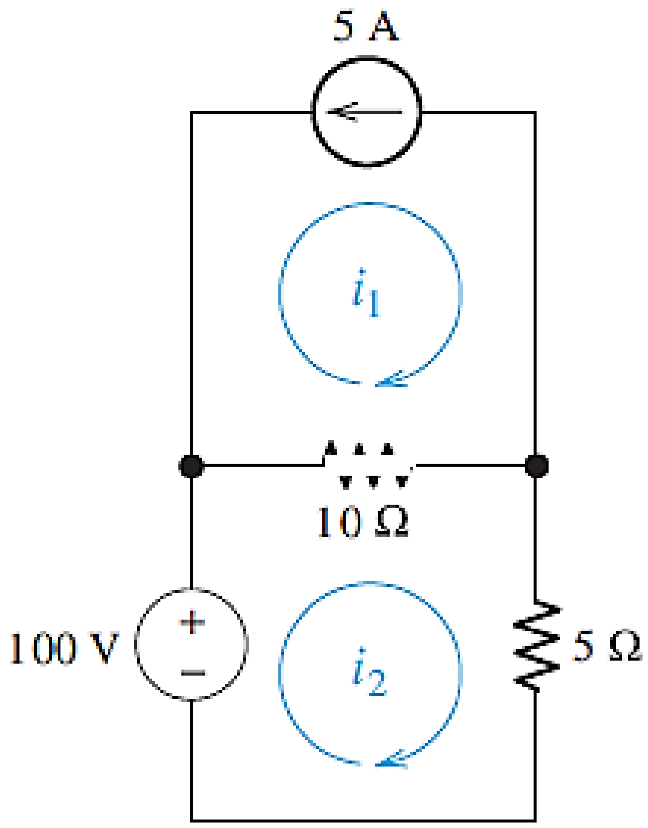
$$3i_3 + 4(i_3 - i_2) + 2(i_3 - i_1) = 0$$

$$i_2 - i_1 = 5$$



2.5 MESH-CURRENT ANALYSIS

- Exercise

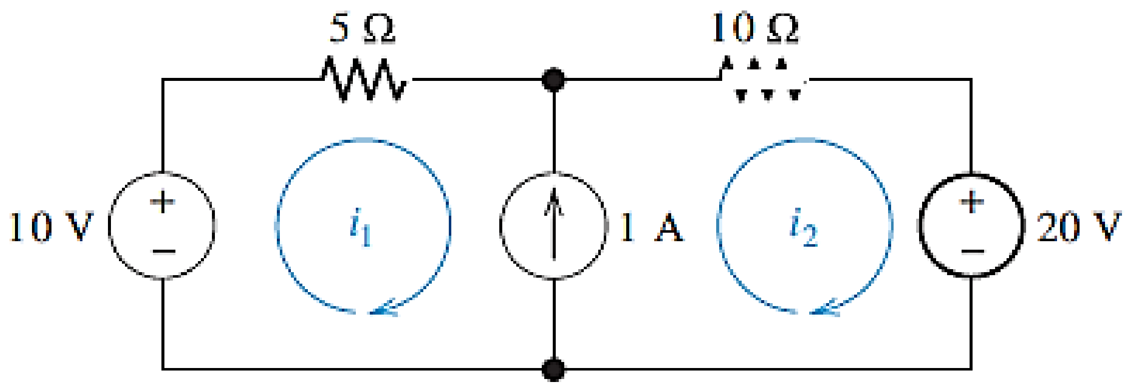


$$i_1 = -5\text{ A}$$

$$10(i_2 - i_1) + 5i_2 - 100 = 0$$

2.5 MESH-CURRENT ANALYSIS

- Exercise



$$5i_1 + 10i_2 + 20 - 10 = 0.$$

$$i_2 - i_1 = 1$$

2.5 MESH-CURRENT ANALYSIS

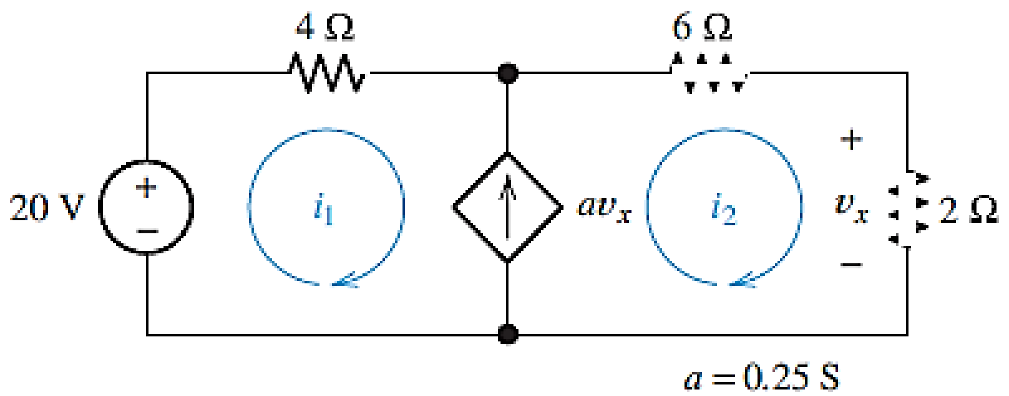
- Circuits with Controlled Sources

Example 2.15 Mesh-Current Analysis with Controlled Sources

$$-20 + 4i_1 + 6i_2 + 2i_2 = 0$$

$$av_x = 0.25v_x = i_2 - i_1$$

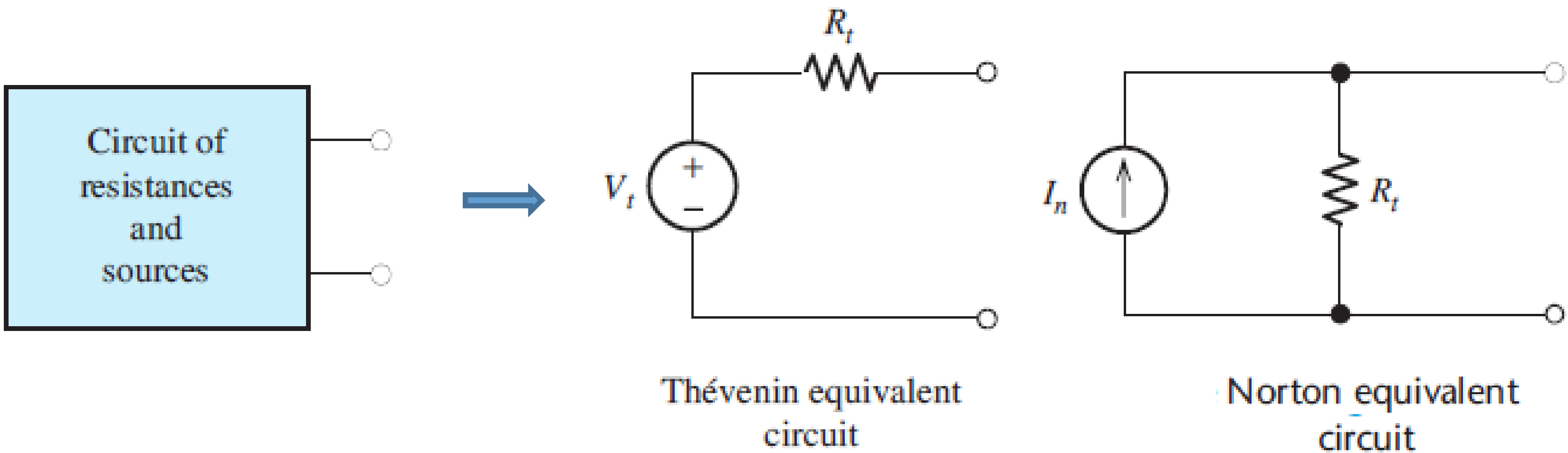
$$v_x = 2i_2$$



$$\Rightarrow \frac{i_2}{2} = i_2 - i_1 \Rightarrow \begin{cases} 4i_1 + 8i_2 = 20 \\ i_1 - \frac{i_2}{2} = 0 \end{cases} \Rightarrow \begin{cases} i_1 = 1\text{ A} \\ i_2 = 2\text{ A} \end{cases}$$

2.6 THÉVENIN AND NORTON EQUIVALENT CIRCUITS

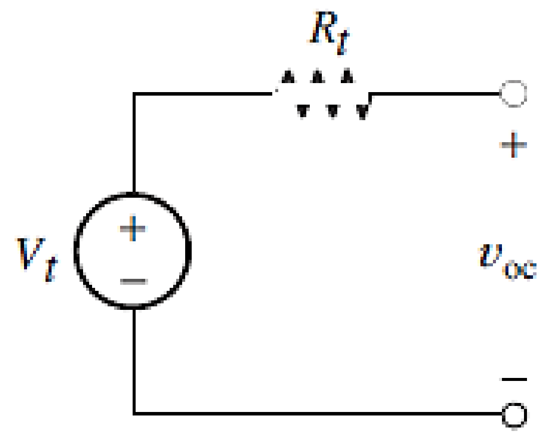
- Replace two-terminal circuits containing resistances and sources



2.6 THÉVENIN AND NORTON EQUIVALENT CIRCUITS

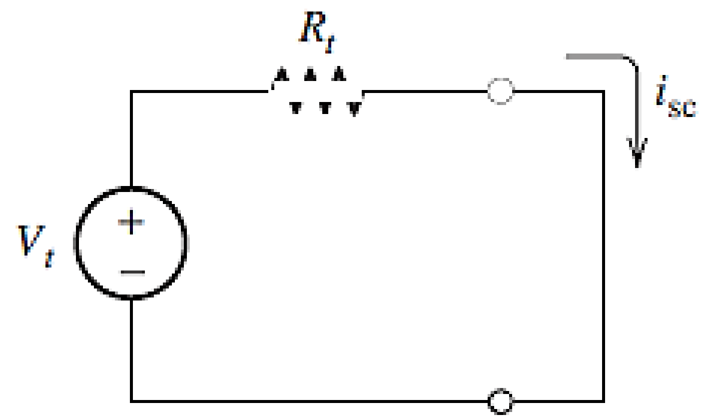
- Thévenin Equivalent Circuits
  - Open circuit

$$V_t = v_{oc}$$



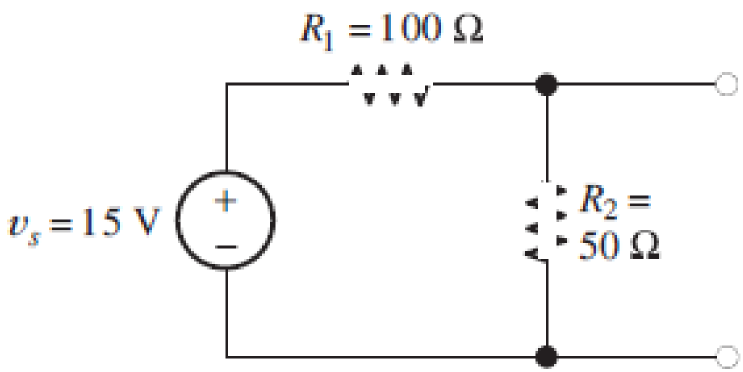
  - Short circuit

$$i_{sc} = \frac{V_t}{R_t} \Rightarrow R_t = \frac{V_t}{i_{sc}} \Rightarrow R_t = \frac{v_{oc}}{i_{sc}}$$

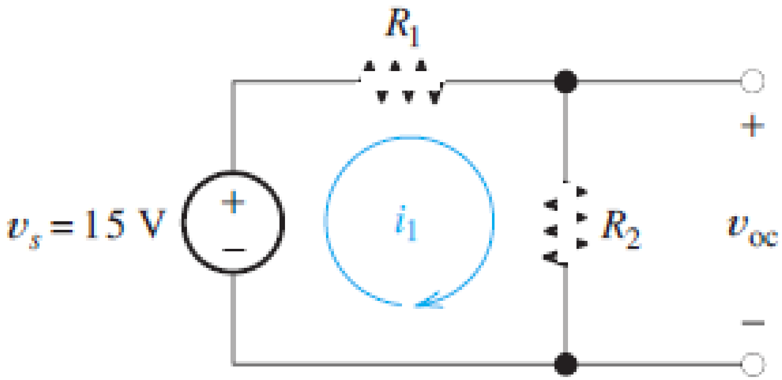


2.6 THÉVENIN AND NORTON EQUIVALENT CIRCUITS

Example 2.16 Determining the Thévenin Equivalent Circuit



(a) Original circuit



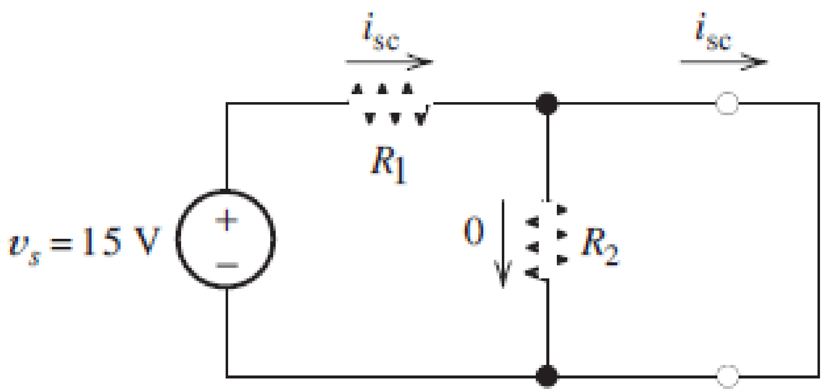
(b) Analysis with an open circuit

$$i_1 = \frac{v_s}{R_1 + R_2} = \frac{15}{100 + 50} = 0.10 \text{ A}$$

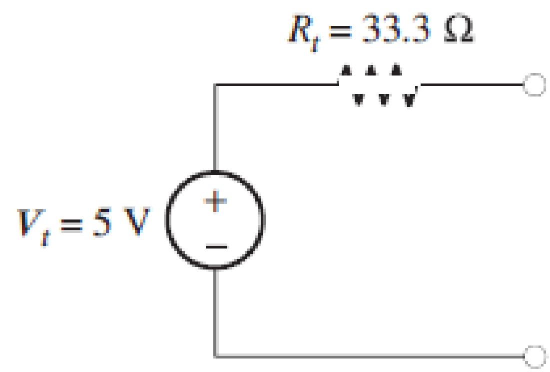
$$v_{oc} = R_2 i_1 = 50 \times 0.10 = 5 \text{ V}$$

2.6 THÉVENIN AND NORTON EQUIVALENT CIRCUITS

Example 2.16 Determining the Thévenin Equivalent Circuit



(c) Analysis with a short circuit



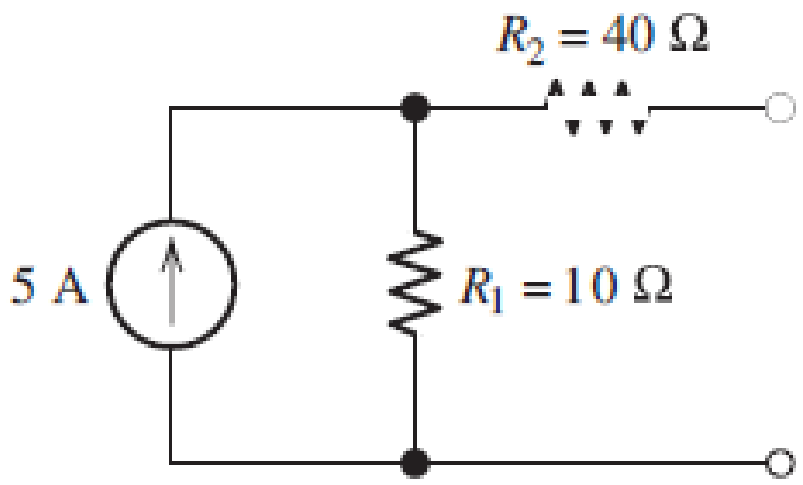
(d) Thévenin equivalent

$$i_{sc} = \frac{v_s}{R_1} = \frac{15}{100} = 0.15 \text{ A}$$

$$R_t = \frac{v_{oc}}{i_{sc}} = \frac{5 \text{ V}}{0.15 \text{ A}} = 33.3\ \Omega$$

2.6 THÉVENIN AND NORTON EQUIVALENT CIRCUITS

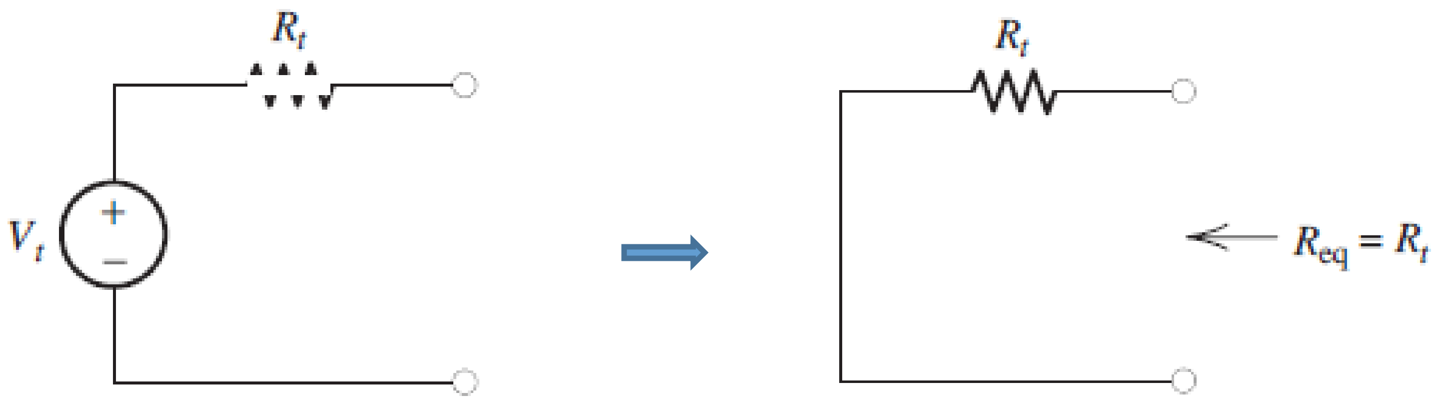
- Exercise



$$V_t = 50\ \text{V},\ R_t = 50\ \Omega.$$

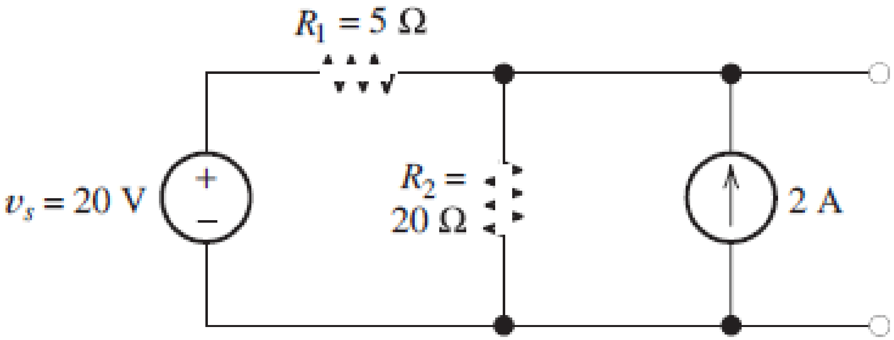
2.6 THÉVENIN AND NORTON EQUIVALENT CIRCUITS

- Finding the Thévenin Resistance Directly
  - If network contains no dependent sources
  - Zero the sources
    - ✓ Voltage source: Short circuit
    - ✓ Current source: Open circuit
  - Find the Req



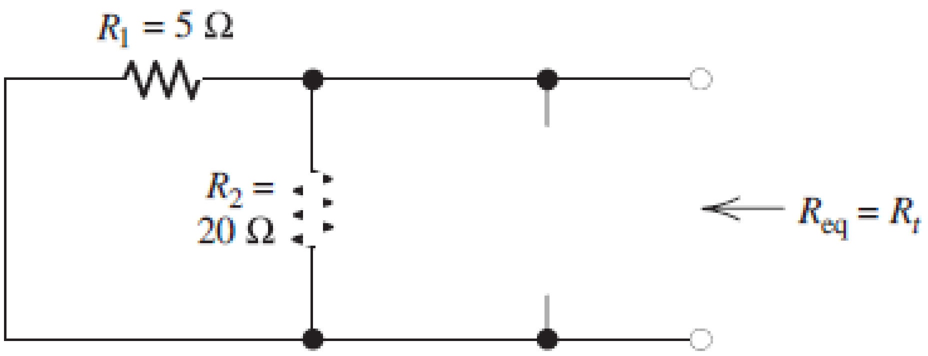
2.6 THÉVENIN AND NORTON EQUIVALENT CIRCUITS

Example 2.17 Zeroing Sources to Find Thévenin Resistance



(a) Original circuit

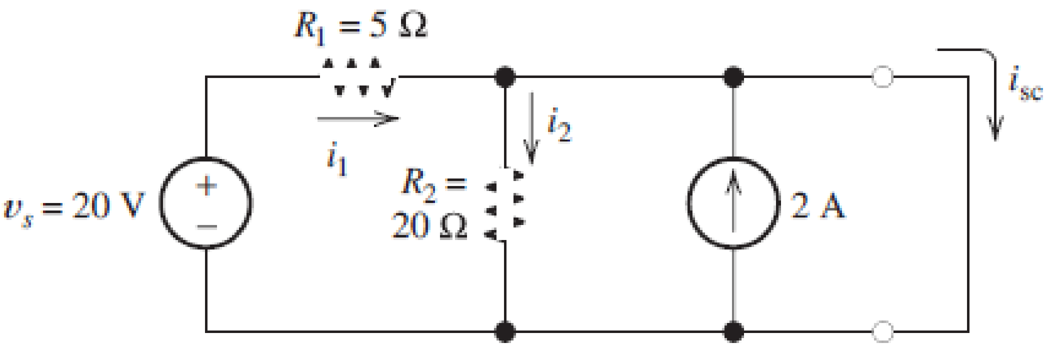
$$R_t = R_{eq} = \frac{1}{1/R_1 + 1/R_2} = \frac{1}{1/5 + 1/20} = 4\ \Omega$$



(b) Circuit with sources zeroed

2.6 THÉVENIN AND NORTON EQUIVALENT CIRCUITS

Example 2.17 Zeroing Sources to Find Thévenin Resistance



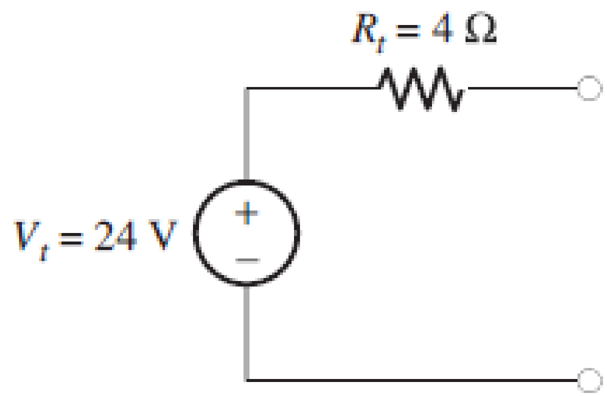
(c) Circuit with a short circuit

$$i_2 = 0$$

$$i_1 = \frac{v_s}{R_1} = \frac{20}{5} = 4\ \text{A}$$

$$i_1 + 2 = i_2 + i_{sc}$$

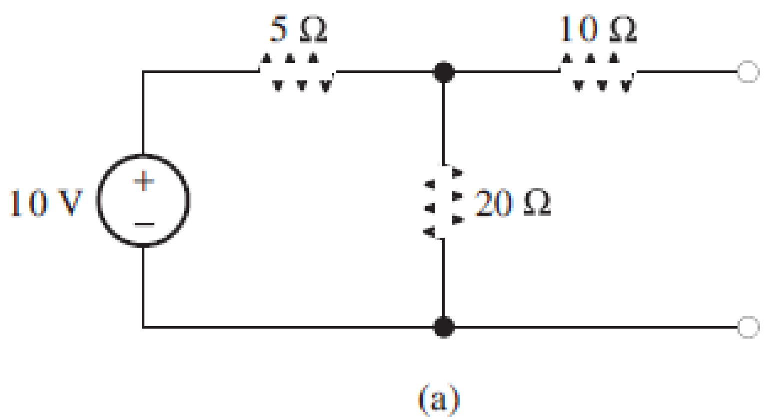
$$V_t = R_t i_{sc} = 4 \times 6 = 24\ \text{V}$$

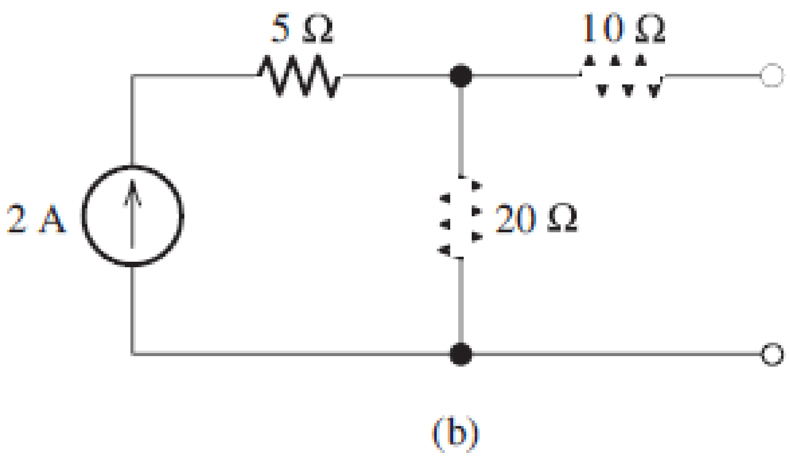


(d) Thévenin equivalent circuit

2.6 THÉVENIN AND NORTON EQUIVALENT CIRCUITS

- Exercise



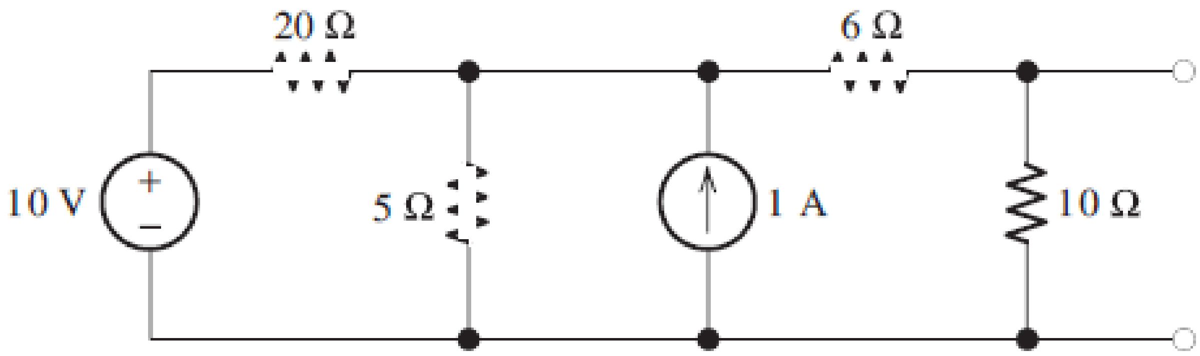


a. $R_t = 14\ \Omega$;

b. $R_t = 30\ \Omega$;

2.6 THÉVENIN AND NORTON EQUIVALENT CIRCUITS

- Exercise

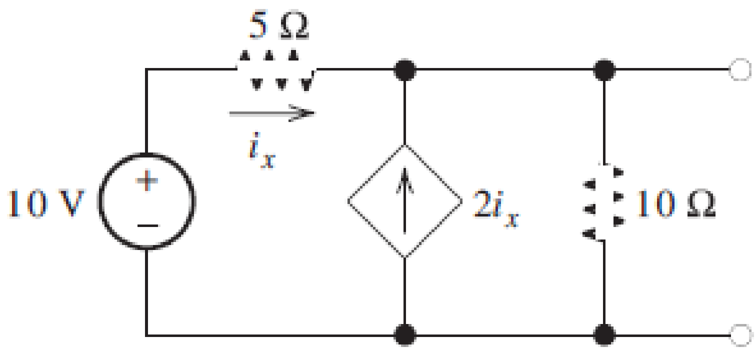


(c)

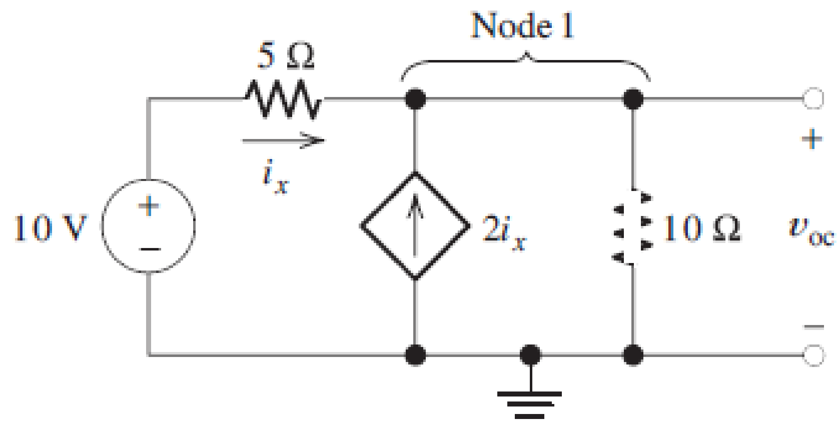
c. $R_t = 5\ \Omega$

2.6 THÉVENIN AND NORTON EQUIVALENT CIRCUITS

Example 2.18 Thévenin Equivalent of a Circuit with a Dependent Source



(a) Original circuit



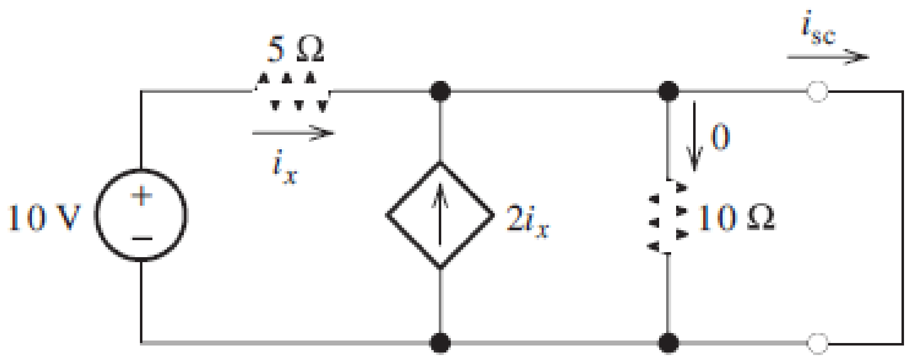
(b) Circuit with an open circuit

$$i_x + 2i_x = \frac{v_{oc}}{10} \qquad i_x = \frac{10 - v_{oc}}{5}$$

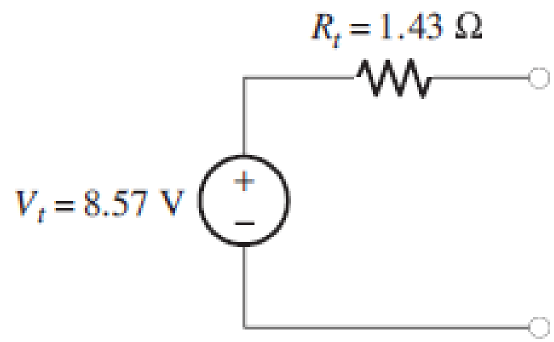
$$\Rightarrow 3\frac{10 - v_{oc}}{5} = \frac{v_{oc}}{10} \quad \Rightarrow v_{oc} = 8.57 \text{ V.}$$

2.6 THÉVENIN AND NORTON EQUIVALENT CIRCUITS

Example 2.18 Thévenin Equivalent of a Circuit with a Dependent Source



(c) Circuit with a short circuit



(d) Thévenin equivalent

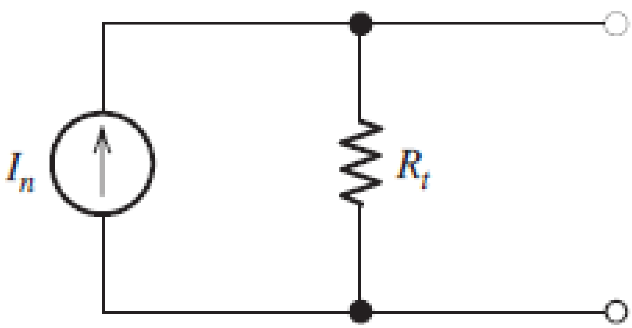
$$i_x = \frac{10\ \text{V}}{5\ \Omega} = 2\ \text{A}$$

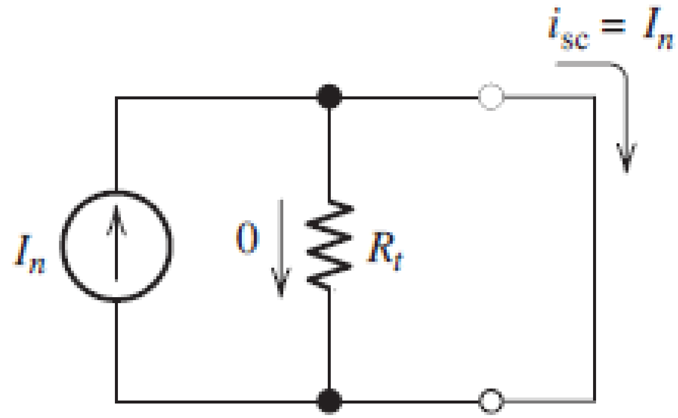
$$i_{sc} = 3i_x = 6\ \text{A}$$

$$\Rightarrow \quad R_t = \frac{v_{oc}}{i_{sc}} = \frac{8.57\ \text{V}}{6\ \text{A}} = 1.43\ \Omega$$

2.6 THÉVENIN AND NORTON EQUIVALENT CIRCUITS

- Norton Equivalent Circuit
  - Resistance in the Norton equivalent is the same as the Thévenin resistance





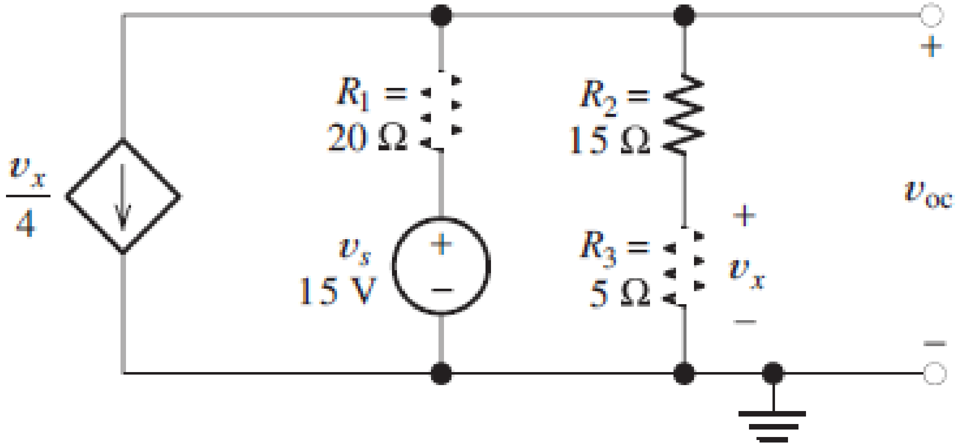
$$I_n = i_{sc}$$

2.6 THÉVENIN AND NORTON EQUIVALENT CIRCUITS

- Step-by-Step Thévenin/Norton-Equivalent-Circuit Analysis
  - 1. Perform two of these:
    - ✓ a. Determine the open-circuit voltage Vt = Voc.
    - ✓ b. Determine the short-circuit current In = Isc.
    - ✓ c. Zero the independent sources and find the Thévenin resistance Rt looking back into the terminals. Do not zero dependent sources.
  - 2. Use the equation Vt = Rt.In to compute the remaining value.
  - 3. The Thévenin equivalent consists of a voltage source Vt in series with Rt
  - 4. The Norton equivalent consists of a current source In in parallel with Rt

2.6 THÉVENIN AND NORTON EQUIVALENT CIRCUITS

Example 2.19 Norton Equivalent Circuit



(a) Original circuit under open-circuit conditions

$$\frac{v_x}{4} + \frac{v_{oc} - 15}{R_1} + \frac{v_{oc}}{R_2 + R_3} = 0$$

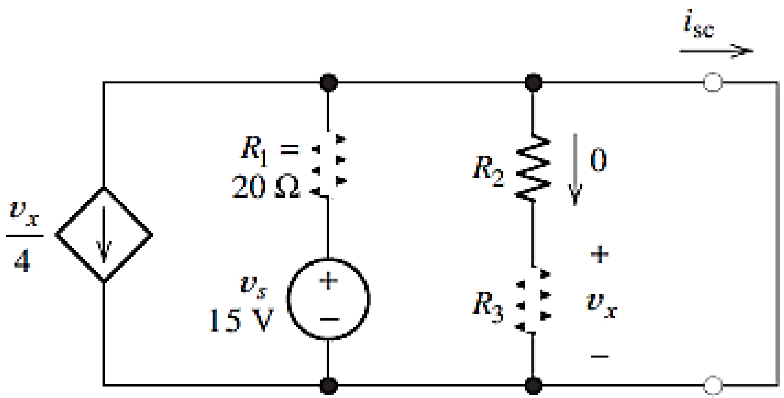
$$v_x = \frac{R_3}{R_2 + R_3} v_{oc} = 0.25 v_{oc}$$

$$\Rightarrow \frac{0.25 v_{oc}}{4} + \frac{v_{oc} - 15}{R_1} + \frac{v_{oc}}{R_2 + R_3} = 0$$

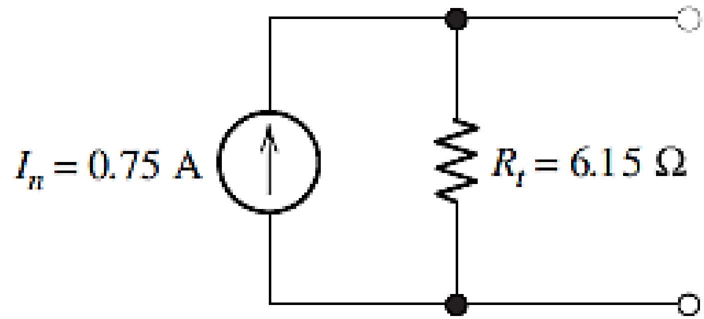
$$\Rightarrow v_{oc} = 4.62 \text{ V.}$$

2.6 THÉVENIN AND NORTON EQUIVALENT CIRCUITS

Example 2.19 Norton Equivalent Circuit



(b) Circuit with a short circuit



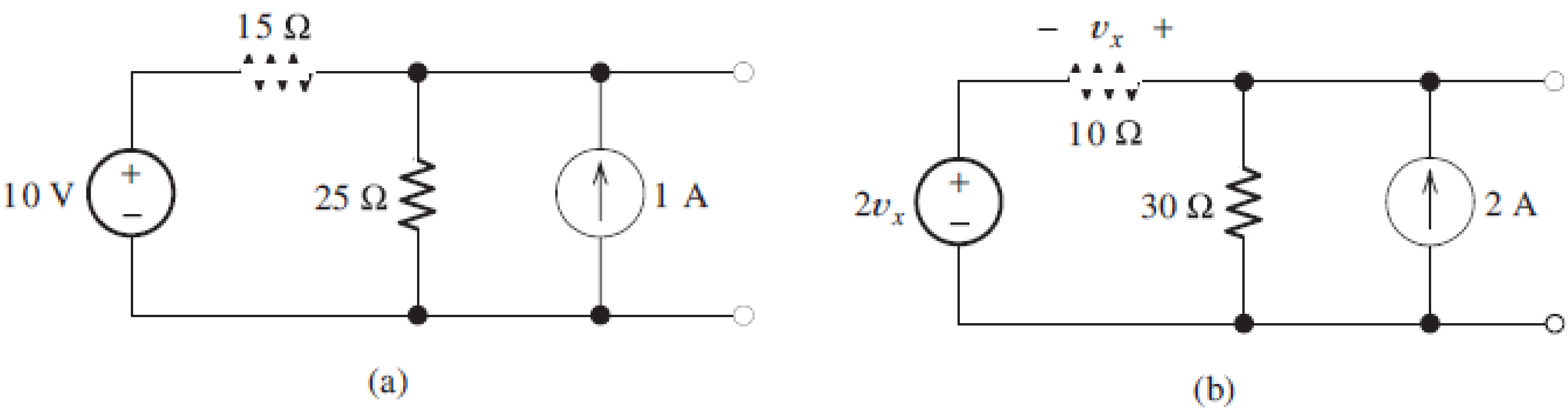
(c) Norton equivalent circuit

$$i_{sc} = \frac{v_s}{R_1} = \frac{15\text{ V}}{20\ \Omega} = 0.75\text{ A}$$

$$R_t = \frac{v_{oc}}{i_{sc}} = \frac{4.62}{0.75} = 6.15\ \Omega$$

2.6 THÉVENIN AND NORTON EQUIVALENT CIRCUITS

- Exercise

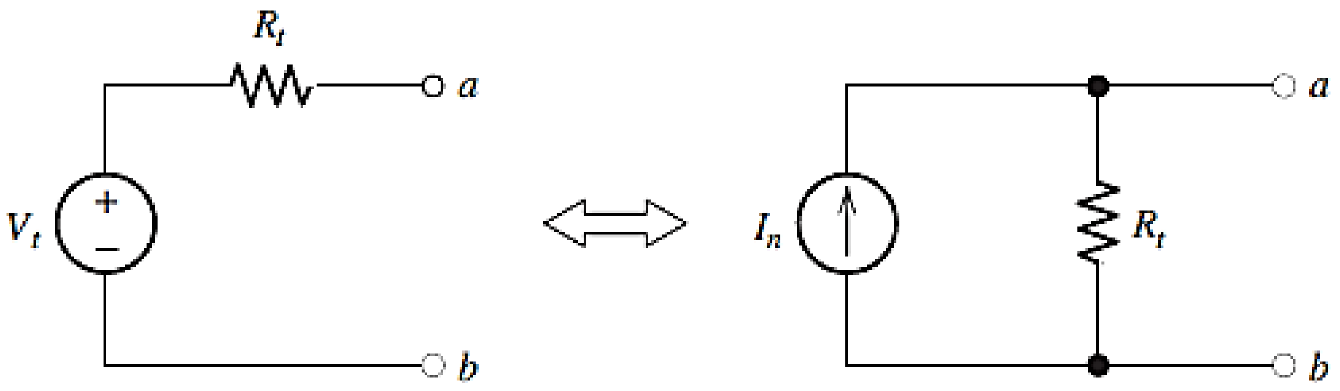


$I_n = 1.67$ A, $R_t = 9.375\ \Omega$

$I_n = 2$ A, $R_t = 15\ \Omega$

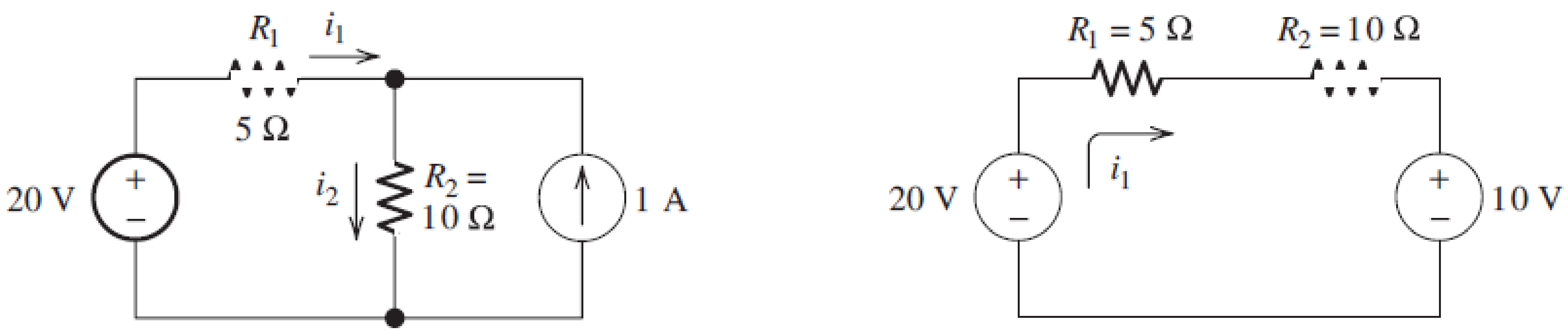
2.6 THÉVENIN AND NORTON EQUIVALENT CIRCUITS

- Source Transformations
  - Replace a voltage source in series with a resistance by a Norton equivalent circuit



2.6 THÉVENIN AND NORTON EQUIVALENT CIRCUITS

Example 2.20 Using Source Transformations



(a) Original circuit

(b) Circuit after transforming the current source into a voltage source

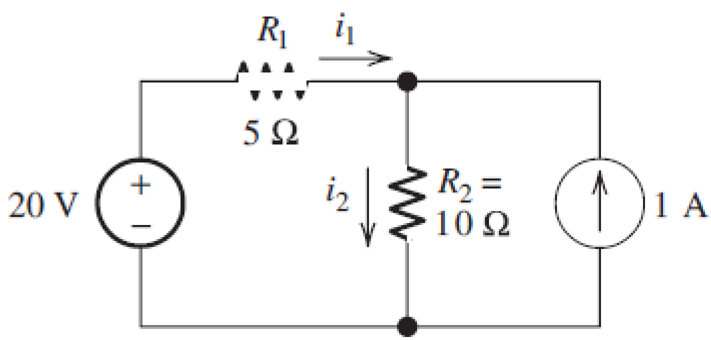
$$R_1 i_1 + R_2 i_1 + 10 - 20 = 0$$

$$i_1 = \frac{10}{R_1 + R_2} = 0.667 \text{ A}$$

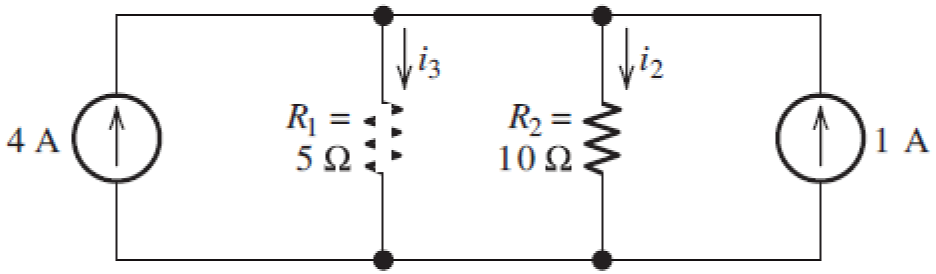
$$i_2 = i_1 + 1 = 1.667 \text{ A}$$

2.6 THÉVENIN AND NORTON EQUIVALENT CIRCUITS

Example 2.20 Using Source Transformations



(a) Original circuit

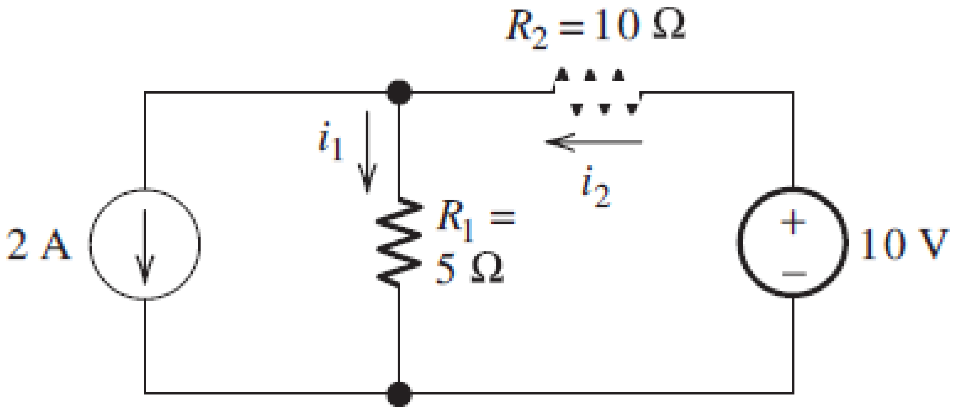


(c) Circuit after transforming the voltage source into a current source

$$i_2 = \frac{R_1}{R_1 + R_2} i_{\text{total}} = \frac{5}{5 + 10}(5) = 1.667 \text{ A}$$

2.6 THÉVENIN AND NORTON EQUIVALENT CIRCUITS

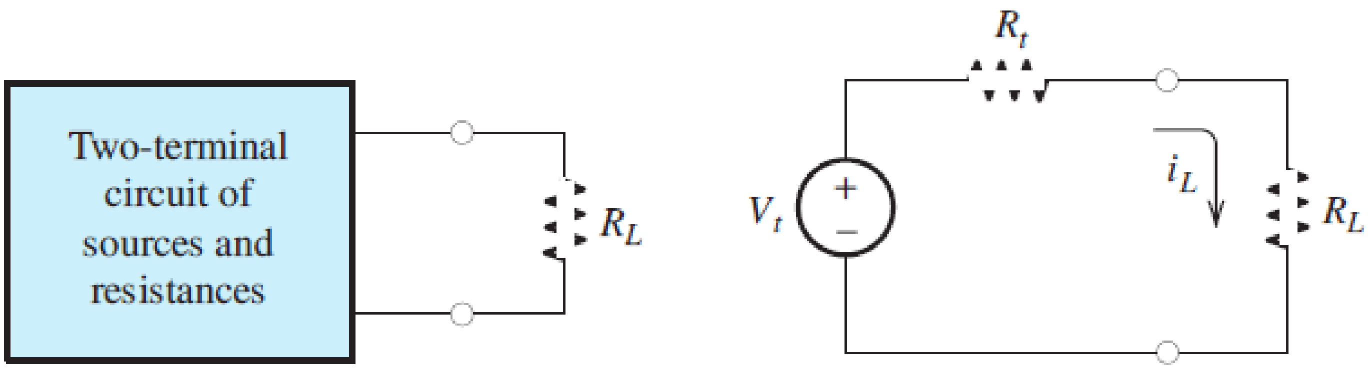
- Exercise



$$i_1 = -0.667\ \text{A},\ i_2 = 1.333\ \text{A}.$$

2.6 THÉVENIN AND NORTON EQUIVALENT CIRCUITS

- Maximum Power Transfer
  - When maximum possible power is delivered to the load?

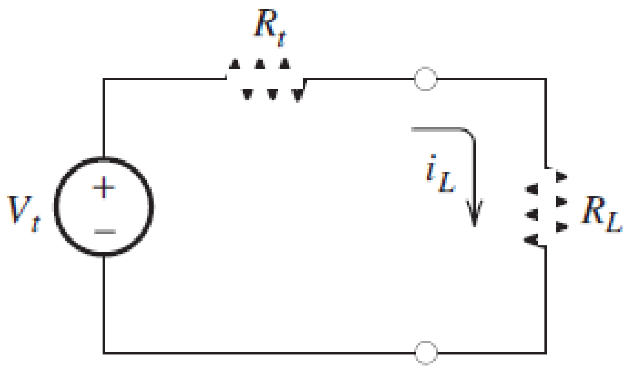


(a) Original circuit with load

(b) Thévenin equivalent circuit with load

2.6 THÉVENIN AND NORTON EQUIVALENT CIRCUITS

- Maximum Power Transfer



$$i_L = \frac{V_t}{R_t + R_L}$$

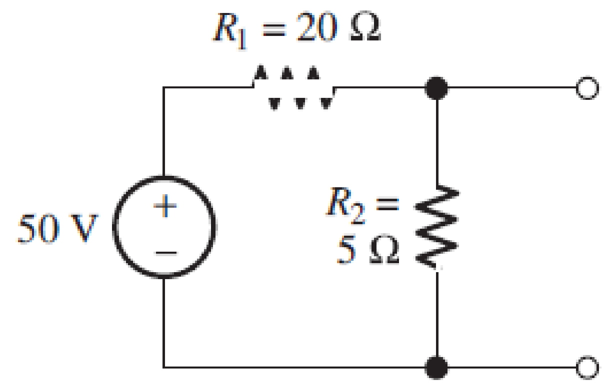
$$p_L = i_L^2 R_L = \frac{V_t^2 R_L}{(R_t + R_L)^2}$$

$$\frac{dp_L}{dR_L} = \frac{V_t^2 (R_t + R_L)^2 - 2V_t^2 R_L (R_t + R_L)}{(R_t + R_L)^4} = 0$$

$$R_L = R_t \qquad P_{L\,\max} = \frac{V_t^2}{4R_t}$$

2.6 THÉVENIN AND NORTON EQUIVALENT CIRCUITS

Example 2.21 Determining Maximum Power Transfer



$$R_t = \frac{1}{1/R_1 + 1/R_2} = \frac{1}{1/20 + 1/5} = 4\ \Omega$$

$$V_t = v_{oc} = \frac{R_2}{R_1 + R_2}(50) = \frac{5}{5+20}(50) = 10\ \text{V}$$

$$R_L = R_t = 4\ \Omega$$

$$P_{L\max} = \frac{V_t^2}{4R_t} = \frac{10^2}{4 \times 4} = 6.25\ \text{W}$$

2.7 SUPERPOSITION PRINCIPLE

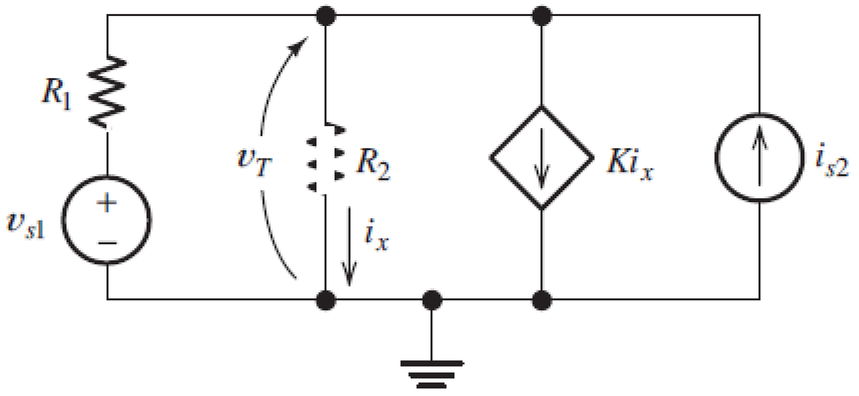
- For a circuit composed of:
  - Resistances
  - Linear dependent sources
  - n independent sources

- Total response:

$$r_T = r_1 + r_2 + \cdots + r_n$$

2.7 SUPERPOSITION PRINCIPLE

- Example:



$$\frac{v_T - v_{s1}}{R_1} + \frac{v_T}{R_2} + Ki_x = i_{s2}$$

$$i_x = \frac{v_T}{R_2}$$

$$\Rightarrow \quad v_T = \frac{R_2}{R_1 + R_2 + KR_1} v_{s1} + \frac{R_1 R_2}{R_1 + R_2 + KR_1} i_{s2}$$

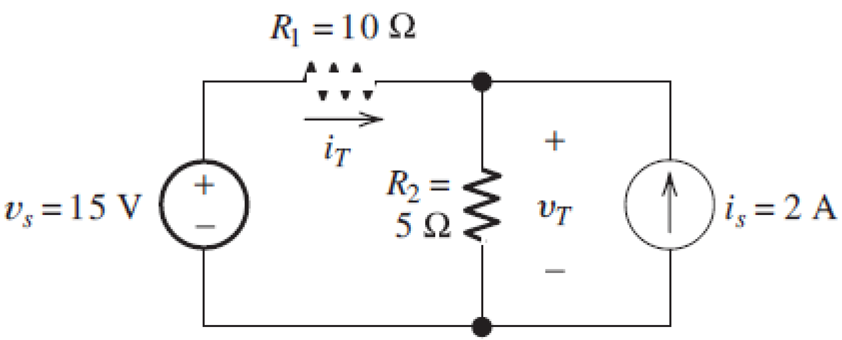
$$v_1 = \frac{R_2}{R_1 + R_2 + KR_1} v_{s1}$$

$$v_2 = \frac{R_1 R_2}{R_1 + R_2 + KR_1} i_{s2}$$

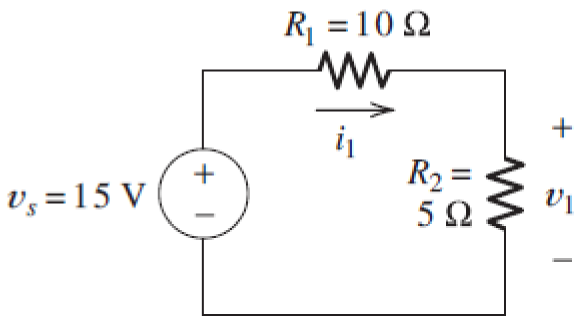
$$\Rightarrow \quad v_T = v_1 + v_2$$

2.7 SUPERPOSITION PRINCIPLE

Example 2.22 Circuit Analysis Using Superposition



(a) Original circuit

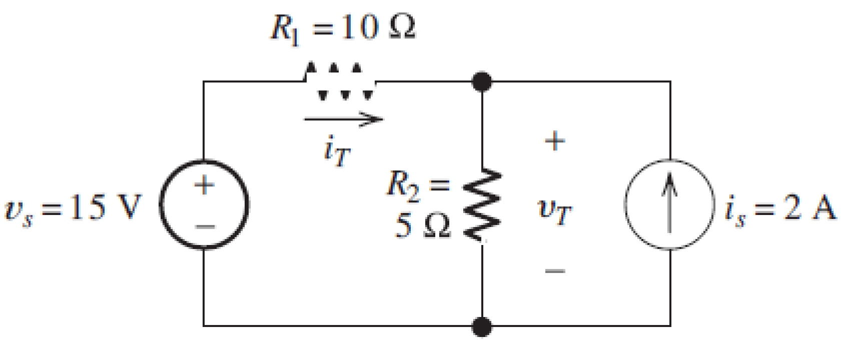


(b) Circuit with only the voltage source active

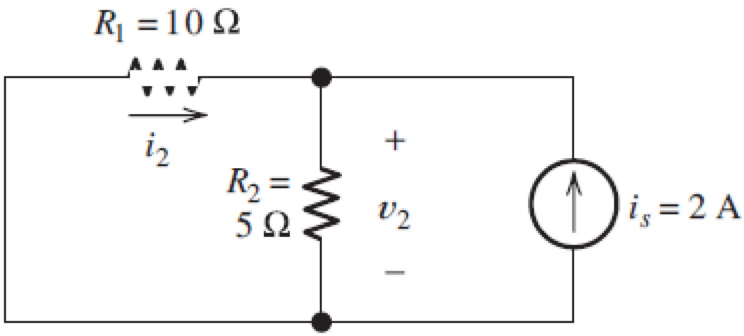
$$v_1 = \frac{R_2}{R_1 + R_2} v_s = \frac{5}{5 + 10}(15) = 5 \text{ V}$$

2.7 SUPERPOSITION PRINCIPLE

Example 2.22 Circuit Analysis Using Superposition



(a) Original circuit



(c) Circuit with only the current source active

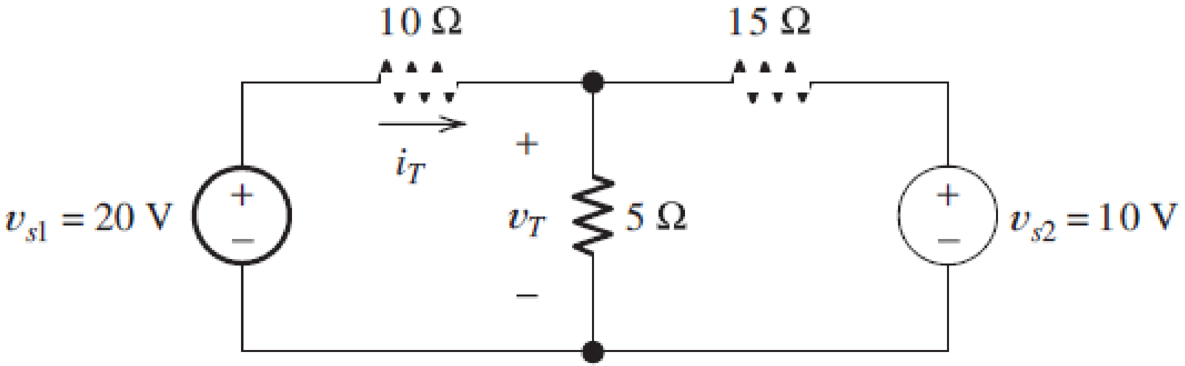
$$R_{eq} = \frac{1}{1/R_1 + 1/R_2} = \frac{1}{1/10 + 1/5} = 3.33\ \Omega$$

$$v_2 = i_s R_{eq} = 2 \times 3.33 = 6.66 \text{ V}$$

$$\Rightarrow \quad v_T = v_1 + v_2 = 5 + 6.66 = 11.66$$

2.7 SUPERPOSITION PRINCIPLE

- Exercise

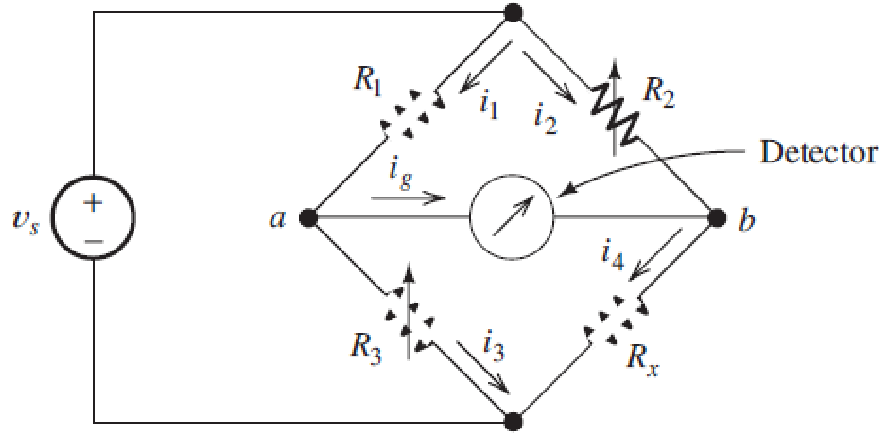


$$v_1 = 5.45 \text{ V},\ v_2 = 1.82 \text{ V},\ v_T = 7.27 \text{ V},$$

$$i_1 = 1.45 \text{ A},\ i_2 = -0.181 \text{ A},\ i_T = 1.27 \text{ A}.$$

2.8 WHEATSTONE BRIDGE

- A circuit used to measure unknown resistances



- When detector indicates zero current: bridge is **balanced**

$$\frac{R_3}{R_1} = \frac{R_x}{R_2}$$

Exercises

- P2.1
- P2.3
- P2.24
- P2.25
- P2.34
- P2.35
- P2.56
- P2.57
- P2.65
- P2.66
- P2.80
- P2.91
- P2.94
- P2.103
- T2.4
- T2.5
- T2.6