دانشگاه سمنان

Semnan University
Faculty of Mechanical Engineering

دانشکده مهندسی مکانیک

دانشکده مهندسی مکانیک

## درس مبانی برق ۱

نیمسال اول ۹۸-۹۹

# Electrical Engineering

## Principles and Applications

Allan R. Hambley

5th Edition

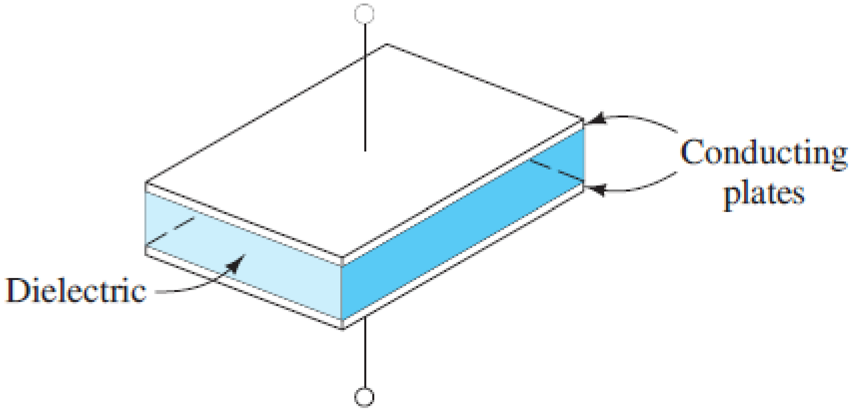
## ❑ *CONTENTS:*

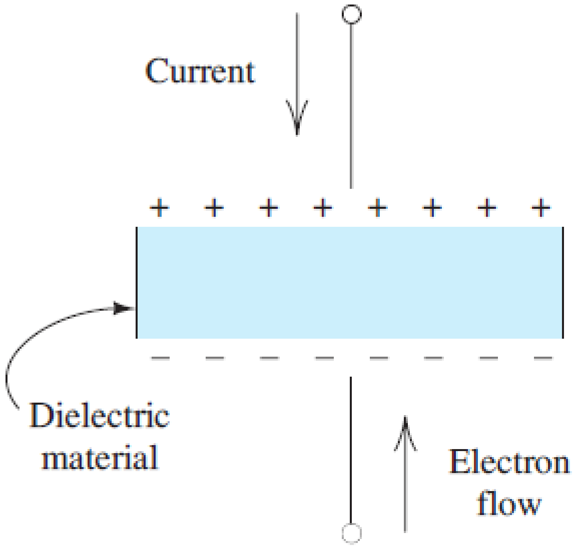
# INTRODUCTION

- Find the I (V) for a capacitance or inductance given the V (I)
- Compute the capacitances of parallel-plate capacitors.
- Compute the energies stored in capacitances or inductances
- Describe typical physical construction of capacitors and inductors
- Mutually coupled inductances

# 3.1 CAPACITANCE

- Separating two sheets of conductor by a thin layer of insulating material



# 3.1 CAPACITANCE

- Stored Charge in Terms of Voltage

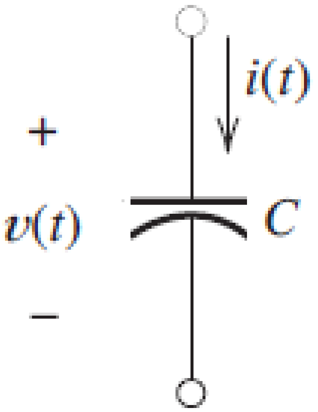
$$q = Cv$$

  - Unit: Farads (F)
  - In most applications: a few picofarads (1 pF = $10^{-12}$ F) up to 0.01 F

- Current in Terms of Voltage

$$i = \frac{dq}{dt} = \frac{d}{dt}(Cv)$$

$$i = C\frac{dv}{dt}$$



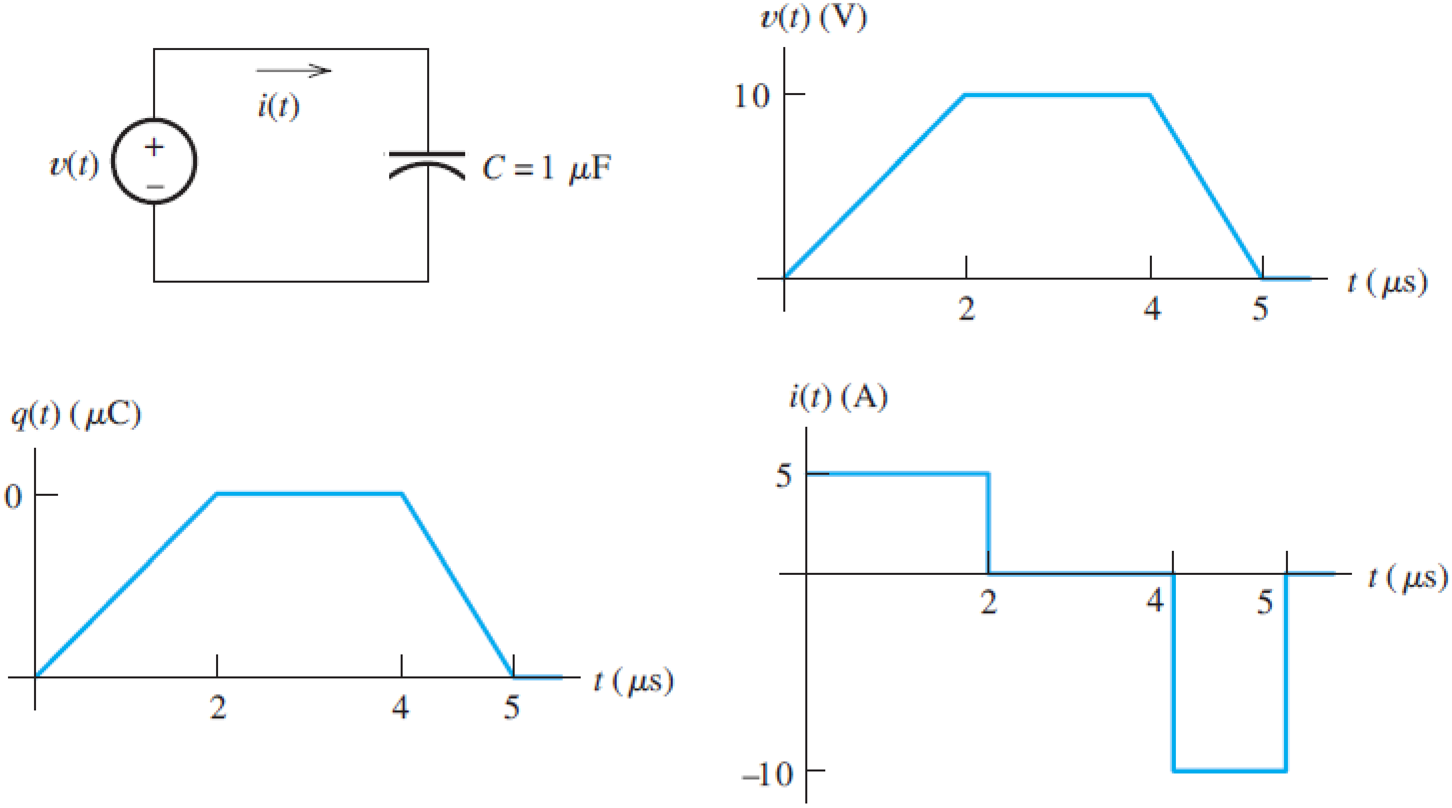

- Voltage in Terms of Current

$$q(t) = \int_{t_0}^{t} i(t)\,dt + q(t_0)$$

$$v(t) = \frac{1}{C}\int_{t_0}^{t} i(t)\,dt + v(t_0)$$

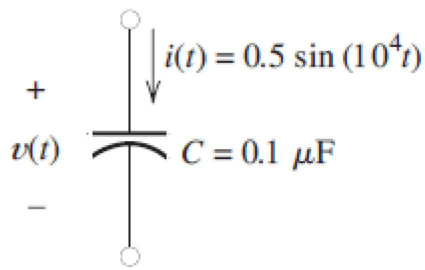
# 3.1 CAPACITANCE

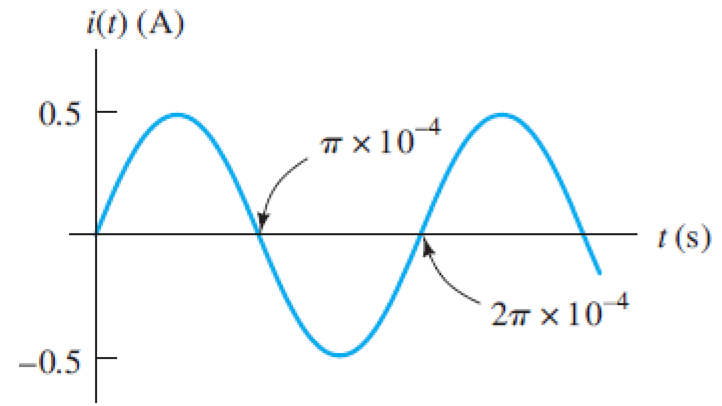
## Example 3.1 Determining Current for a Capacitance Given Voltage



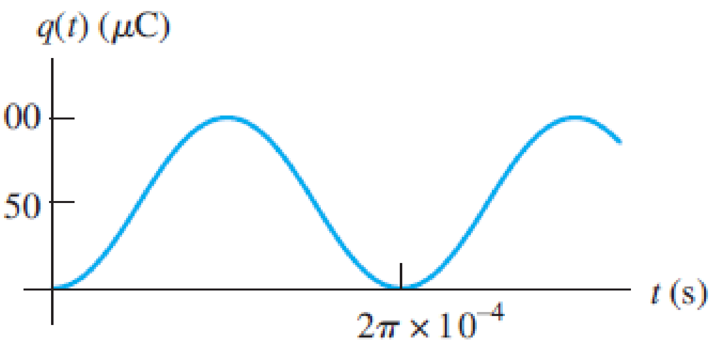
# 3.1 CAPACITANCE

## Example 3.2 Determining Voltage for a Capacitance Given Current



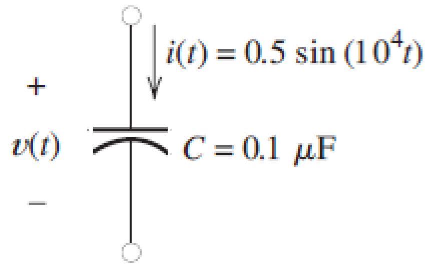




$$
\begin{aligned}
q(t) &= \int_0^t i(t)\,dt + q(0) \\
&= \int_0^t 0.5\sin(10^4 t)\,dt \\
&= -0.5\times 10^{-4}\cos(10^4 t)\Big|_0^t \\
&= 0.5\times 10^{-4}[1-\cos(10^4 t)]
\end{aligned}
$$



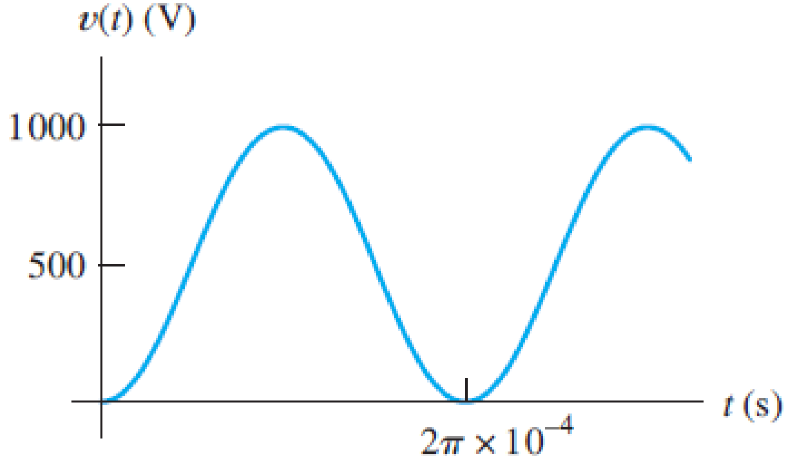
# 3.1 CAPACITANCE

## Example 3.2 Determining Voltage for a Capacitance Given Current




$$v(t) = \frac{q(t)}{C} = \frac{q(t)}{10^{-7}}$$

$$= 500[1 - \cos(10^4 t)]$$



# 3.1 CAPACITANCE

- Stored Energy

$$p(t) = v(t)i(t) = Cv\frac{dv}{dt}$$

$$w(t) = \int_{t_0}^{t} p(t)\,dt = \int_{t_0}^{t} Cv\frac{dv}{dt}\,dt = \int_{0}^{v(t)} Cv\,dv$$

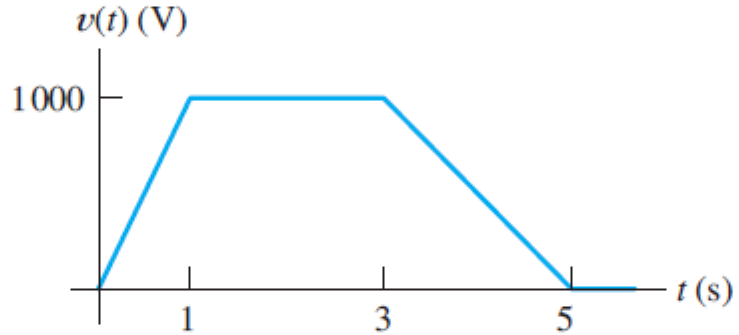
$$\Rightarrow \quad w(t) = \frac{1}{2}Cv^2(t) \qquad w(t) = \frac{1}{2}v(t)q(t) \qquad w(t) = \frac{q^2(t)}{2C}$$

# 3.1 CAPACITANCE

## Example 3.3 Current, Power, and Energy for a Capacitance

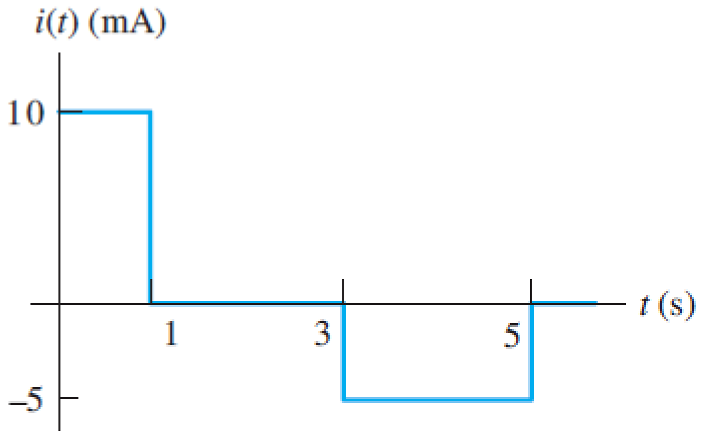
10-$\mu$F capacitance

$$v(t) = \begin{cases} 1000t \text{ V} & \text{for } 0 < t < 1 \\ 1000 \text{ V} & \text{for } 1 < t < 3 \\ 500(5 - t) \text{ V} & \text{for } 3 < t < 5 \end{cases}$$




$$i(t) = C\frac{dv(t)}{dt}$$

$$i(t) = \begin{cases} 10 \times 10^{-3} \text{ A} & \text{for } 0 < t < 1 \\ 0 \text{ A} & \text{for } 1 < t < 3 \\ -5 \times 10^{-3} \text{ A} & \text{for } 3 < t < 5 \end{cases}$$

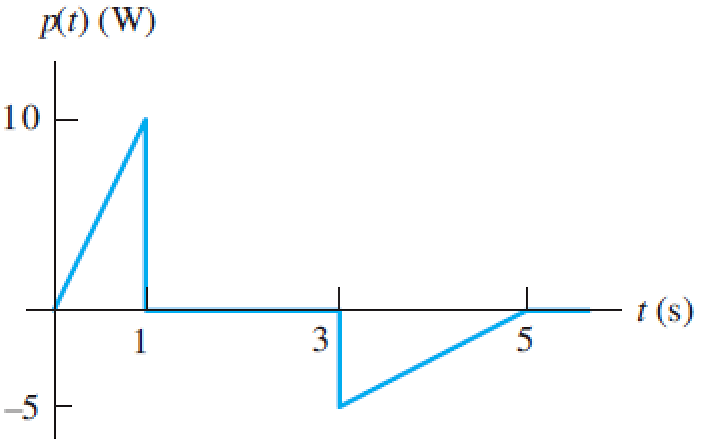


# 3.1 CAPACITANCE

## Example 3.3 Current, Power, and Energy for a Capacitance

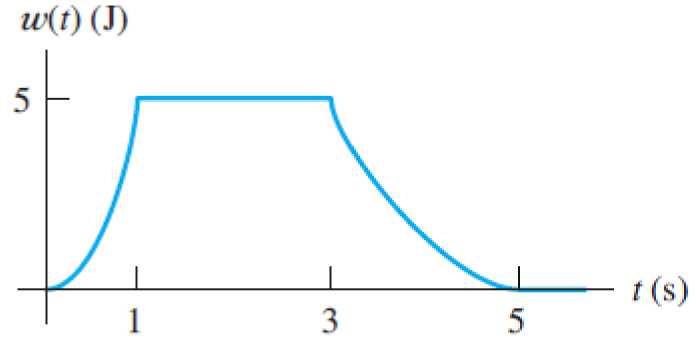
$$p(t) = v(t)i(t)$$

$$p(t) = \begin{cases} 10t \text{ W} & \text{for } 0 < t < 1 \\ 0 \text{ W} & \text{for } 1 < t < 3 \\ 2.5(t-5) \text{ W} & \text{for } 3 < t < 5 \end{cases}$$




$$w(t) = \frac{1}{2}Cv^2(t)$$

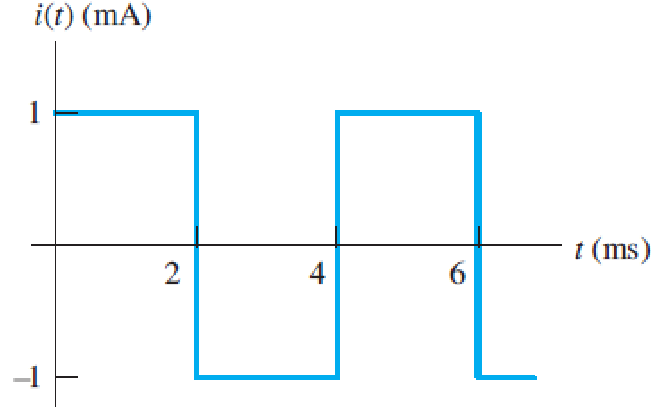
$$w(t) = \begin{cases} 5t^2 \text{ J} & \text{for } 0 < t < 1 \\ 5 \text{ J} & \text{for } 1 < t < 3 \\ 1.25(5-t)^2 \text{ J} & \text{for } 3 < t < 5 \end{cases}$$

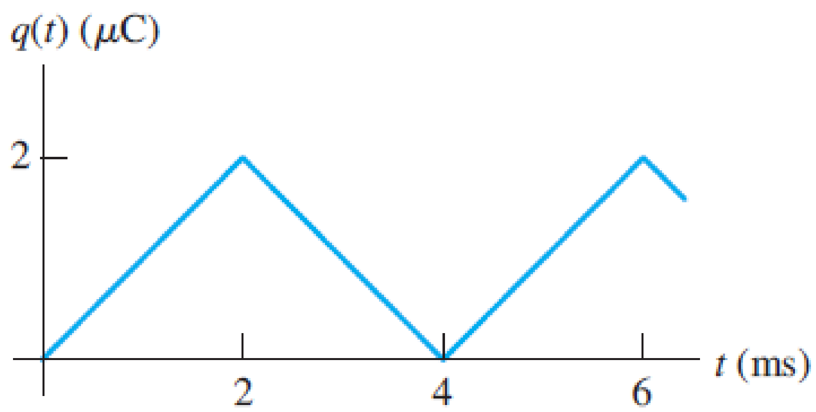


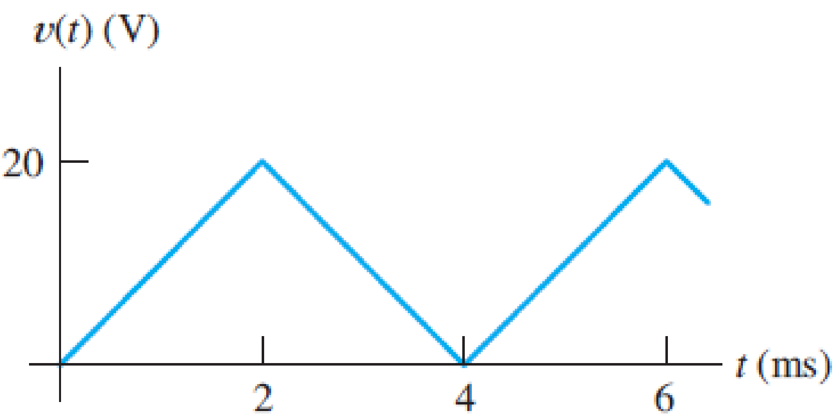
# 3.1 CAPACITANCE

- Exercise

0.1-$\mu$F capacitor



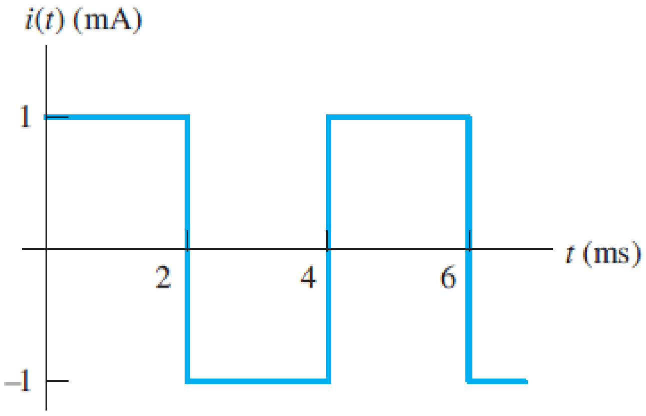



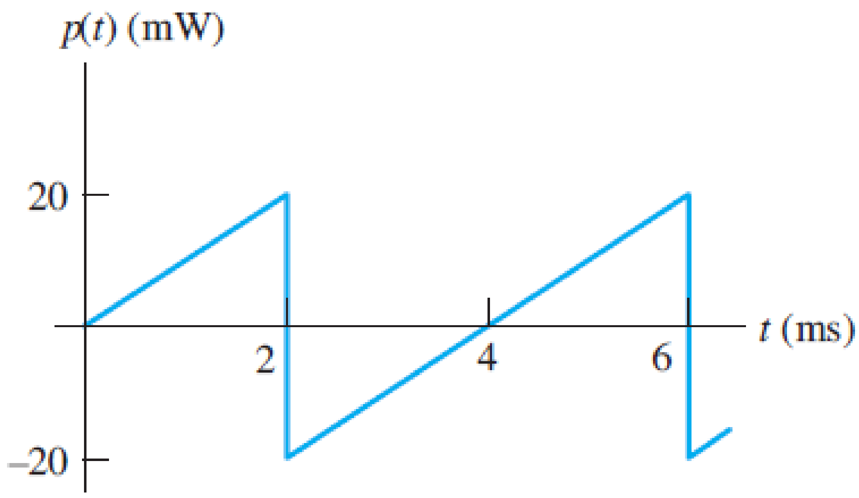



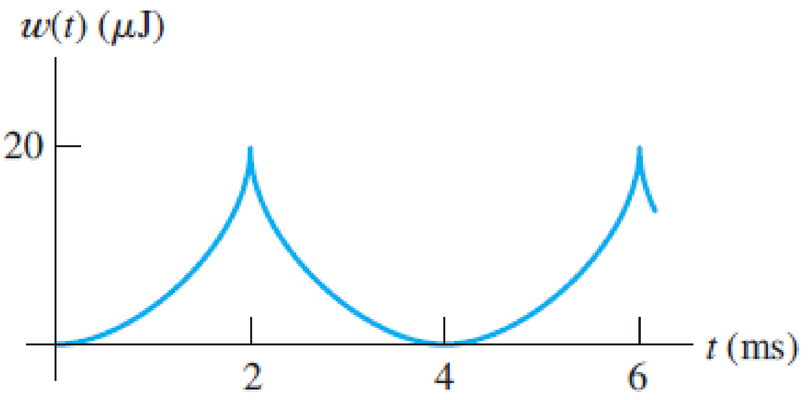
# 3.1 CAPACITANCE

- Exercise

0.1-$\mu$F capacitor



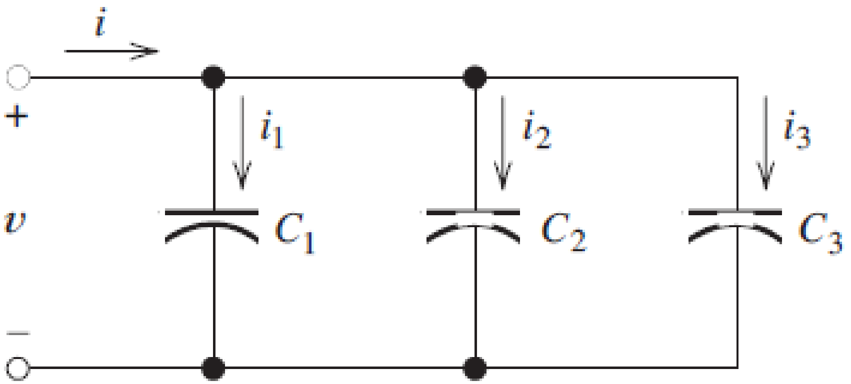
# 3.2 CAPACITANCES IN SERIES AND PARALLEL

- Capacitances in Parallel

$$i_1 = C_1 \frac{dv}{dt}$$

$$i_2 = C_2 \frac{dv}{dt}$$

$$i_3 = C_3 \frac{dv}{dt}$$



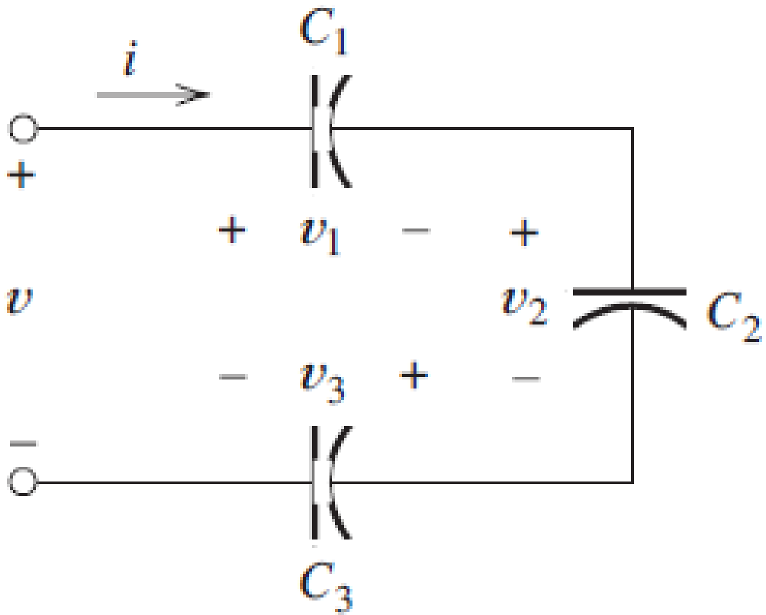

$$i = i_1 + i_2 + i_3 = C_1 \frac{dv}{dt} + C_2 \frac{dv}{dt} + C_3 \frac{dv}{dt} = (C_1 + C_2 + C_3)\frac{dv}{dt}$$

$$\Rightarrow \quad C_{eq} = C_1 + C_2 + C_3 \qquad i = C_{eq}\frac{dv}{dt}$$

# 3.2 CAPACITANCES IN SERIES AND PARALLEL

- Capacitances in Series

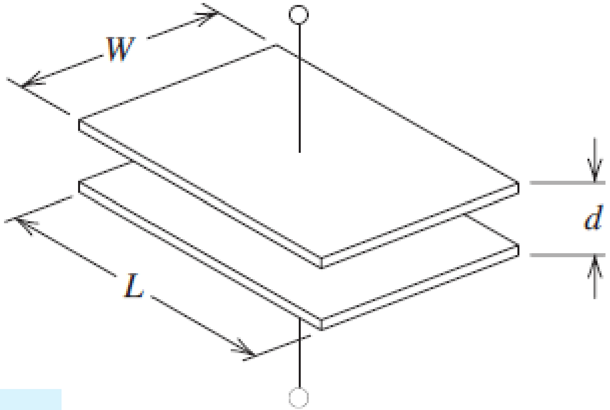
$$C_{eq} = \frac{1}{1/C_1 + 1/C_2 + 1/C_3}$$



# 3.3 PHYSICAL CHARACTERISTICS OF CAPACITORS

- Capacitance of the Parallel-Plate Capacitor

$$C = \frac{\epsilon A}{d}$$




$\epsilon$ is the **dielectric constant**

For vacuum $\epsilon = \epsilon_0 \cong 8.85 \times 10^{-12}$ F/m

For other materials $\epsilon = \epsilon_r \epsilon_0$

$\epsilon_r$ is the **relative dielectric constant**.

# 3.3 PHYSICAL CHARACTERISTICS OF CAPACITORS

- Relative dielectric constant

**Table 3.1.** Relative Dielectric Constants for Selected Materials

| | |
|---|---|
| Air | 1.0 |
| Diamond | 5.5 |
| Mica | 7.0 |
| Polyester | 3.4 |
| Quartz | 4.3 |
| Silicon dioxide | 3.9 |
| Water | 78.5 |

# 3.3 PHYSICAL CHARACTERISTICS OF CAPACITORS

## Example 3.4 Calculating Capacitance Given Physical Parameters

rectangular plates 10 cm by 20 cm
distance of 0.1 mm
dielectric is air.

$$A = L \times W = (10 \times 10^{-2}) \times (20 \times 10^{-2}) = 0.02\ \text{m}^2$$

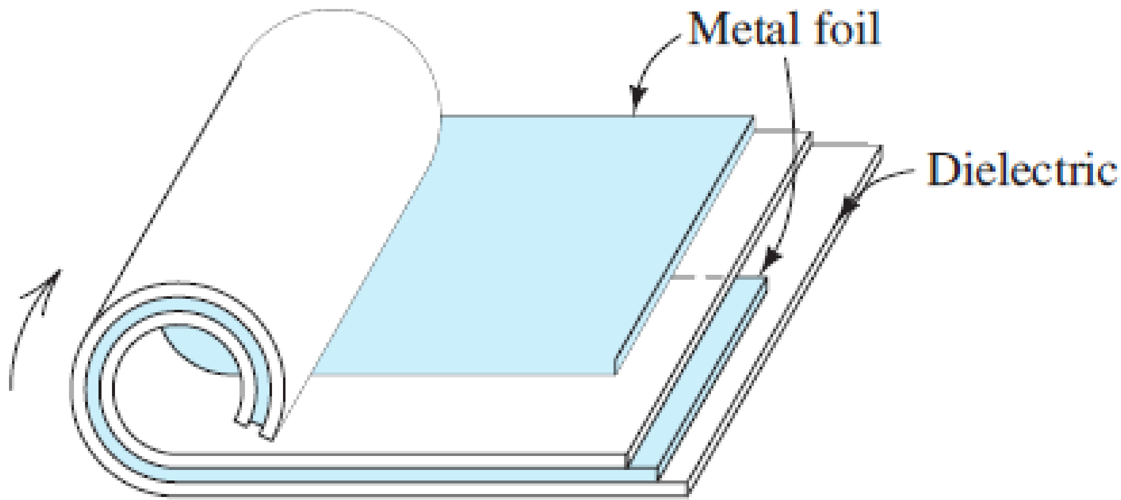
relative dielectric constant of air is 1.00

$$\epsilon = \epsilon_r \epsilon_0 = 1.00 \times 8.85 \times 10^{-12}\ \text{F/m}$$

$$C = \frac{\epsilon A}{d} = \frac{8.85 \times 10^{-12} \times 0.02}{10^{-4}} = 1770 \times 10^{-12}\ \text{F}$$

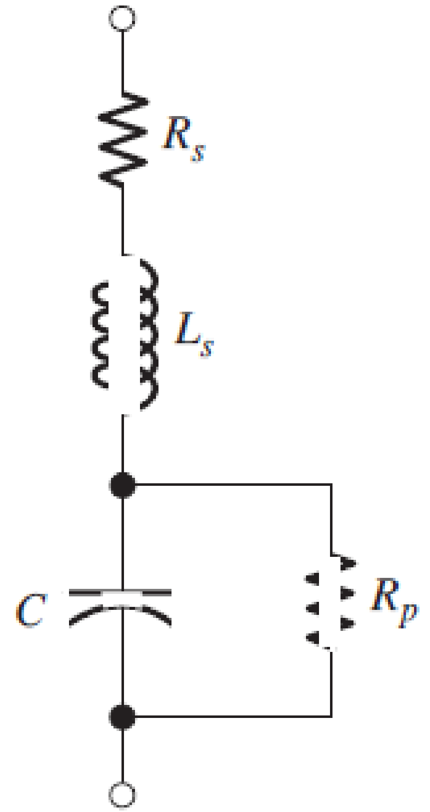
# 3.3 PHYSICAL CHARACTERISTICS OF CAPACITORS

- Practical Capacitors
  - The dimensions of parallel-plate capacitors are too large
  - Rolled to fit in a smaller area
  - Real capacitors have maximum voltage ratings



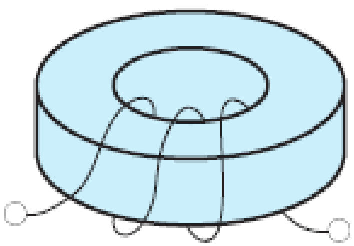
# 3.3 PHYSICAL CHARACTERISTICS OF CAPACITORS

- Parasitic Effects
  - Resistivity of the material composing the plates
  - Magnetic field in capacitor
  - No practical material is a perfect insulator

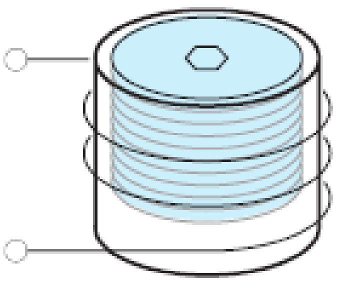


# 3.4 INDUCTANCE

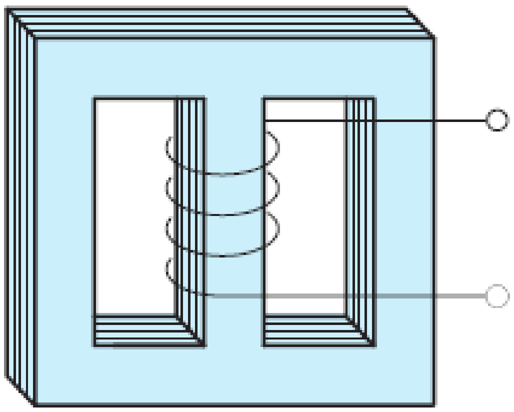
- Coiling a wire around some type of form
- Coil creates a magnetic field or flux
- Faraday’s law of electromagnetic induction



(a) Toroidal inductor



(b) Coil with an iron-oxide slug that can be screwed in or out to adjust the inductance

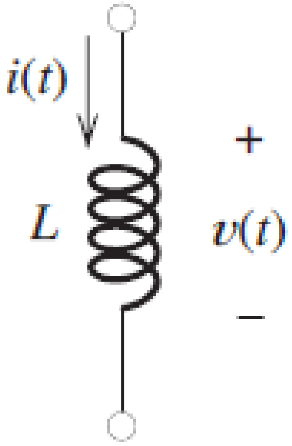


(c) Inductor with a laminated iron core

# 3.4 INDUCTANCE

- Voltage and current relation

$$v(t) = L\frac{di}{dt}$$




  - Unit: Henries (H) (volt seconds per ampere)
  - Typically: from µH to several tens of Henries

- Current in Terms of Voltage

$$di = \frac{1}{L}v(t)\,dt$$

$$i(t) = \frac{1}{L}\int_{t_0}^{t} v(t)\,dt + i(t_0)$$

# 3.4 INDUCTANCE

- Stored energy

$$p(t) = v(t)i(t) = Li(t)\frac{di}{dt}$$

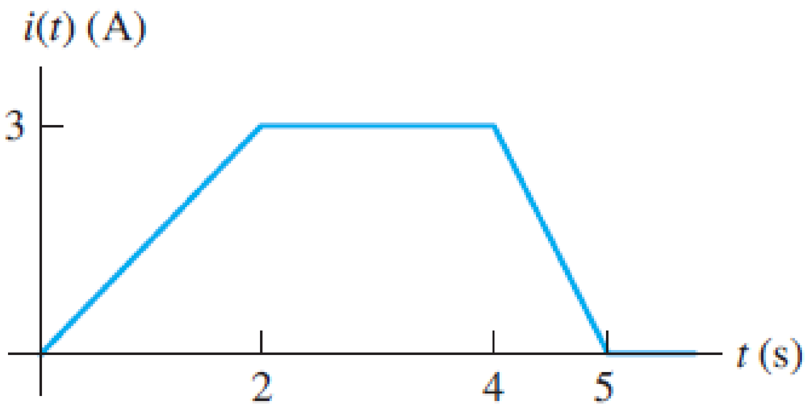
$$w(t) = \int_{t_0}^{t} p(t)\,dt = \int_{t_0}^{t} Li\frac{di}{dt}\,dt = \int_{0}^{i(t)} Li\,di$$

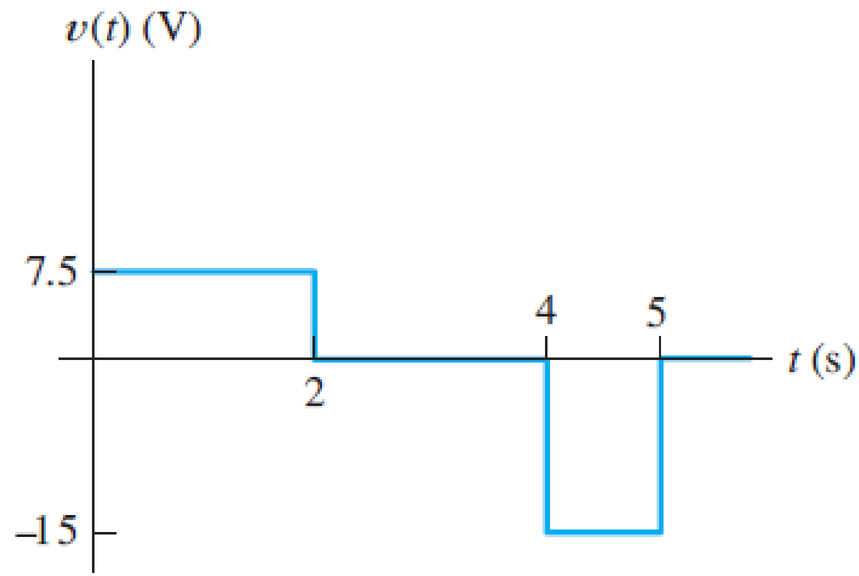
$$w(t) = \frac{1}{2}Li^2(t)$$

# 3.4 INDUCTANCE

## Example 3.6 Voltage, Power, and Energy for an Inductance

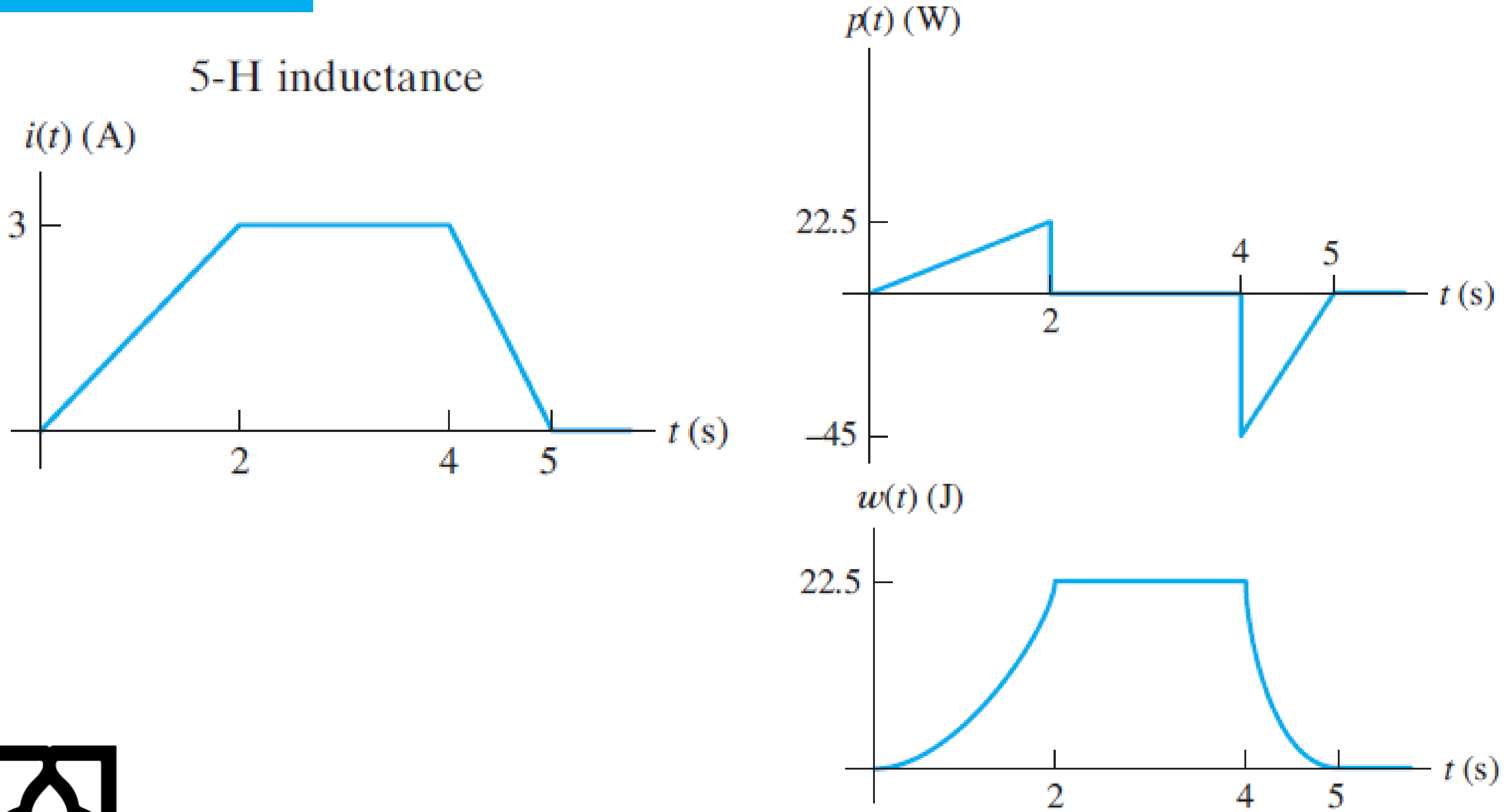
5-H inductance



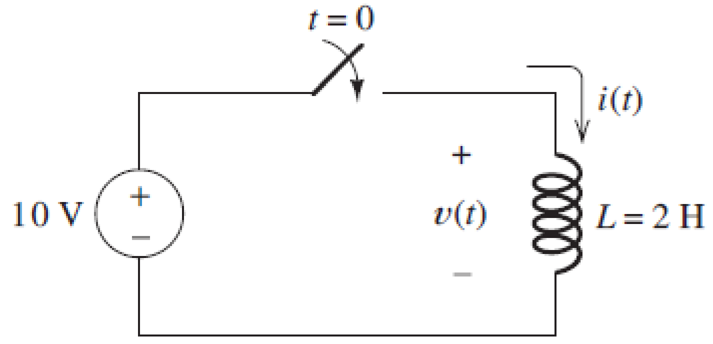
# 3.4 INDUCTANCE

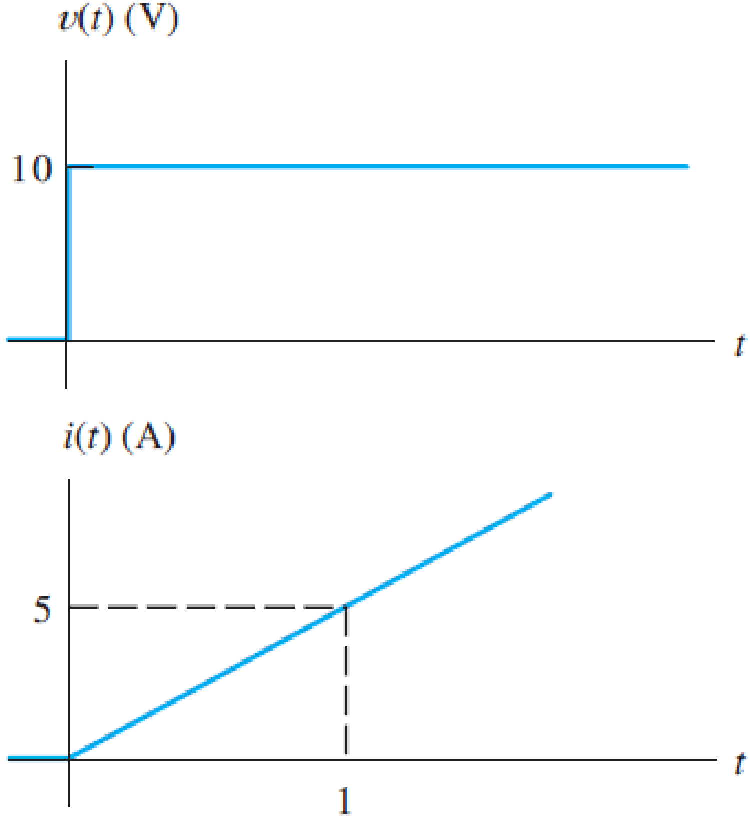
## Example 3.6 Voltage, Power, and Energy for an Inductance



# 3.4 INDUCTANCE

## Example 3.7 Inductor Current with Constant Applied Voltage




$$i(t) = \frac{1}{L}\int_{t_0}^{t} v(t)\,dt + i(t_0) = \frac{1}{2}\int_{0}^{t} 10\,dt$$

$$\Longrightarrow \quad i(t) = 5t \text{ A} \qquad \text{for } t > 0$$

- What happens if we open switch at t=1s??

# 3.4 INDUCTANCE

- Exercise

10-mH inductance

$i(t) = 0.1\cos(10^4 t)$ A.

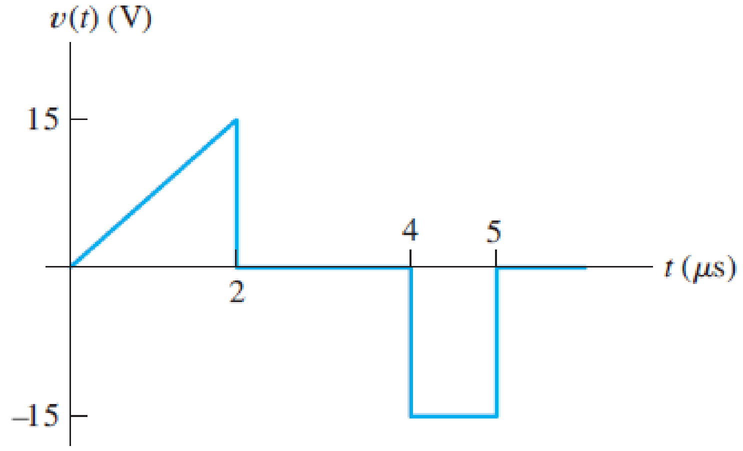
$$v(t) = L\frac{di(t)}{dt} = (10\times10^{-3})\frac{d}{dt}\left[0.1\cos\left(10^4 t\right)\right] = -10\sin\left(10^4 t\right) \quad \text{V}$$

$$w(t) = \tfrac{1}{2}Li^2(t) = 5\times10^{-3}\times\left[0.1\cos\left(10^4 t\right)\right]^2 = 50\times10^{-6}\cos^2\left(10^4 t\right) \quad \text{J}$$

# 3.4 INDUCTANCE

- Exercise

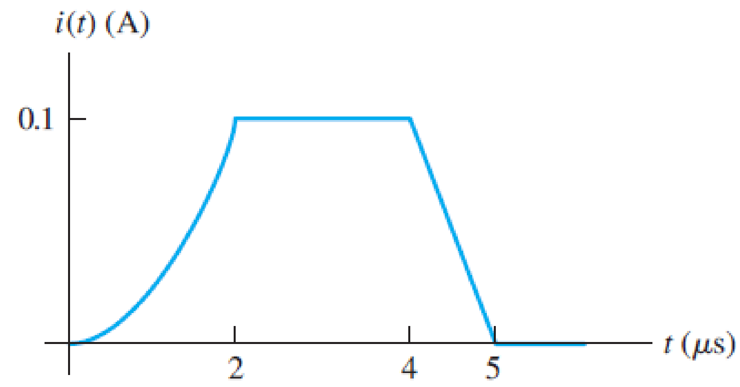
150-$\mu$H inductance




$$i(t)=\frac{1}{L}\int_0^t v(x)dx+i(0)=\frac{1}{150\times10^{-6}}\int_0^t v(x)dx$$

for $0\le t\le 2\ \mu s$

$$=6667\int_0^t 7.5\times10^6\,xdx=25\times10^9 t^2\ \text{V}$$




for $2\mu s\le t\le 4\ \mu s$

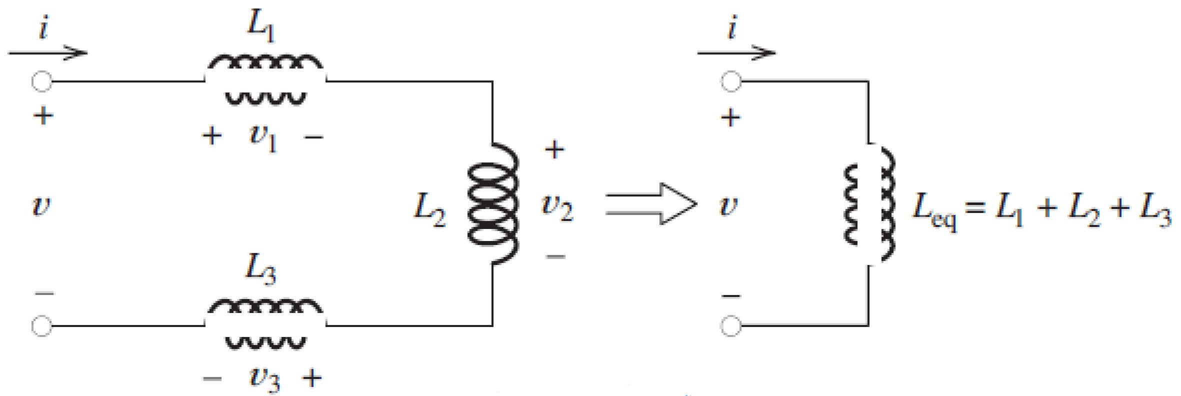
$$=6667\int_0^{2E-6} 7.5\times10^6\,xdx=0.1\,\text{V}$$

for $4\mu s\le t\le 5\ \mu s$

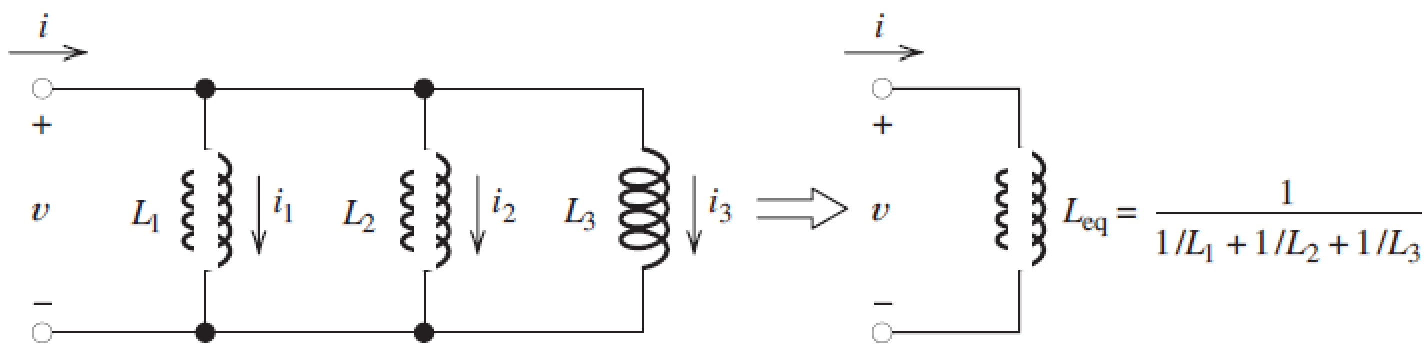
$$=6667\left(\int_0^{2E-6} 7.5\times10^6\,xdx+\int_{4E-6}^{t}(-15)dx\right)=0.5-10^5 t\ \ \text{V}$$

# 3.5 INDUCTANCES IN SERIES AND PARALLEL

- Series inductances

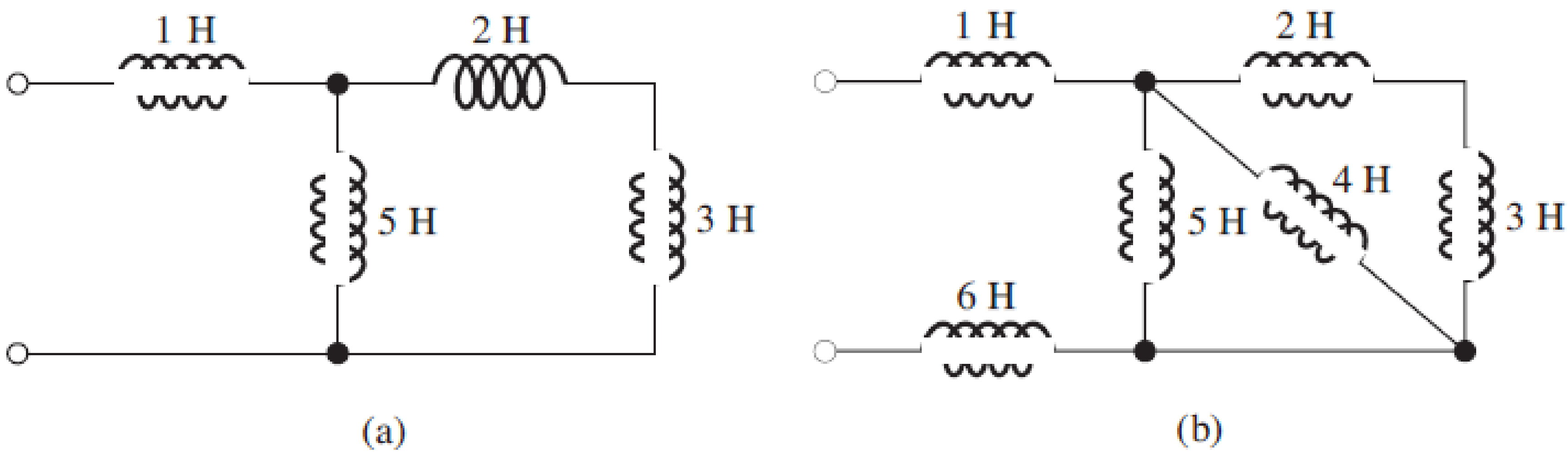



- Parallel inductances



# 3.5 INDUCTANCES IN SERIES AND PARALLEL

- Exercise

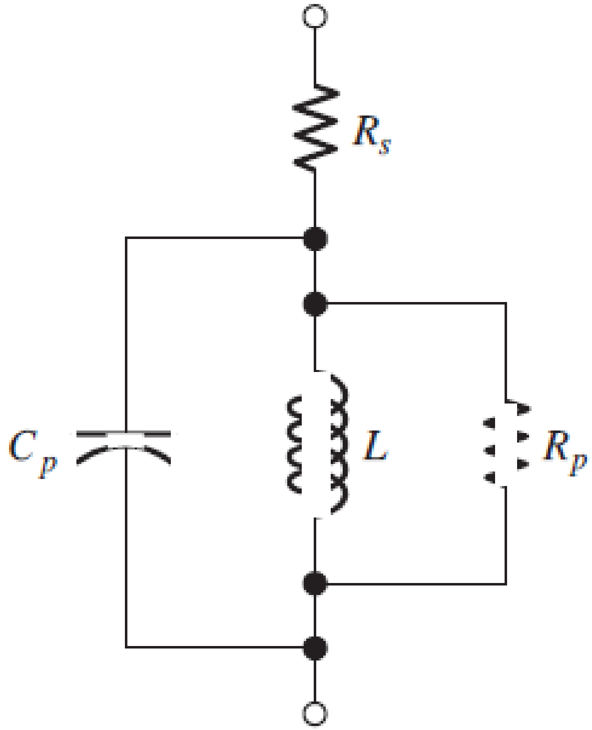



**a.** 3.5 H; **b.** 8.54 H.

# 3.6 PRACTICAL INDUCTORS

- Parasitic Effects

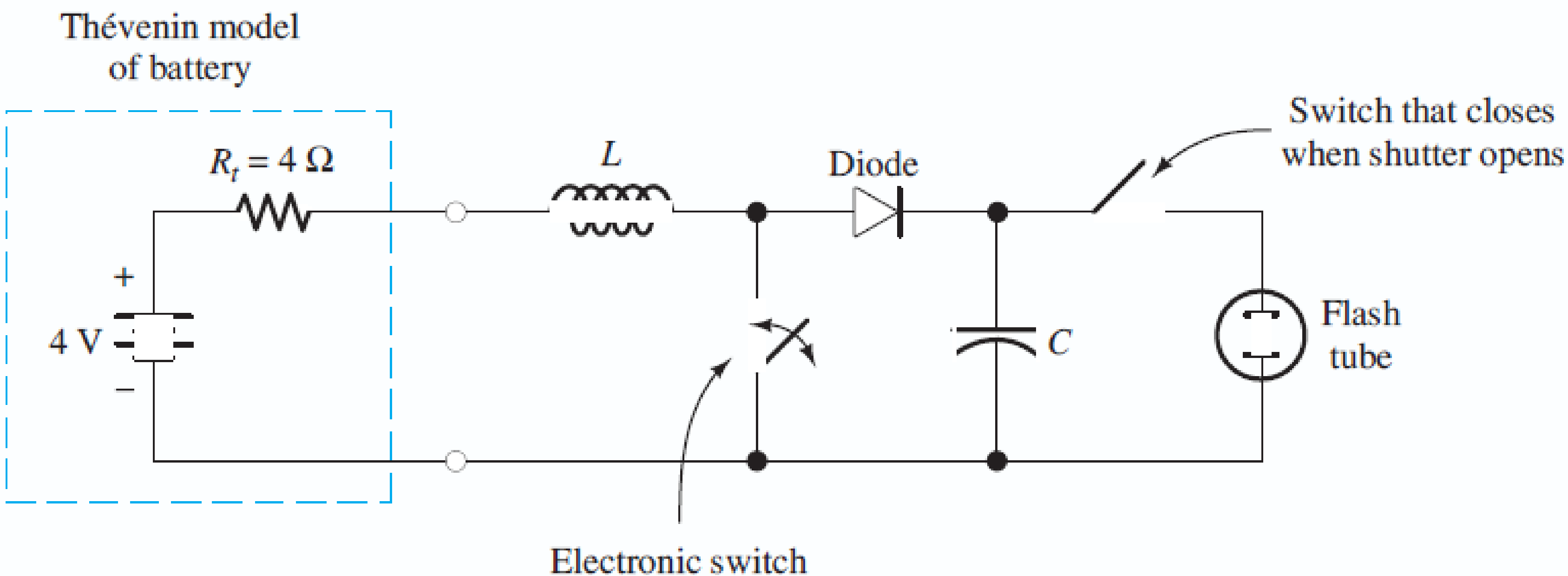
  - Resistivity of the material composing the wire
  - Electric field in the dielectric between the coils
  - Core loss (e.g. eddy currents)



# 3.6 PRACTICAL INDUCTORS

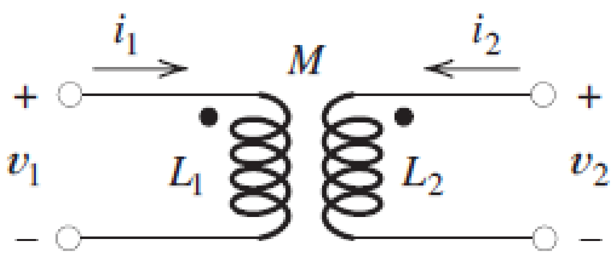
## PRACTICAL APPLICATION 3.1

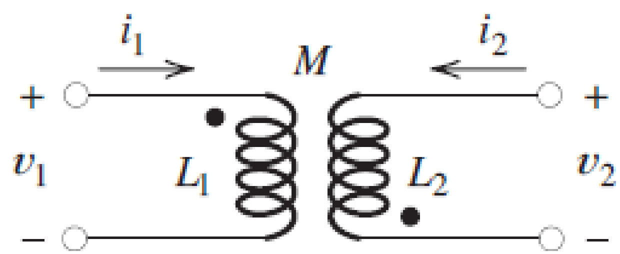
### Electronic Photo Flash



# 3.7 MUTUAL INDUCTANCE

- Several coils are wound on the same form
- Magnetic flux produced by one coil links the others
- Time-varying current in one coil induces voltages in the others
- Can either aid or oppose the flux produced by the other coil
  - **Self inductances: L** **Mutual inductances: M**




$$v_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

$$v_2 = M \frac{di_1}{dt} + L_2 \frac{di_2}{dt}$$

$$v_1 = L_1 \frac{di_1}{dt} - M \frac{di_2}{dt}$$

$$v_2 = -M \frac{di_1}{dt} + L_2 \frac{di_2}{dt}$$

# 3.7 MUTUAL INDUCTANCE

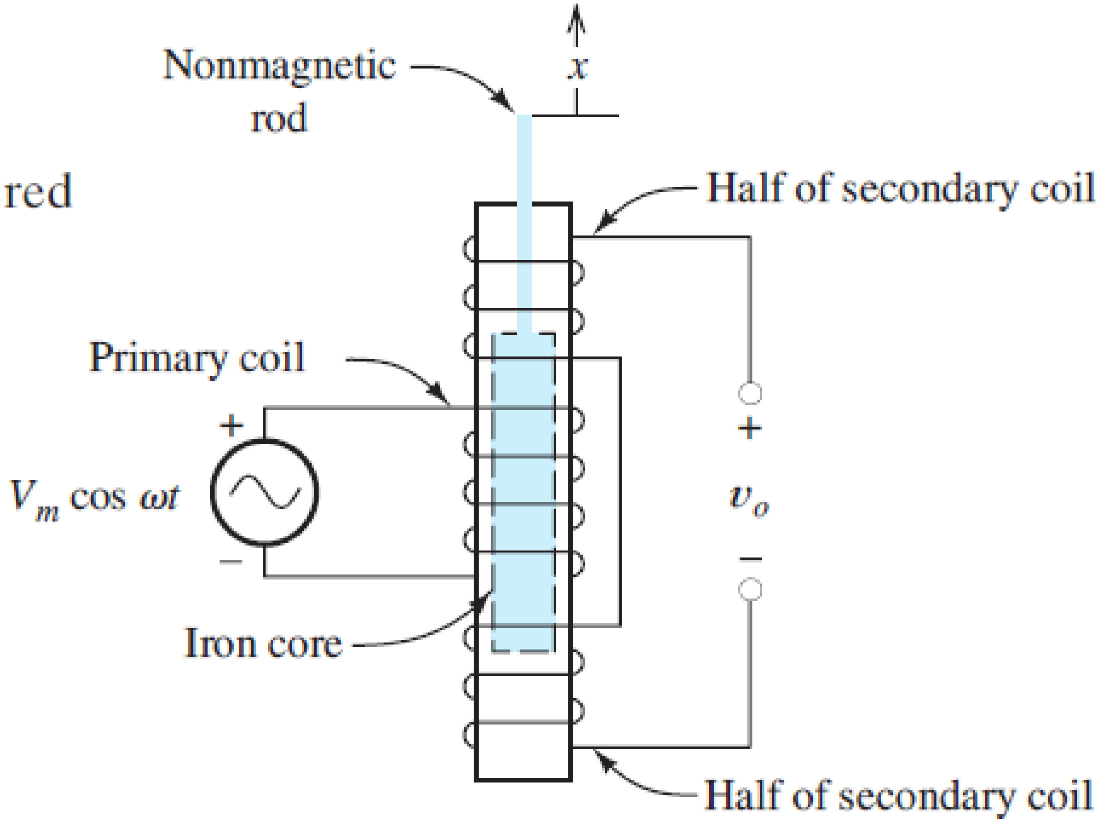
- Linear Variable Differential Transformer (LVDT)
  - Application of mutual inductance in a position transducer

When the iron core is centered

$v_o(t) = 0.$

As the core moves

$v_o(t) = Kx\cos(\omega t)$



# EXERCISES

- P3.5
- P3.16
- P3.24
- P3.32
- P3.43
- P3.44
- P3.45
- P3.60
- P3.61
- P3.72
- T3.1
- T3.2
- T3.3
- T3.4
- T3.5
- T3.6