دانشگاه سمنان

Semnan University
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دانشکده مهندسی مکانیک

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درس استاتیک

STATICS

Chapter 6 - Friction
Class Lecture

❑ CONTENTS:

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6.1 INTRODUCTION

- We have usually assumed that the forces of action and reaction between contacting surfaces act normal to the surfaces. This assumption characterizes the interaction between smooth surfaces.
- Although this ideal assumption often involves only a relatively small error, there are many problems in which we must consider the ability of contacting surfaces to support tangential as well as normal forces.
- Tangential forces generated between contacting surfaces are called friction forces and occur to some degree in the interaction between all real surfaces.
- Whenever a tendency exists for one contacting surface to slide along another surface, the friction forces developed are always in a direction to oppose this tendency

6.1 INTRODUCTION

- In some types of machines and processes we want to minimize the retarding effect of friction forces.
  - Examples are bearings of all types, power screws, gears, the flow of fluids in pipes, and the propulsion of aircraft and missiles through the atmosphere.

- In other situations we wish to maximize the effects of friction.
  - Examples are as in brakes, clutches, belt drives, and wedges.

6.1 INTRODUCTION

- Friction forces are present throughout nature and exist in all machines no matter how accurately constructed or carefully lubricated.
- A machine or process in which friction is small enough to be neglected is said to be ideal. When friction must be taken into account, the machine or process is termed real.
- In all cases where there is sliding motion between parts, the friction forces result in a loss of energy which is dissipated in the form of heat.
- Wear is another effect of friction.

6.1 Introduction

- Section A: Frictional Phenomena
  - 6/2 Types of Friction
  - 6/3 Dry Friction

- Section B: Applications of Friction in Machines
  - 6/4 Wedges
  - 6/5 Screws
  - 6/6 Journal Bearings
  - 6/7 Thrust Bearings; Disk Friction
  - 6/8 Flexible Belts
  - 6/9 Rolling Resistance

6.2 TYPES OF FRICTION

- (a) Dry Friction:
  - Dry friction occurs when the unlubricated surfaces of two solids are in contact under a condition of sliding or a tendency to slide.
- (b) Fluid Friction:
  - Fluid friction occurs when adjacent layers in a fluid (liquid or gas) are moving at different velocities.
- (c) Internal Friction:
  - Internal friction occurs in all solid materials which are subjected to cyclical loading.

6.2 Types of Friction

- **(a) Dry Friction.**
  - Dry friction occurs when the unlubricated surfaces of two solids are in contact under a condition of sliding or a tendency to slide.
  - A friction force tangent to the surfaces of contact occurs both during the interval leading up to impending slippage and while slippage takes place.
  - The direction of this friction force always opposes the motion or impending motion.
  - This type of friction is also called **Coulomb friction**.

6.2 TYPES OF FRICTION

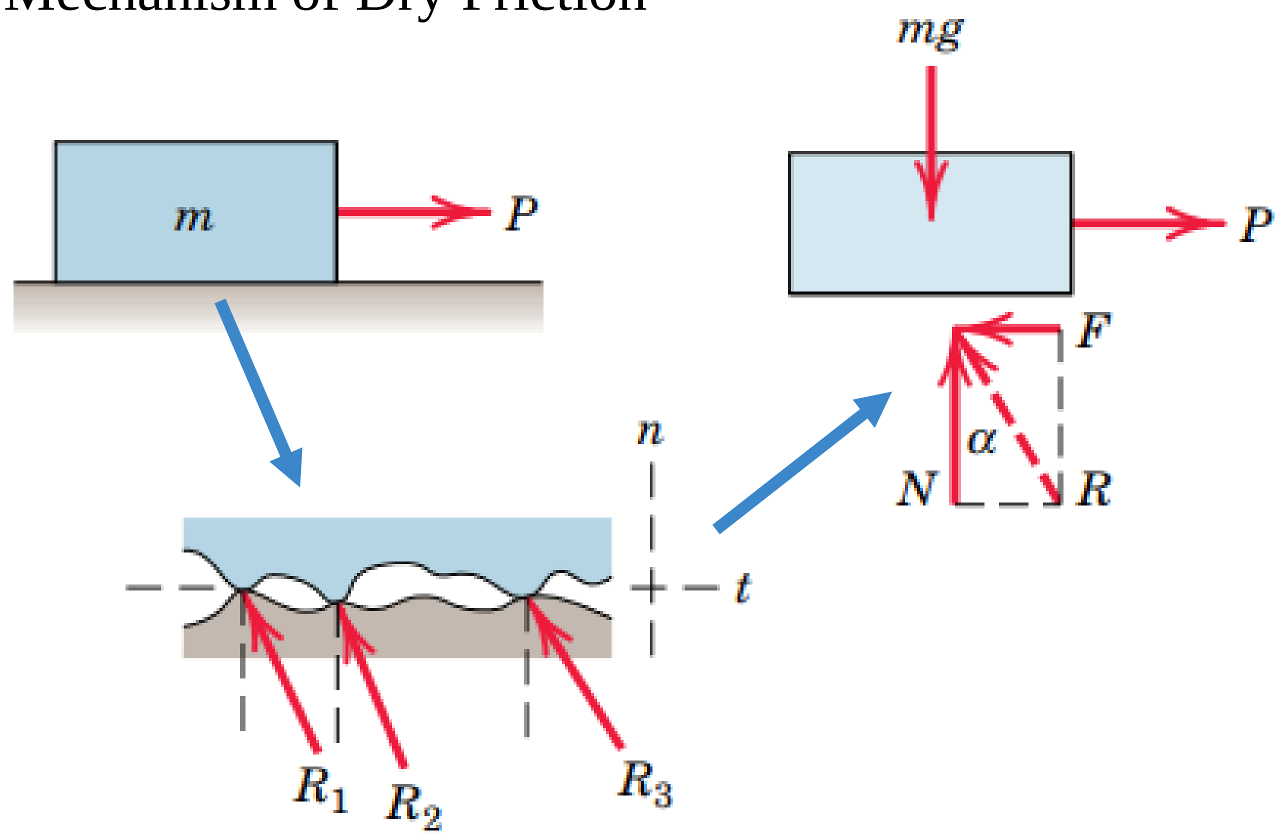
- **(b) Fluid Friction.**
  - Fluid friction occurs when adjacent layers in a fluid (liquid or gas) are moving at different velocities.
  - This motion causes frictional forces between fluid elements, and these forces depend on the relative velocity between layers.
  - When there is no relative velocity, there is no fluid friction.
  - Fluid friction depends not only on the velocity gradients within the fluid but also on the viscosity of the fluid, which is a measure of its resistance to shearing action between fluid layers.
  - Fluid friction is treated in the study of fluid mechanics and will not be discussed further.

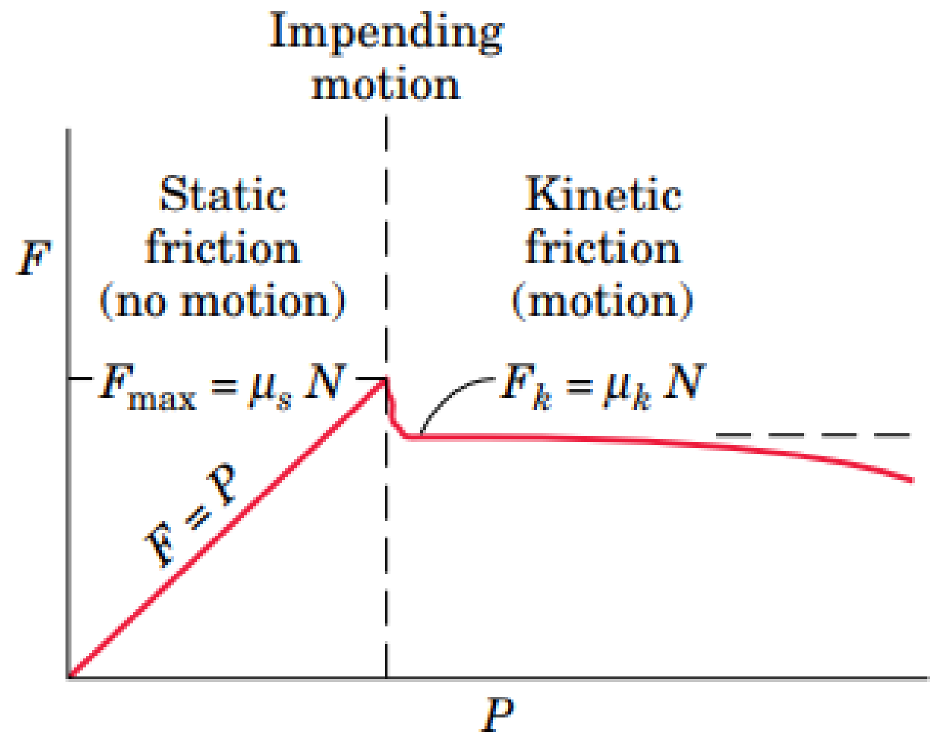
6.2 TYPES OF FRICTION

- **(c) Internal Friction.**
  - Internal friction occurs in all solid materials which are subjected to cyclical loading.
  - For highly elastic materials the recovery from deformation occurs with very little loss of energy due to internal friction.
  - For materials which have low limits of elasticity and which undergo appreciable plastic deformation during loading, a considerable amount of internal friction may accompany this deformation.
  - The mechanism of internal friction is associated with the action of shear deformation, which is discussed in references on materials science.

6.3 DRY FRICTION

- Mechanism of Dry Friction





6.3 DRY FRICTION

- Static Friction
  - The region up to the point of slippage or impending motion is called the range of static friction
  - In this range the value of the friction force is determined by the equations of equilibrium.
  - This friction force may have any value from zero up to and including the maximum value.
  - For a given pair of mating surfaces the experiment shows that this maximum value of static friction F_{max} is proportional to the normal force N.

$$F_{max} = \mu_s N$$

✓ μ_s : the coefficient of static friction

6.3 Dry Friction

- Static Friction

$$F_{\max} = \mu_s N$$

- This Equation describes only the limiting or maximum value of the static friction force and not any lesser value.
- Thus, the equation applies only to cases where motion is impending with the friction force at its peak value.
- For a condition of static equilibrium when motion is not impending, the static friction force is:

$$\longrightarrow \quad F < \mu_s N$$

6.3 DRY FRICTION

- Kinetic Friction
  - After slippage occurs, a condition of kinetic friction accompanies the ensuing motion.
  - Kinetic friction force is usually somewhat less than the maximum static friction force.
  - The kinetic friction force F_k is also proportional to the normal force.

$$F_k = \mu_k N$$

✓ μ_k : the coefficient of kinetic friction

6.3 Dry Friction

- Factors Affecting Friction
  - Friction force is essentially independent of the apparent or projected area of contact.
  - Generation of high local temperatures and adhesion at contact points
  - Relative hardness of mating surfaces
  - Presence of thin surface films of oxide, oil, dirt, or other substances

- A comprehensive theory of dry friction must go beyond the mechanical explanation presented here. For example, there is evidence that molecular attraction may be an important cause of friction under conditions where the mating surfaces are in very close contact.

6.3 Dry Friction

- Some typical values of coefficients

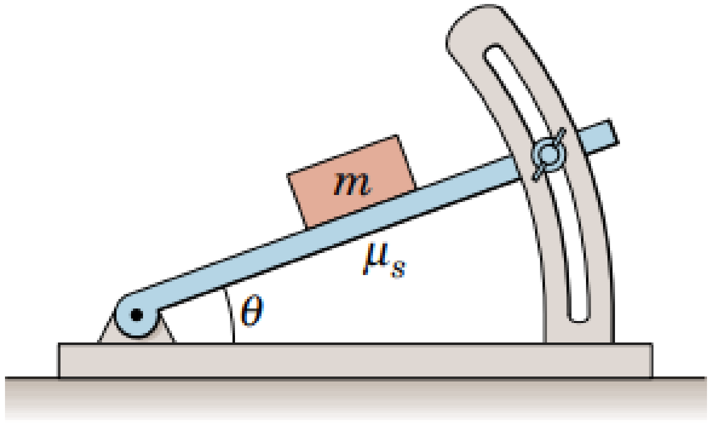
CONTACTING SURFACE	STATIC, μ_s	KINETIC, μ_k
Steel on steel (dry)	0.6	0.4
Steel on steel (greasy)	0.1	0.05
Teflon on steel	0.04	0.04
Steel on babbitt (dry)	0.4	0.3
Steel on babbitt (greasy)	0.1	0.07
Brass on steel (dry)	0.5	0.4
Brake lining on cast iron	0.4	0.3
Rubber tires on smooth pavement (dry)	0.9	0.8
Wire rope on iron pulley (dry)	0.2	0.15
Hemp rope on metal	0.3	0.2
Metal on ice		0.02

6.3 Dry Friction

- Types of Friction Problems
  - 1) The condition of impending motion is known to exist.
    - ✓ Body is in equilibrium and on the verge of slipping
    - ✓ Friction force equals the limiting static friction: $F_{max} = \mu_s N$
  - 2) Neither the condition of impending motion nor the condition of motion is known to exist.
    - ✓ First assume static equilibrium and then solve for the friction force F
      - (a) $F < (F_{max} = \mu_s N)$: The body is in static equilibrium as assumed.
      - (b) $F = (F_{max} = \mu_s N)$: Motion impends, the assumption of static equilibrium is valid.
      - (c) $F > (F_{max} = \mu_s N)$: The assumption of equilibrium is therefore invalid, and motion occurs.
  - 3) Relative motion is known to exist between the contacting surfaces
    - ✓ Kinetic coefficient of friction clearly applies $F = \mu_k N$

Sample Problem 6/1

Determine the maximum angle θ which the adjustable incline may have with the horizontal before the block of mass m begins to slip. The coefficient of static friction between the block and the inclined surface is μ_s.



$$[\Sigma F_x = 0] \qquad mg \sin\theta - F = 0 \qquad F = mg\sin\theta$$

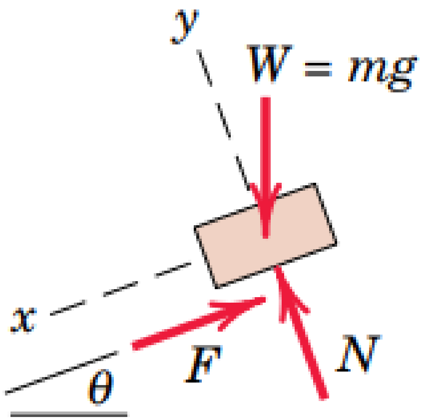
$$[\Sigma F_y = 0] \qquad -mg\cos\theta + N = 0 \qquad N = mg\cos\theta$$

$$\rightarrow \quad F/N = \tan\theta$$

$$F = F_{max} = \mu_s N$$

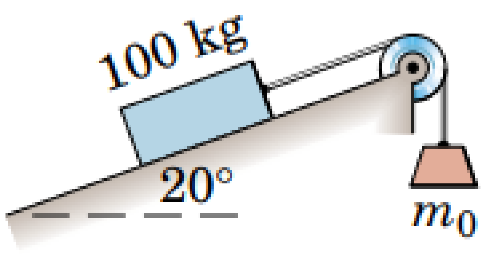
$$\rightarrow \quad \mu_s = \tan\theta_{max}$$

$$\rightarrow \quad \theta_{max} = \tan^{-1}\mu_s$$



Sample Problem 6/2

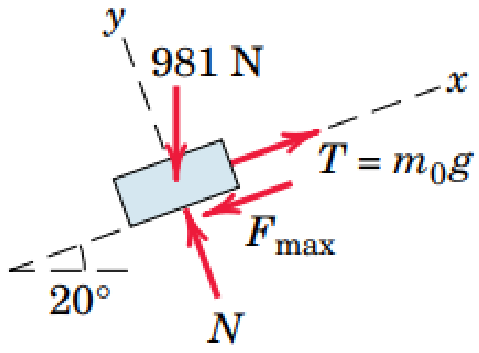
Determine the range of values which the mass m_0 may have so that the 100-kg block shown in the figure will neither start moving up the plane nor slip down the plane. The coefficient of static friction for the contact surfaces is 0.30.



$[\Sigma F_y = 0] \qquad N - 981 \cos 20° = 0 \qquad N = 922 \text{ N}$

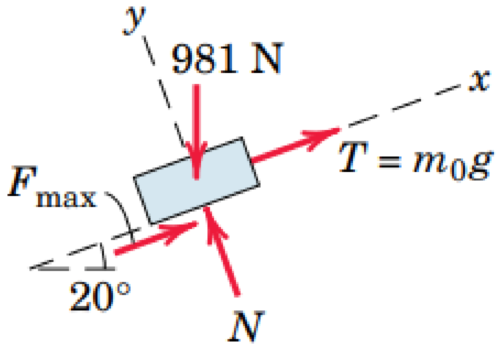
$[F_{max} = \mu_s N] \qquad F_{max} = 0.30(922) = 277 \text{ N}$

$[\Sigma F_x = 0] \qquad m_0(9.81) - 277 - 981 \sin 20° = 0 \qquad m_0 = 62.4 \text{ kg}$



Case I

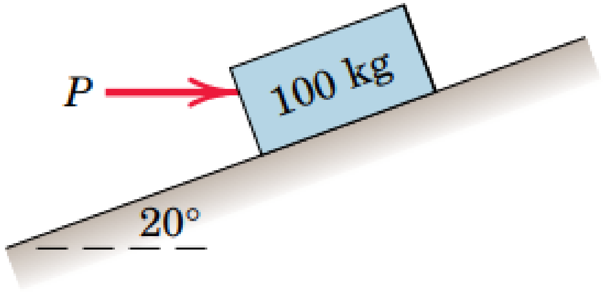
$[\Sigma F_x = 0] \qquad m_0(9.81) + 277 - 981 \sin 20° = 0 \qquad m_0 = 6.01 \text{ kg}$



Case II

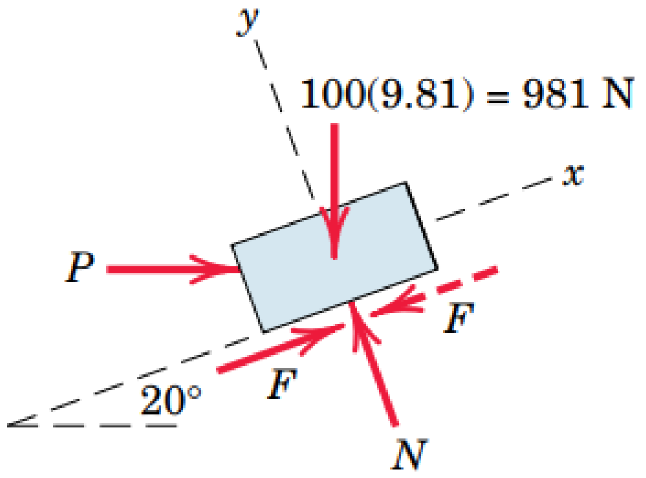
Sample Problem 6/3

Determine the magnitude and direction of the friction force acting on the 100-kg block shown if, first, $P = 500$ N and, second, $P = 100$ N. The coefficient of static friction is 0.20, and the coefficient of kinetic friction is 0.17. The forces are applied with the block initially at rest.



$[\Sigma F_x = 0]$ $\quad P \cos 20° + F - 981 \sin 20° = 0$

$[\Sigma F_y = 0]$ $\quad N - P \sin 20° - 981 \cos 20° = 0$



Case I. $P = 500$ N → $F = -134.3$ N $\quad N = 1093$ N

$[F_{max} = \mu_s N]$ $\quad F_{max} = 0.20(1093) = 219$ N

→ $F = 134.3$ N down the plane

Case II. $P = 100$ N → $F = 242$ N $\quad N = 956$ N

$[F_{max} = \mu_s N]$ $\quad F_{max} = 0.20(956) = 191.2$ N

→ $[F_k = \mu_k N]$ $\quad F = 0.17(956) = 162.5$ N up the plane

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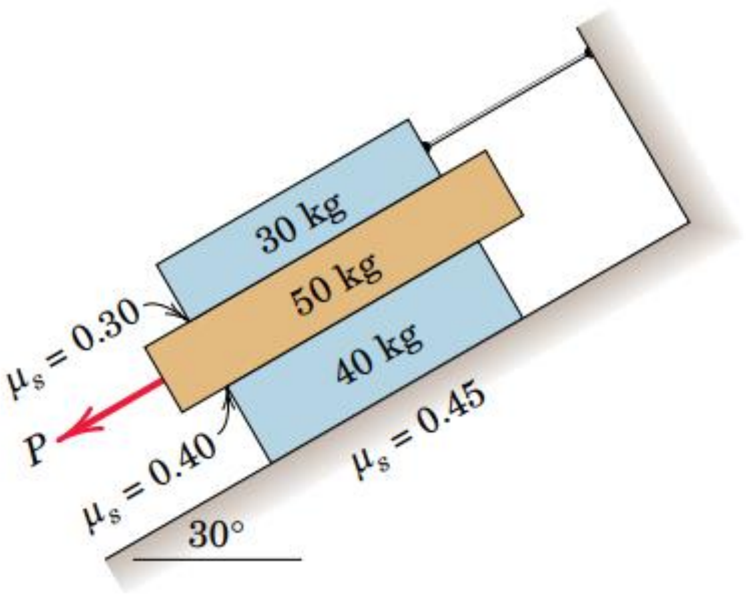
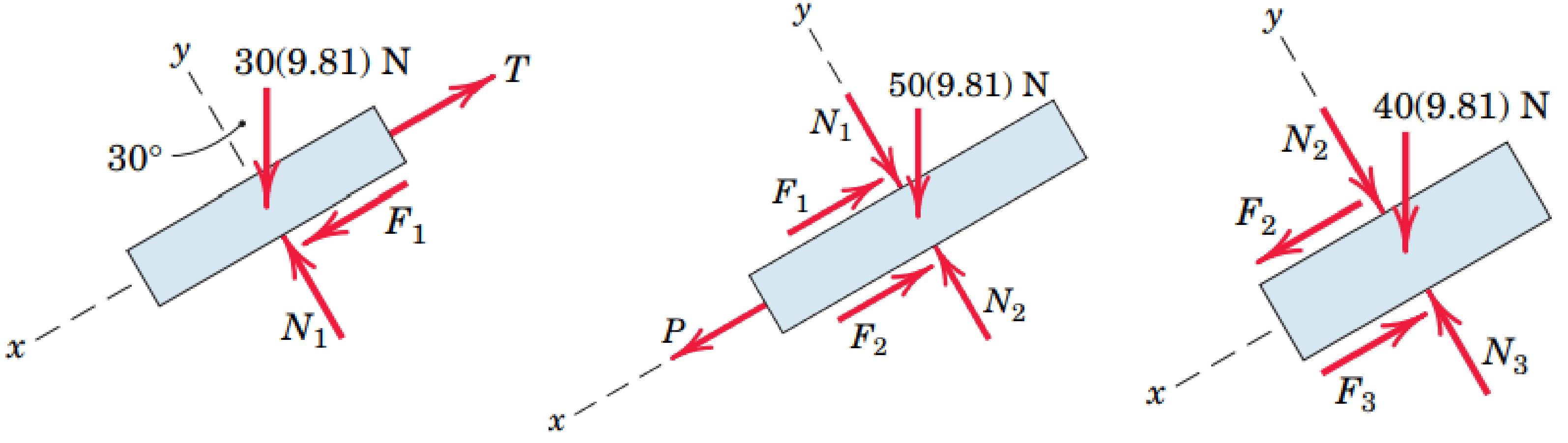
Sample Problem 6/5

The three flat blocks are positioned on the 30° incline as shown, and a force P parallel to the incline is applied to the middle block. The upper block is prevented from moving by a wire which attaches it to the fixed support. The coefficient of static friction for each of the three pairs of mating surfaces is shown. Determine the maximum value which P may have before any slipping takes place.





$[\Sigma F_y = 0]$	(30-kg)	$N_1 - 30(9.81)\cos 30° = 0$	$N_1 = 255$ N
	(50-kg)	$N_2 - 50(9.81)\cos 30° - 255 = 0$	$N_2 = 680$ N
	(40-kg)	$N_3 - 40(9.81)\cos 30° - 680 = 0$	$N_3 = 1019$ N

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There are two possible conditions for impending motion:

- ❑ 50-kg block slips and the 40-kg block remains in place
- ❑ 50- and 40-kg blocks move together with slipping occurring between the 40-kg block and the incline.

We will assume arbitrarily that only the 50-kg block slips, so that the 40-kg block remains in place.

$[F_{max} = \mu_s N] \qquad F_1 = 0.30(255) = 76.5\text{ N} \qquad F_2 = 0.40(680) = 272\text{ N}$

$[\Sigma F_x = 0] \qquad P - 76.5 - 272 + 50(9.81)\sin 30° = 0 \qquad P = 103.1\text{ N}$

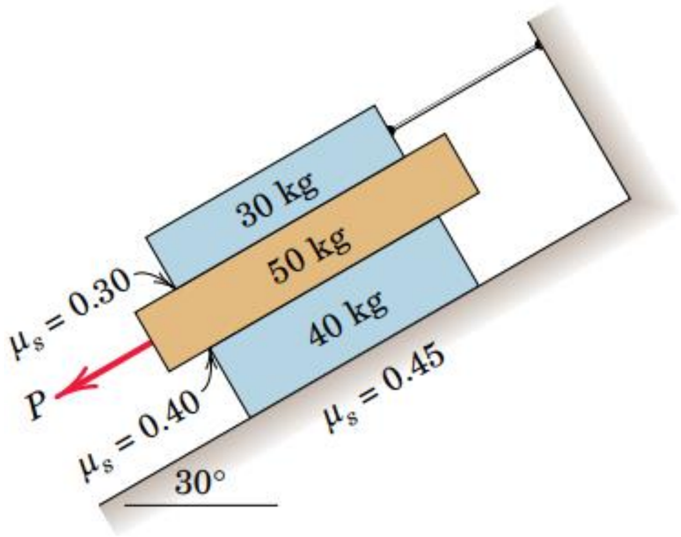
$[\Sigma F_x = 0] \qquad 272 + 40(9.81)\sin 30° - F_3 = 0 \qquad F_3 = 468\text{ N}$

maximum possible value of F_3 is $F_3 = \mu_s N_3 = 0.45(1019) = 459$ N.

$[\Sigma F_x = 0] \qquad F_2 + 40(9.81)\sin 30° - 459 = 0 \qquad F_2 = 263\text{ N}$

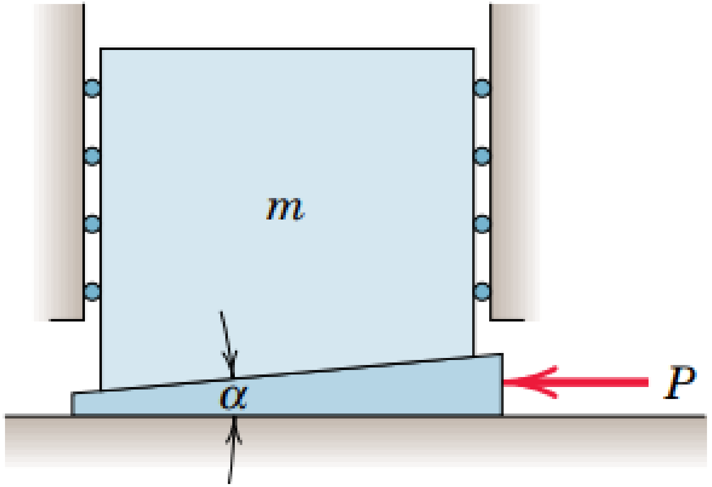
$[\Sigma F_x = 0] \qquad P + 50(9.81)\sin 30° - 263 - 76.5 = 0$

$P = 93.8\text{ N}$



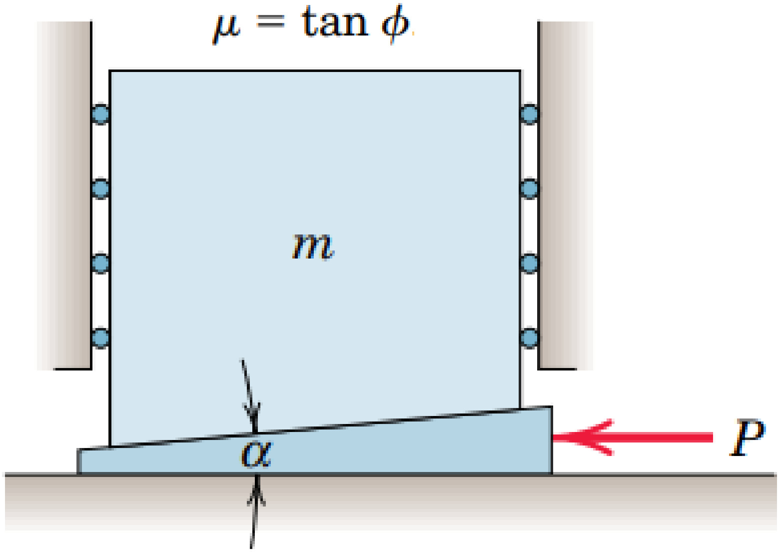
6.4 WEDGES

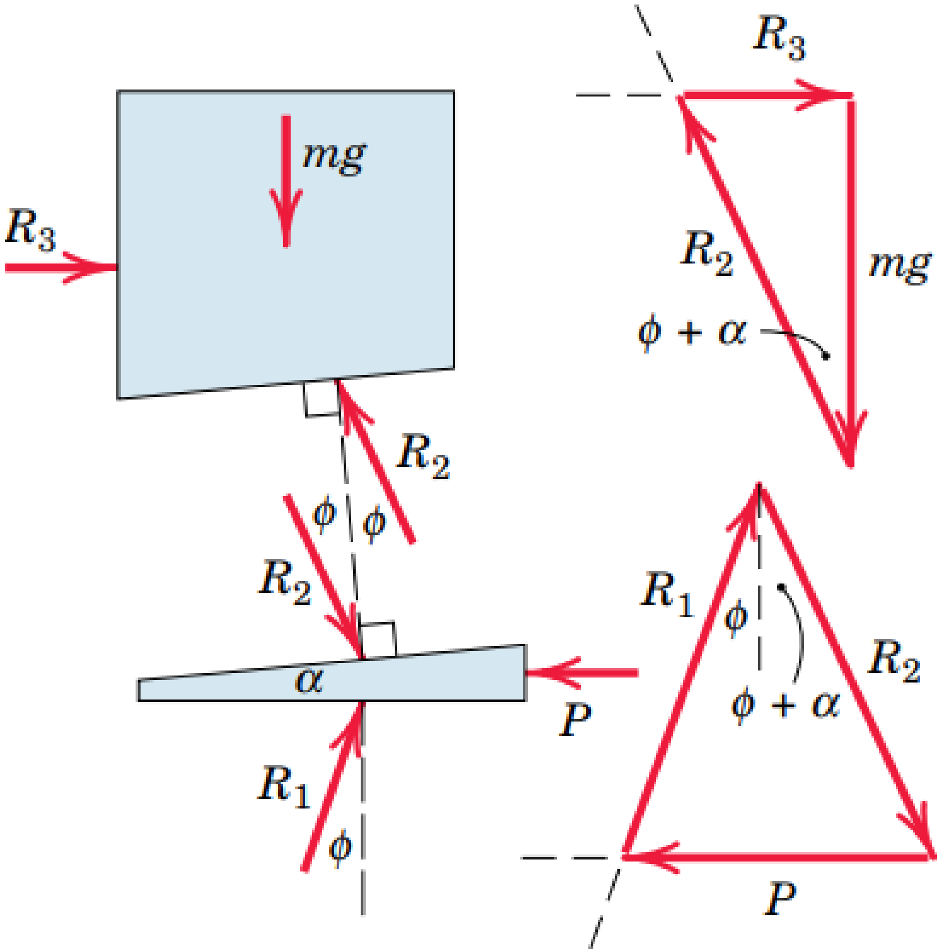
- A wedge is one of the simplest and most useful machines.
- A wedge is used to produce small adjustments in the position of a body or to apply large forces.
- Wedges largely depend on friction to function.



6.4 WEDGES

- Forces to raise load

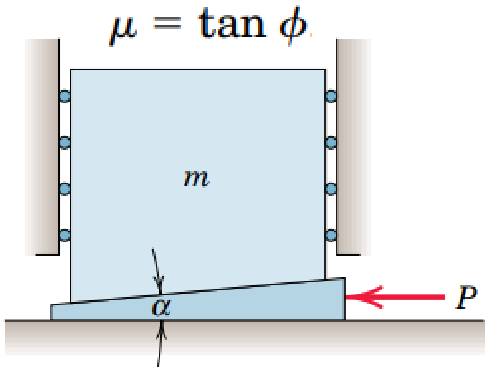


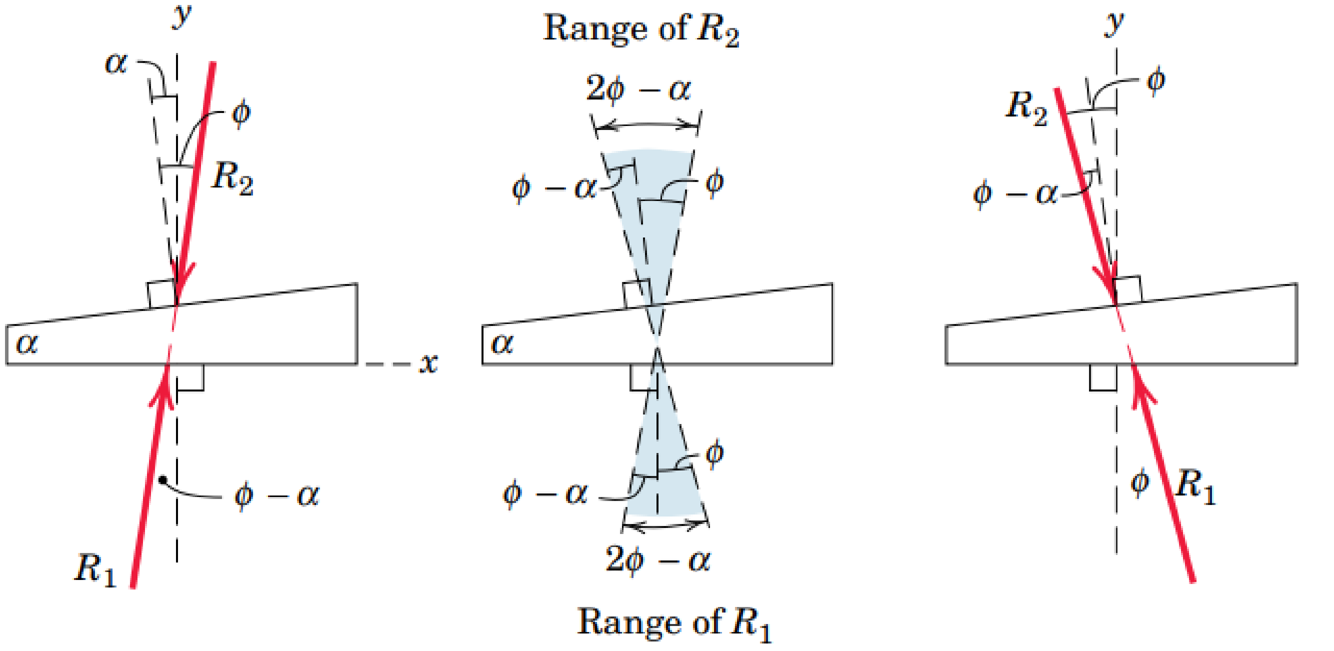


6.4 WEDGES

- When P is removed
  - In order for the wedge to slide out of its space, slippage must occur at both surfaces simultaneously; otherwise, the wedge is self-locking.
  - Simultaneous slippage is not possible if:

$$\rightarrow \alpha < 2\phi$$

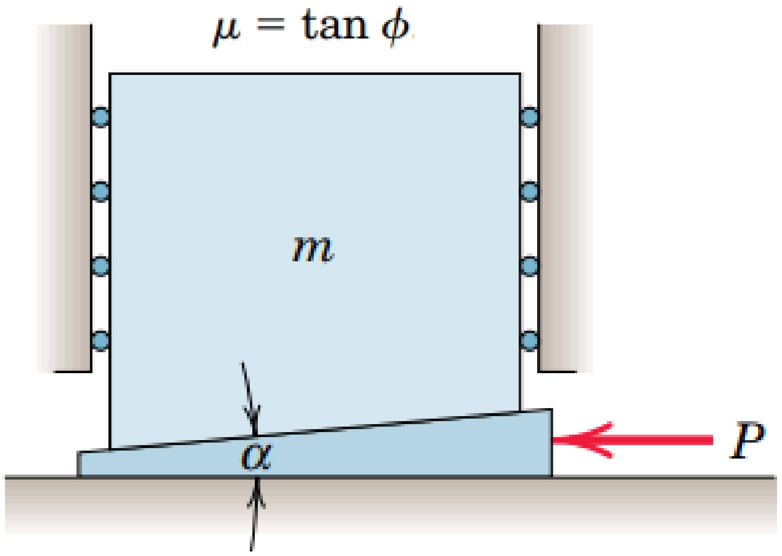


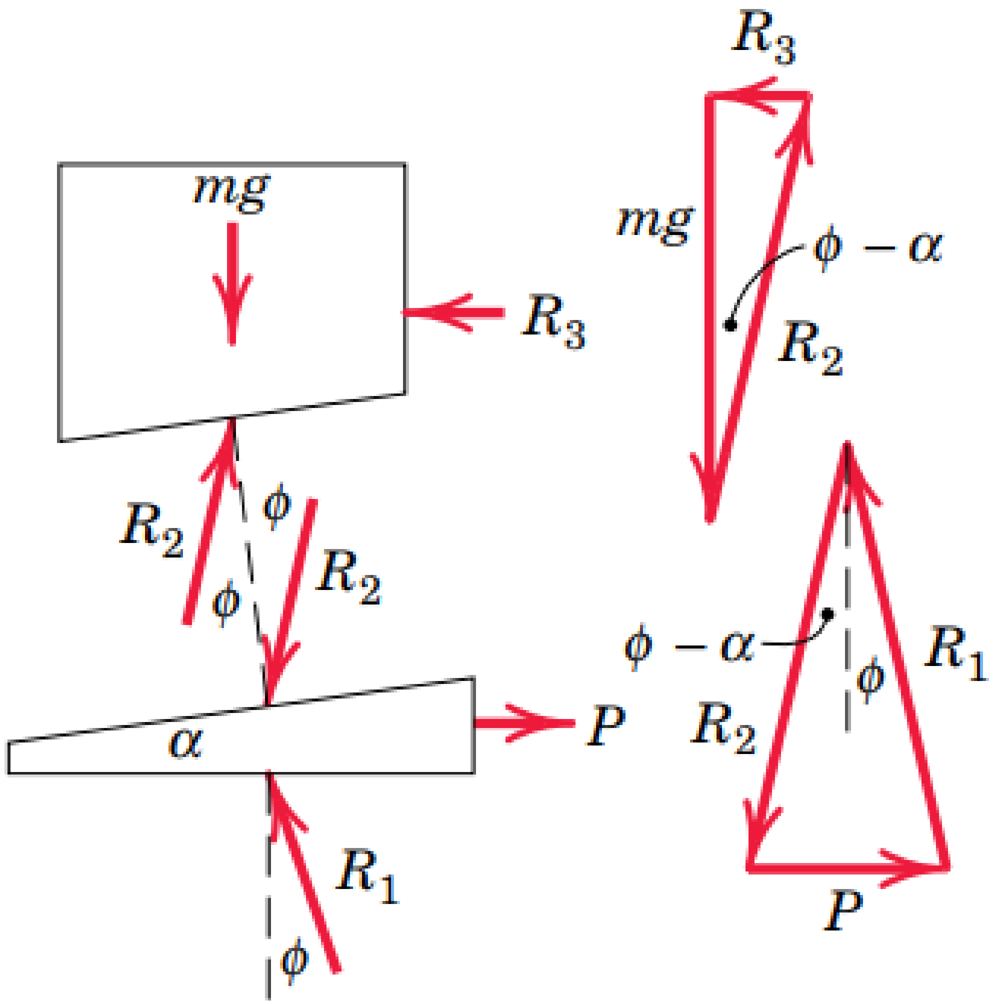


(a) Slipping impending at upper surface (b) Range of $R_1 = R_2$ for no slip (c) Slipping impending at lower surface

6.4 Wedges

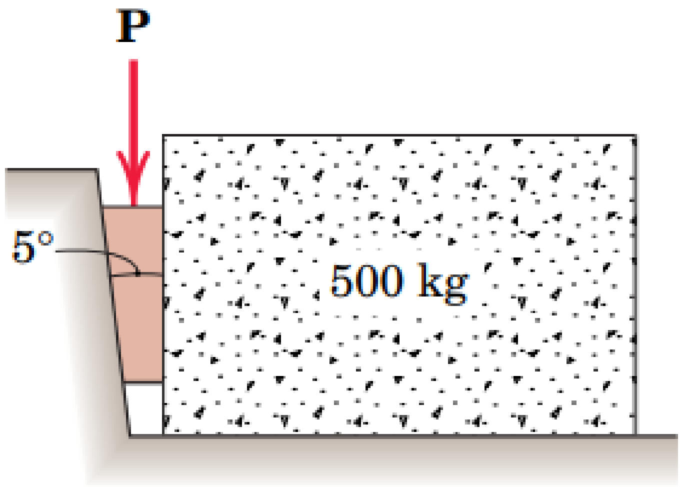
- Forces to lower load

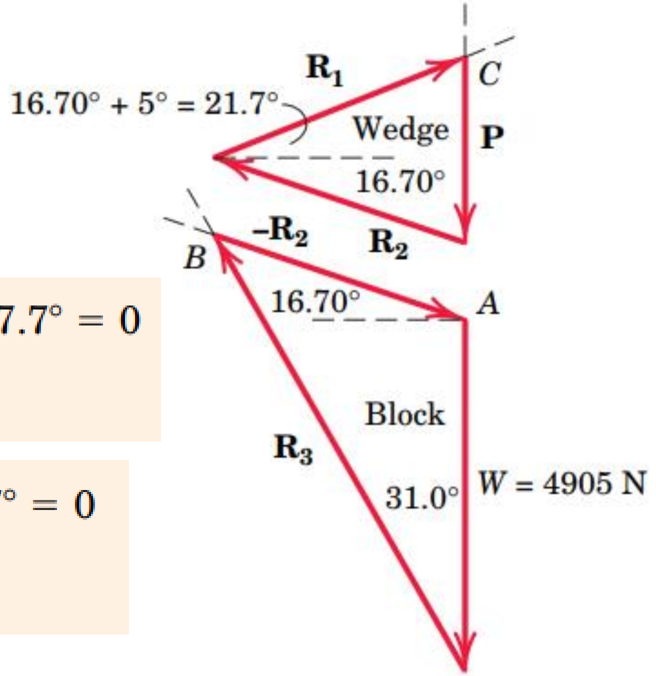


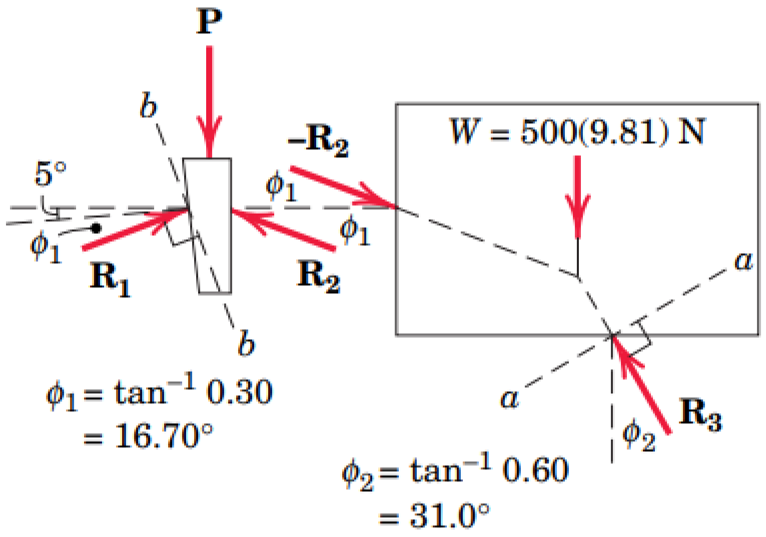


Sample Problem 6/6

The horizontal position of the 500-kg rectangular block of concrete is adjusted by the 5° wedge under the action of the force **P**. If the coefficient of static friction for both wedge surfaces is 0.30 and if the coefficient of static friction between the block and the horizontal surface is 0.60, determine the least force P required to move the block.







$[\Sigma F_a = 0]$ $\quad 500(9.81)\sin 31.0^\circ - R_2 \cos 47.7^\circ = 0$

$$R_2 = 3750 \text{ N}$$

$[\Sigma F_b = 0]$ $\quad 3750 \cos 51.6^\circ - P \cos 21.7^\circ = 0$

$$P = 2500 \text{ N}$$

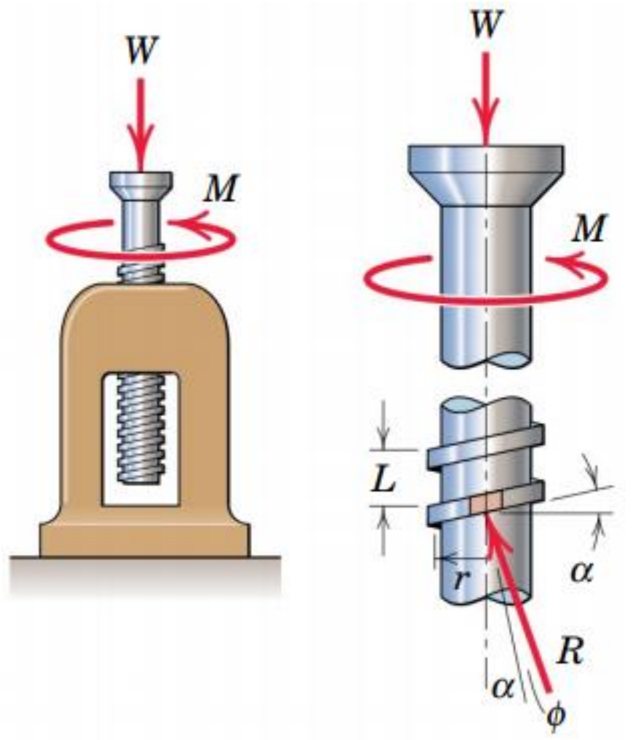
6.5 SCREWS

- Force analysis

$$M = [r \sin(\alpha + \phi)]\, \Sigma R$$

$$W = \Sigma R \cos(\alpha + \phi) = [\cos(\alpha + \phi)]\, \Sigma R$$

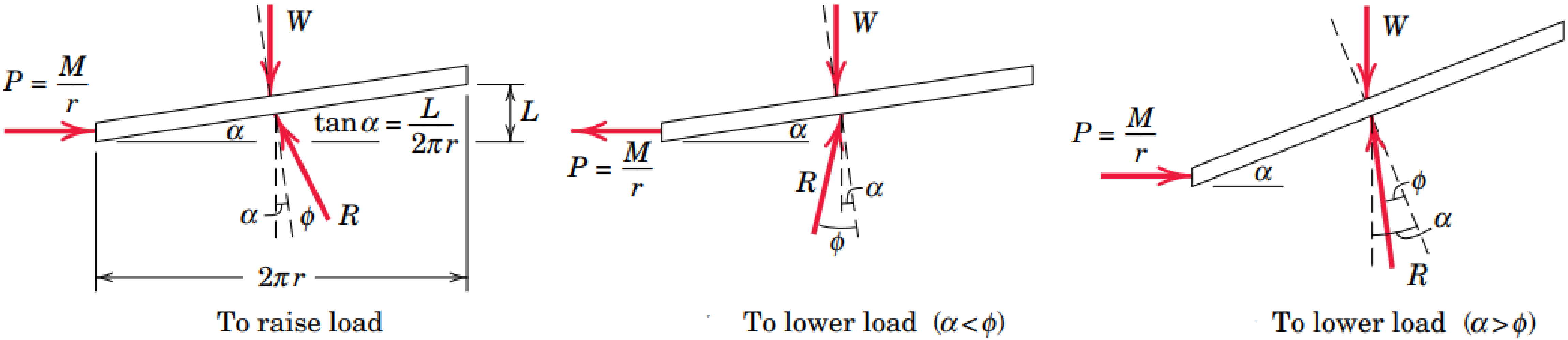
$$\longrightarrow \quad M = Wr \tan(\alpha + \phi)$$



$$\tan \phi = \mu$$

6.5 SCREWS

- Unwrap the thread of the screw



6.5 SCREWS

- Conditions for Unwinding
  - If the moment M is removed, the friction force changes direction
    - ✓ The screw will remain in place and be self-locking: $\alpha < \phi$
    - ✓ The screw will be on the verge of unwinding: $\alpha = \phi$
      - The moment required to lower the screw:

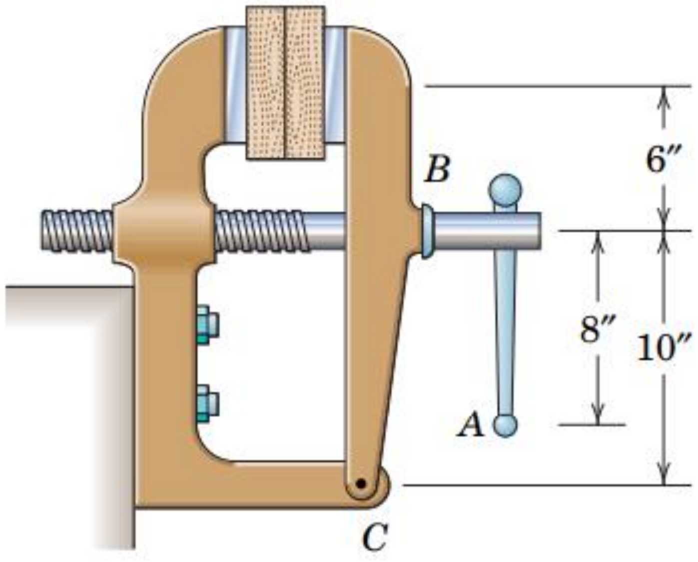
$$M = Wr \tan(\phi - \alpha)$$

    - ✓ The screw will unwind by itself: $\alpha > \phi$
      - The moment required to prevent unwinding:

$$M = Wr \tan(\alpha - \phi)$$

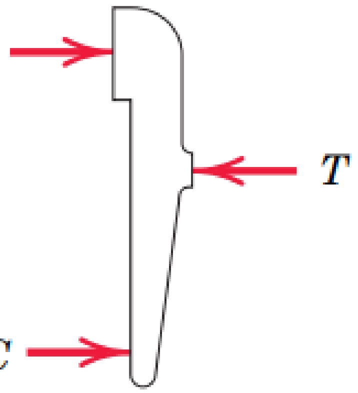
Sample Problem 6/7

The single-threaded screw of the vise has a mean diameter of 1 in. and has 5 square threads per inch. The coefficient of static friction in the threads is 0.20. A 60-lb pull applied normal to the handle at A produces a clamping force of 1000 lb between the jaws of the vise. (a) Determine the frictional moment M_B, developed at B, due to the thrust of the screw against the body of the jaw. (b) Determine the force Q applied normal to the handle at A required to loosen the vise.



$$\alpha = \tan^{-1}\frac{L}{2\pi r} = \tan^{-1}\frac{1/5}{2\pi(0.5)} = 3.64°$$

$$\phi = \tan^{-1}\mu = \tan^{-1} 0.20 = 11.31°$$



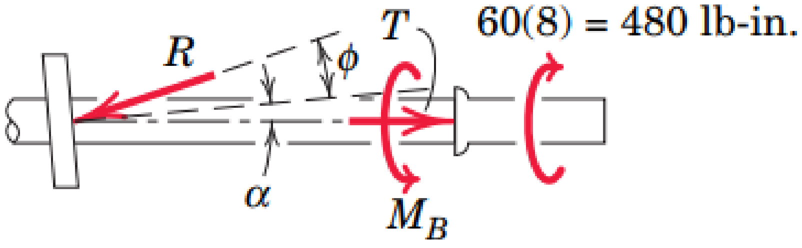
$$[\Sigma M_C = 0] \qquad 1000(16) - 10T = 0 \qquad T = 1600 \text{ lb}$$

(a) To tighten.

$$M = Tr\tan(\alpha + \phi)$$

$$480 - M_B = 1600(0.5)\tan(3.64° + 11.31°)$$

$$M_B = 266 \text{ lb-in.}$$



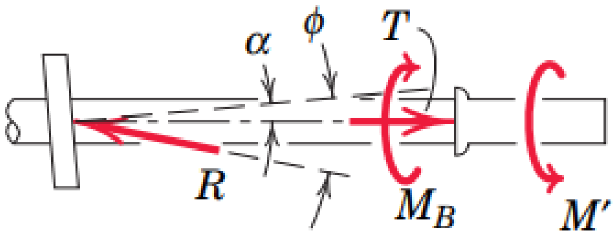
To tighten

(b) To loosen.

$$M = Tr\tan(\phi - \alpha)$$

$$M' - 266 = 1600(0.5)\tan(11.31° - 3.64°)$$

$$M' = 374 \text{ lb-in.}$$



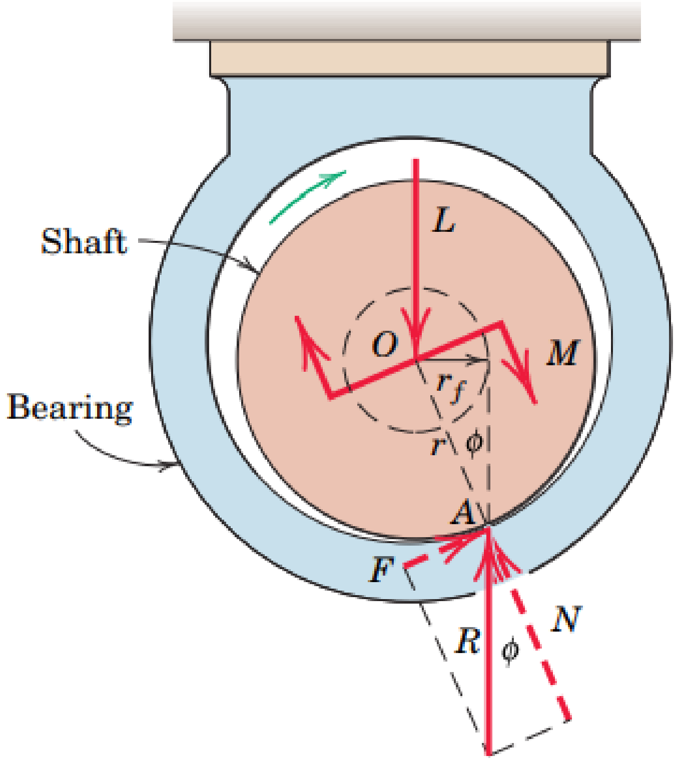
To loosen

→ $$Q = M'/d = 374/8 = 46.8 \text{ lb}$$

6.6 Journal Bearings

- A journal bearing is one which gives lateral support to a shaft in contrast to axial or thrust support.
- For dry bearings and for many partially lubricated bearings we may apply the principles of dry friction.
- As the shaft begins to turn in the direction shown, it will roll up the inner surface of the bearing until it slips.

$$M = Lr_f = Lr \sin \phi$$



- For a small coefficient of friction:

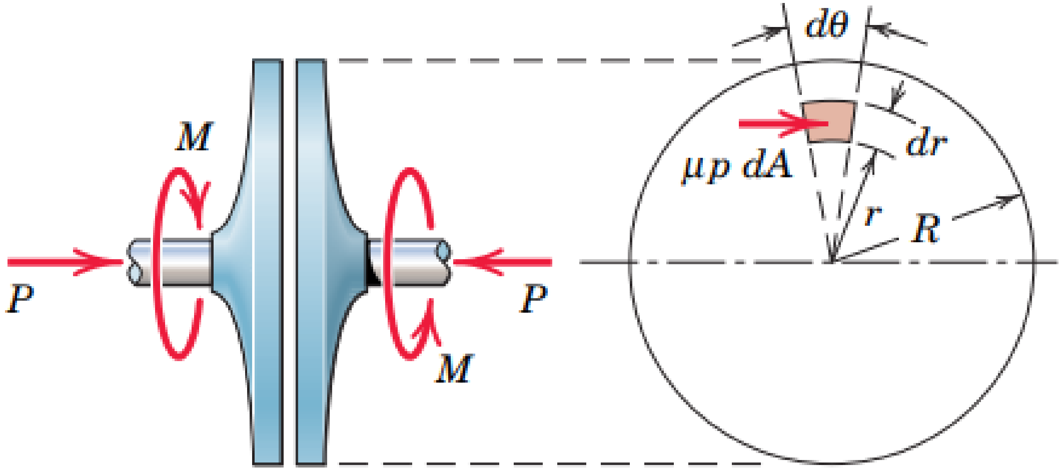
$$\rightarrow \quad M = \mu L r$$

6.7 Thrust Bearings; Disk Friction

- Friction between circular surfaces under distributed normal pressure occurs in pivot bearings, clutch plates, and disk brakes.
- The maximum torque which this clutch can transmit is equal to the torque M required to slip one disk against the other.

$$M = \int \mu p r \, dA$$

$$\pi R^2 p = P \rightarrow M = \frac{\mu P}{\pi R^2} \int_0^{2\pi} \int_0^R r^2 \, dr \, d\theta = \frac{2}{3}\mu P R$$

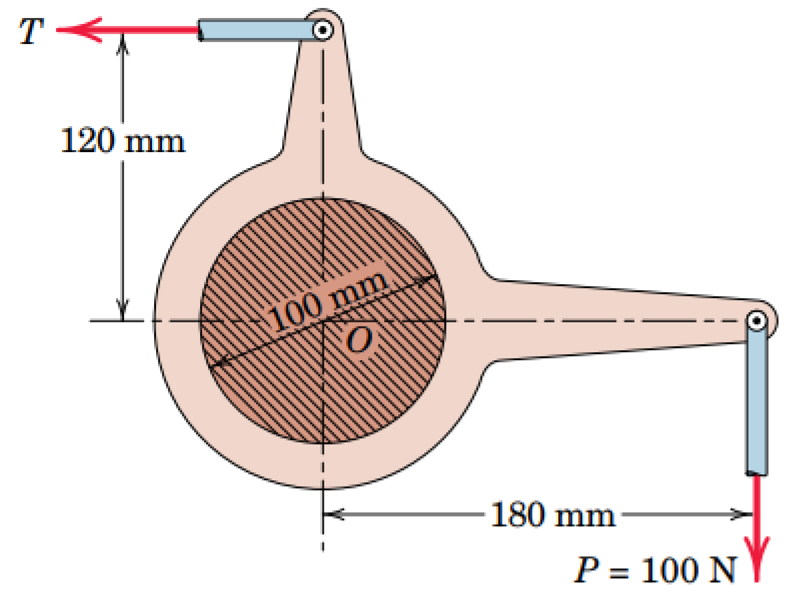


- If the friction disks are rings:

$$\rightarrow M = \frac{2}{3}\mu P \frac{R_o^3 - R_i^3}{R_o^2 - R_i^2}$$

Sample Problem 6/8

The bell crank fits over a 100-mm-diameter shaft which is fixed and cannot rotate. The horizontal force T is applied to maintain equilibrium of the crank under the action of the vertical force $P = 100$ N. Determine the maximum and minimum values which T may have without causing the crank to rotate in either direction. The coefficient of static friction μ between the shaft and the bearing surface of the crank is 0.20.

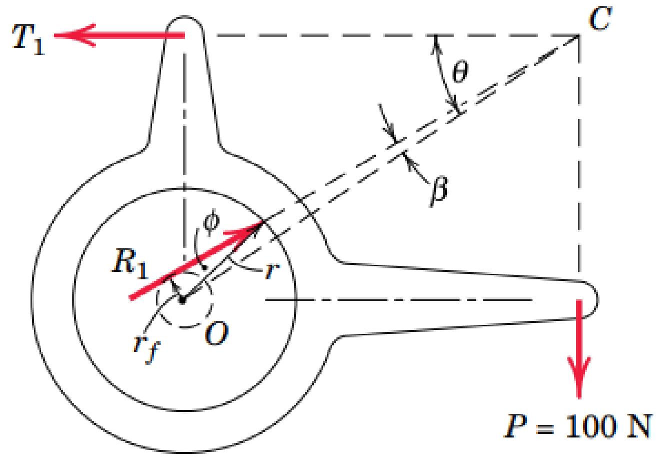


Friction angle $\phi = \tan^{-1} \mu = \tan^{-1} 0.20 = 11.31°$

Radius of friction circle $r_f = r \sin \phi = 50 \sin 11.31° = 9.81$ mm

Angle $\theta = \tan^{-1} \dfrac{120}{180} = 33.7°$

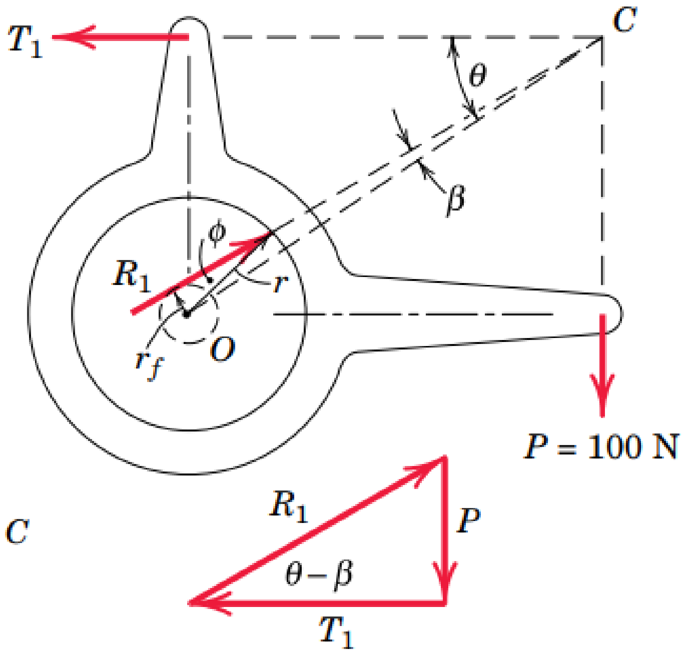
Angle $\beta = \sin^{-1} \dfrac{r_f}{\overline{OC}} = \sin^{-1} \dfrac{9.81}{\sqrt{(120)^2 + (180)^2}} = 2.60°$



(a) Impending counterclockwise motion.

$$T_1 = P \cot(\theta - \beta) = 100 \cot(33.7^\circ - 2.60^\circ)$$

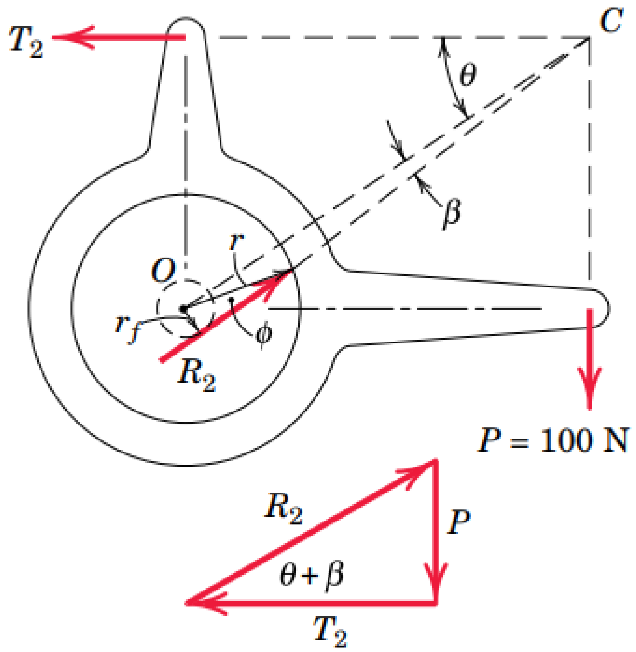
$$T_1 = T_{max} = 165.8 \text{ N}$$



(b) Impending clockwise motion.

$$T_2 = P \cot(\theta + \beta) = 100 \cot(33.7^\circ + 2.60^\circ)$$

$$T_2 = T_{min} = 136.2 \text{ N}$$



6.8 Flexible Belts

- The impending slippage of flexible cables, belts, and ropes over sheaves and drums is important in the design of belt drives

$$T\cos\frac{d\theta}{2} + \mu\, dN = (T + dT)\cos\frac{d\theta}{2}$$

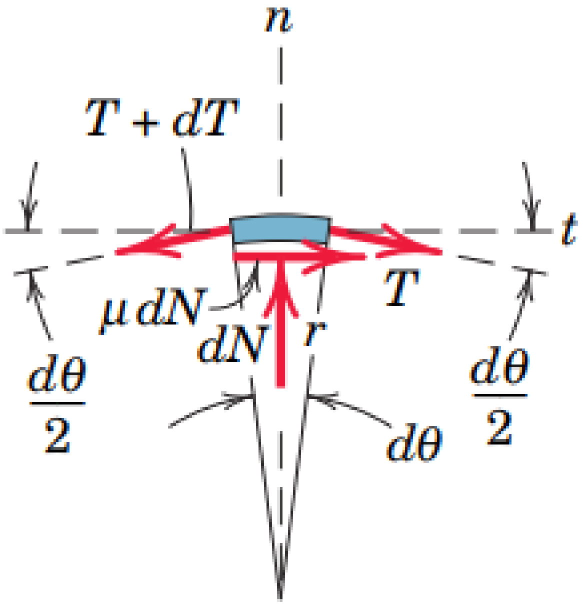
$$\rightarrow \quad \mu\, dN = dT$$

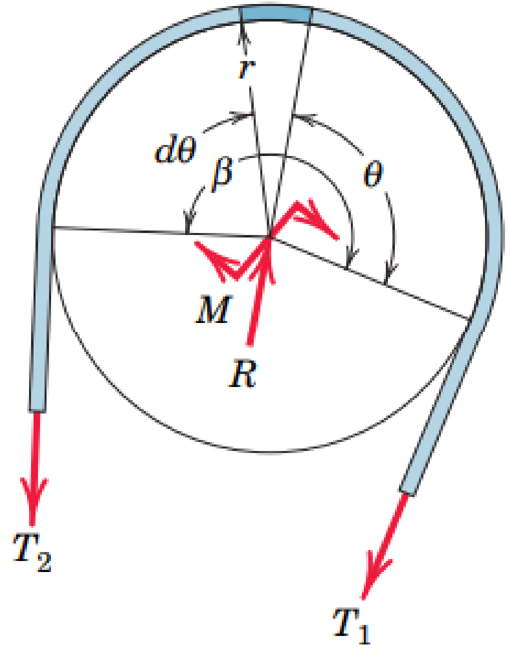
$$dN = (T + dT)\sin\frac{d\theta}{2} + T\sin\frac{d\theta}{2}$$

$$\rightarrow \quad dN = T\, d\theta$$

$$\rightarrow \quad \frac{dT}{T} = \mu\, d\theta \quad \rightarrow \quad \int_{T_1}^{T_2}\frac{dT}{T} = \int_0^{\beta}\mu\, d\theta$$

$$\rightarrow \quad \ln\frac{T_2}{T_1} = \mu\beta \quad \rightarrow \quad \boxed{T_2 = T_1 e^{\mu\beta}}$$



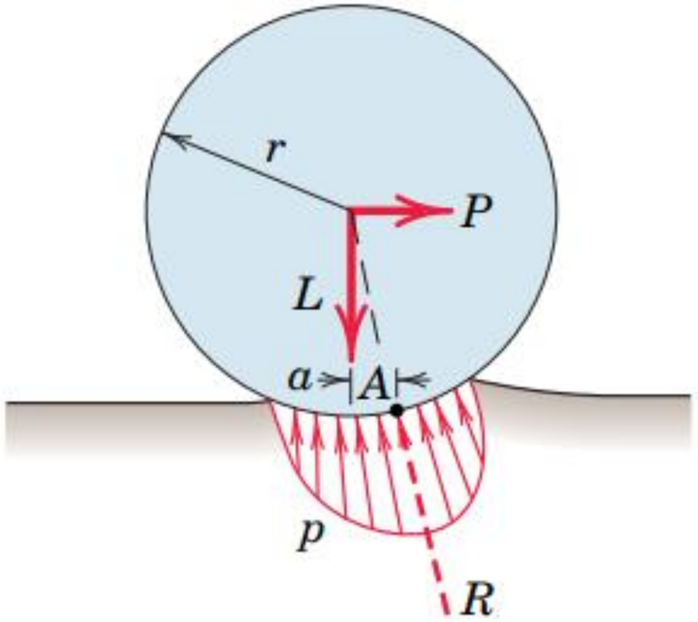


6.9 ROLLING RESISTANCE

- Deformation at the point of contact between a rolling wheel and its supporting surface introduces a resistance to rolling.
- This resistance is not due to tangential friction forces and therefore is an entirely different phenomenon from that of dry friction.

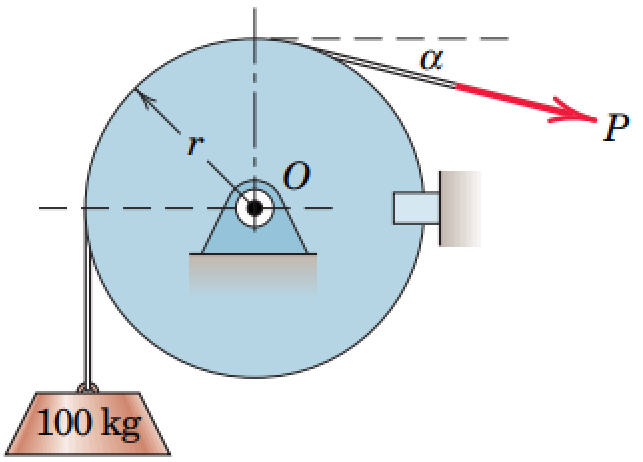
  - The force P necessary to maintain rolling at constant speed:

$$\rightarrow \quad P = \frac{a}{r} L = \mu_r L \qquad \text{(coefficient of rolling resistance)}$$



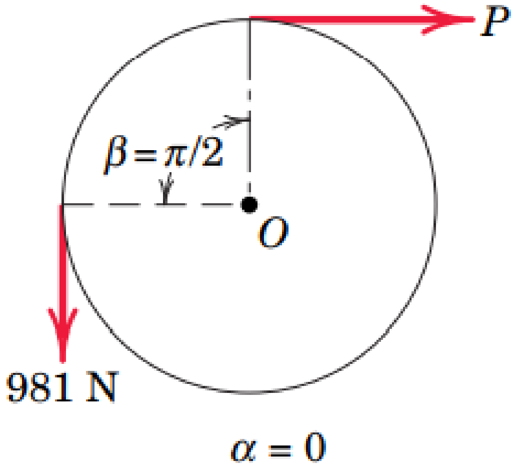
Sample Problem 6/9

A flexible cable which supports the 100-kg load is passed over a fixed circular drum and subjected to a force P to maintain equilibrium. The coefficient of static friction μ between the cable and the fixed drum is 0.30. (a) For $\alpha = 0$, determine the maximum and minimum values which P may have in order not to raise or lower the load. (b) For $P = 500$ N, determine the minimum value which the angle α may have before the load begins to slip.



$$P_{max}/981 = e^{0.30(\pi/2)} \qquad P_{max} = 981(1.602) = 1572 \text{ N}$$

$$981/P_{min} = e^{0.30(\pi/2)} \qquad P_{min} = 981/1.602 = 612 \text{ N}$$



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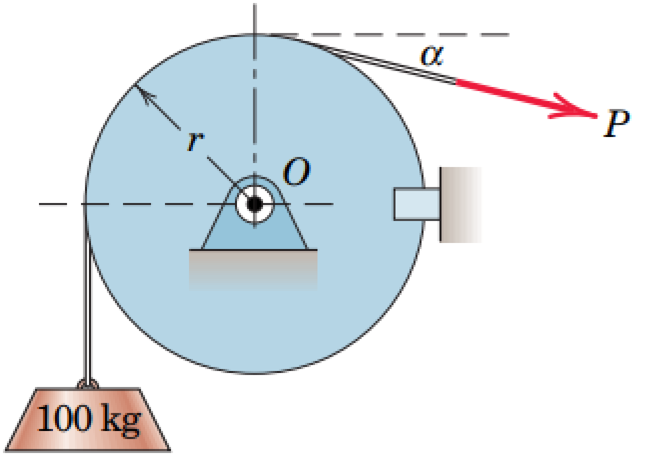
Sample Problem 6/9

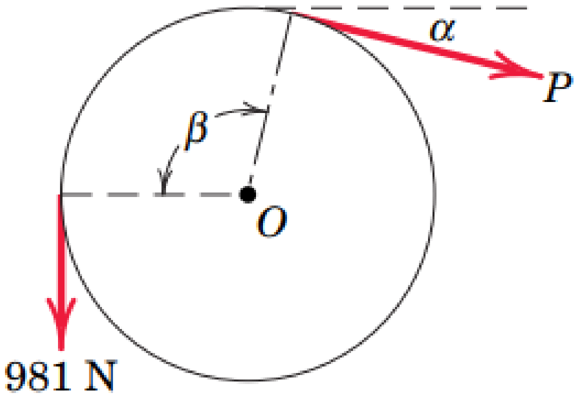
A flexible cable which supports the 100-kg load is passed over a fixed circular drum and subjected to a force P to maintain equilibrium. The coefficient of static friction μ between the cable and the fixed drum is 0.30. (a) For $\alpha = 0$, determine the maximum and minimum values which P may have in order not to raise or lower the load. (b) For $P = 500$ N, determine the minimum value which the angle α may have before the load begins to slip.



$$981/500 = e^{0.30\beta} \qquad 0.30\beta = \ln(981/500) = 0.674$$

$$\beta = 2.25 \text{ rad} \qquad \text{or} \qquad \beta = 2.25\left(\frac{360}{2\pi}\right) = 128.7^\circ$$

$$\alpha = 128.7^\circ - 90^\circ = 38.7^\circ$$



$P = 500$ N