دانشگاه سمنان

Semnan University
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درس استاتیک

STATICS

Chapter 4 - Structures
Class Lecture

❑ CONTENTS:

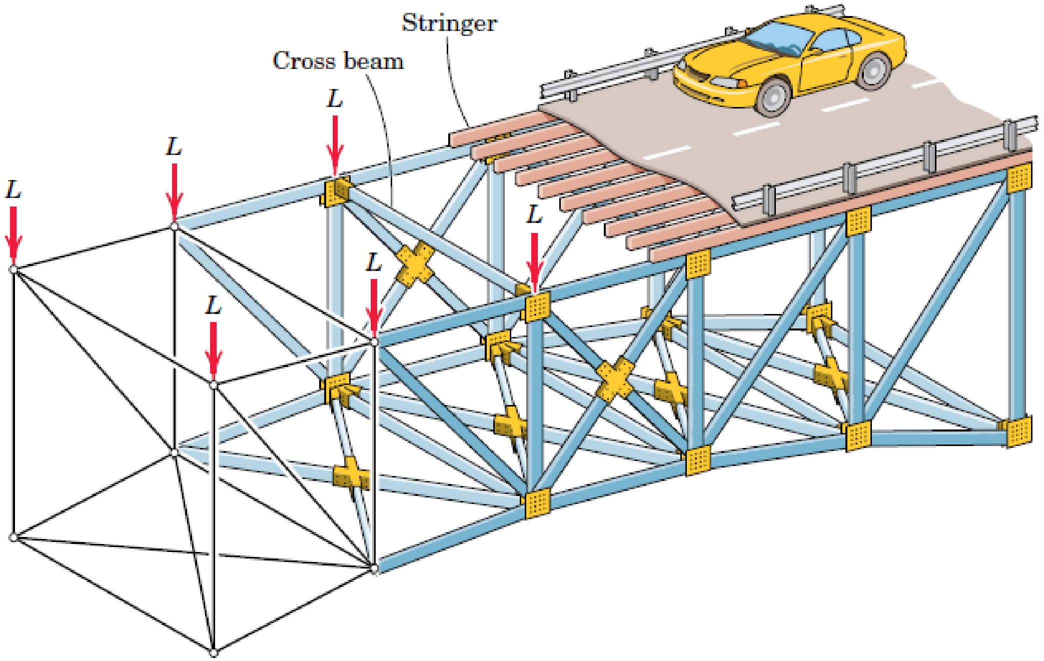
4.1 INTRODUCTION

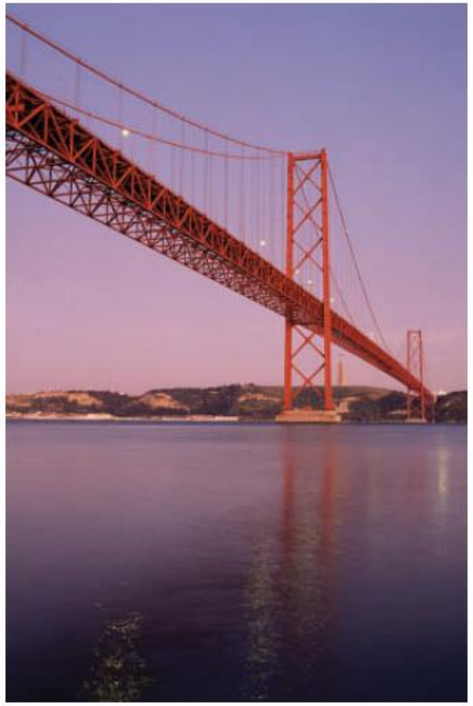
- An engineering structure:
  - Any connected system of members built to support or transfer forces and to safely withstand the loads applied to it.
- To determine the forces internal to an engineering structure;
  - dismember the structure and analyze separate free body diagrams of individual members or combinations of members.

- In Chapter 4 we focus on the determination of the forces internal to a structure.
  - We consider only *statically determinate* structures, which do not have more supporting constraints than are necessary to maintain an equilibrium configuration.

4.2 Plane Trusses

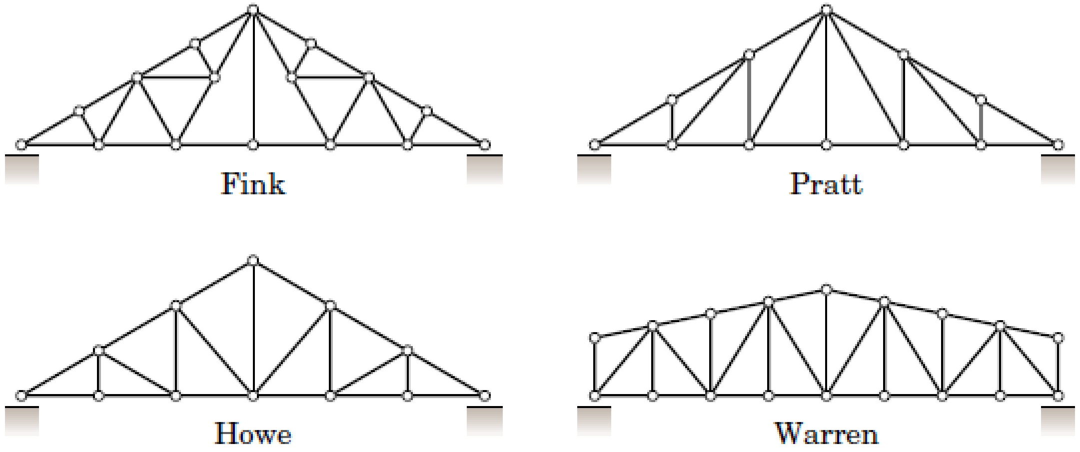
- A framework composed of members joined at their ends to form a rigid structure is called a *truss*.
  - Bridges, roof supports, and other such structures are common examples of trusses.



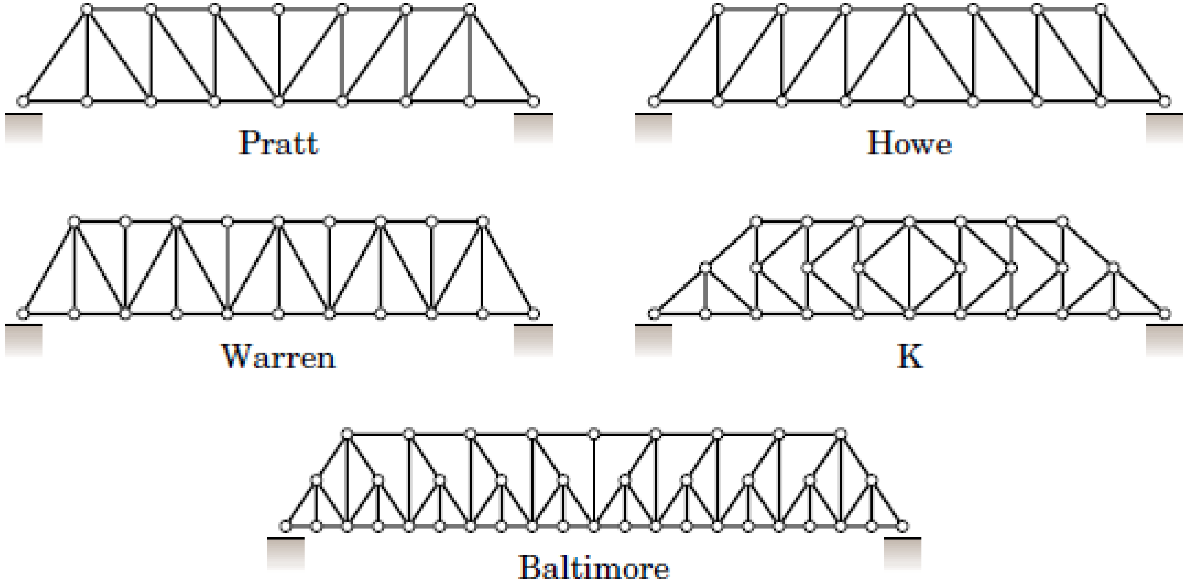


4.2 Plane Trusses

- Bridge and Roof Plane Trusses



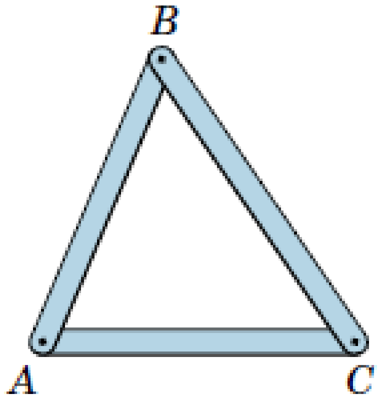
Commonly Used Roof Trusses



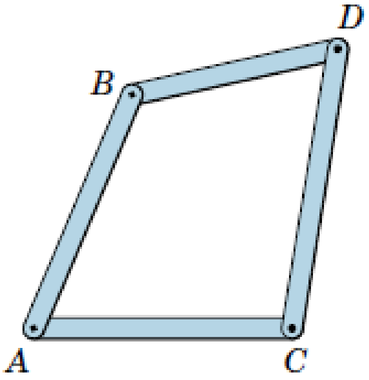
Commonly Used Bridge Trusses

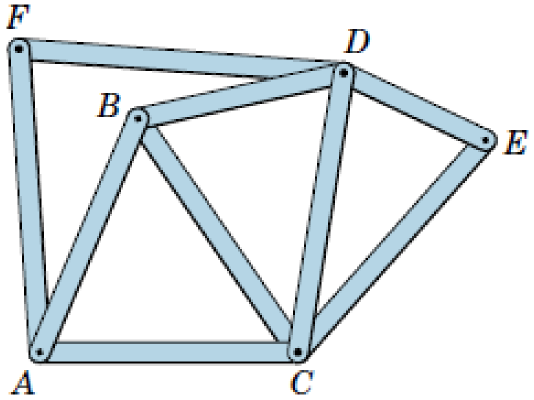
4.2 PLANE TRUSSES

- The basic element of a plane truss is the triangle.
  - Three bars joined by pins at their ends constitute a rigid frame.
    - ✓ Noncollapsible
    - ✓ Negligible internal strains deformation



  - Four or more bars pin-jointed to form a polygon of as many sides constitute a nonrigid frame.
  - We can make the nonrigid frame rigid, or stable, by adding a diagonal bar





4.2 PLANE TRUSSES

- Structures built from a basic triangle in the manner described are known as *simple trusses*.

- When more members are present than are needed to prevent collapse, the truss is statically indeterminate.

- A statically indeterminate truss cannot be analyzed by the equations of equilibrium alone.

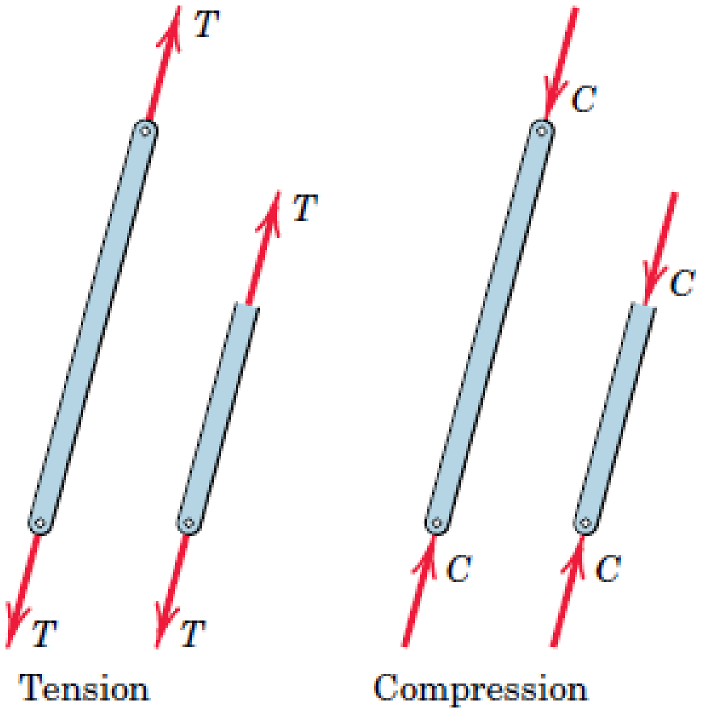
4.2 PLANE TRUSSES

- To design a truss:
  - Determine the forces in the various members
  - Select appropriate sizes and structural shapes to withstand the forces.

- Several assumptions are made in the force analysis of simple trusses:
  - All members to be *two-force members*.
    - ✓ A two-force member is one in equilibrium under the action of two forces only
  - Each member is normally a straight link joining the two points of application of force.
  - The two forces are applied at the ends and are necessarily equal, opposite, and *collinear.*
  - The weight of the member is small compared with the force it supports.

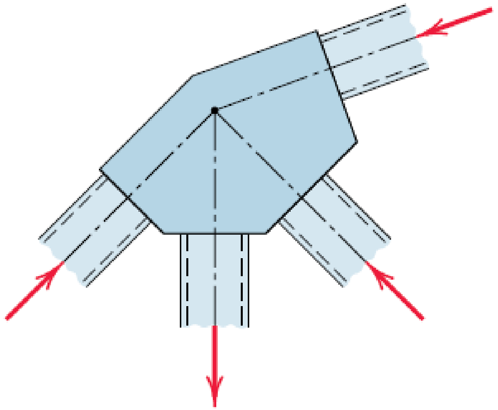
4.2 Plane Trusses

- The member may be in tension or compression:



Two-Force Members

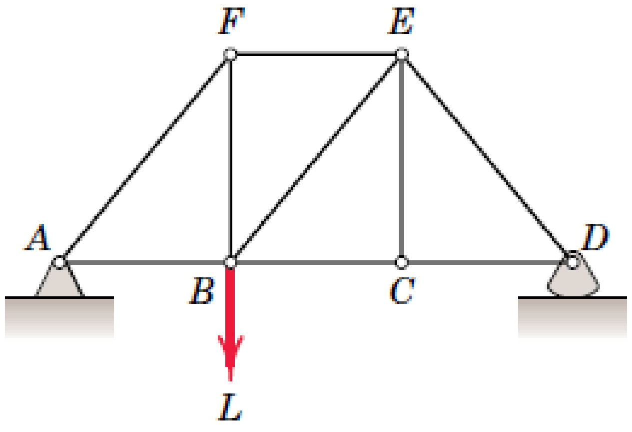
- Truss Connections and Supports:
  - ✓ When welded or riveted connections are used to join structural members, we may usually assume that the connection is a pin joint if the centerlines of the members are concurrent at the joint

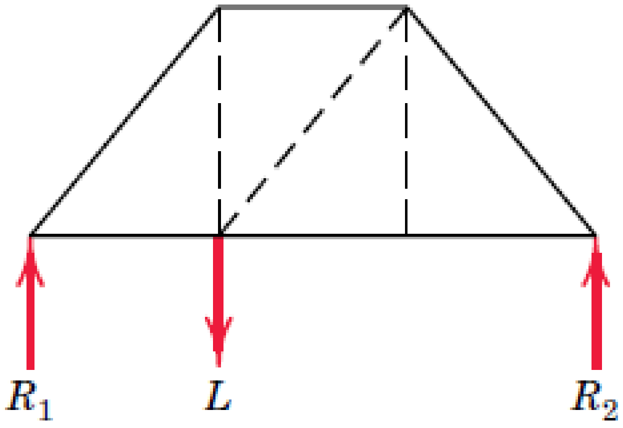


4.2 Plane Trusses

- Two methods for the force analysis of simple trusses will be given:
  - ✓*Method of Joints*
  - ✓*Method of Sections*

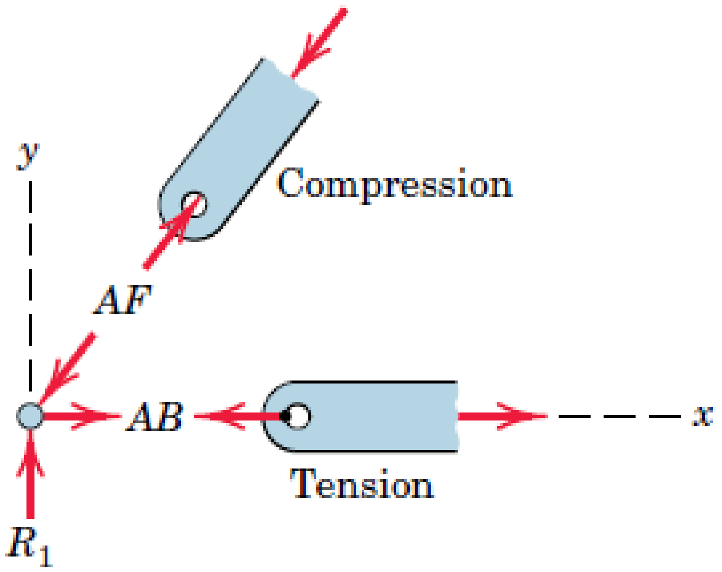
- The external reactions are usually determined first, by applying the equilibrium equations to the truss as a whole. Then the force analysis of the remainder of the truss is performed.

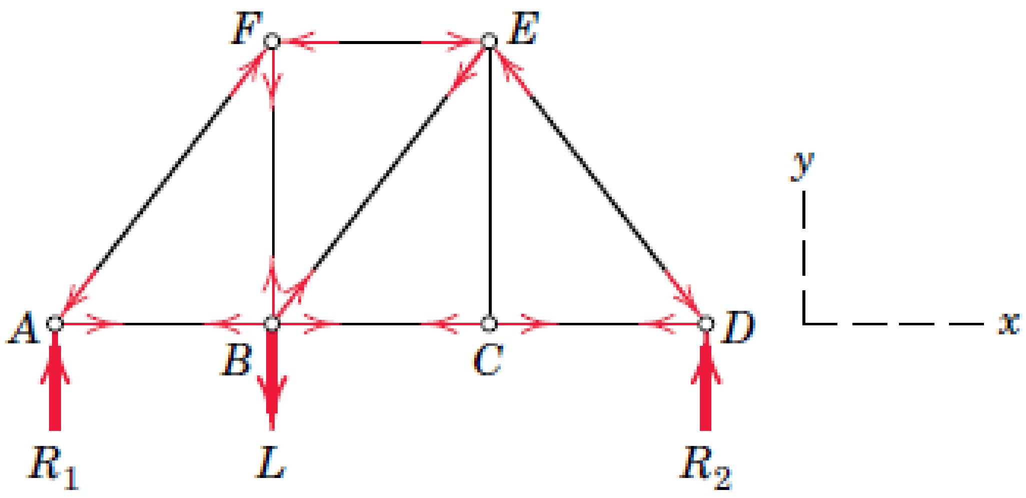




4.3 METHOD OF JOINTS

- Satisfying the conditions of equilibrium for the forces acting on the connecting pin of each joint.
  - The method therefore deals with the equilibrium of concurrent forces, and only two independent equilibrium equations are involved.

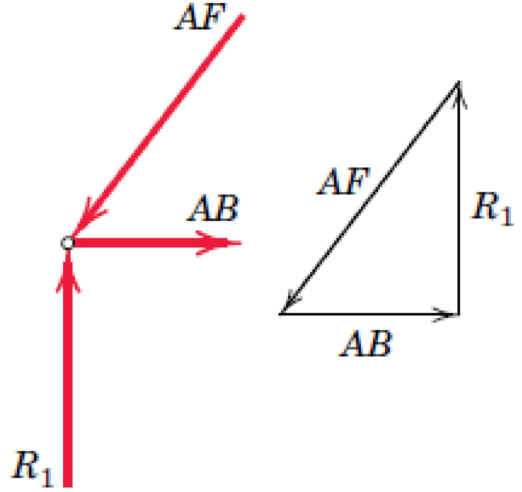




4.3 METHOD OF JOINTS

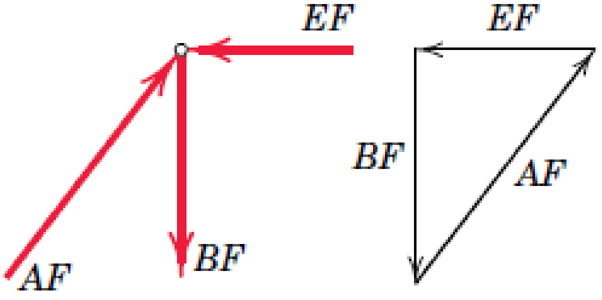
- Analyzing the Truss

1



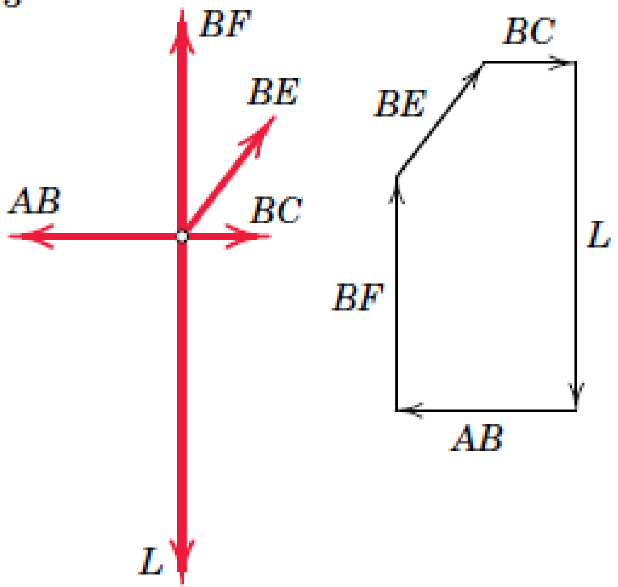
Joint A

2

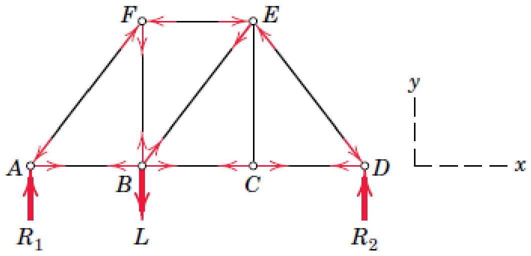


Joint F

3



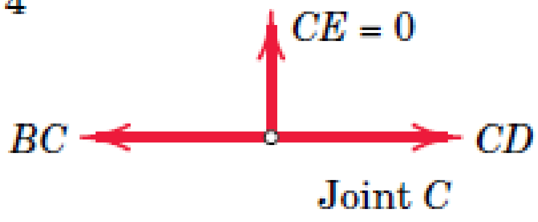
Joint B



4.3 METHOD OF JOINTS

- Analyzing the Truss

4



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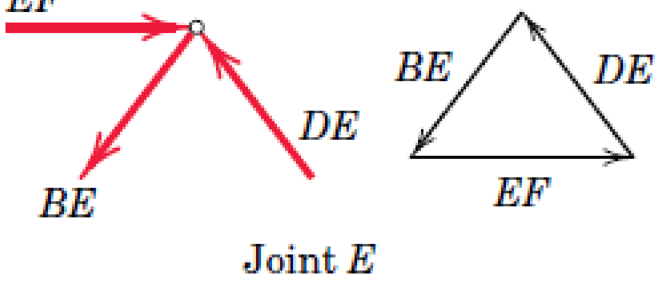

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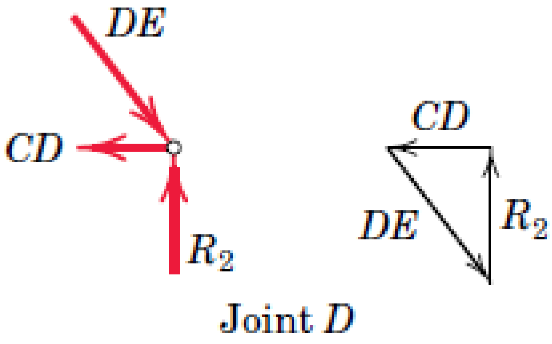
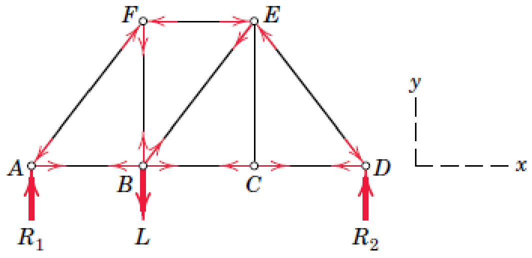




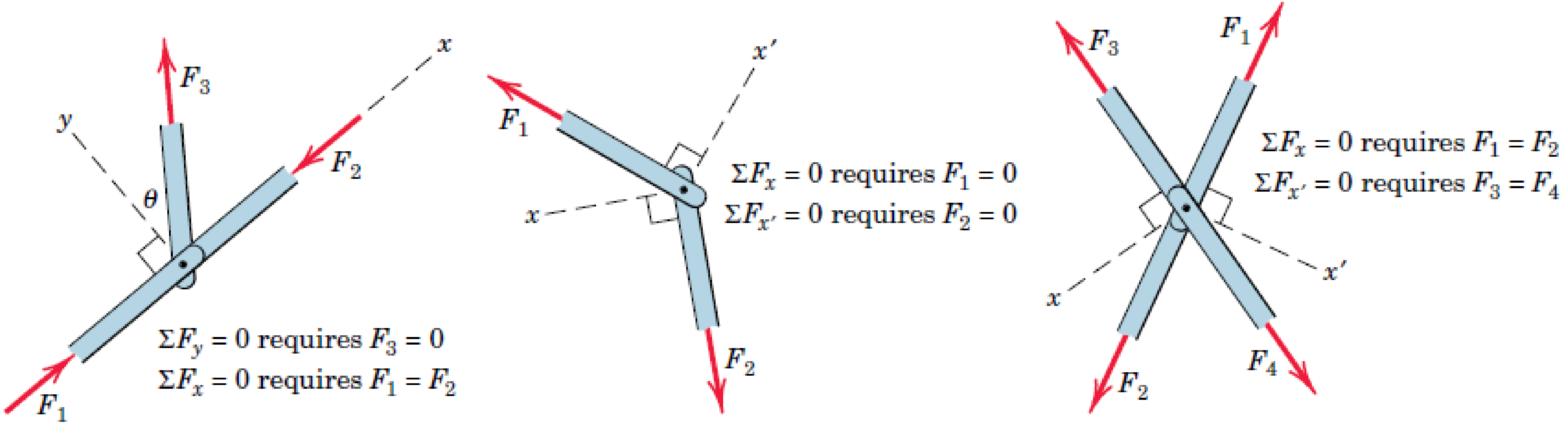
4.3 METHOD OF JOINTS

- Internal and External Redundancy
  - If a plane truss has more external supports than are necessary to ensure a stable equilibrium configuration, the truss as a whole is statically indeterminate, and the extra supports constitute *external* redundancy.
  - If a truss has more internal members than are necessary to prevent collapse when the truss is removed from its supports, then the extra members constitute *internal* redundancy and the truss is again statically indeterminate.
  - For a statically determinate externally m two-force members and j joints:
    - ✓ If the truss is statically determinate internally:

$$m + 3 = 2j$$

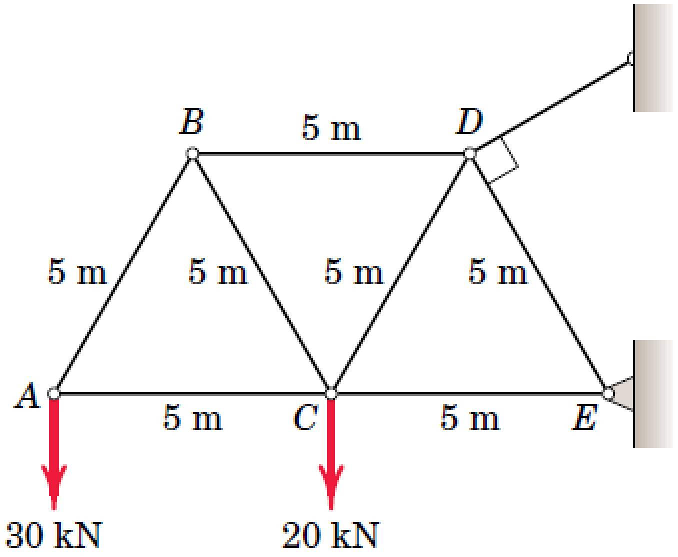
4.3 Method of Joints

- Special Conditions

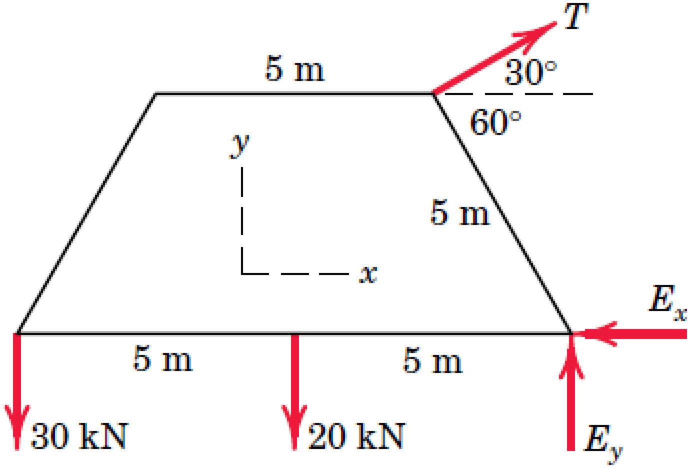


Sample Problem 4/1

Compute the force in each member of the loaded cantilever truss by the method of joints.



$[\Sigma M_E = 0]$	$5T - 20(5) - 30(10) = 0$	$T = 80\text{ kN}$
$[\Sigma F_x = 0]$	$80 \cos 30° - E_x = 0$	$E_x = 69.3\text{ kN}$
$[\Sigma F_y = 0]$	$80 \sin 30° + E_y - 20 - 30 = 0$	$E_y = 10\text{ kN}$



$$[\Sigma F_y = 0] \qquad 0.866AB - 30 = 0 \qquad AB = 34.6 \text{ kN } T$$

$$[\Sigma F_x = 0] \qquad AC - 0.5(34.6) = 0 \qquad AC = 17.32 \text{ kN } C$$

$$[\Sigma F_y = 0] \qquad 0.866BC - 0.866(34.6) = 0 \qquad BC = 34.6 \text{ kN } C$$

$$[\Sigma F_x = 0] \qquad BD - 2(0.5)(34.6) = 0 \qquad BD = 34.6 \text{ kN } T$$

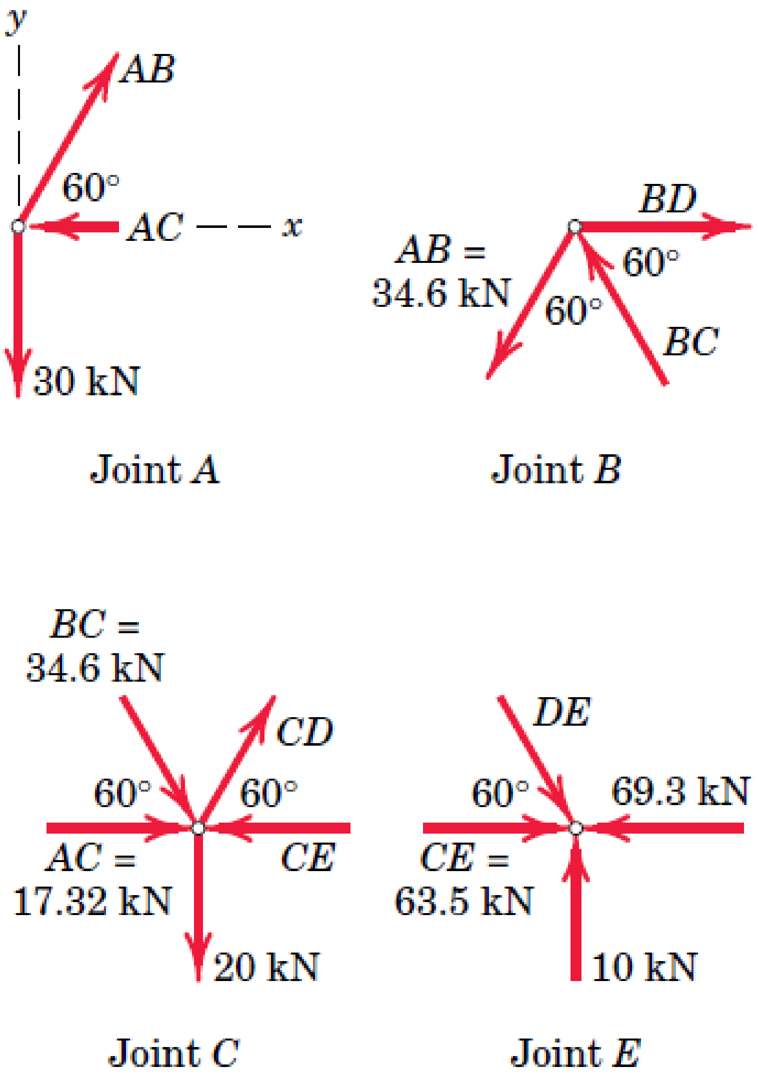
$$[\Sigma F_y = 0] \qquad 0.866CD - 0.866(34.6) - 20 = 0$$

$$CD = 57.7 \text{ kN } T$$

$$[\Sigma F_x = 0] \qquad CE - 17.32 - 0.5(34.6) - 0.5(57.7) = 0$$

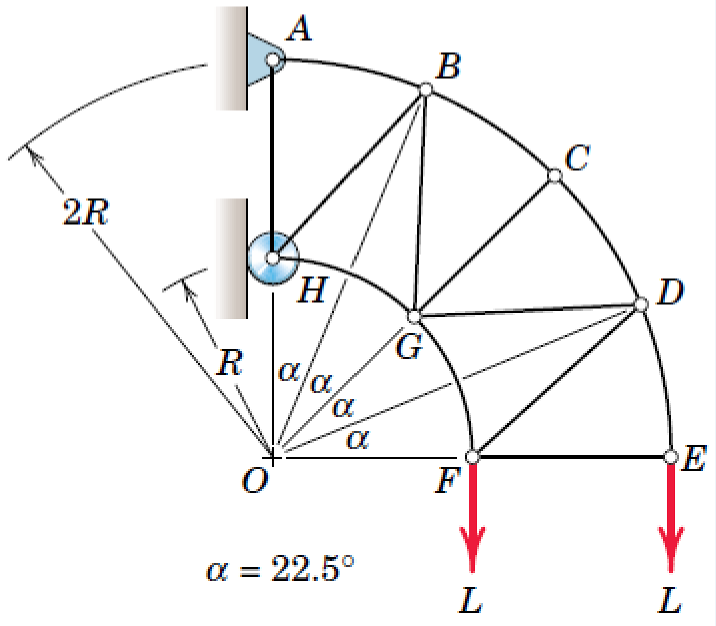
$$CE = 63.5 \text{ kN } C$$

$$[\Sigma F_y = 0] \qquad 0.866DE = 10 \qquad DE = 11.55 \text{ kN } C$$



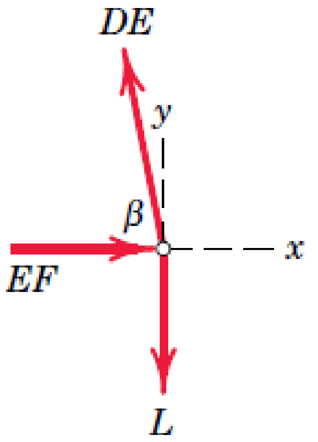
Sample Problem 4/2

The simple truss shown supports the two loads, each of magnitude L. Determine the forces in members DE, DF, DG, and CD.

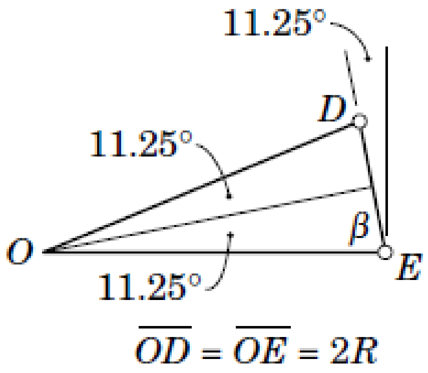


$$\beta = 180^\circ - 11.25^\circ - 90^\circ = 78.8^\circ$$

$[\Sigma F_y = 0]$	$DE \sin 78.8^\circ - L = 0$	$DE = 1.020L\ T$
$[\Sigma F_x = 0]$	$EF - DE \cos 78.8^\circ = 0$	$EF = 0.1989L\ C$



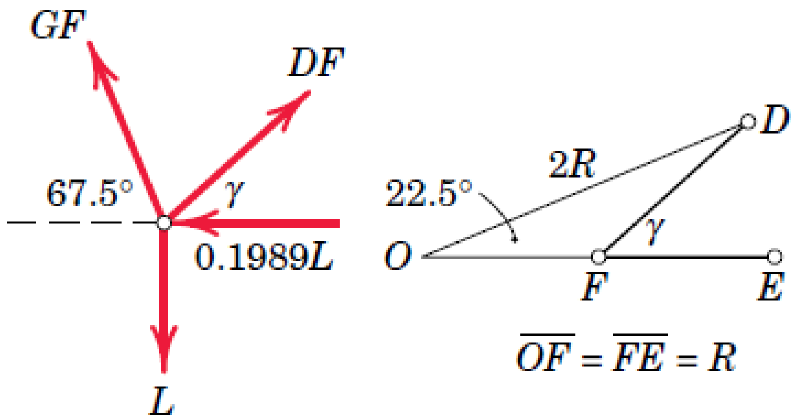
Joint E



$$\gamma = \tan^{-1}\left[\frac{2R \sin 22.5^\circ}{2R \cos 22.5^\circ - R}\right] = 42.1^\circ$$

$$[\Sigma F_x = 0] \qquad -GF \cos 67.5^\circ + DF \cos 42.1^\circ - 0.1989L = 0$$

$$[\Sigma F_y = 0] \qquad GF \sin 67.5^\circ + DF \sin 42.1^\circ - L = 0$$

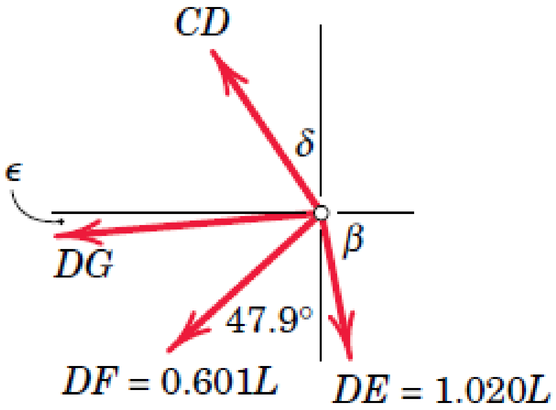


Joint F

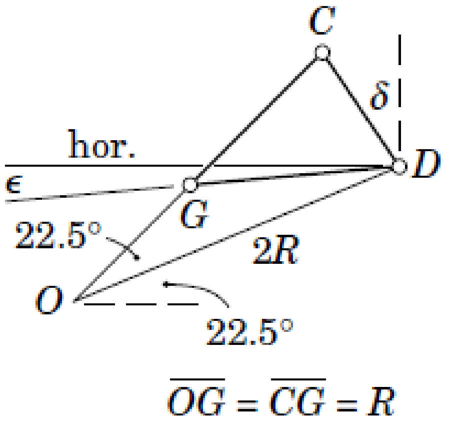
→ $GF = 0.646L\ T \qquad DF = 0.601L\ T$

$$\delta = \tan^{-1}\left[\frac{2R\cos 22.5^\circ - 2R\cos 45^\circ}{2R\sin 45^\circ - 2R\sin 22.5^\circ}\right] = 33.8^\circ$$

$$\epsilon = \tan^{-1}\left[\frac{2R\sin 22.5^\circ - R\sin 45^\circ}{2R\cos 22.5^\circ - R\cos 45^\circ}\right] = 2.92^\circ$$



Joint D



$$[\Sigma F_x = 0]\ -DG\cos 2.92^\circ - CD\sin 33.8^\circ - 0.601L\sin 47.9^\circ + 1.020L\cos 78.8^\circ = 0$$

$$[\Sigma F_y = 0]\ -DG\sin 2.92^\circ + CD\cos 33.8^\circ - 0.601L\cos 47.9^\circ - 1.020L\sin 78.8^\circ = 0$$

$$CD = 1.617L\ T \qquad DG = -1.147L \text{ or } DG = 1.147L\ C$$

4.4 METHOD OF SECTIONS

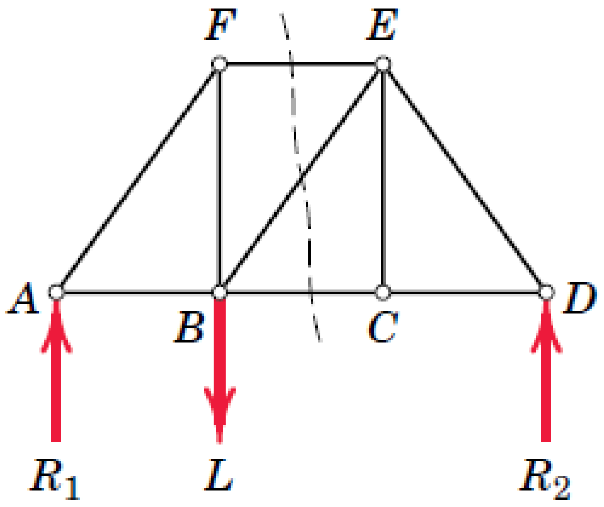
- We can take advantage of the third or moment equation of equilibrium by selecting an entire section of the truss for the free body in equilibrium.

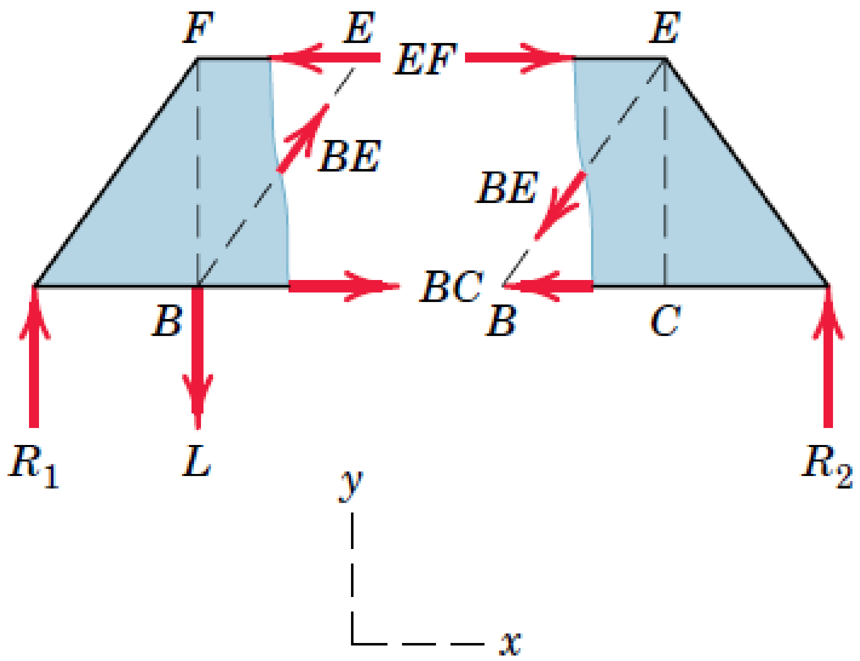
- The force in almost any desired member may be found directly from an analysis of a section which has cut that member.

- In choosing a section of the truss, in general, not more than three members whose forces are unknown should be cut.

4.4 METHOD OF SECTIONS

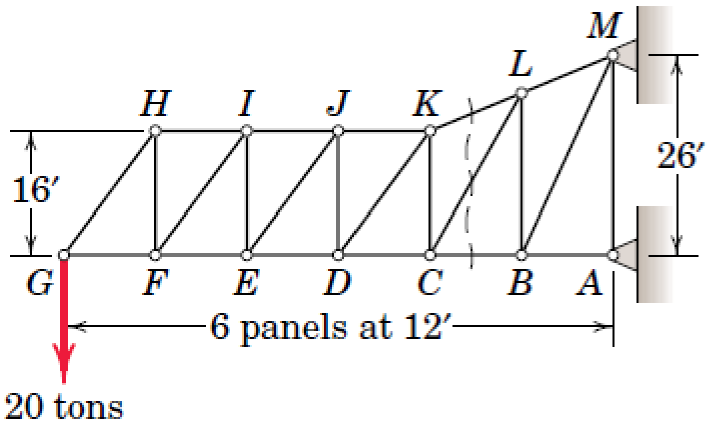
- Illustration of the Method of Sections





Sample Problem 4/3

Calculate the forces induced in members *KL*, *CL*, and *CB* by the 20-ton load on the cantilever truss.



$$\overline{BL} = 16 + (26 - 16)/2 = 21 \text{ ft}$$

$[\Sigma M_L = 0]$ $\quad 20(5)(12) - CB(21) = 0 \quad CB = 57.1 \text{ tons } C$

$\theta = \tan^{-1}(5/12)$ → $\cos\theta = 12/13$

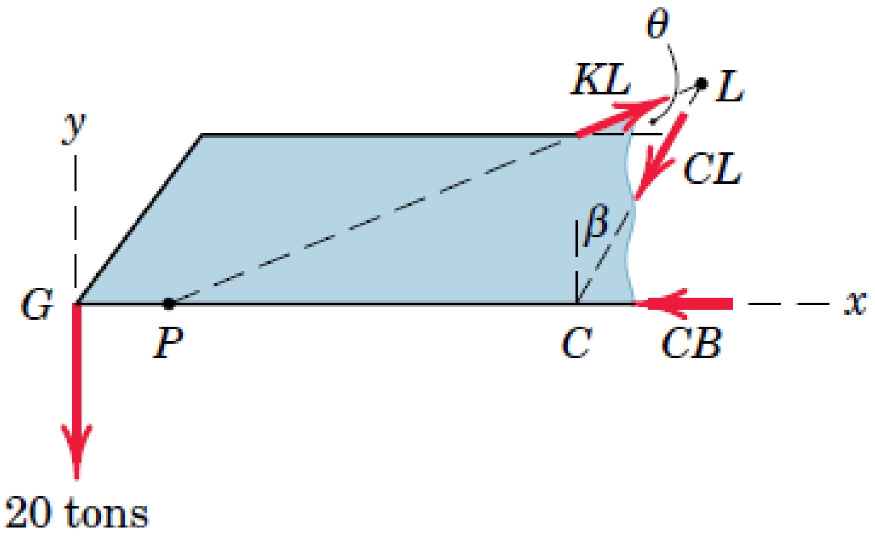
$[\Sigma M_C = 0]$ $\quad 20(4)(12) - \frac{12}{13}KL(16) = 0 \quad KL = 65 \text{ tons } T$

$$\overline{PC}/16 = 24/(26 - 16) = 38.4 \text{ ft}$$

$\beta = \tan^{-1}(\overline{CB}/\overline{BL}) = \tan^{-1}(12/21) = 29.7°$ → $\cos\beta = 0.868$

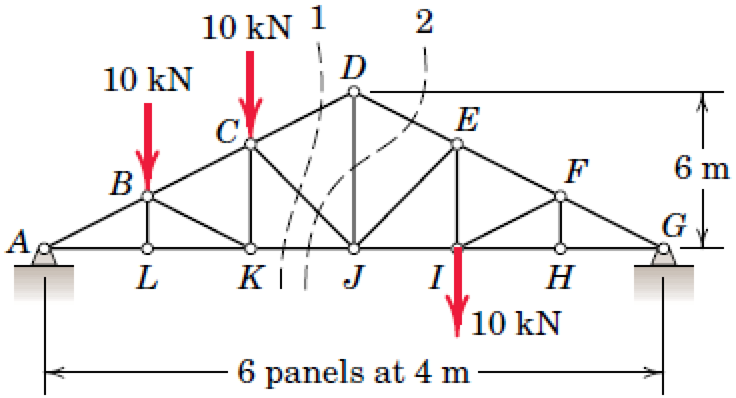
→ $[\Sigma M_P = 0]$ $\quad 20(48 - 38.4) - CL(0.868)(38.4) = 0$

$CL = 5.76 \text{ tons } C$

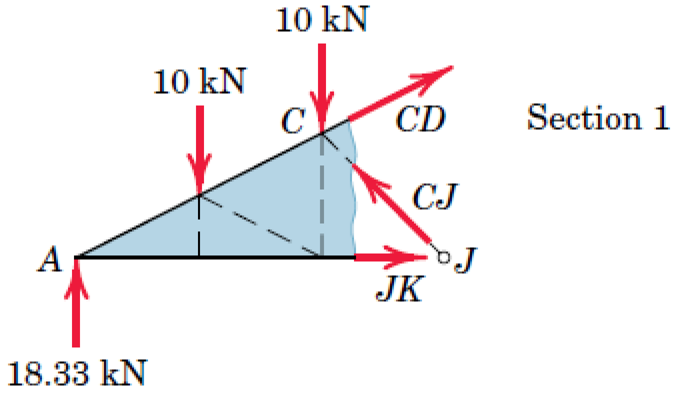


Sample Problem 4/4

Calculate the force in member *DJ* of the Howe roof truss illustrated. Neglect any horizontal components of force at the supports.



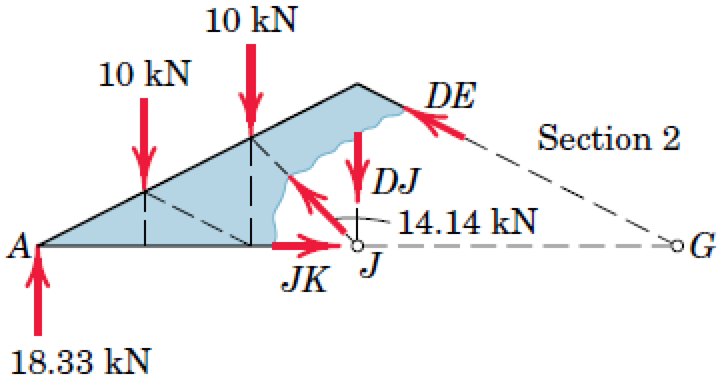
$[\Sigma M_A = 0]$ $\quad 0.707CJ(12) - 10(4) - 10(8) = 0 \qquad CJ = 14.14 \text{ kN } C$



$[\Sigma M_J = 0]$ $\quad 0.894CD(6) + 18.33(12) - 10(4) - 10(8) = 0$

$CD = -18.63 \text{ kN}$

$CD = 18.63 \text{ kN } C$

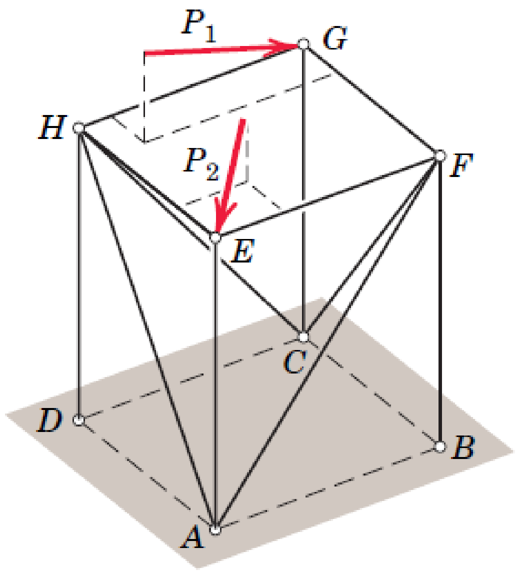


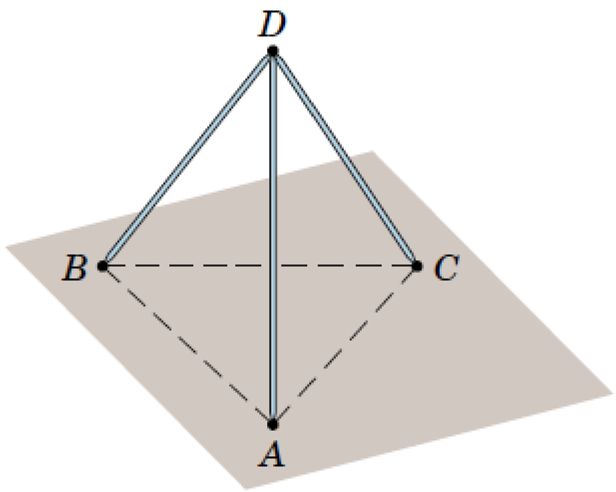
$[\Sigma M_G = 0]$ $\quad 12DJ + 10(16) + 10(20) - 18.33(24) - 14.14(0.707)(12) = 0$

$DJ = 16.67 \text{ kN } T$

4.5 SPACE TRUSSES

- A *space truss* is the three-dimensional counterpart of the plane truss.
  - The idealized space truss consists of rigid links connected at their ends by ball-and-socket joints.
  - A space truss requires six bars joined at their ends to form the edges of a tetrahedron as the basic noncollapsible unit.





- Statically determinate space trusses:

$$m + 6 = 3j$$

4.5 SPACE TRUSSES

- *Method of Joints for Space Trusses*
  - *For each joint:*

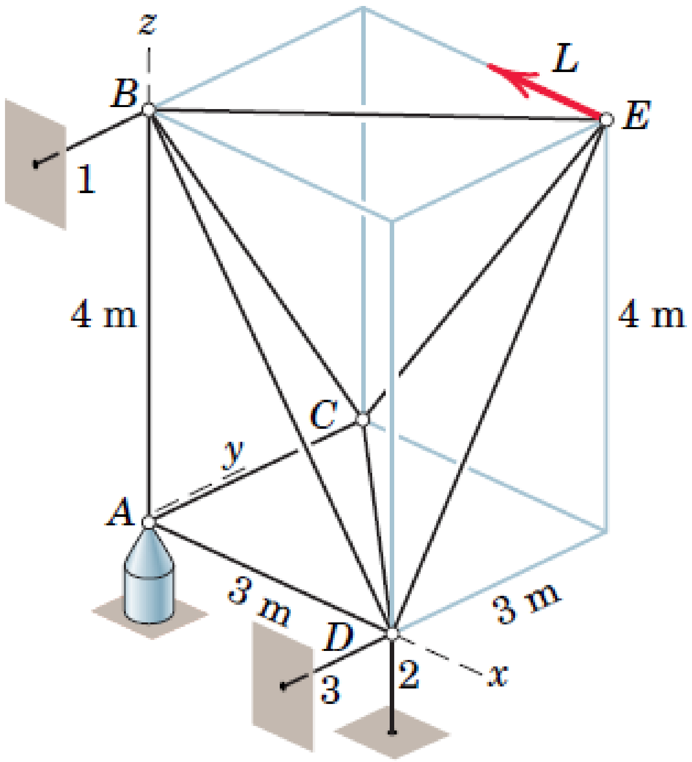
$$\Sigma \mathbf{F} = 0$$

- *Method of Sections for Space Trusses*
  - *For any section of the truss:*

$$\Sigma \mathbf{F} = 0 \qquad \text{and} \qquad \Sigma \mathbf{M} = 0$$

Sample Problem 4/5

The space truss consists of the rigid tetrahedron *ABCD* anchored by a ball-and-socket connection at *A* and prevented from any rotation about the *x*-, *y*-, or *z*-axes by the respective links 1, 2, and 3. The load *L* is applied to joint *E*, which is rigidly fixed to the tetrahedron by the three additional links. Solve for the forces in the members at joint *E* and indicate the procedure for the determination of the forces in the remaining members of the truss.



Solution. We note first that the truss is supported with six properly placed constraints, which are the three at *A* and the links 1, 2, and 3. Also, with $m = 9$ members and $j = 5$ joints, the condition $m + 6 = 3j$ for a sufficiency of members to provide a noncollapsible structure is satisfied.

The external reactions at *A*, *B*, and *D* can be calculated easily as a first step, although their values will be determined from the solution of all forces on each of the joints in succession.

$$\mathbf{A}_x = L\mathbf{i},\ \mathbf{A}_y = L\mathbf{j},\ \mathbf{A}_z = (4L/3)\mathbf{k},\ \mathbf{B}_y = 0,\ \mathbf{D}_y = -L\mathbf{j},\ \mathbf{D}_z = -(4L/3)\mathbf{k}.$$

$$\mathbf{F}_{EB} = \frac{F_{EB}}{\sqrt{2}}(-\mathbf{i} - \mathbf{j}), \qquad \mathbf{F}_{EC} = \frac{F_{EC}}{5}(-3\mathbf{i} - 4\mathbf{k}), \qquad \mathbf{F}_{ED} = \frac{F_{ED}}{5}(-3\mathbf{j} - 4\mathbf{k})$$

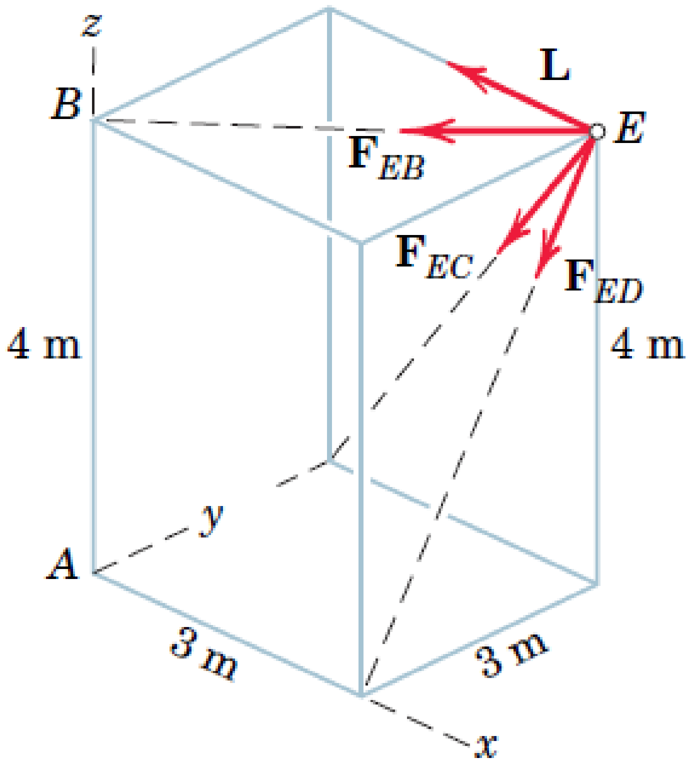
$$[\Sigma\mathbf{F} = \mathbf{0}] \qquad \mathbf{L} + \mathbf{F}_{EB} + \mathbf{F}_{EC} + \mathbf{F}_{ED} = \mathbf{0}$$

$$-L\mathbf{i} + \frac{F_{EB}}{\sqrt{2}}(-\mathbf{i} - \mathbf{j}) + \frac{F_{EC}}{5}(-3\mathbf{i} + 4\mathbf{k}) + \frac{F_{ED}}{5}(-3\mathbf{j} - 4\mathbf{k}) = \mathbf{0}$$

$$\left(-L - \frac{F_{EB}}{\sqrt{2}} - \frac{3F_{EC}}{5}\right)\mathbf{i} + \left(-\frac{F_{EB}}{\sqrt{2}} - \frac{3F_{ED}}{5}\right)\mathbf{j} + \left(-\frac{4F_{EC}}{5} - \frac{4F_{ED}}{5}\right)\mathbf{k} = \mathbf{0}$$

$$\frac{F_{EB}}{\sqrt{2}} + \frac{3F_{EC}}{5} = -L \qquad \frac{F_{EB}}{\sqrt{2}} + \frac{3F_{ED}}{5} = 0 \qquad F_{EC} + F_{ED} = 0$$

$$F_{EB} = -L/\sqrt{2} \qquad F_{EC} = -5L/6 \qquad F_{ED} = 5L/6$$



4.6 Frames and Machines

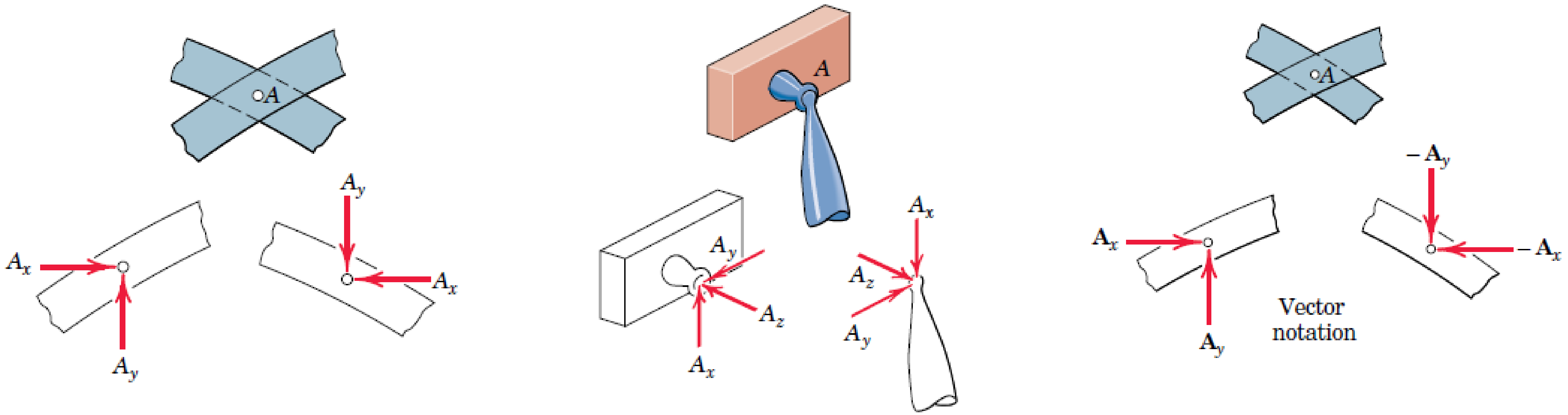
- *Frame* or *Machine:*
  - A structure which at least one of its individual members is a *multiforce member*.
    - ✓ A multiforce member is defined as one with three or more forces acting on it, or one with two or more forces and one or more couples acting on it.
    - ✓ Frames are structures which are designed to support applied loads and are usually fixed in position.
    - ✓ Machines are structures which contain moving parts and are designed to transmit input forces or couples to output forces or couples.
    - ✓ Because frames and machines contain multiforce members, the forces in these members in general will *not* be in the directions of the members.
    - ✓ Therefore, we cannot analyze these structures by the methods developed.

4.6 FRAMES AND MACHINES

- Interconnected Rigid Bodies with Multiforce Members
  - The forces acting on each member of a connected system are found by isolating the member with a free-body diagram and applying the equations of equilibrium.
  - The *principle of action and reaction* must be carefully observed.

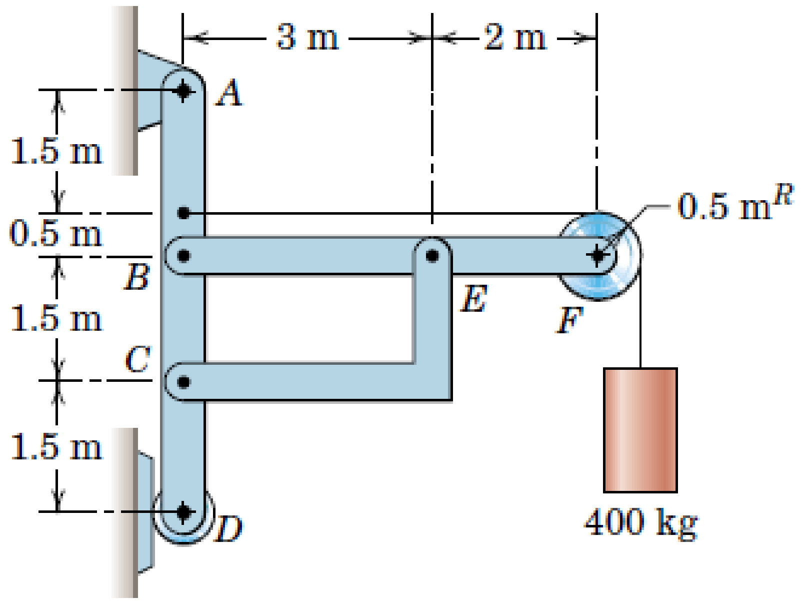
4.6 FRAMES AND MACHINES

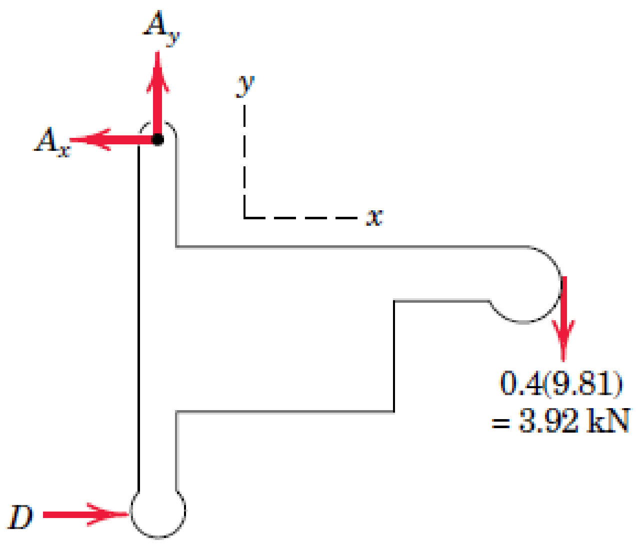
- Force Representation and Free-Body Diagrams



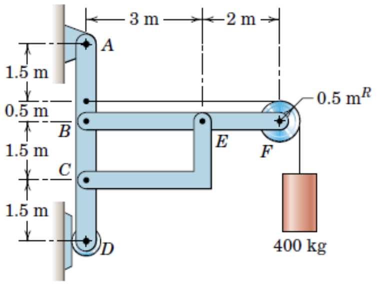
Sample Problem 4/6

The frame supports the 400-kg load in the manner shown. Neglect the weights of the members compared with the forces induced by the load and compute the horizontal and vertical components of all forces acting on each of the members.





$[\Sigma M_A = 0]$	$5.5(0.4)(9.81) - 5D = 0$	$D = 4.32$ kN
$[\Sigma F_x = 0]$	$A_x - 4.32 = 0$	$A_x = 4.32$ kN
$[\Sigma F_y = 0]$	$A_y - 3.92 = 0$	$A_y = 3.92$ kN

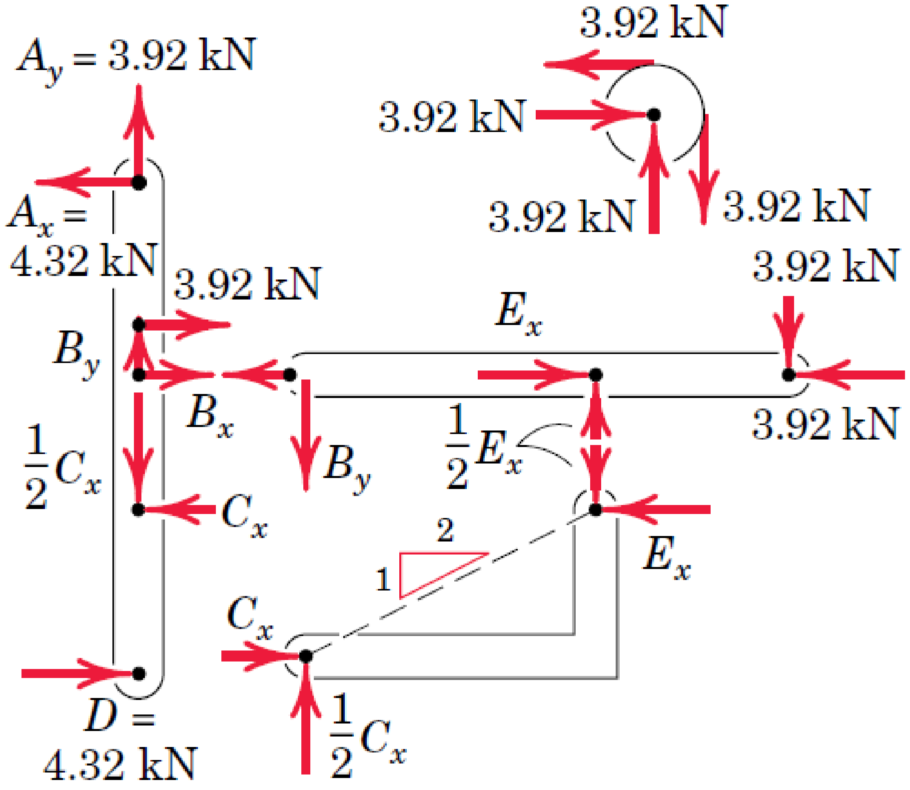


❖ Member BF:

$[\Sigma M_B = 0]$	$3.92(5) - \frac{1}{2}E_x(3) = 0$	$E_x = 13.08$ kN
$[\Sigma F_y = 0]$	$B_y + 3.92 - 13.08/2 = 0$	$B_y = 2.62$ kN
$[\Sigma F_x = 0]$	$B_x + 3.92 - 13.08 = 0$	$B_x = 9.15$ kN

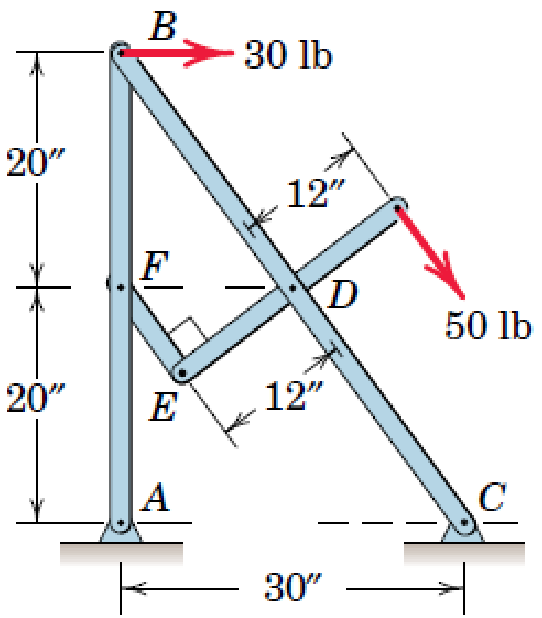
❖ Member AD:

$[\Sigma M_C = 0]$	$4.32(3.5) + 4.32(1.5) - 3.92(2) - 9.15(1.5) = 0$
$[\Sigma F_x = 0]$	$4.32 - 13.08 + 9.15 + 3.92 + 4.32 = 0$
$[\Sigma F_y = 0]$	$-13.08/2 + 2.62 + 3.92 = 0$



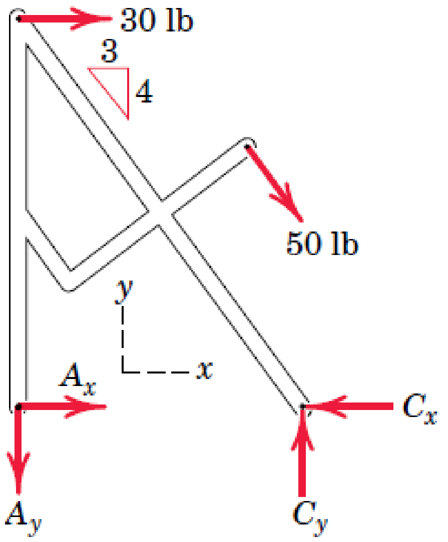
Sample Problem 4/7

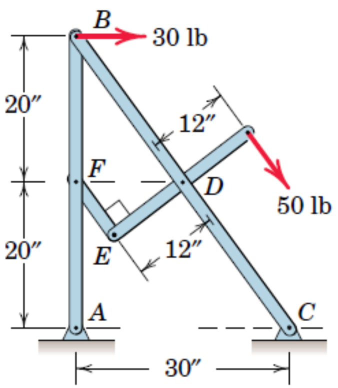
Neglect the weight of the frame and compute the forces acting on all of its members.



Solution. We note first that the frame is not a rigid unit when removed from its supports since *BDEF* is a movable quadrilateral and not a rigid triangle. Consequently, the external reactions cannot be completely determined until the individual members are analyzed. However, we can determine the vertical components of the reactions at *A* and *C* from the free-body diagram of the frame as a whole. Thus,

$[\Sigma M_C = 0]$	$50(12) + 30(40) - 30A_y = 0$	$A_y = 60$ lb
$[\Sigma F_y = 0]$	$C_y - 50(4/5) - 60 = 0$	$C_y = 100$ lb





❖ Member EF:

Clearly F is equal and opposite to E with the magnitude of 50 lb.

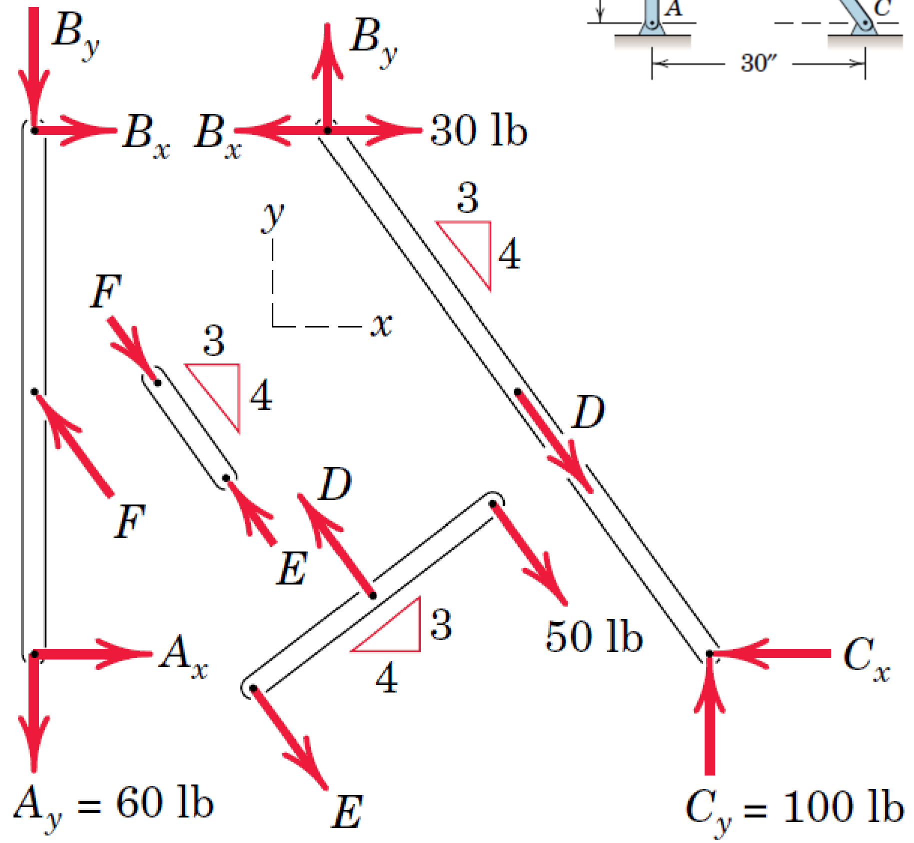
❖ Member AB:

$[\Sigma M_A = 0]$	$50(3/5)(20) - B_x(40) = 0$	$B_x = 15$ lb
$[\Sigma F_x = 0]$	$A_x + 15 - 50(3/5) = 0$	$A_x = 15$ lb
$[\Sigma F_y = 0]$	$50(4/5) - 60 - B_y = 0$	$B_y = -20$ lb

❖ Member BC:

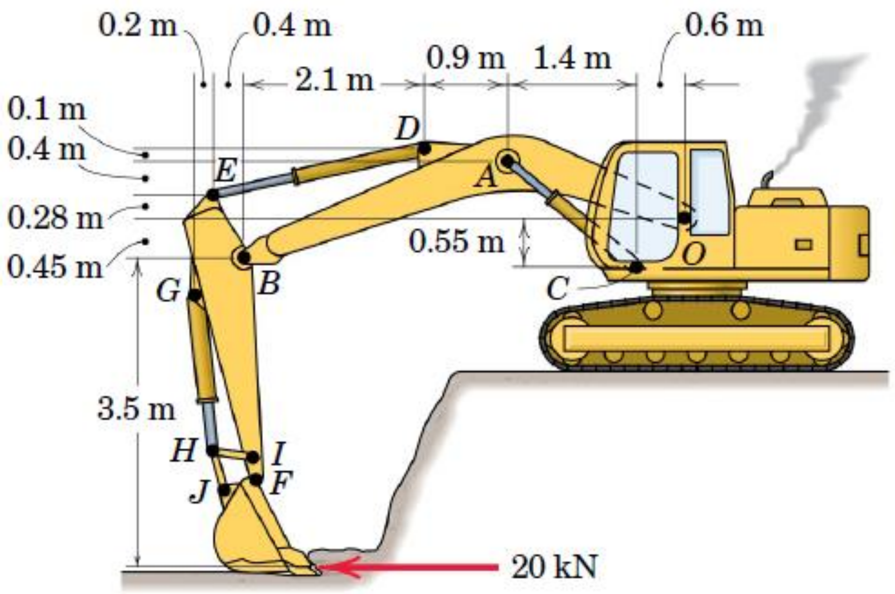
$[\Sigma F_x = 0]$	$30 + 100(3/5) - 15 - C_x = 0$	$C_x = 75$ lb

$[\Sigma F_y = 0]$	$100 + (-20) - 100(4/5) = 0$
$[\Sigma M_C = 0]$	$(30 - 15)(40) + (-20)(30) = 0$



Sample Problem 4/9

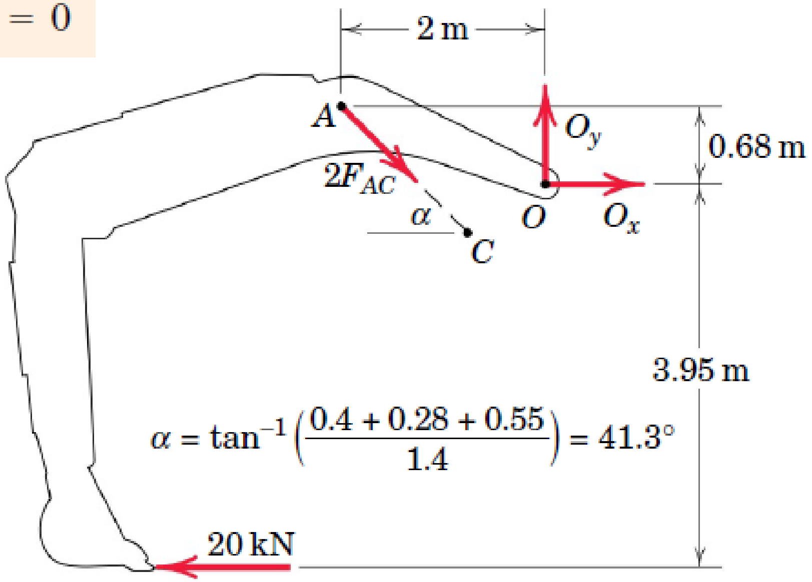
In the particular position shown, the excavator applies a 20-kN force parallel to the ground. There are two hydraulic cylinders *AC* to control the arm *OAB* and a single cylinder *DE* to control arm *EBIF*. (*a*) Determine the force in the hydraulic cylinders *AC* and the pressure p_{AC} against their pistons, which have an effective diameter of 95 mm. (*b*) Also determine the force in hydraulic cylinder *DE* and the pressure p_{DE} against its 105-mm-diameter piston. Neglect the weights of the members compared with the effects of the 20-kN force.



$$[\Sigma M_O = 0] \qquad -20\,000(3.95) - 2F_{AC}\cos 41.3°(0.68) + 2F_{AC}\sin 41.3°(2) = 0$$

$$F_{AC} = 48\,800 \text{ N or } 48.8 \text{ kN}$$

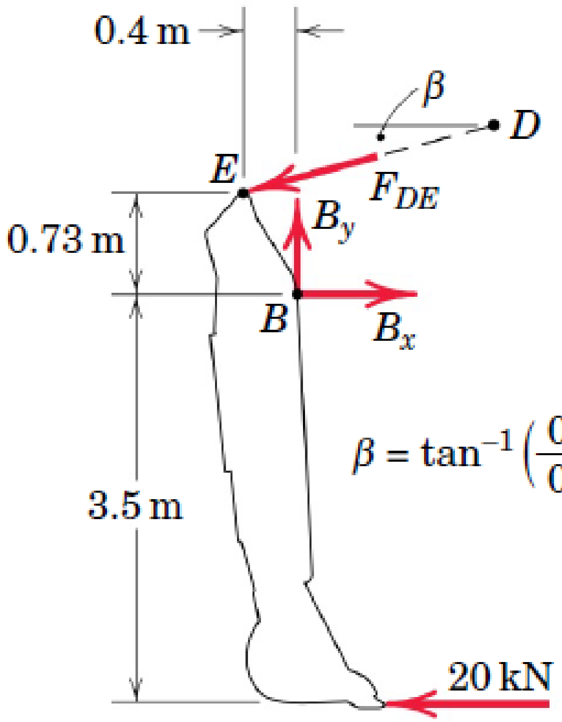
$$p_{AC} = \frac{F_{AC}}{A_{AC}} = \frac{48\,800}{\left(\pi\dfrac{0.095^2}{4}\right)} = 6.89(10^6) \text{ Pa or } 6.89 \text{ MPa}$$



$[\Sigma M_B = 0] \qquad -20\,000(3.5) + F_{DE}\cos 11.31°(0.73) + F_{DE}\sin 11.31°(0.4) = 0$

$F_{DE} = 88\,100 \text{ N or } 88.1 \text{ kN}$

$p_{DE} = \dfrac{F_{DE}}{A_{DE}} = \dfrac{88\,100}{\left(\pi\dfrac{0.105^2}{4}\right)} = 10.18(10^6) \text{ Pa or } 10.18 \text{ MPa}$



$\beta = \tan^{-1}\left(\dfrac{0.1 + 0.4}{0.4 + 2.1}\right) = 11.31°$