دانشگاه سمنان

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دانشکده مهندسی مکانیک

دانشکده مهندسی مکانیک

درس مبانی برق ۱

نیمسال اول ۹۸-۹۹

Electrical Engineering

Principles and Applications

Allan R. Hambley

5th Edition

❑ *CONTENTS:*

INTRODUCTION

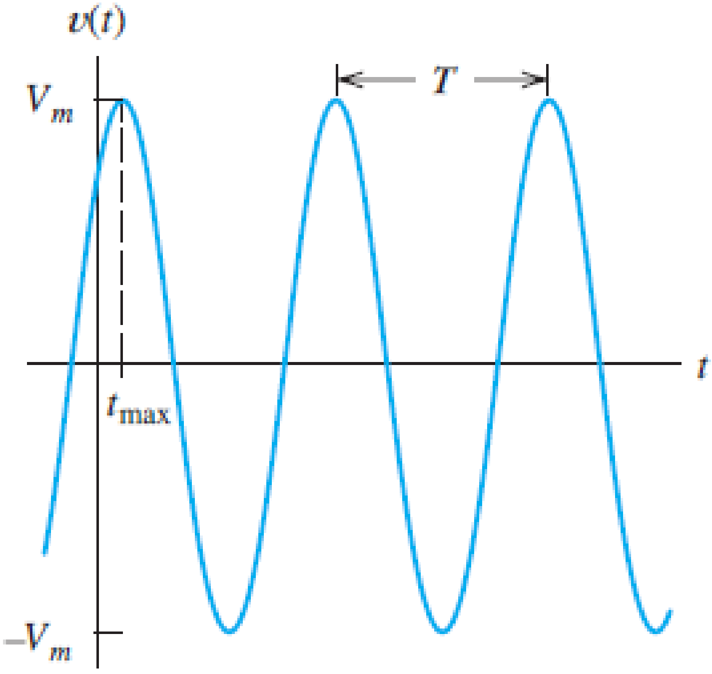
- Identify sinusoidal signal properties
- Analyze steady-state ac circuits (using phasors and complex impedances)
- Power of steady-state ac circuits
- Thévenin and Norton equivalent circuits
- Determine load impedances for maximum power transfer
- Introduction to three-phase power distribution
- Analyze balanced three-phase circuits

5.1 SINUSOIDAL CURRENTS AND VOLTAGES

- Sinusoidal voltage

$$v(t) = V_m \cos(\omega t + \theta)$$

V_m is the **peak value**
ω is the **angular frequency**
θ is the **phase angle**.



period T $\quad \omega T = 2\pi$

frequency $\quad f = \dfrac{1}{T}$ hertz (Hz).

angular frequency $\quad \omega = \dfrac{2\pi}{T}$ → $\omega = 2\pi f$

radians per second (rad/s)

5.1 SINUSOIDAL CURRENTS AND VOLTAGES

- For uniformity:
  - express sinusoidal functions by using the cosine function

$$\sin(z) = \cos(z - 90^\circ)$$

$$v_x(t) = 10\sin(200t + 30^\circ) \quad \Longrightarrow \quad v_x(t) = 10\cos(200t + 30^\circ - 90^\circ)$$

$$= 10\cos(200t - 60^\circ)$$

5.1 SINUSOIDAL CURRENTS AND VOLTAGES

- Root-Mean-Square Values (RMS)
  - Applying a periodic voltage $v(t)$ with period T to a resistance R
    - ✓ Power $$p(t) = \frac{v^2(t)}{R}$$
    - ✓ Energy (in one T) $$E_T = \int_0^T p(t)\,dt$$
    - ✓ Average power $$P_{avg} = \frac{E_T}{T} = \frac{1}{T}\int_0^T p(t)\,dt = \frac{1}{T}\int_0^T \frac{v^2(t)}{R}\,dt$$

$$\Rightarrow \quad P_{avg} = \frac{\left[\sqrt{\frac{1}{T}\int_0^T v^2(t)\,dt}\right]^2}{R}$$

5.1 SINUSOIDAL CURRENTS AND VOLTAGES

- Root-Mean-Square Values (RMS)
  - **root-mean-square** (rms) value of the periodic voltage $v(t)$

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2(t)\,dt}$$

$$\Rightarrow \quad P_{avg} = \frac{V_{rms}^2}{R}$$

  - rms value is also called the **effective value**

5.1 SINUSOIDAL CURRENTS AND VOLTAGES

- Root-Mean-Square Values (RMS)
  - Similarly for current:

$$I_{\text{rms}} = \sqrt{\frac{1}{T}\int_0^T i^2(t)\,dt}$$

$$P_{\text{avg}} = I_{\text{rms}}^2 R$$

5.1 SINUSOIDAL CURRENTS AND VOLTAGES

- RMS Value of a Sinusoid

$$v(t) = V_m \cos(\omega t + \theta)$$

$$\Longrightarrow V_{\text{rms}} = \sqrt{\frac{1}{T}\int_0^T V_m^2 \cos^2(\omega t + \theta)\, dt}$$

$$\cos^2(z) = \frac{1}{2} + \frac{1}{2}\cos(2z)$$

$$\Longrightarrow V_{\text{rms}} = \sqrt{\frac{V_m^2}{2T}\int_0^T [1 + \cos(2\omega t + 2\theta)]\, dt}$$

$$\Longrightarrow V_{\text{rms}} = \sqrt{\frac{V_m^2}{2T}\left[t + \frac{1}{2\omega}\sin(2\omega t + 2\theta)\right]_0^T}$$

5.1 SINUSOIDAL CURRENTS AND VOLTAGES

- RMS Value of a Sinusoid

$$\Rightarrow \quad V_{\text{rms}} = \sqrt{\frac{V_m^2}{2T}\left[T + \frac{1}{2\omega}\sin(2\omega T + 2\theta) - \frac{1}{2\omega}\sin(2\theta)\right]}$$

$$\begin{aligned}\frac{1}{2\omega}\sin(2\omega T + 2\theta) - \frac{1}{2\omega}\sin(2\theta) &= \frac{1}{2\omega}\sin(4\pi + 2\theta) - \frac{1}{2\omega}\sin(2\theta) \\ &= \frac{1}{2\omega}\sin(2\theta) - \frac{1}{2\omega}\sin(2\theta) \\ &= 0\end{aligned}$$

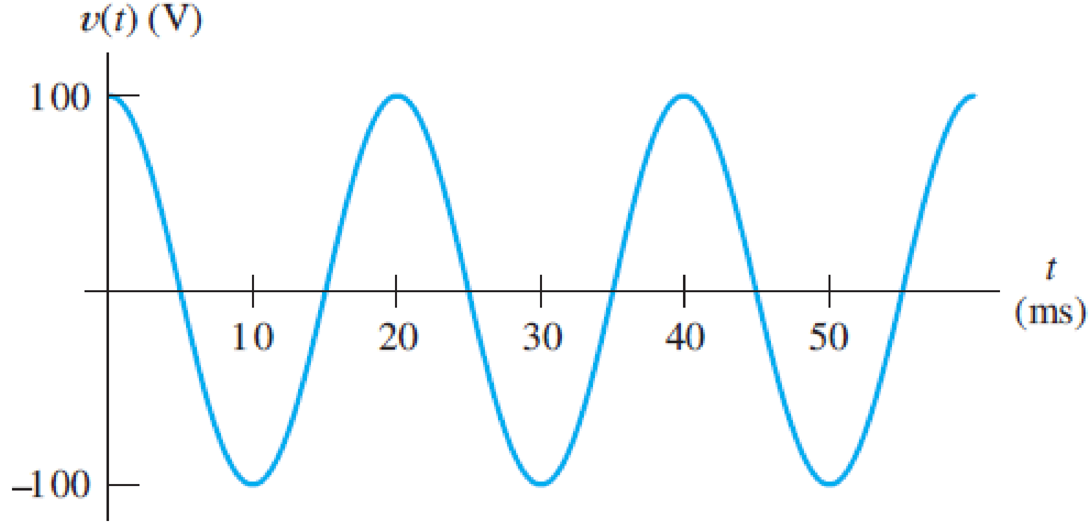
$$\Rightarrow \quad V_{\text{rms}} = \frac{V_m}{\sqrt{2}}$$

5.1 SINUSOIDAL CURRENTS AND VOLTAGES

Example 5.1 Power Delivered to a Resistance by a Sinusoidal Source

$v(t) = 100\cos(100\pi t)$ V

50-Ω resistance



$\omega = 100\pi$

$f = \omega/2\pi = 50$ Hz

$T = 1/f = 20$ ms

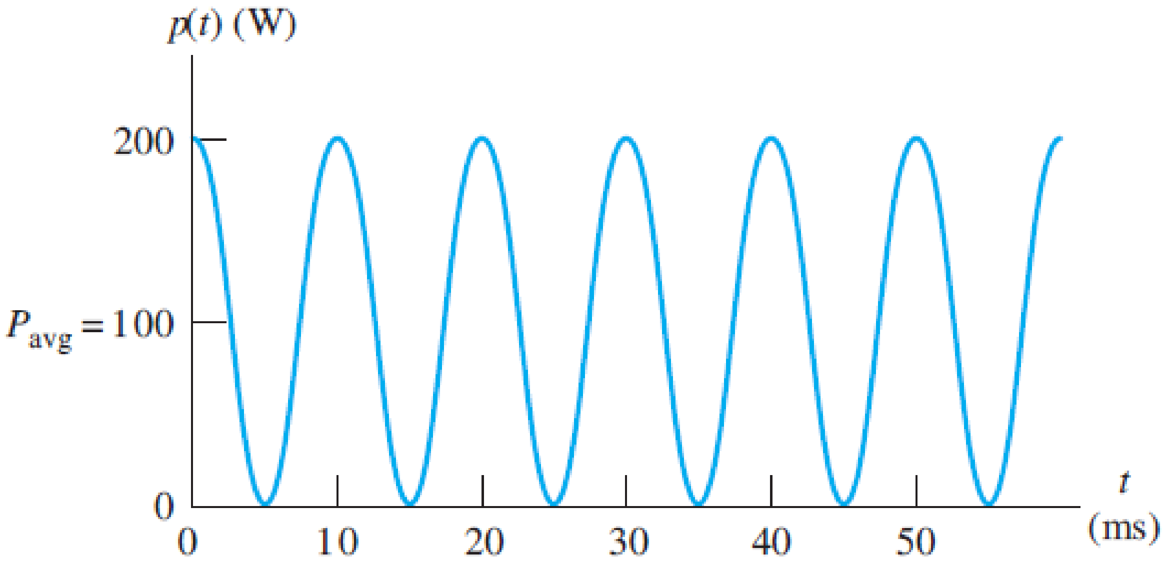
$V_m = 100$ V. $\Rightarrow$ $V_{rms} = V_m/\sqrt{2} = 70.71$ V.

5.1 SINUSOIDAL CURRENTS AND VOLTAGES

Example 5.1 Power Delivered to a Resistance by a Sinusoidal Source

$$\Rightarrow P_{avg} = \frac{V_{rms}^2}{R} = \frac{(70.71)^2}{50} = 100\ \text{W}$$

$$p(t) = \frac{v^2(t)}{R} = \frac{100^2 \cos^2(100\pi t)}{50} = 200 \cos^2(100\pi t)\ \text{W}$$

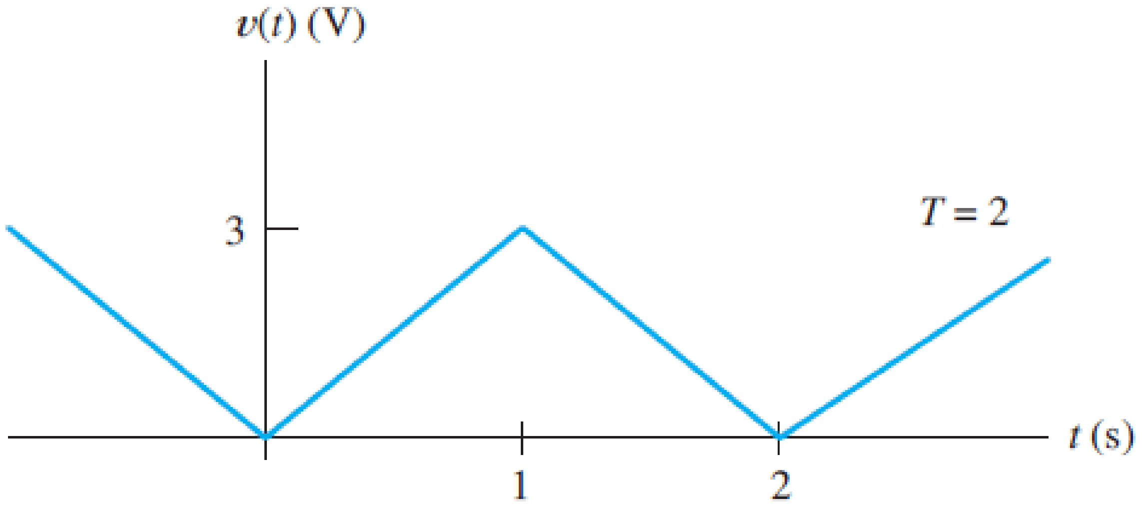


5.1 SINUSOIDAL CURRENTS AND VOLTAGES

- RMS Values of Nonsinusoidal Voltages or Currents

Example 5.2 RMS Value of a Triangular Voltage

$t = 0$ and $t = T = 2$ s

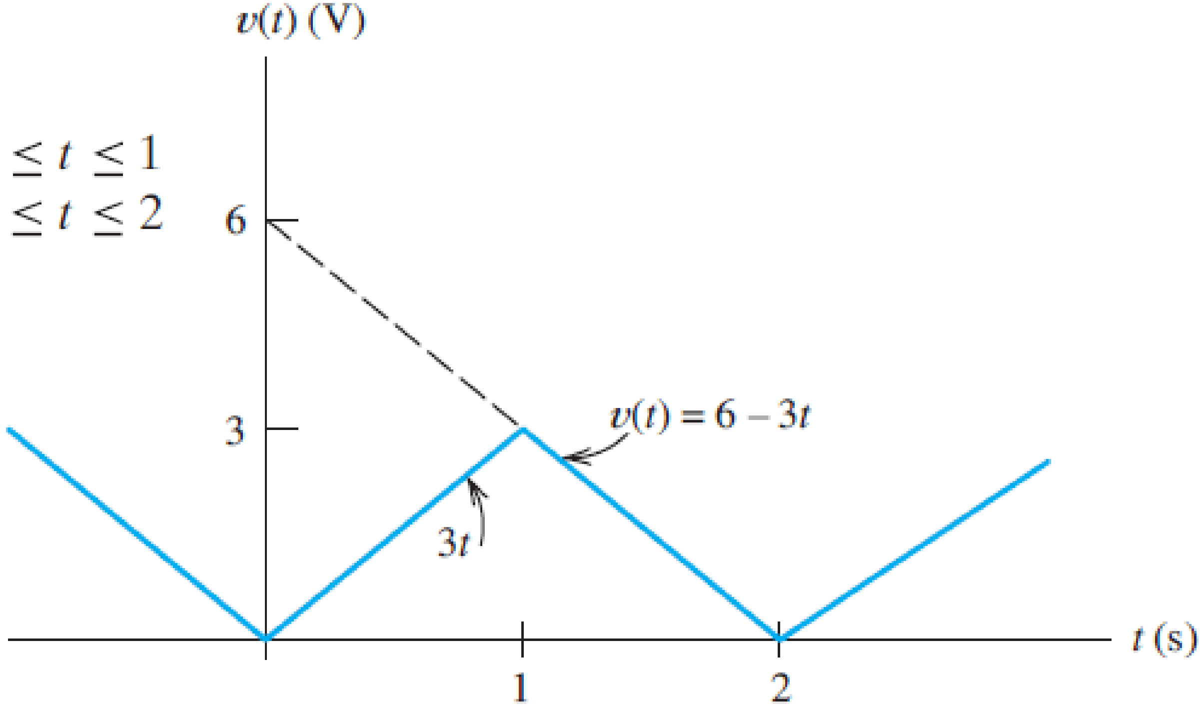


5.1 SINUSOIDAL CURRENTS AND VOLTAGES

- RMS Values of Nonsinusoidal Voltages or Currents

Example 5.2 RMS Value of a Triangular Voltage

$$v(t) = \begin{cases} 3t & for \quad 0 \le t \le 1 \\ 6 - 3t & for \quad 1 \le t \le 2 \end{cases}$$



5.1 SINUSOIDAL CURRENTS AND VOLTAGES

- RMS Values of Nonsinusoidal Voltages or Currents

Example 5.2 RMS Value of a Triangular Voltage

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2(t)dt}$$

$$= \sqrt{\frac{1}{2}\left[\int_0^1 9t^2 dt + \int_1^2 (6-3t)^2 dt\right]}$$

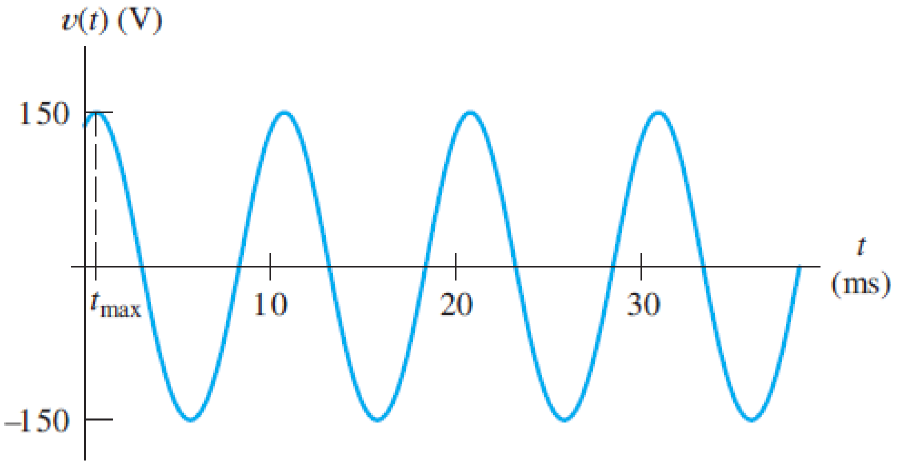
$$= \sqrt{\frac{1}{2}\left[3t^3\Big|_{t=0}^{t=1} + (36t - 18t^2 + 3t^3)\Big|_{t=1}^{t=2}\right]}$$

$$= \sqrt{\frac{1}{2}[3 + (72 - 36 - 72 + 18 + 24 - 3)]} = \sqrt{3}\ \text{V}$$

5.1 SINUSOIDAL CURRENTS AND VOLTAGES

- Exercise

$$v(t) = 150\cos(200\pi t - 30°)\ \text{V}$$



$\omega = 200\pi,\ f = 100\ \text{Hz},\ T = 10\ \text{ms},$

$V_m = 150\ \text{V},\ V_{\text{rms}} = 106.1\ \text{V},$

- First t_{max} after $t = 0\ s$:

$$t_{\text{max}} = \frac{30°}{360°} \times T = 0.833\ \text{ms}$$

5.1 SINUSOIDAL CURRENTS AND VOLTAGES

- Exercise

Express $v(t) = 100\sin(300\pi t + 60°)$ V as a cosine function

$$\Longrightarrow \quad v(t) = 100\cos(300\pi t - 30°)\ \text{V}.$$

- Exercise
  - rms value: 110V
  - Frequency: 60Hz
  - Peak voltage at $t = 5$ms

$$\Longrightarrow \quad v(t) = 155.6\cos(377t - 108°)\ \text{V}.$$

5.2 PHASORS

- Vector representation of sinusoidal signals in complex-number plane

- Convenient method for ac circuits:
  - For example: KVL leads to

$$v(t) = 10\cos(\omega t) + 5\sin(\omega t + 60^\circ) + 5\cos(\omega t + 90^\circ)$$

  - Put into form:

$$v(t) = V_m \cos(\omega t + \theta)$$

  - Using standard trigonometric identities…

5.2 PHASORS

- Phasor Definition
  - For sinusoidal voltages:

$$v_1(t) = V_1 \cos(\omega t + \theta_1) \quad \Longrightarrow \quad \mathbf{V}_1 = V_1 \angle \theta_1$$

$$v_2(t) = V_2 \sin(\omega t + \theta_2)$$

$$\sin(z) = \cos(z - 90°) \quad \Longrightarrow \quad v_2(t) = V_2 \cos(\omega t + \theta_2 - 90°)$$

$$\Longrightarrow \quad \mathbf{V}_2 = V_2 \angle \theta_2 - 90°$$

5.2 PHASORS

- Phasor Definition
  - For sinusoidal currents:

$$i_1(t) = I_1 \cos(\omega t + \theta_1) \Longrightarrow \mathbf{I}_1 = I_1 \angle \theta_1$$

$$i_2(t) = I_2 \sin(\omega t + \theta_2) \Longrightarrow \mathbf{I}_2 = I_2 \angle \theta_2 - 90°$$

5.2 PHASORS

- Adding Sinusoids Using Phasors

$$v(t) = 10\cos(\omega t) + 5\sin(\omega t + 60^\circ) + 5\cos(\omega t + 90^\circ)$$

$$\Longrightarrow v(t) = 10\cos(\omega t) + 5\cos(\omega t + 60^\circ - 90^\circ) + 5\cos(\omega t + 90^\circ)$$

$$\Longrightarrow v(t) = 10\cos(\omega t) + 5\cos(\omega t - 30^\circ) + 5\cos(\omega t + 90^\circ)$$

Euler's formula $\cos(\theta) = \mathrm{Re}\left(e^{j\theta}\right) = \mathrm{Re}[\cos(\theta) + j\sin(\theta)]$

$$\Longrightarrow v(t) = 10\,\mathrm{Re}\left[e^{j\omega t}\right] + 5\,\mathrm{Re}\left[e^{j(\omega t - 30^\circ)}\right] + 5\,\mathrm{Re}\left[e^{j(\omega t + 90^\circ)}\right]$$

$$\Longrightarrow v(t) = \mathrm{Re}\left[10e^{j\omega t}\right] + \mathrm{Re}\left[5e^{j(\omega t - 30^\circ)}\right] + \mathrm{Re}\left[5e^{j(\omega t + 90^\circ)}\right]$$

5.2 PHASORS

- Adding Sinusoids Using Phasors

$$\Longrightarrow \; v(t) = \text{Re}\left[10e^{j\omega t} + 5e^{j(\omega t - 30^\circ)} + 5e^{j(\omega t + 90^\circ)}\right]$$

$$\Longrightarrow \; v(t) = \text{Re}\left[\left(10 + 5e^{-j30^\circ} + 5^{j90^\circ}\right) e^{j\omega t}\right]$$

$$\Longrightarrow \; v(t) = \text{Re}\left[(10 \,\underline{/0^\circ} + 5 \,\underline{/-30^\circ} + 5 \,\underline{/90^\circ})e^{j\omega t}\right]$$

$$\Longrightarrow \; 10 \,\underline{/0^\circ} + 5 \,\underline{/-30^\circ} + 5 \,\underline{/90^\circ} = 10 + 4.33 - j2.50 + j5$$

$$= 14.33 + j2.5$$

$$= 14.54 \,\underline{/9.90^\circ}$$

$$= 14.54e^{j9.90^\circ}$$

5.2 PHASORS

- Adding Sinusoids Using Phasors

$$\Longrightarrow \quad v(t) = \mathrm{Re}\left[\left(14.54e^{j9.90°}\right)e^{j\omega t}\right]$$

$$\Longrightarrow \quad v(t) = \mathrm{Re}\left[14.54e^{j(\omega t + 9.90°)}\right]$$

$$\Longrightarrow \quad v(t) = 14.54\cos(\omega t + 9.90°)$$

- Streamlined Procedure for Adding Sinusoids
  - Write the phasor for each term
  - Add the phasors by using complex-number arithmetic

5.2 PHASORS

Example 5.3 Using Phasors to Add Sinusoids

$$v_1(t) = 20\cos(\omega t - 45^\circ)$$
$$v_2(t) = 10\sin(\omega t + 60^\circ)$$

$\Rightarrow$ $v_s(t) = v_1(t) + v_2(t)$

$\Rightarrow$
$$\mathbf{V}_1 = 20\ \angle -45^\circ$$
$$\mathbf{V}_2 = 10\ \angle -30^\circ$$

$\Rightarrow$
$$\begin{aligned}\mathbf{V}_s &= \mathbf{V}_1 + \mathbf{V}_2 \\ &= 20\ \angle -45^\circ + 10\ \angle -30^\circ \\ &= 14.14 - j14.14 + 8.660 - j5 \\ &= 22.80 - j19.14 \\ &= 29.77\ \angle -40.01^\circ\end{aligned}$$

$\Rightarrow$ $v_s(t) = 29.77\cos(\omega t - 40.01^\circ)$

5.2 PHASORS

- Exercise

$$v_1(t) = 10\cos(\omega t) + 10\sin(\omega t)$$

$$i_1(t) = 10\cos(\omega t + 30^\circ) + 5\sin(\omega t + 30^\circ)$$

$$i_2(t) = 20\sin(\omega t + 90^\circ) + 15\cos(\omega t - 60^\circ)$$

$$\Longrightarrow \quad v_1(t) = 14.14\cos(\omega t - 45^\circ)$$

$$i_1(t) = 11.18\cos(\omega t + 3.44^\circ)$$

$$i_2(t) = 30.4\cos(\omega t - 25.3^\circ)$$

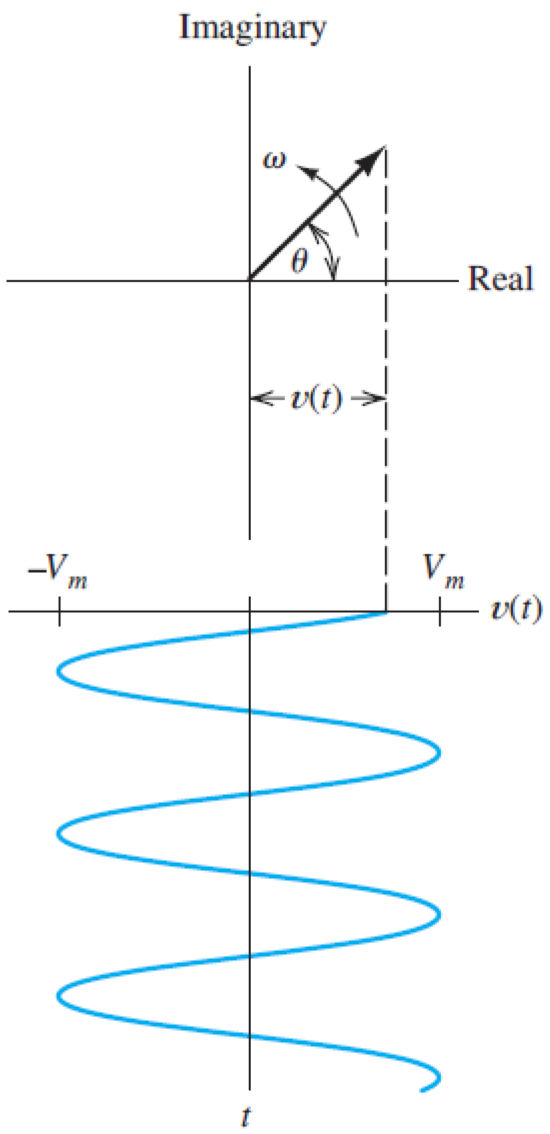
5.2 PHASORS

- Phasors as Rotating Vectors

$$v(t) = V_m \cos(\omega t + \theta)$$

$$\Rightarrow v(t) = \operatorname{Re}\left[V_m e^{j(\omega t+\theta)}\right]$$

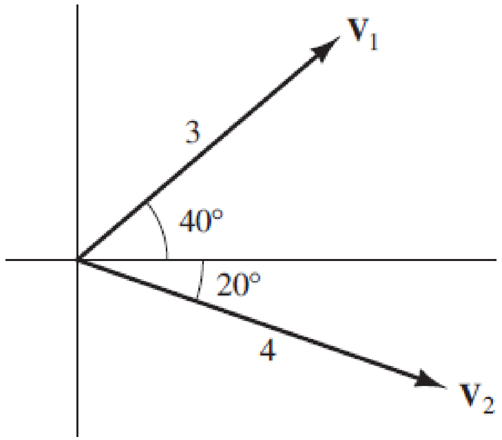
$$\Rightarrow V_m e^{j(\omega t+\theta)} = V_m \underline{/\omega t + \theta}$$



5.2 PHASORS

- Phase Relationships

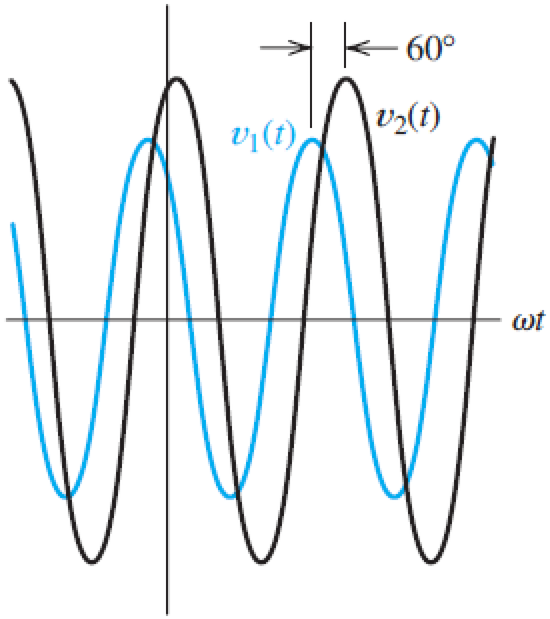
$$v_1(t) = 3\cos(\omega t + 40^\circ)$$
$$v_2(t) = 4\cos(\omega t - 20^\circ)$$



$$\Rightarrow \quad \mathbf{V}_1 = 3\,\angle 40^\circ \qquad \mathbf{V}_2 = 4\,\angle -20^\circ$$

$\Rightarrow$ $\mathbf{V}_1$ leads $\mathbf{V}_2$ by 60°

$\mathbf{V}_2$ lags $\mathbf{V}_1$ by 60°



5.2 PHASORS

- Exercise

$$v_1(t) = 10\cos(\omega t - 30^\circ)$$

$$v_2(t) = 10\cos(\omega t + 30^\circ)$$

$$v_3(t) = 10\sin(\omega t + 45^\circ)$$

v_1 lags v_2 by 60° (or v_2 leads v_1 by 60°)

⟹ v_1 leads v_3 by 15° (or v_3 lags v_1 by 15°)

v_2 leads v_3 by 75° (or v_3 lags v_2 by 75°)

5.3 COMPLEX IMPEDANCES

- Solve sinusoidal steady-state circuit problems using Phasors
- Use complex arithmetic

- Inductance

$$i_L(t) = I_m \sin(\omega t + \theta)$$

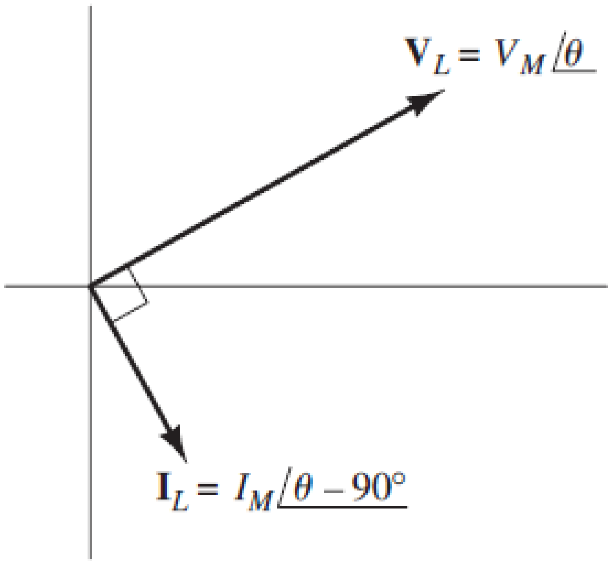
$$\Longrightarrow v_L(t) = L\frac{di_L(t)}{dt}$$

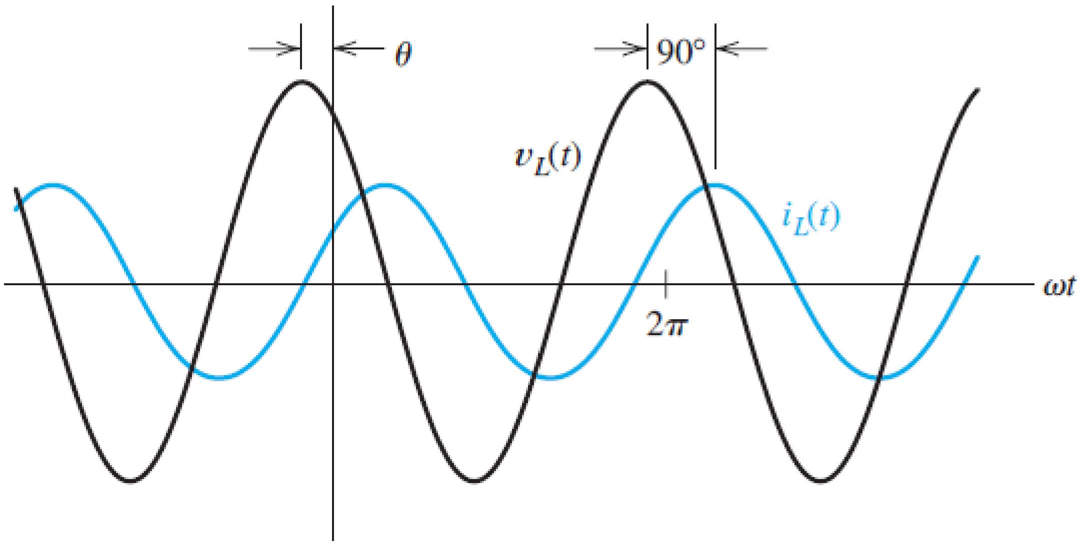
$$\Longrightarrow v_L(t) = \omega L I_m \cos(\omega t + \theta)$$

$$\mathbf{I}_L = I_m \angle \theta - 90^\circ \Longrightarrow \mathbf{V}_L = \omega L I_m \angle \theta = V_m \angle \theta$$

5.3 COMPLEX IMPEDANCES

- Inductance
  - Phasor diagram





- ✓ Current lags voltage by 90 degrees for a pure inductance

5.3 COMPLEX IMPEDANCES

- Inductance

$$\mathbf{V}_L = \omega L I_m \angle\theta = V_m \angle\theta$$

$$\Longrightarrow \mathbf{V}_L = (\omega L \angle 90^\circ) \times I_m \angle \theta - 90^\circ$$

$$\Longrightarrow \mathbf{V}_L = (\omega L \angle 90^\circ) \times \mathbf{I}_L$$

$$\Longrightarrow \mathbf{V}_L = j\omega L \times \mathbf{I}_L$$

impedance of the inductance

$$Z_L = j\omega L = \omega L \angle 90^\circ$$

$$\mathbf{V}_L = Z_L \mathbf{I}_L$$

5.3 COMPLEX IMPEDANCES

- Inductance
  - Ohm's law in Phasor form

$$\mathbf{V}_L = Z_L \mathbf{I}_L$$

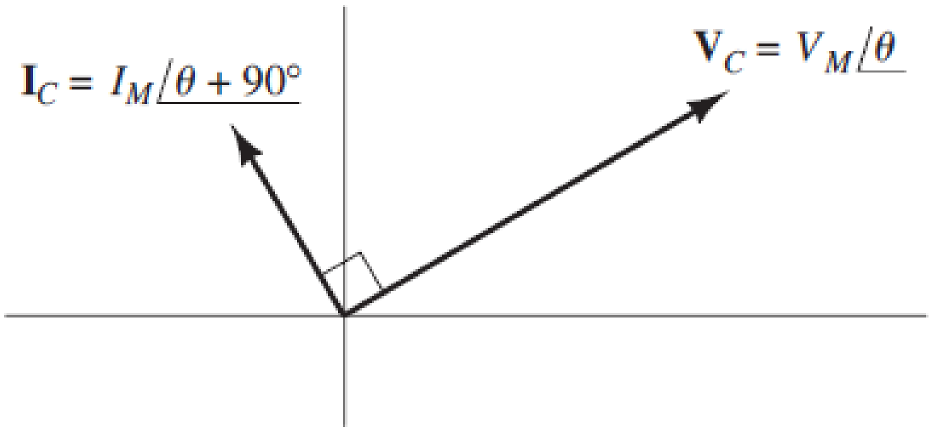
- ✓ For an inductance, the impedance is an imaginary number
- ✓ For a resistance, the impedance is a real number
- ✓ Impedances that are pure imaginary are also called **reactances**

5.3 COMPLEX IMPEDANCES

- Capacitance

$$\mathbf{V}_C = Z_C \mathbf{I}_C$$

$$Z_C = -j\frac{1}{\omega C} = \frac{1}{j\omega C} = \frac{1}{\omega C}\angle -90^\circ$$

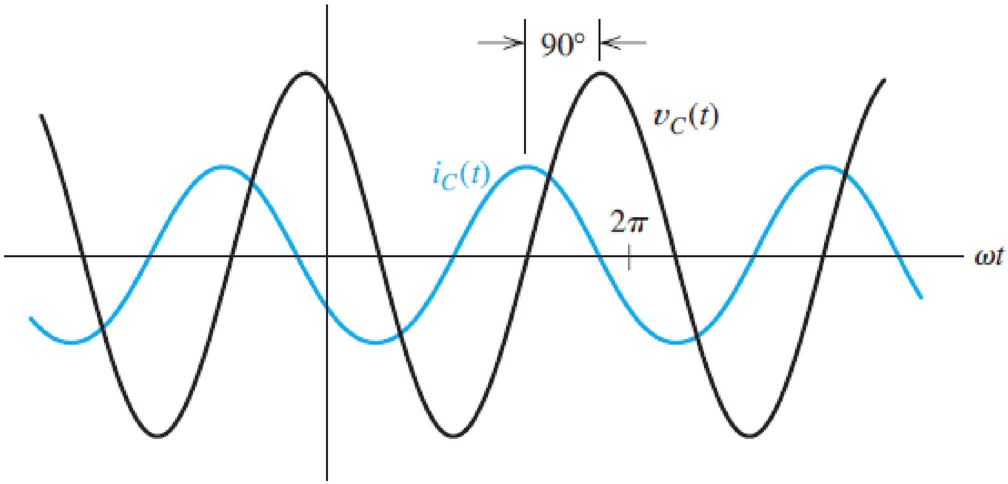


5.3 COMPLEX IMPEDANCES

- Capacitance

$$\mathbf{V}_C = V_m \underline{\angle \theta} \quad \Longrightarrow \quad \mathbf{I}_C = \frac{\mathbf{V}_C}{Z_C} = \frac{V_m \underline{\angle \theta}}{(1/\omega C) \underline{\angle -90^\circ}} = \omega C V_m \underline{\angle \theta + 90^\circ}$$

$$\Longrightarrow \quad \mathbf{I}_C = I_m \underline{\angle \theta + 90^\circ}$$

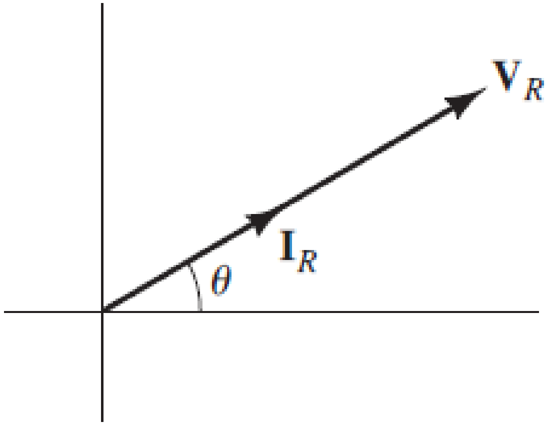


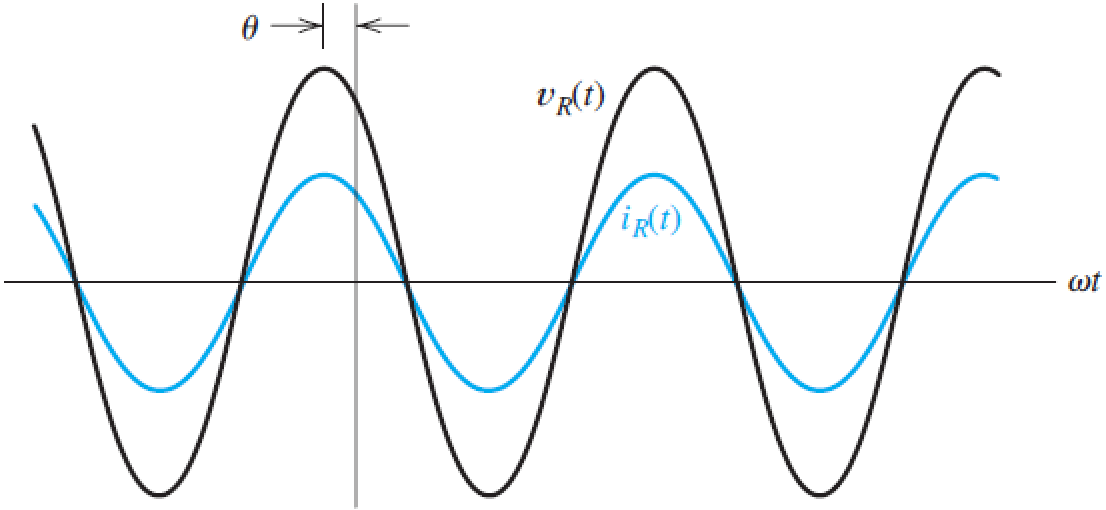
✓ Current leads voltage by 90 degrees for a pure capacitance

5.3 COMPLEX IMPEDANCES

- Resistance

$$\mathbf{V}_R = R\mathbf{I}_R$$





✓ Current and voltage are in phase for a pure resistance

5.3 COMPLEX IMPEDANCES

- Exercise

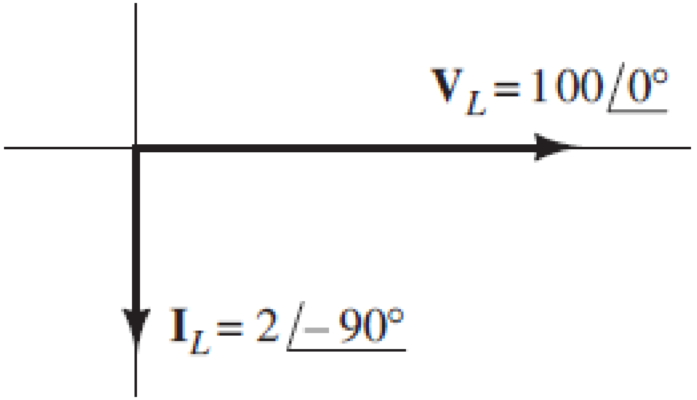
0.25-H inductance

$$v_L(t) = 100\cos(200t)$$

$\Rightarrow$ $Z_L = j50 = 50\angle 90^\circ$

$\Rightarrow$ $\mathbf{V}_L = 100\angle 0^\circ$

$\Rightarrow$ $\mathbf{I}_L = 2\angle -90^\circ$



5.3 COMPLEX IMPEDANCES

- Exercise

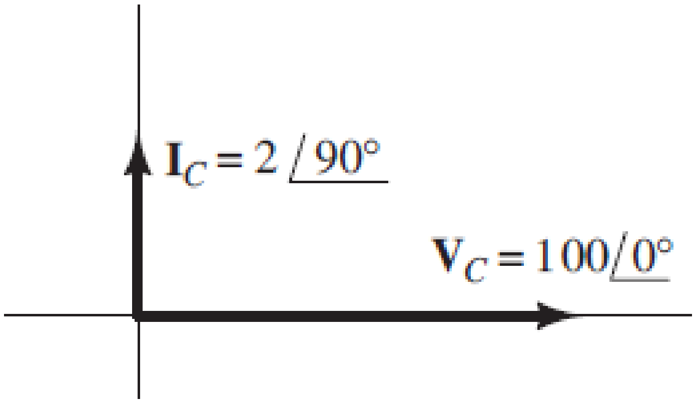
100-μF capacitance

$$v_L(t) = 100\cos(200t)$$

$$\Rightarrow \quad Z_C = -j50 = 50\angle -90^\circ$$

$$\Rightarrow \quad \mathbf{V}_L = 100\angle 0^\circ$$

$$\Rightarrow \quad \mathbf{I}_C = 2\angle 90^\circ$$



5.3 COMPLEX IMPEDANCES

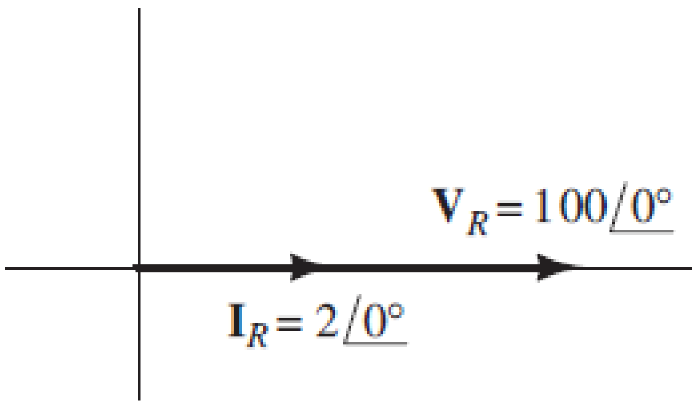
- Exercise

50-Ω resistance

$$v_L(t) = 100\cos(200t)$$

$$\Rightarrow \mathbf{V}_L = 100\angle 0^\circ$$

$$\Rightarrow \mathbf{I}_R = 2\angle 0^\circ$$



5.4 CIRCUIT ANALYSIS WITH PHASORS AND COMPLEX IMPEDANCES

- Kirchhoff's Laws in Phasor Form
  - KVL equation:

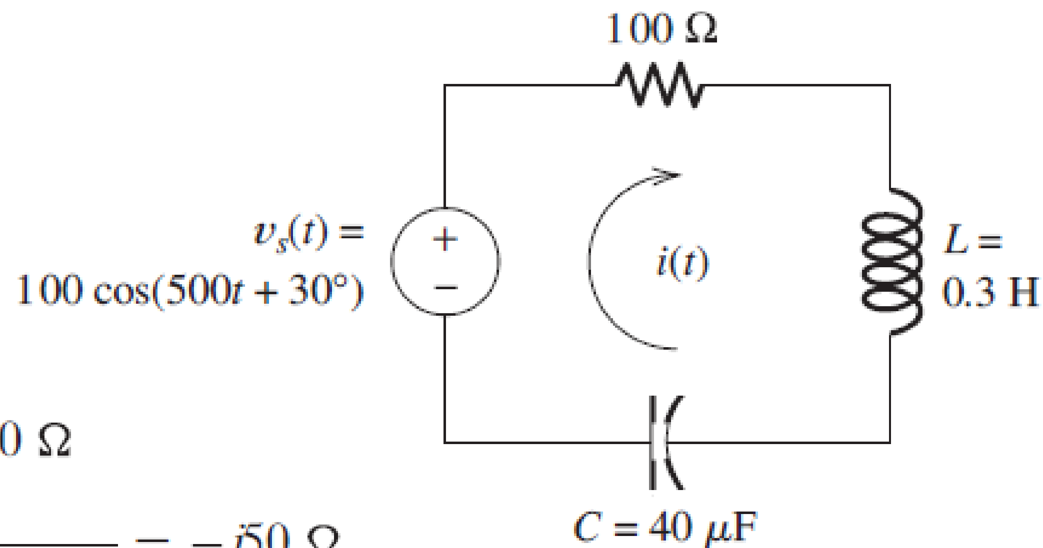
$$v_1(t) + v_2(t) - v_3(t) = 0 \quad \Longrightarrow \quad \mathbf{V}_1 + \mathbf{V}_2 - \mathbf{V}_3 = 0$$

  - Similarly for KCL

- Procedure for steady-state analysis of circuits with sinusoidal sources:
  - Replace time descriptions with corresponding phasors
  - Replace inductances and capacitances by their complex impedances
  - Analyze the circuit by using any of previous techniques with complex arithmetic

5.4 CIRCUIT ANALYSIS WITH PHASORS AND COMPLEX IMPEDANCES

Example 5.4 Steady-State AC Analysis of a Series Circuit



$\omega = 500$

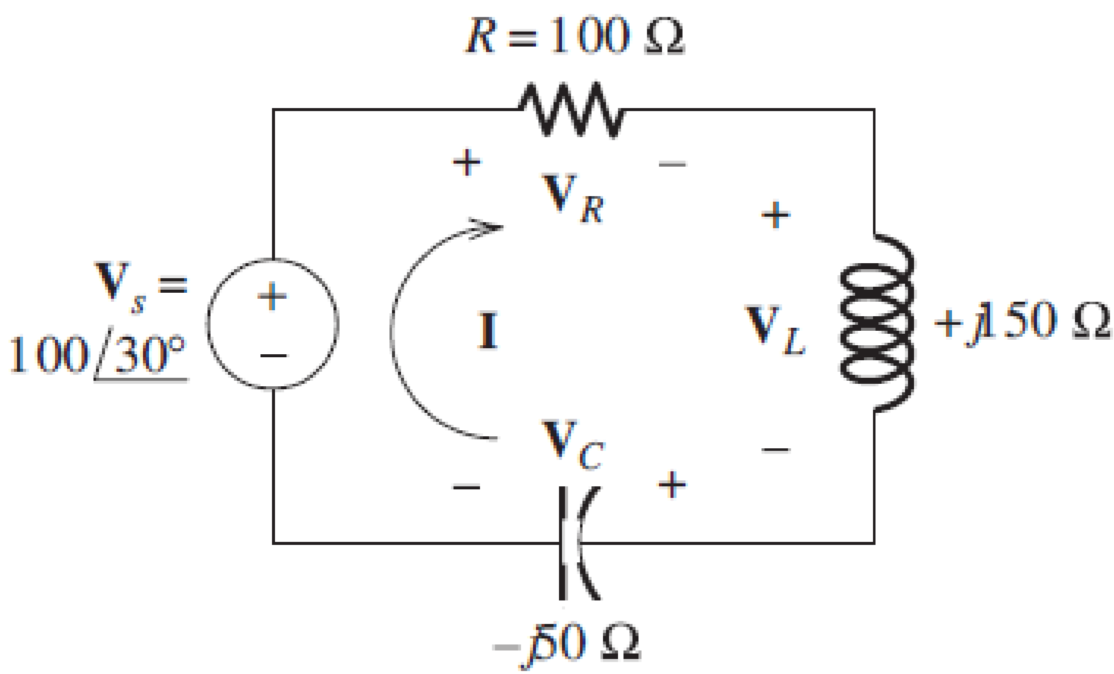
$\mathbf{V}_s = 100\ \angle 30°$

$$Z_L = j\omega L = j500 \times 0.3 = j150\ \Omega$$

$$Z_C = -j\frac{1}{\omega C} = -j\frac{1}{500 \times 40 \times 10^{-6}} = -j50\ \Omega$$

5.4 CIRCUIT ANALYSIS WITH PHASORS AND COMPLEX IMPEDANCES

Example 5.4 Steady-State AC Analysis of a Series Circuit



5.4 CIRCUIT ANALYSIS WITH PHASORS AND COMPLEX IMPEDANCES

Example 5.4 Steady-State AC Analysis of a Series Circuit

$$Z_{eq} = R + Z_L + Z_C \quad \Rightarrow \quad Z_{eq} = 100 + j150 - j50 = 100 + j100$$

$$\Rightarrow \quad Z_{eq} = 141.4 \angle 45^\circ$$

$$\Rightarrow \quad \mathbf{I} = \frac{\mathbf{V}_s}{Z} = \frac{100 \angle 30^\circ}{141.4 \angle 45^\circ} = 0.707 \angle -15^\circ$$

$$\Rightarrow \quad i(t) = 0.707 \cos(500t - 15^\circ)$$

5.4 CIRCUIT ANALYSIS WITH PHASORS AND COMPLEX IMPEDANCES

Example 5.4 Steady-State AC Analysis of a Series Circuit

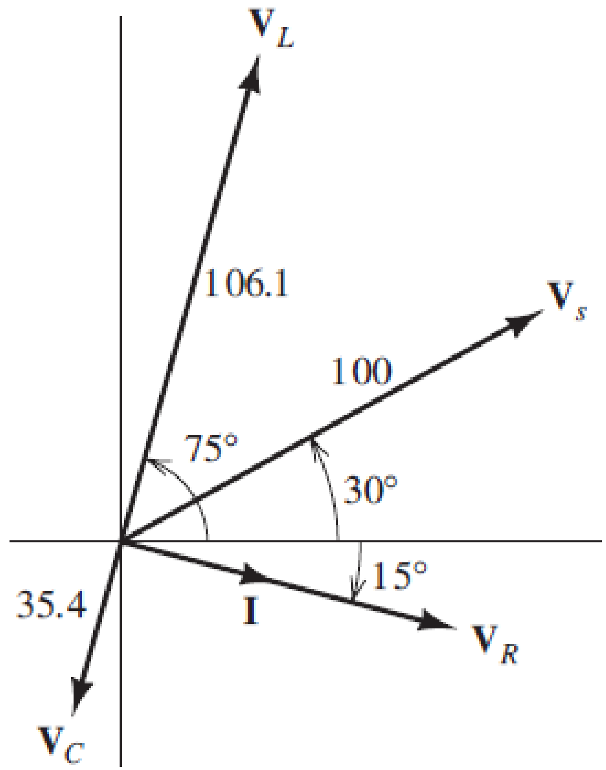
$$\mathbf{V}_R = R \times \mathbf{I} = 100 \times 0.707 \angle -15^\circ = 70.7 \angle -15^\circ$$

$$\mathbf{V}_L = j\omega L \times \mathbf{I} = \omega L \angle 90^\circ \times \mathbf{I} = 150 \angle 90^\circ \times 0.707 \angle -15^\circ$$
$$= 106.1 \angle 75^\circ$$

$$\mathbf{V}_C = -j\frac{1}{\omega C} \times \mathbf{I} = \frac{1}{\omega C} \angle -90^\circ \times \mathbf{I} = 50 \angle -90^\circ \times 0.707 \angle -15^\circ$$
$$= 35.4 \angle -105^\circ$$

5.4 CIRCUIT ANALYSIS WITH PHASORS AND COMPLEX IMPEDANCES

Example 5.4 Steady-State AC Analysis of a Series Circuit

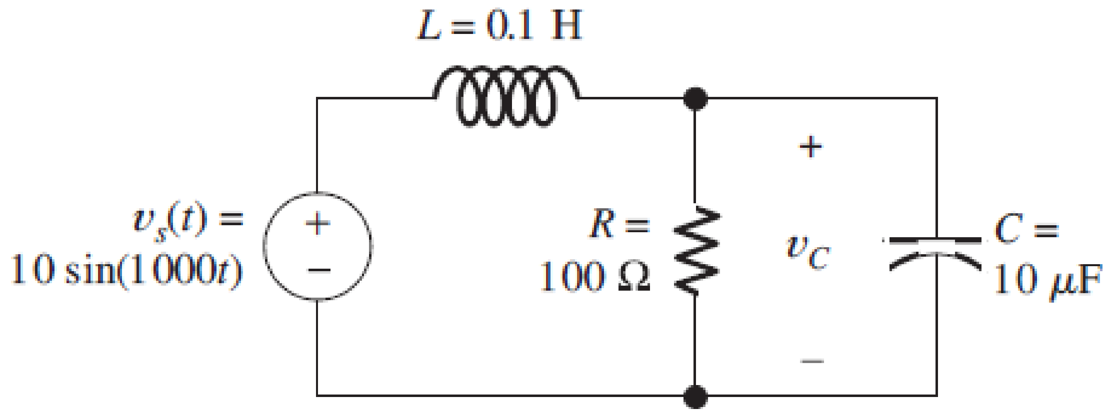


5.4 CIRCUIT ANALYSIS WITH PHASORS AND COMPLEX IMPEDANCES

Example 5.5 Series and Parallel Combinations of Complex Impedances

$\omega = 1000$

$\mathbf{V}_s = 10\angle -90^\circ.$

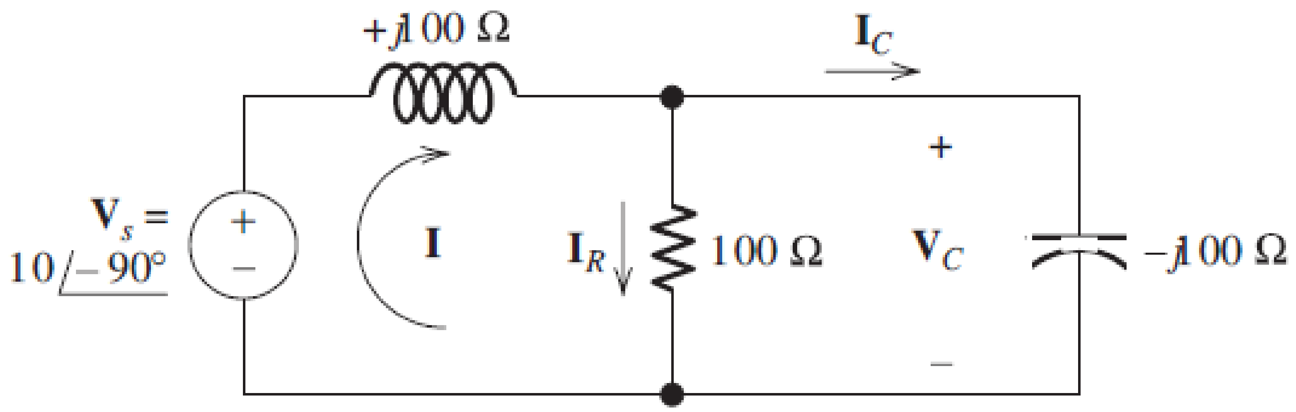


$$Z_L = j\omega L = j1000 \times 0.1 = j100\ \Omega$$

$$Z_C = -j\frac{1}{\omega C} = -j\frac{1}{1000 \times 10 \times 10^{-6}} = -j100\ \Omega$$

5.4 CIRCUIT ANALYSIS WITH PHASORS AND COMPLEX IMPEDANCES

Example 5.5 Series and Parallel Combinations of Complex Impedances



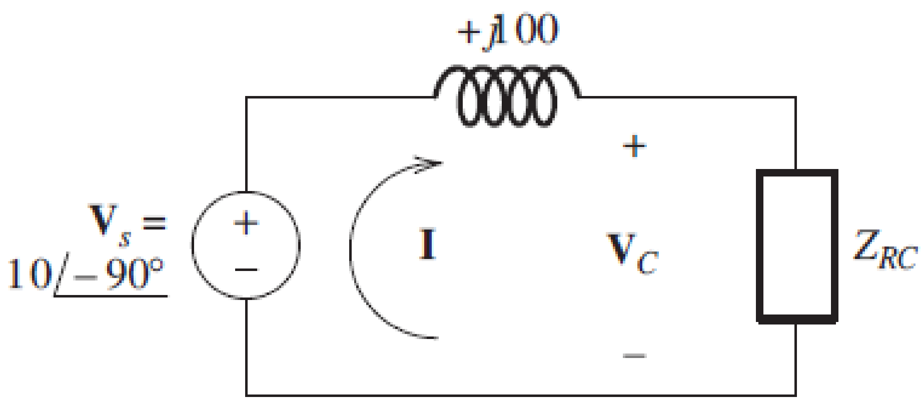
$$Z_{RC} = \frac{1}{1/R + 1/Z_C} = \frac{1}{1/100 + 1/(-j100)}$$

$$= \frac{1}{0.01 + j0.01} = \frac{1\ \angle 0^\circ}{0.01414\ \angle 45^\circ} = 70.71\ \angle{-45^\circ}$$

5.4 CIRCUIT ANALYSIS WITH PHASORS AND COMPLEX IMPEDANCES

Example 5.5 Series and Parallel Combinations of Complex Impedances

$$\Longrightarrow \quad Z_{RC} = 50 - j50$$



5.4 CIRCUIT ANALYSIS WITH PHASORS AND COMPLEX IMPEDANCES

Example 5.5 Series and Parallel Combinations of Complex Impedances

$$\mathbf{V}_C = \mathbf{V}_s \frac{Z_{RC}}{Z_L + Z_{RC}} = 10\,\underline{/-90^\circ}\, \frac{70.71\,\underline{/-45^\circ}}{j100 + 50 - j50}$$

$$= 10\,\underline{/-90^\circ}\, \frac{70.71\,\underline{/-45^\circ}}{50 + j50} = 10\,\underline{/-90^\circ}\, \frac{70.71\,\underline{/-45^\circ}}{70.71\,\underline{/45^\circ}}$$

$$= 10\,\underline{/-180^\circ}$$

$$\Longrightarrow \quad v_C(t) = 10\cos(1000t - 180^\circ) = -10\cos(1000t)$$

5.4 CIRCUIT ANALYSIS WITH PHASORS AND COMPLEX IMPEDANCES

Example 5.5 Series and Parallel Combinations of Complex Impedances

$$\mathbf{I} = \frac{\mathbf{V}_s}{Z_L + Z_{RC}} = \frac{10\angle -90^\circ}{j100 + 50 - j50} = \frac{10\angle -90^\circ}{50 + j50}$$

$$= \frac{10\angle -90^\circ}{70.71\angle 45^\circ} = 0.1414\angle -135^\circ$$

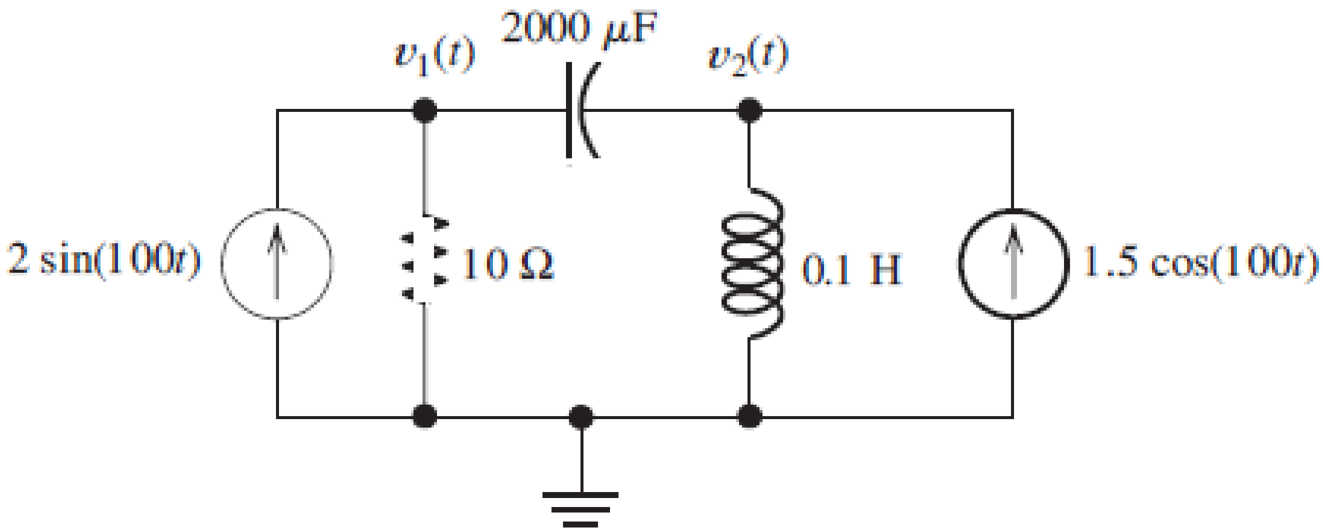
$$\mathbf{I}_R = \frac{\mathbf{V}_C}{R} = \frac{10\angle -180^\circ}{100} = 0.1\angle -180^\circ$$

$$\mathbf{I}_C = \frac{\mathbf{V}_C}{Z_C} = \frac{10\angle -180^\circ}{-j100} = \frac{10\angle -180^\circ}{100\angle -90^\circ} = 0.1\angle -90^\circ$$

5.4 CIRCUIT ANALYSIS WITH PHASORS AND COMPLEX IMPEDANCES

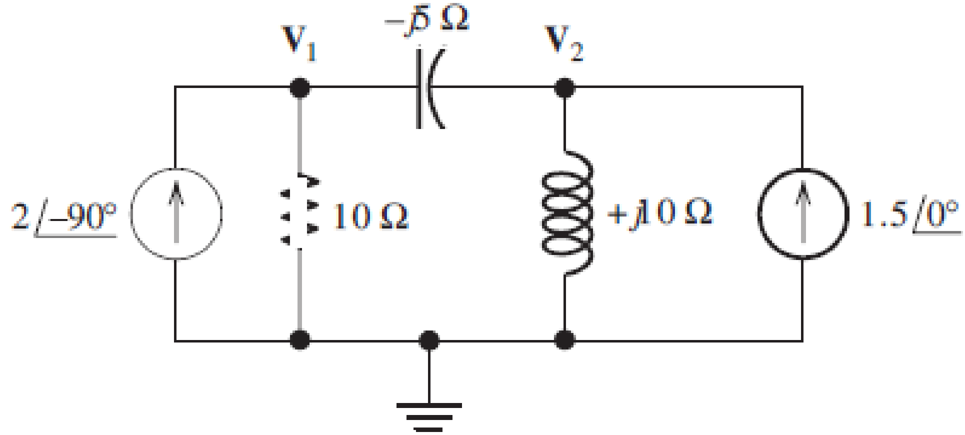
- Node-Voltage Analysis

Example 5.6 Steady-State AC Node-Voltage Analysis



5.4 CIRCUIT ANALYSIS WITH PHASORS AND COMPLEX IMPEDANCES

Example 5.6 Steady-State AC Node-Voltage Analysis



$$\frac{\mathbf{V}_1}{10} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{-j5} = 2\angle{-90^\circ}$$

$$\frac{\mathbf{V}_2}{j10} + \frac{\mathbf{V}_2 - \mathbf{V}_1}{-j5} = 1.5\angle{0^\circ}$$

$$\Rightarrow \quad (0.1 + j0.2)\mathbf{V}_1 - j0.2\mathbf{V}_2 = -j2$$

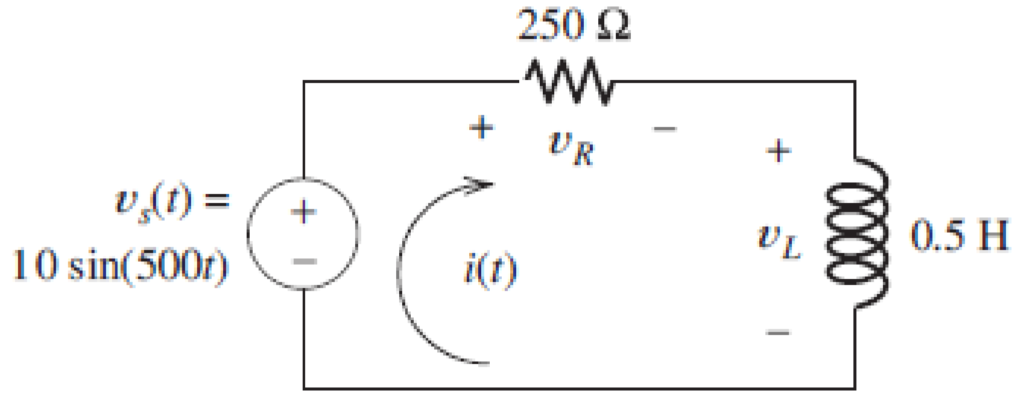
$$-j0.2\mathbf{V}_1 + j0.1\mathbf{V}_2 = 1.5$$

$$\Rightarrow \quad \mathbf{V}_1 = 16.1\angle{29.7^\circ}$$

$$\Rightarrow \quad v_1(t) = 16.1\cos(100t + 29.7^\circ)$$

5.4 CIRCUIT ANALYSIS WITH PHASORS AND COMPLEX IMPEDANCES

- Exercise

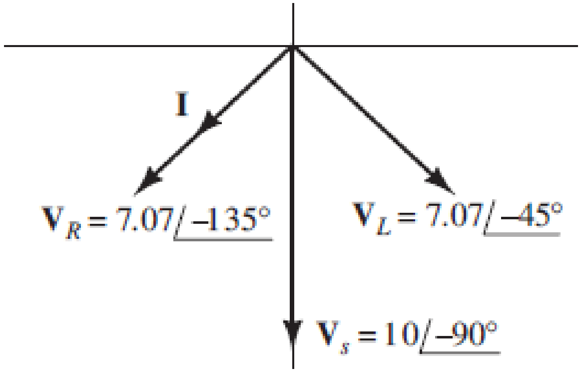


$$\mathbf{I} = \frac{\mathbf{V}_s}{Z} = \frac{10\angle -90^\circ}{250 + j250} = 28.28\angle -135^\circ \ \text{mA}$$

$$\Rightarrow \quad i(t) = 28.28\cos(500t - 135^\circ)\ \text{mA}$$

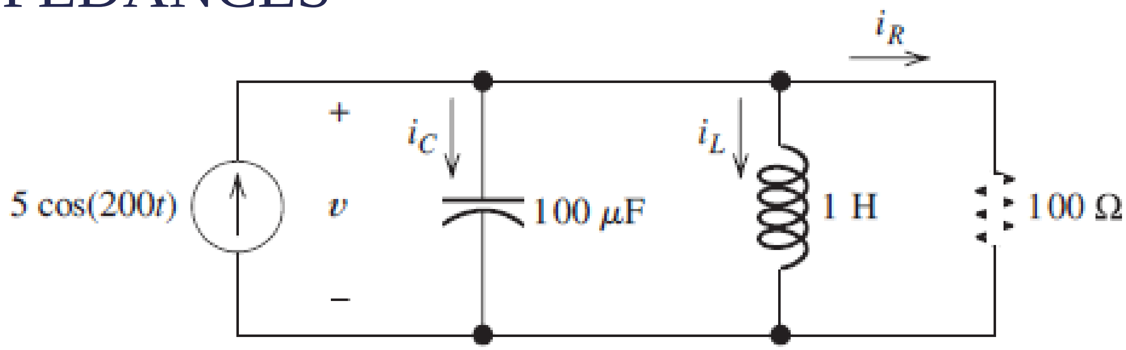
$$\Rightarrow \quad \mathbf{V}_R = R\mathbf{I} = 7.07\angle -135^\circ$$

$$\Rightarrow \quad \mathbf{V}_L = j\omega L\mathbf{I} = 7.07\angle -45^\circ$$



5.4 CIRCUIT ANALYSIS WITH PHASORS AND COMPLEX IMPEDANCES

- Exercise



$$Z = \frac{1}{1/100 + 1/(-j50) + 1/(+j200)} = 55.47\angle -56.31^\circ \ \Omega$$

$$\Longrightarrow \mathbf{V} = Z\mathbf{I} = 277.4\angle -56.31^\circ \ \text{V}$$

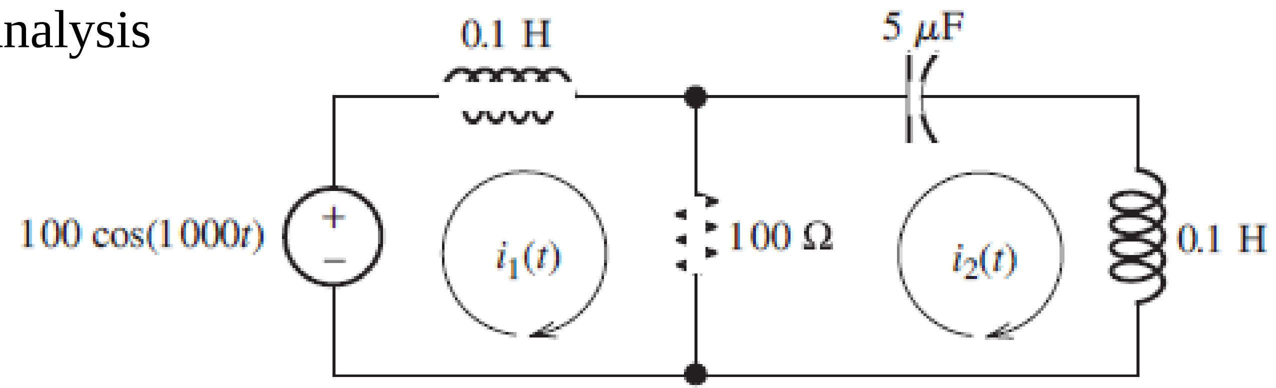
$$\Longrightarrow \mathbf{I}_C = \mathbf{V}/(-j50) = 5.547\angle 33.69^\circ \ A$$

$$\Longrightarrow \mathbf{I}_L = \mathbf{V}/(j200) = 1.387\angle -146.31^\circ \ A$$

$$\Longrightarrow \mathbf{I}_R = \mathbf{V}/(100) = 2.774\angle -56.31^\circ \ A$$

5.4 CIRCUIT ANALYSIS WITH PHASORS AND COMPLEX IMPEDANCES

- Mesh-Current Analysis



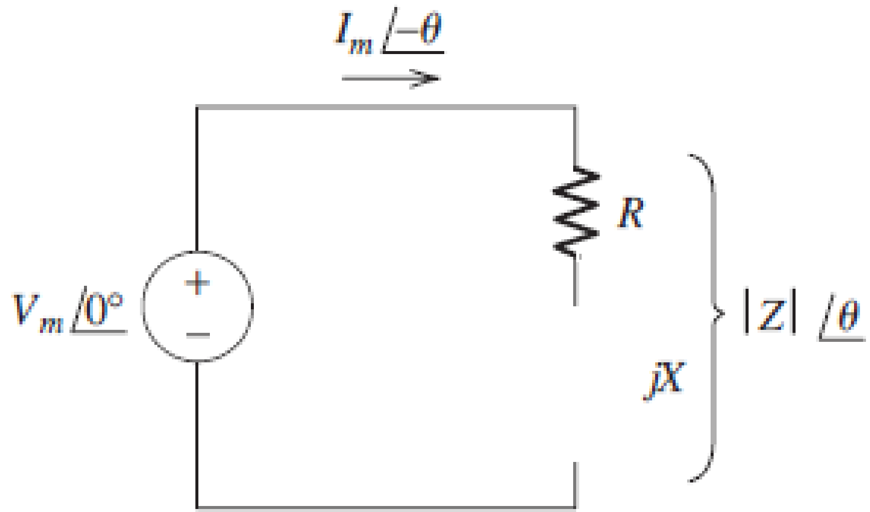
$$j100\mathbf{I}_1 + 100(\mathbf{I}_1 - \mathbf{I}_2) = 100$$
$$-j200\mathbf{I}_2 + j100\mathbf{I}_2 + 100(\mathbf{I}_2 - \mathbf{I}_1) = 0$$

$$\Rightarrow \quad (100 + j100)\mathbf{I}_1 - 100\mathbf{I}_2 = 100$$
$$-100\mathbf{I}_1 + (100 - j100)\mathbf{I}_2 = 0$$

$$\Rightarrow \quad \mathbf{I}_1 = 1.414\angle -45^\circ\ A$$
$$\mathbf{I}_2 = 1\angle 0^\circ\ A$$

5.5 POWER IN AC CIRCUITS

- Power delivered to a general load



$$v(t) = V_m \cos(\omega t)$$

$$\Rightarrow \quad \mathbf{V} = V_m \angle 0°$$

$$Z = |Z| \angle \theta = R + jX$$

$$\Rightarrow \quad \mathbf{I} = \frac{\mathbf{V}}{Z} = \frac{V_m \angle 0°}{|Z| \angle \theta} = I_m \angle -\theta$$

$$I_m = \frac{V_m}{|Z|}$$

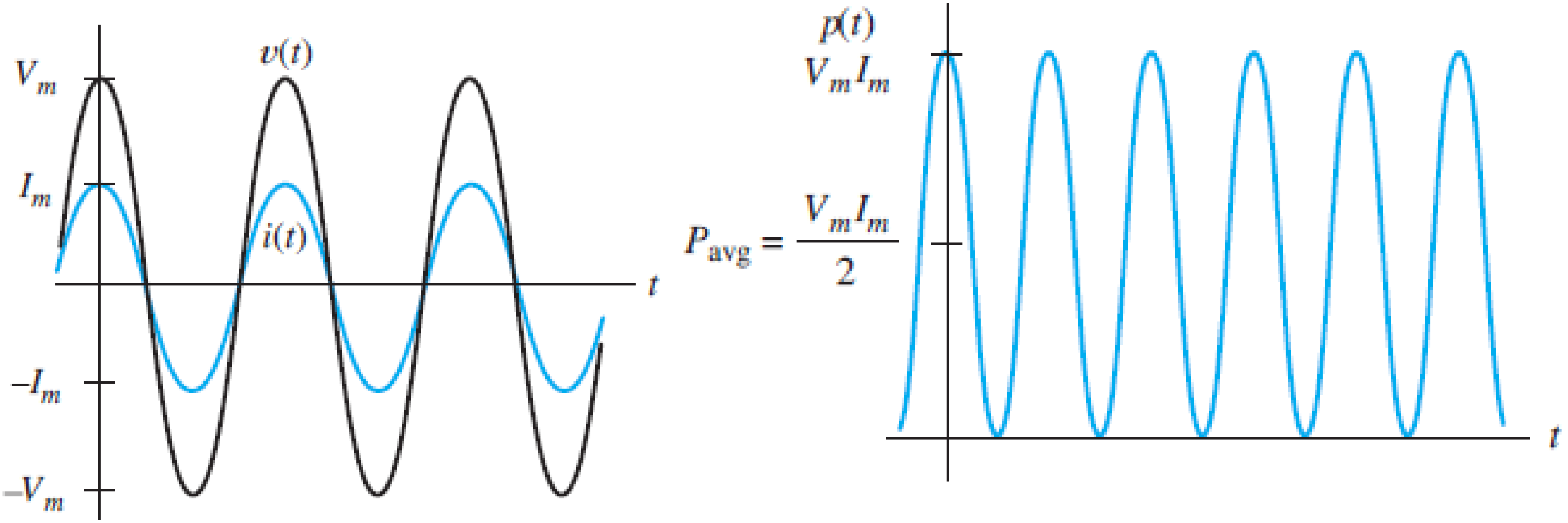
5.5 POWER IN AC CIRCUITS

- Current, Voltage, and Power for a Resistive Load

$$v(t) = V_m \cos(\omega t)$$

$$i(t) = I_m \cos(\omega t)$$

$$p(t) = v(t)i(t) = V_m I_m \cos^2(\omega t)$$



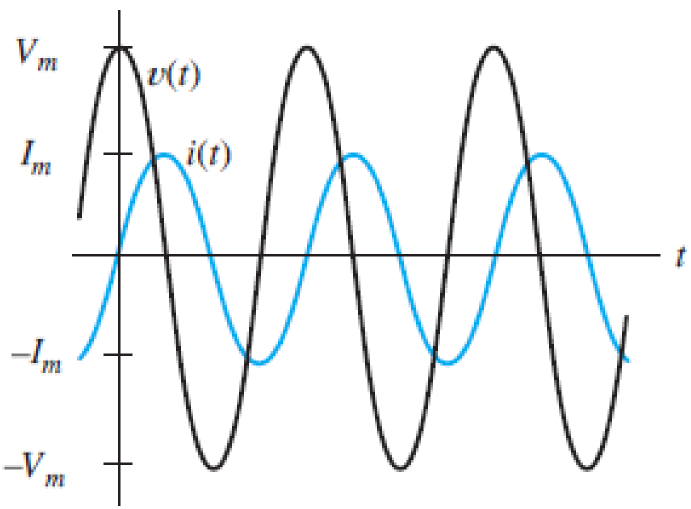
5.5 POWER IN AC CIRCUITS

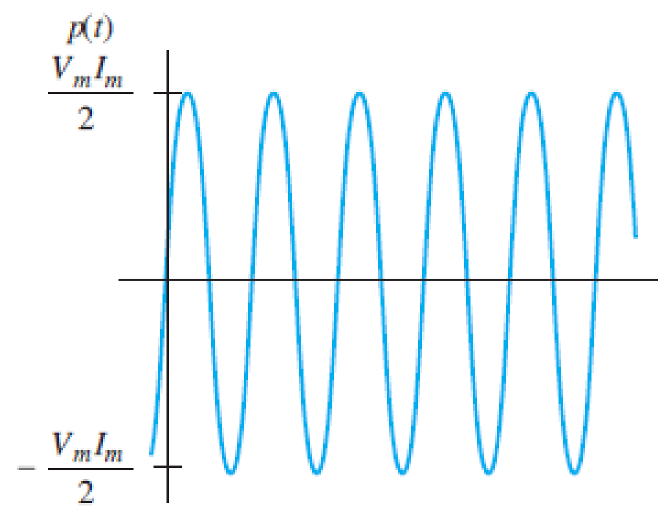
- Current, Voltage, and Power for a Inductive Load $Z = \omega L \angle 90^\circ$

$$v(t) = V_m \cos(\omega t)$$

$$i(t) = I_m \cos(\omega t - 90^\circ) = I_m \sin(\omega t)$$

$$p(t) = v(t)i(t) = V_m I_m \cos(\omega t)\sin(\omega t) = \frac{V_m I_m}{2}\sin(2\omega t)$$





Reactive Power

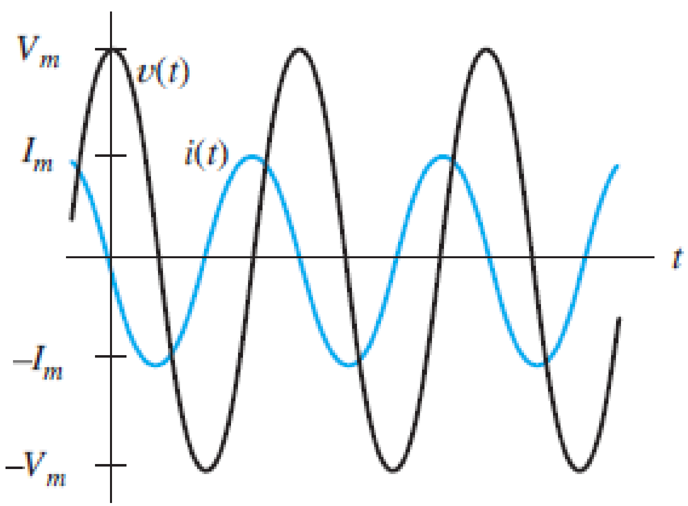
5.5 POWER IN AC CIRCUITS

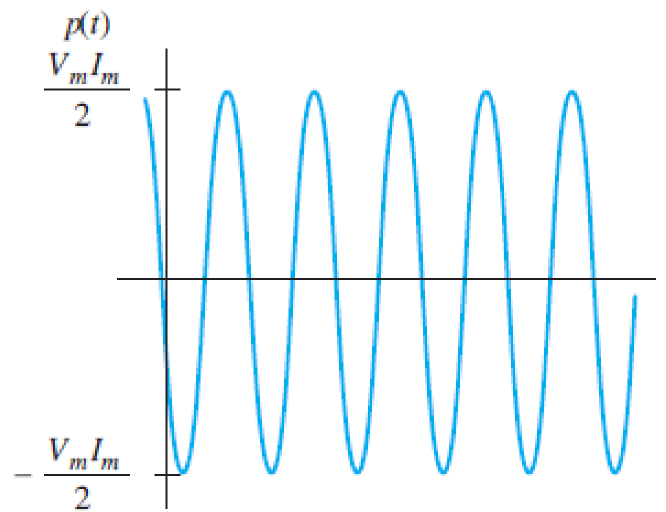
- Current, Voltage, and Power for a Capacitive Load $Z = (1/\omega C)\ \angle -90^\circ$

$$v(t) = V_m \cos(\omega t)$$

$$i(t) = I_m \cos(\omega t + 90^\circ) = -I_m \sin(\omega t)$$

$$p(t) = v(t)i(t) = -V_m I_m \cos(\omega t)\sin(\omega t) = -\frac{V_m I_m}{2}\sin(2\omega t)$$





Reactive Power

5.5 POWER IN AC CIRCUITS

- Power Calculations for a General RLC Load

$$v(t) = V_m \cos(\omega t)$$

$$i(t) = I_m \cos(\omega t - \theta)$$

$$p(t) = V_m I_m \cos(\omega t) \cos(\omega t - \theta)$$

$$\cos(\omega t - \theta) = \cos(\theta)\cos(\omega t) + \sin(\theta)\sin(\omega t)$$

$$\Longrightarrow \quad p(t) = V_m I_m \cos(\theta)\cos^2(\omega t) + V_m I_m \sin(\theta)\cos(\omega t)\sin(\omega t)$$

5.5 POWER IN AC CIRCUITS

- Power Calculations for a General RLC Load

$$\cos^2(\omega t) = \frac{1}{2} + \frac{1}{2}\cos(2\omega t)$$

$$\cos(\omega t)\sin(\omega t) = \frac{1}{2}\sin(2\omega t)$$

$$\Longrightarrow p(t) = \frac{V_m I_m}{2}\cos(\theta)[1 + \cos(2\omega t)] + \frac{V_m I_m}{2}\sin(\theta)\sin(2\omega t)$$

average power P $\Longrightarrow$ $P = \frac{V_m I_m}{2}\cos(\theta)$

$$\Longrightarrow P = V_{\text{rms}} I_{\text{rms}} \cos(\theta)$$

Power Factor

5.5 POWER IN AC CIRCUITS

- Power Factor

$$\mathrm{PF} = \cos(\theta)$$

phase of the voltage θ_v minus the phase of the current θ_i.

$$\theta = \theta_v - \theta_i$$

θ is called the **power angle**

5.5 POWER IN AC CIRCUITS

- Reactive Power

$$Q = V_{\text{rms}} I_{\text{rms}} \sin(\theta)$$

  - ✓ units are usually given as *V*olt *A*mperes *R*eactive (VARs)

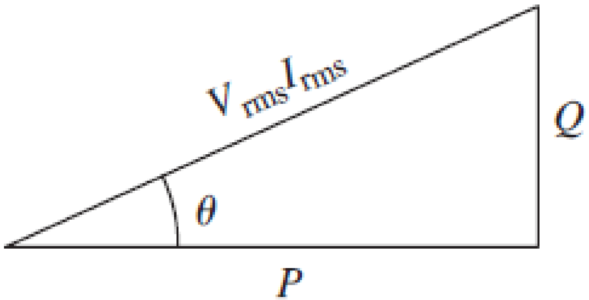
- Apparent Power

$$\text{apparent power} = V_{\text{rms}} I_{\text{rms}}$$

  - ✓ units are volt-amperes (VA)

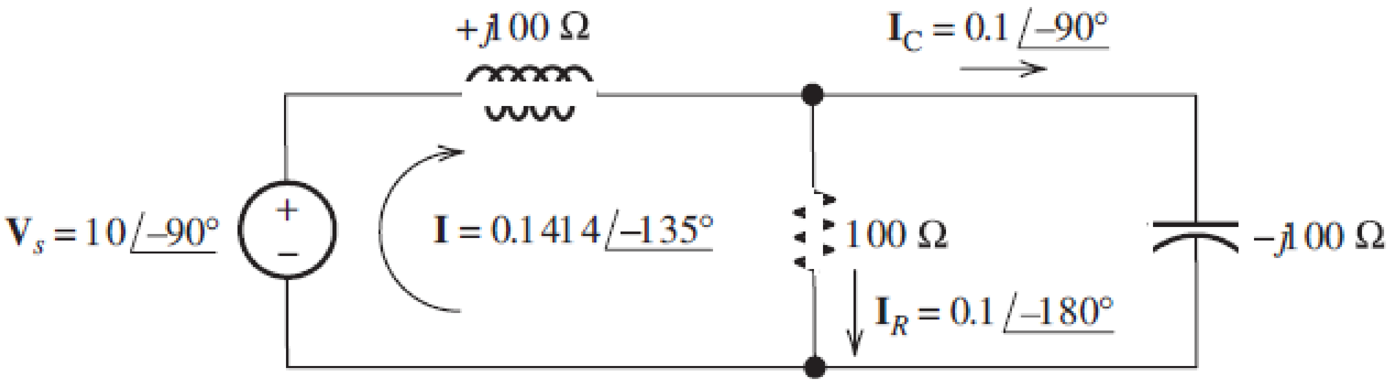
- Power Triangle

$$\Rightarrow \quad P^2 + Q^2 = (V_{\text{rms}} I_{\text{rms}})^2$$



5.5 POWER IN AC CIRCUITS

Example 5.7 AC Power Calculations



$$\theta = -90^\circ - (-135^\circ) = 45^\circ$$

$$V_{srms} = \frac{|\mathbf{V}_s|}{\sqrt{2}} = \frac{10}{\sqrt{2}} = 7.071 \text{ V}$$

$$I_{rms} = \frac{|\mathbf{I}|}{\sqrt{2}} = \frac{0.1414}{\sqrt{2}} = 0.1 \text{ A}$$

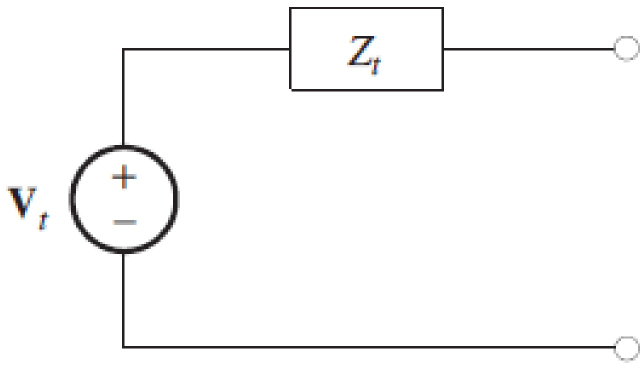
$$\Rightarrow P = V_{srms} I_{rms} \cos(\theta) = 7.071 \times 0.1 \cos(45^\circ) = 0.5 \text{ W}$$

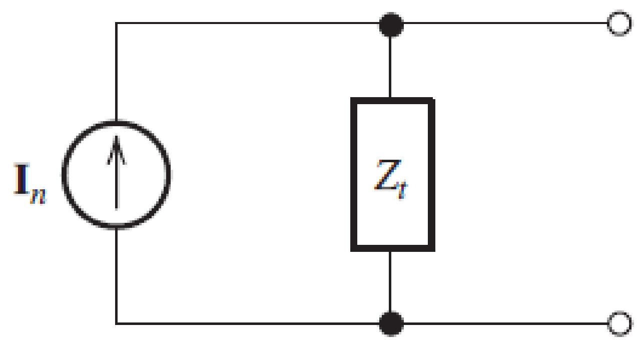
$$\Rightarrow Q = V_{srms} I_{rms} \sin(\theta) = 7.071 \times 0.1 \sin(45^\circ) = 0.5 \text{ VAR}$$

5.6 THÉVENIN AND NORTON EQUIVALENT CIRCUITS

- Valid for only steady-state operation
- Same as before (with Complex Impedance)

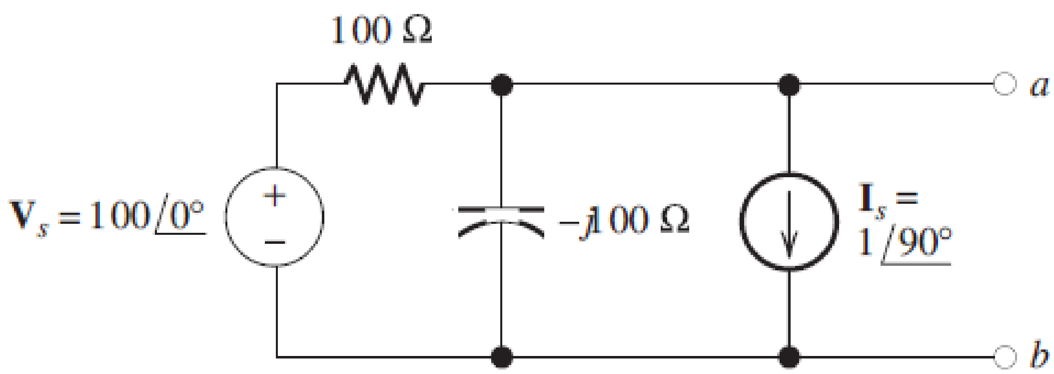
$$\mathbf{V}_t = \mathbf{V}_{oc} \qquad \mathbf{I}_n = \mathbf{I}_{sc} \qquad Z_t = \frac{\mathbf{V}_{oc}}{\mathbf{I}_{sc}} = \frac{\mathbf{V}_t}{\mathbf{I}_{sc}}$$





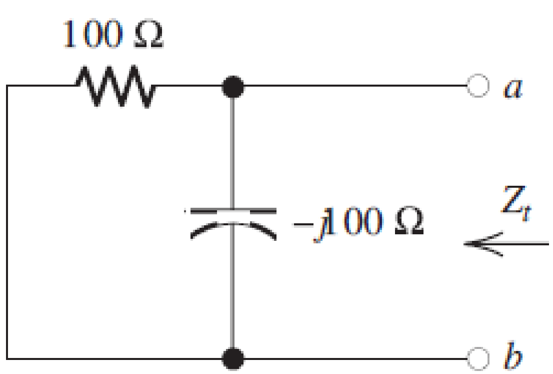
5.6 THÉVENIN AND NORTON EQUIVALENT CIRCUITS

Example 5.10 Thévenin and Norton Equivalents



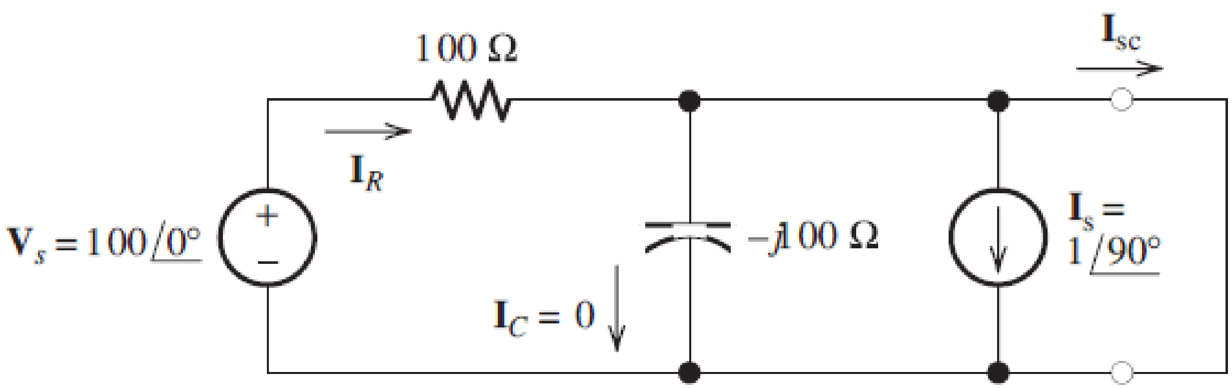
$$Z_t = \frac{1}{1/100 + 1/(-j100)} = \frac{1}{0.01 + j0.01}$$

$$= \frac{1}{0.01414\angle 45°} = 70.71\angle -45° = 50 - j50\ \Omega$$



5.6 THÉVENIN AND NORTON EQUIVALENT CIRCUITS

Example 5.10 Thévenin and Norton Equivalents



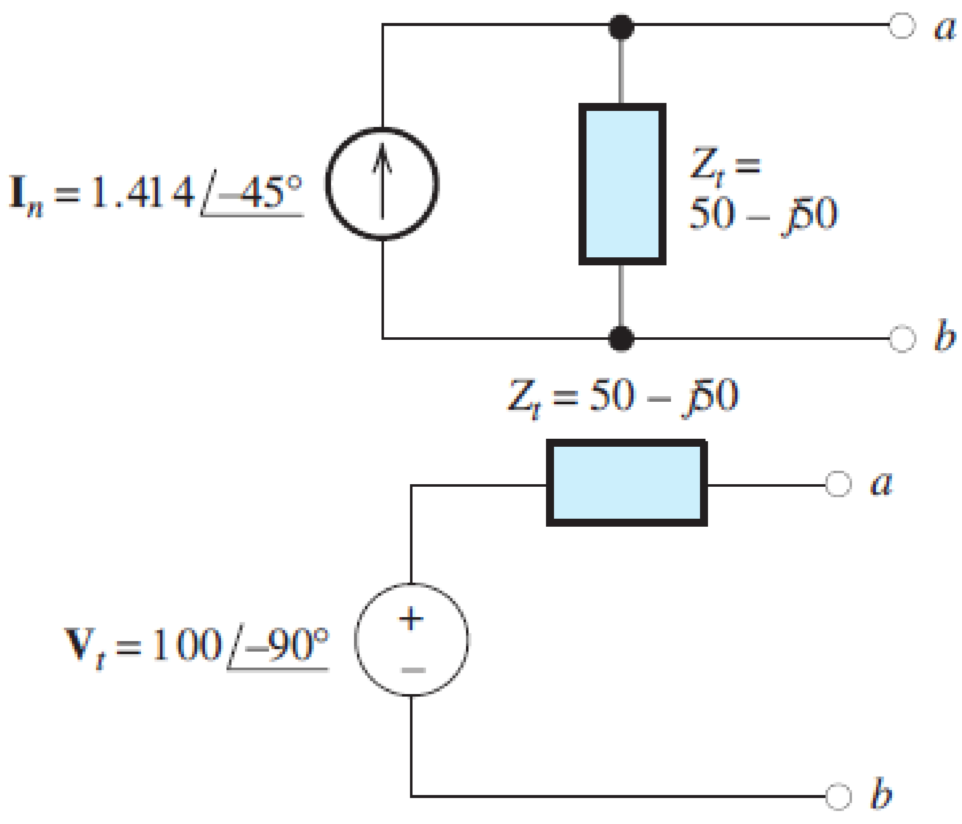
$$\mathbf{I}_R = \frac{\mathbf{V}_s}{100} = \frac{100}{100} = 1\ \angle 0°\ \text{A}$$

$$\Rightarrow \mathbf{I}_{sc} = \mathbf{I}_R - \mathbf{I}_s = 1 - 1\ \angle 90° = 1 - j = 1.414\ \angle -45°\ \text{A}$$

$$\Rightarrow \mathbf{V}_t = \mathbf{I}_{sc} Z_t = 1.414\ \angle -45° \times 70.71\ \angle -45° = 100\ \angle -90°\ \text{V}$$

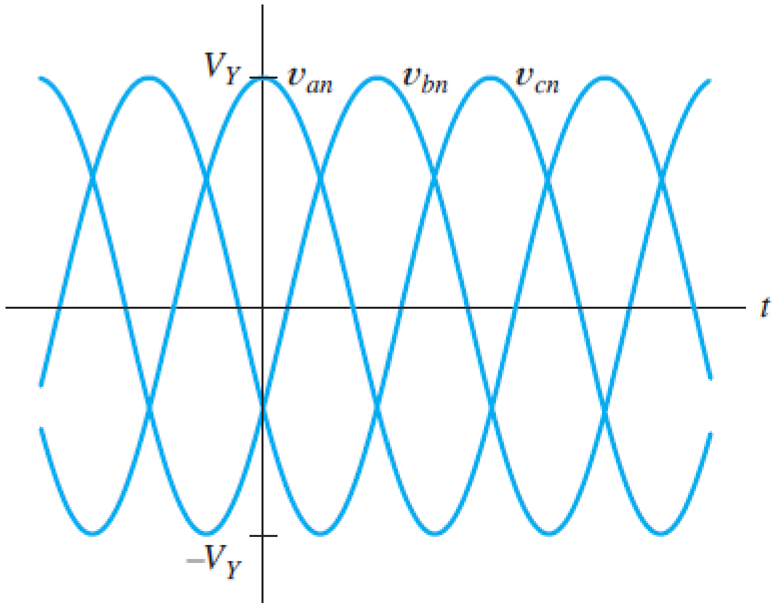
5.6 THÉVENIN AND NORTON EQUIVALENT CIRCUITS

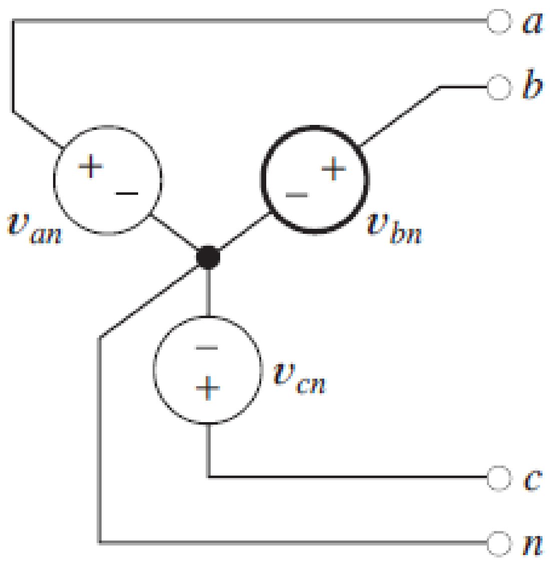
Example 5.10 Thévenin and Norton Equivalents



5.7 BALANCED THREE-PHASE CIRCUITS

- Balanced three-phase source
  - three equal-amplitude ac voltages having phases that are 120 degrees apart
  - wye connected (Y connected)





$$v_{an}(t) = V_Y \cos(\omega t)$$

$$v_{bn}(t) = V_Y \cos(\omega t - 120^\circ)$$

$$v_{cn}(t) = V_Y \cos(\omega t + 120^\circ)$$

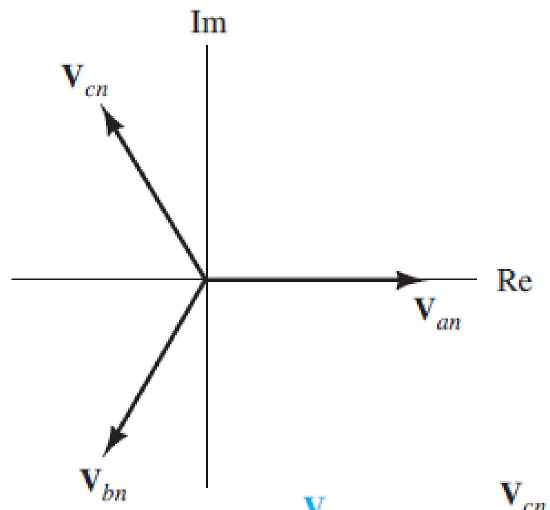
5.7 BALANCED THREE-PHASE CIRCUITS

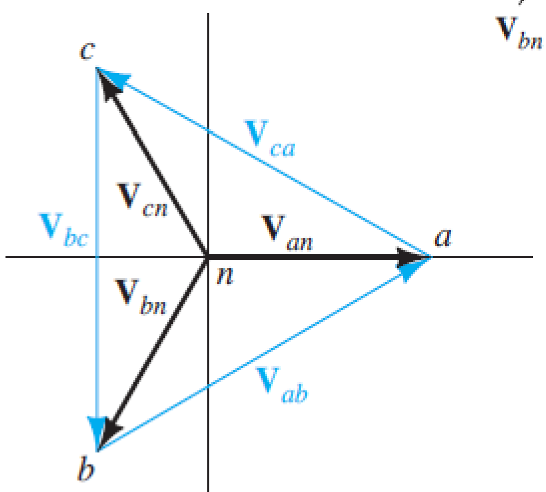
- Phasor Notation

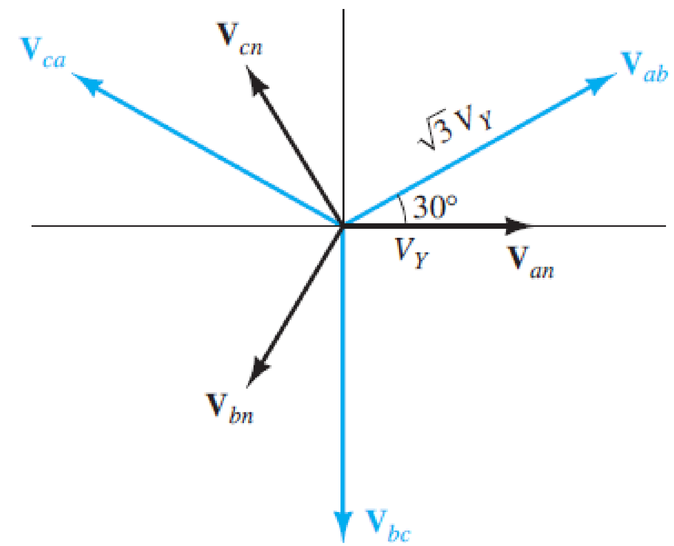
$$\mathbf{V}_{an} = V_Y \angle 0^\circ$$

$$\mathbf{V}_{bn} = V_Y \angle -120^\circ$$

$$\mathbf{V}_{cn} = V_Y \angle 120^\circ$$

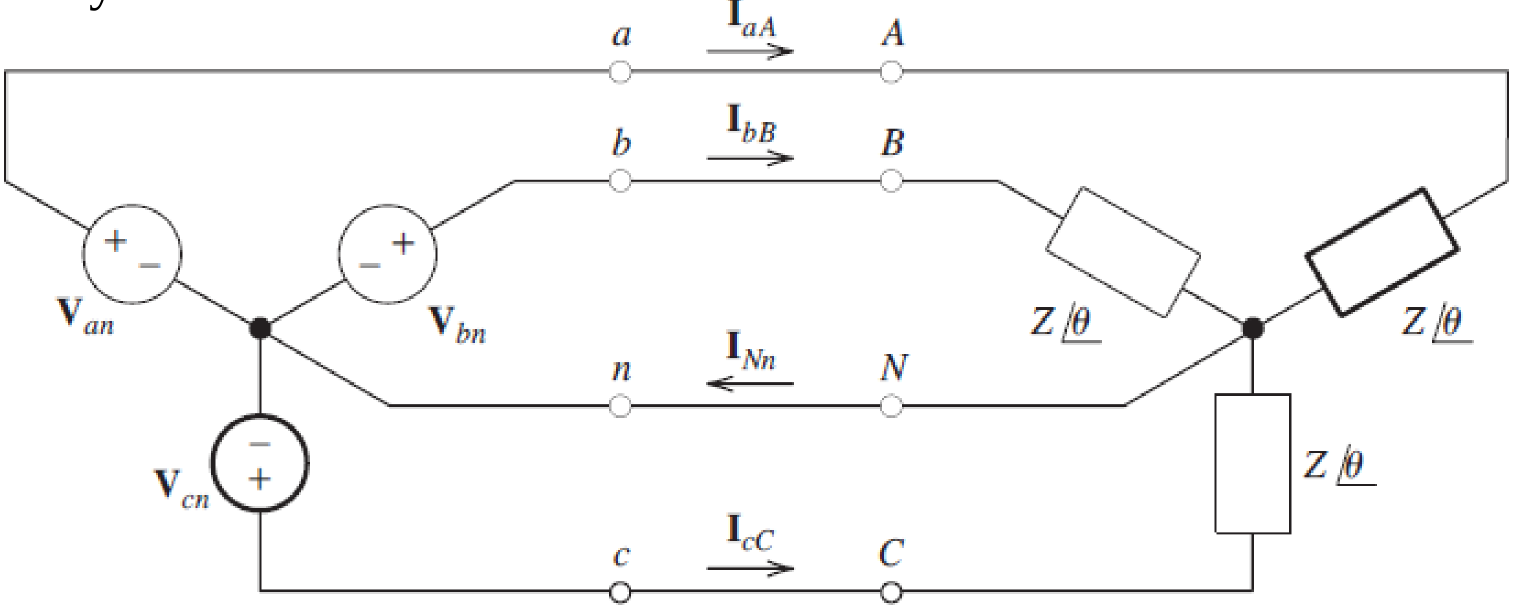






5.7 BALANCED THREE-PHASE CIRCUITS

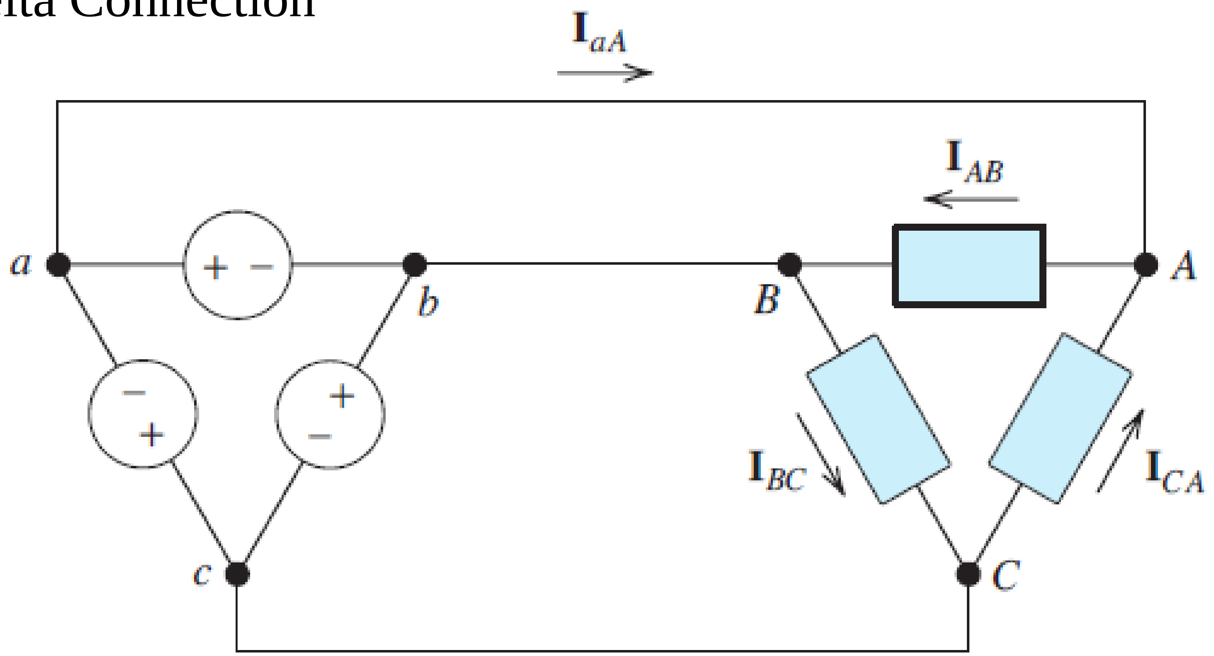
- Wye-Wye Connection



$$\mathbf{I}_{aA} = \frac{\mathbf{V}_{an}}{Z\angle\theta} = \frac{V_Y\angle 0^\circ}{Z\angle\theta} = I_L\angle -\theta$$

5.7 BALANCED THREE-PHASE CIRCUITS

- Delta-Delta Connection



$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{AB}}{Z_\Delta \angle\theta} = \frac{\mathbf{V}_{ab}}{Z_\Delta \angle\theta} = \frac{V_L \angle 30°}{Z_\Delta \angle\theta} = \frac{V_L}{Z_\Delta} \angle 30° - \theta$$

Exercises

- P5.6
- P5.12
- P5.13
- P5.23
- P5.24
- P5.25
- P5.35
- P5.37
- P5.44
- P5.46
- P5.49
- P5.52
- P5.67
- P5.78
- P5.87
- P5.91
- P5.95
- P5.96
- P5.99
- T5.1
- T5.2
- T5.3
- T5.4
- T5.5
- T5.6