دانشکده مهندسی مکانیک

درس استاتیک

STATICS

Chapter 3 - Equilibrium
Class Lecture

❑ CONTENTS:

- Chapter 1: Introduction to Statics
- Chapter 2: Force Systems
- Chapter 3: **Equilibrium**
- Chapter 4: Structures
- Chapter 5: Distributed Forces
- Chapter 6: Friction

3.1 INTRODUCTION

- Statics:
  - Description of the force conditions necessary and sufficient to maintain the equilibrium
- Procedures developed here form the basis for solving problems in both statics and dynamics
- When a body is in equilibrium, the resultant of *all* forces acting on it is zero.

$$\mathbf{R} = \Sigma\mathbf{F} = 0 \qquad \mathbf{M} = \Sigma\mathbf{M} = 0$$

3.2 SYSTEM ISOLATION AND THE FREE-BODY DIAGRAM

- First, Define the system to be analyzed and represent all forces acting on the body
  - A *mechanical system* is defined as a body or group of bodies which can be conceptually isolated from all other bodies
- ***Free-Body Diagram* (FBD)**:
  - A diagrammatic representation of the isolated system treated as a single body which shows all forces applied to the system containing

3.2 System Isolation and the Free-Body Diagram

- Modeling the Action of Forces

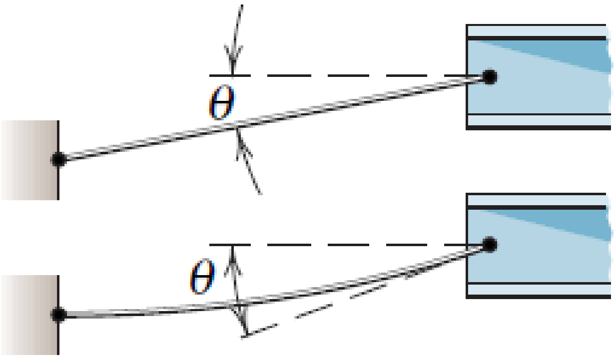
Type of Contact and Force Origin

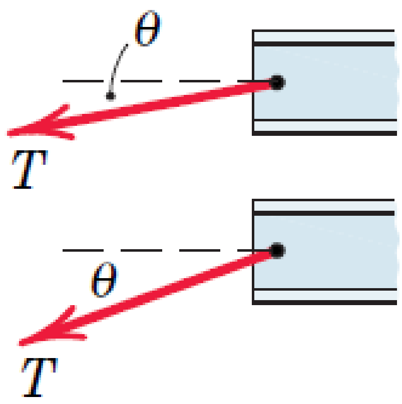
Action on Body to Be Isolated

1. Flexible cable, belt, chain, or rope

Weight of cable negligible

Weight of cable not negligible





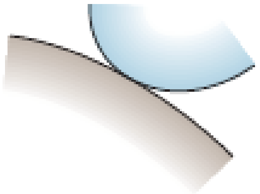
Force exerted by a flexible cable is always a tension away from the body in the direction of the cable.

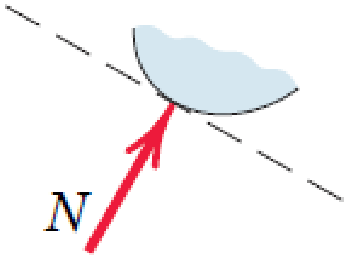
3.2 SYSTEM ISOLATION AND THE FREE-BODY DIAGRAM

- Modeling the Action of Forces

Type of Contact and Force Origin | Action on Body to Be Isolated

2. Smooth surfaces

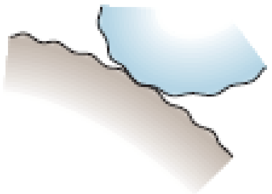
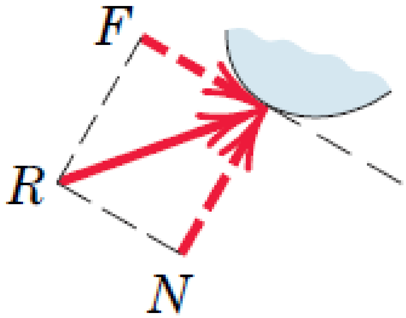




Contact force is compressive and is normal to the surface.

3.2 SYSTEM ISOLATION AND THE FREE-BODY DIAGRAM

- Modeling the Action of Forces

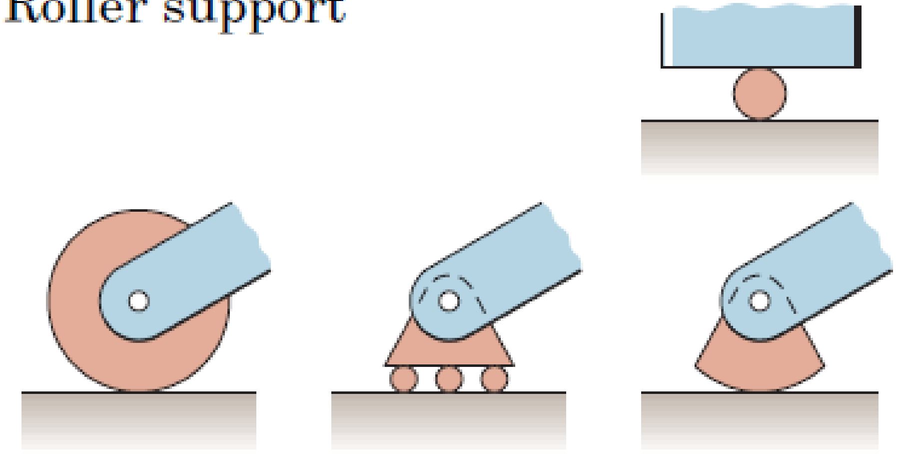
Type of Contact and Force Origin	Action on Body to Be Isolated
3. Rough surfaces 	 Rough surfaces are capable of supporting a tangential component F (frictional force) as well as a normal component N of the resultant contact force R.

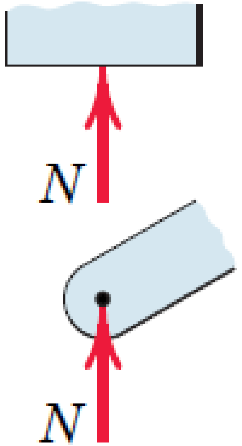
3.2 System Isolation and the Free-Body Diagram

- Modeling the Action of Forces

Type of Contact and Force Origin	Action on Body to Be Isolated

4. Roller support





Roller, rocker, or ball support transmits a compressive force normal to the supporting surface.

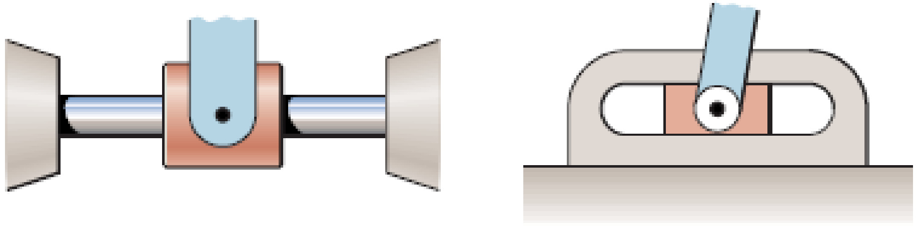
3.2 System Isolation and the Free-Body Diagram

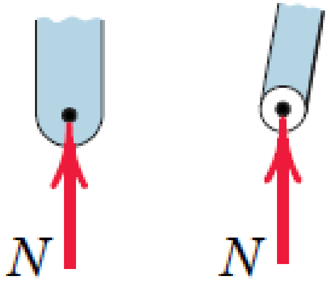
- Modeling the Action of Forces

Type of Contact and Force Origin

Action on Body to Be Isolated

5. Freely sliding guide





Collar or slider free to move along smooth guides; can support force normal to guide only.

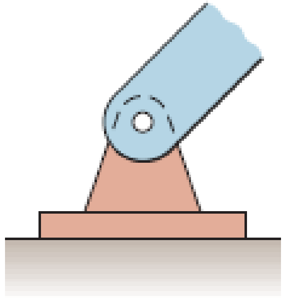
3.2 System Isolation and the Free-Body Diagram

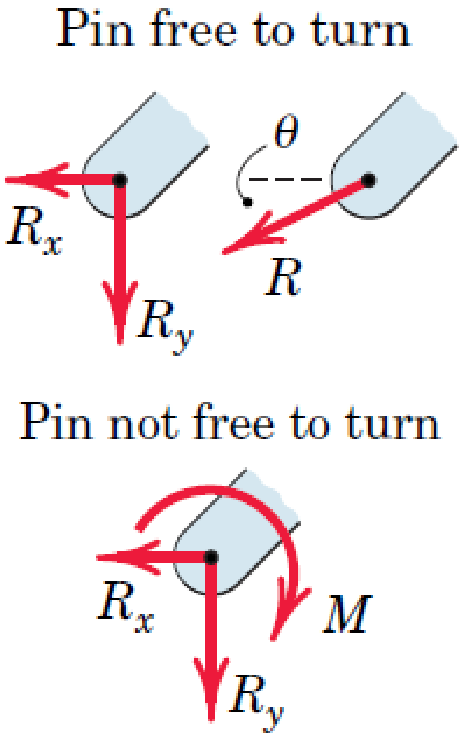
- Modeling the Action of Forces

Type of Contact and Force Origin

Action on Body to Be Isolated

6. Pin connection





A freely hinged pin connection is capable of supporting a force in any direction in the plane normal to the pin axis. We may either show two components R_x and R_y or a magnitude R and direction θ. A pin not free to turn also supports a couple M.

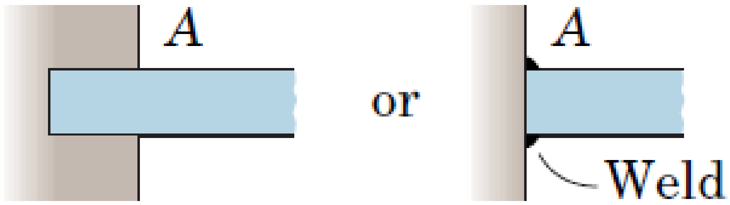
3.2 System Isolation and the Free-Body Diagram

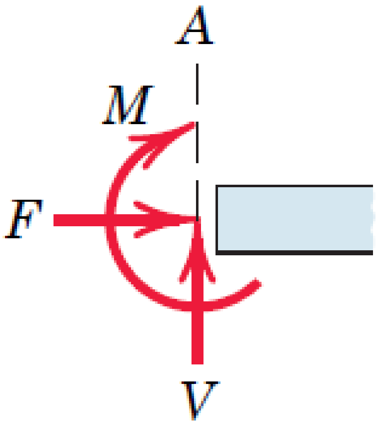
- Modeling the Action of Forces

Type of Contact and Force Origin

Action on Body to Be Isolated

7. Built-in or fixed support





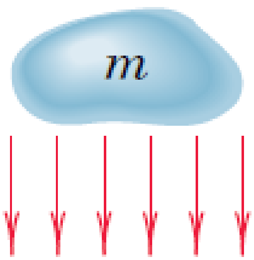
A built-in or fixed support is capable of supporting an axial force F, a transverse force V (shear force), and a couple M (bending moment) to prevent rotation.

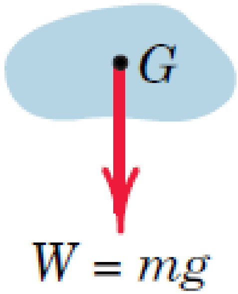
3.2 System Isolation and the Free-Body Diagram

- Modeling the Action of Forces

Type of Contact and Force Origin | Action on Body to Be Isolated

8. Gravitational attraction





The resultant of gravitational attraction on all elements of a body of mass m is the weight $W = mg$ and acts toward the center of the earth through the center of gravity G.

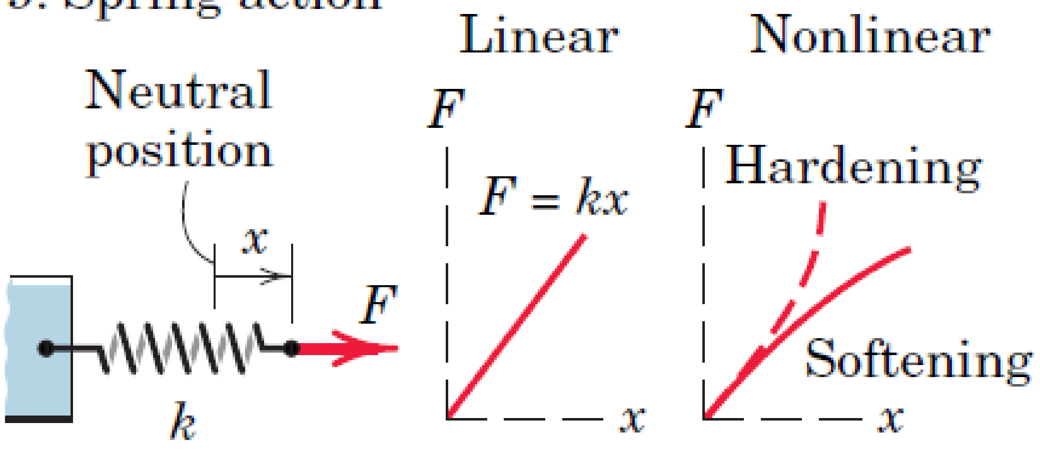
3.2 System Isolation and the Free-Body Diagram

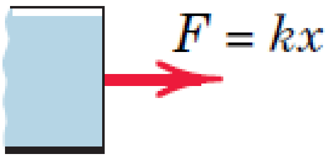
- Modeling the Action of Forces

Type of Contact and Force Origin

Action on Body to Be Isolated

9. Spring action





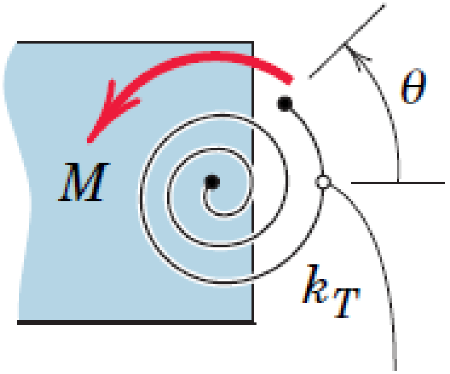
Spring force is tensile if the spring is stretched and compressive if compressed. For a linearly elastic spring the stiffness k is the force required to deform the spring a unit distance.

3.2 System Isolation and the Free-Body Diagram

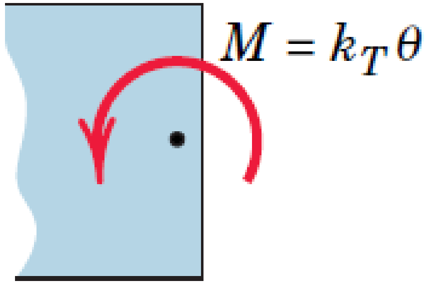
- Modeling the Action of Forces

Type of Contact and Force Origin | Action on Body to Be Isolated

10. Torsional spring action



Neutral position



For a linear torsional spring, the applied moment M is proportional to the angular deflection θ from the neutral position. The stiffness k_T is the moment required to deform the spring one radian.

3.2 System Isolation and the Free-Body Diagram

- Construction of Free-Body Diagrams

  - Step1. Decide which system to isolate. (involve one or more of the desired unknown quantities)

  - Step2. Isolate the system by drawing a diagram which represents its complete external boundary

  - Step 3. Identify all forces acting as applied by the removed contacting and attracting bodies

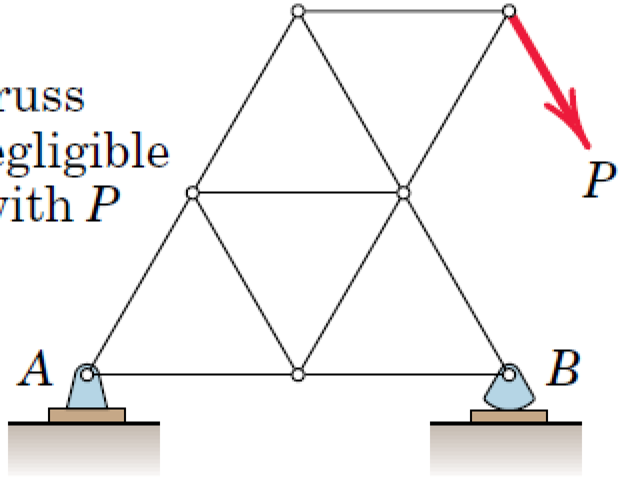
  - Step 4. Show the choice of coordinate axes directly on the diagram

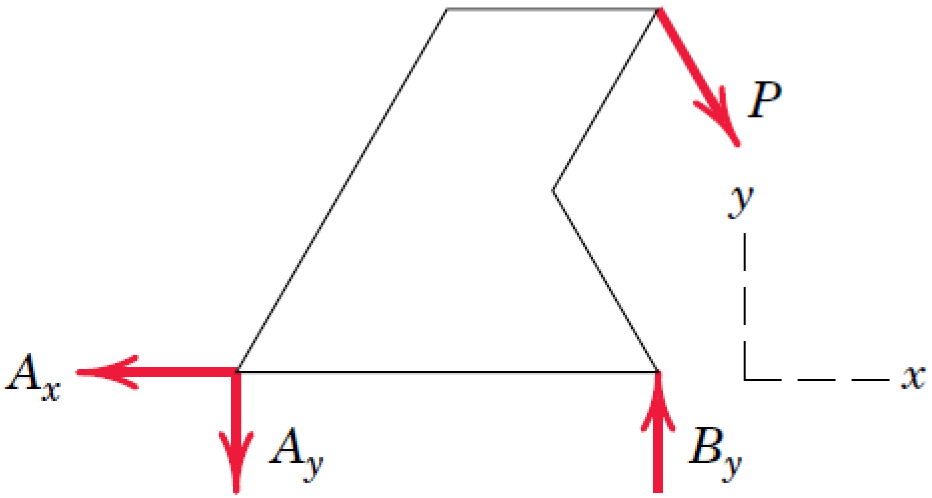
3.2 System Isolation and the Free-Body Diagram

- Examples of Free-Body Diagrams

1. Plane truss

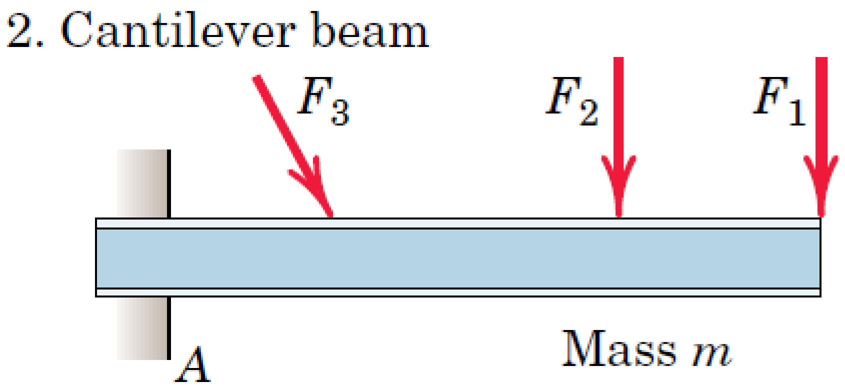
Weight of truss assumed negligible compared with P

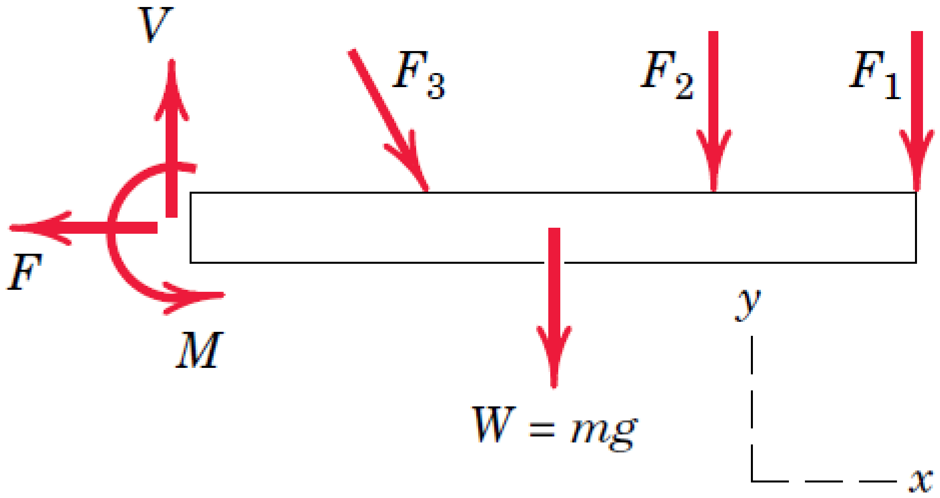




3.2 System Isolation and the Free-Body Diagram

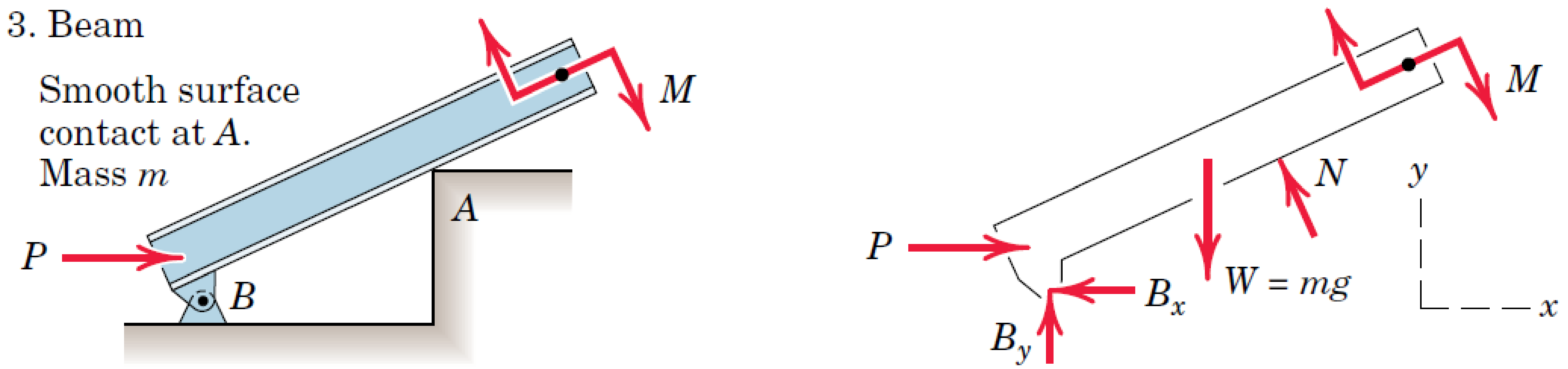
- Examples of Free-Body Diagrams





3.2 SYSTEM ISOLATION AND THE FREE-BODY DIAGRAM

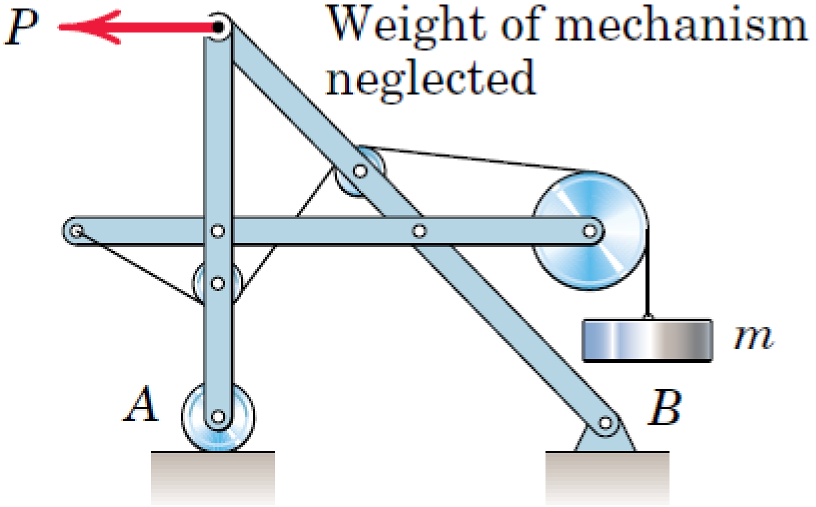
- Examples of Free-Body Diagrams

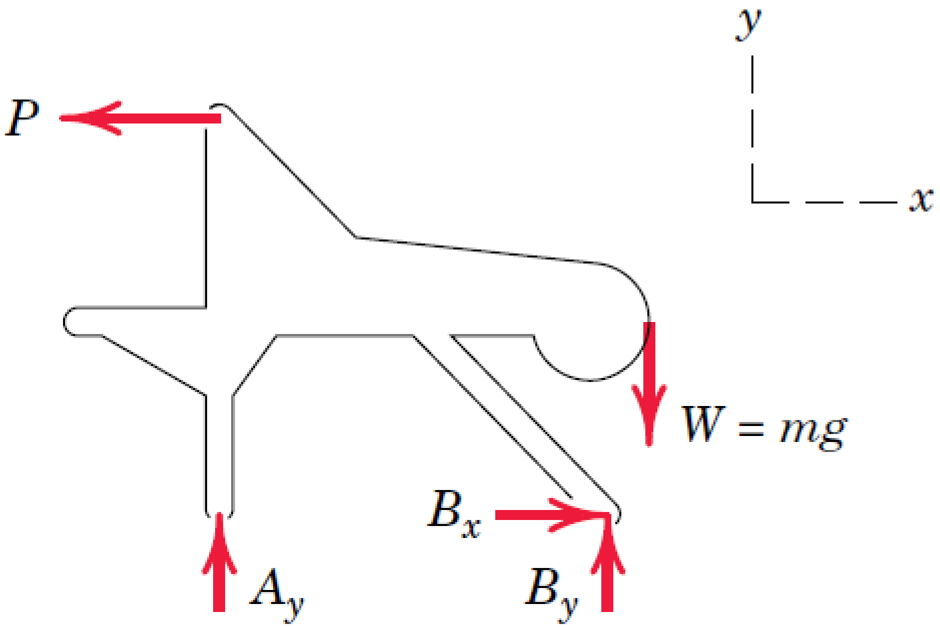


3.2 System Isolation and the Free-Body Diagram

- Examples of Free-Body Diagrams

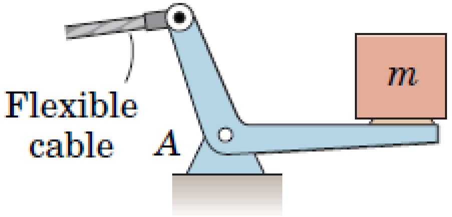
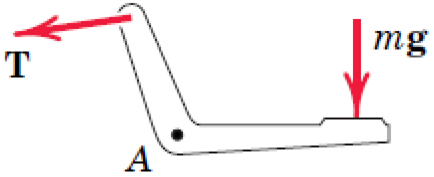
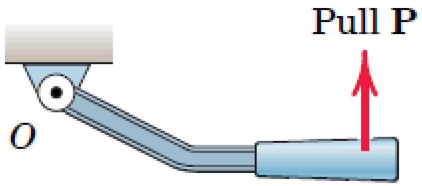
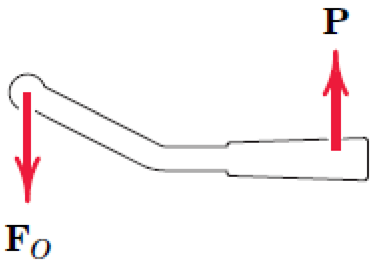
4. Rigid system of interconnected bodies analyzed as a single unit





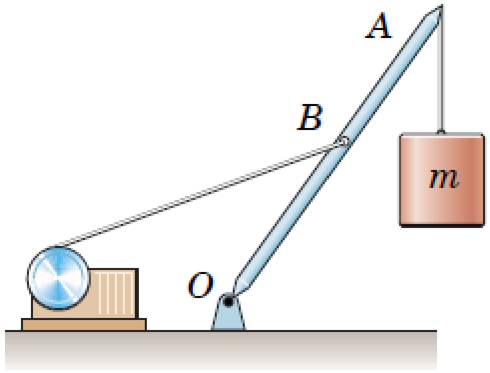
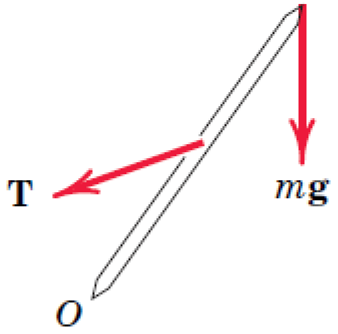
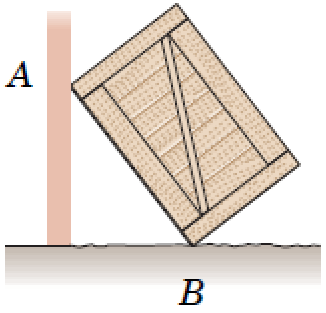
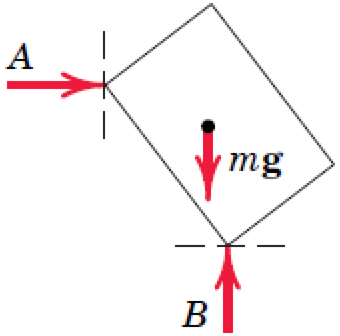
3.2 System Isolation and the Free-Body Diagram

- Examples ----- Incomplete P3.A

	Body	Incomplete FBD
1. Bell crank supporting mass m with pin support at A.		
2. Control lever applying torque to shaft at O.		

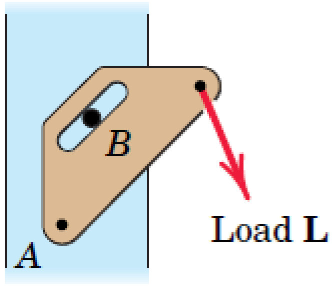
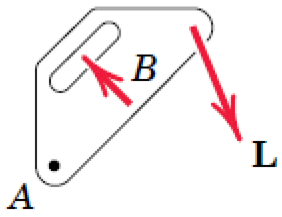
3.2 System Isolation and the Free-Body Diagram

- Examples ----- Incomplete

3. Boom OA, of negligible mass compared with mass m. Boom hinged at O and supported by hoisting cable at B.		
4. Uniform crate of mass m leaning against smooth vertical wall and supported on a rough horizontal surface.		

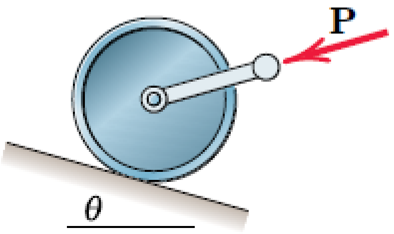
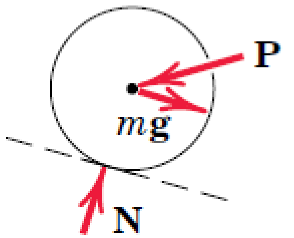
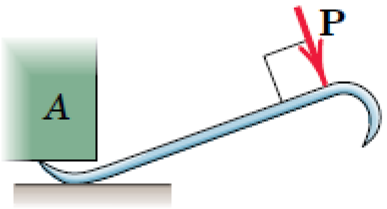
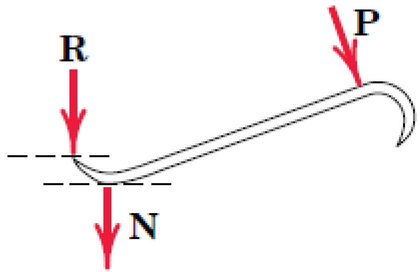
3.2 System Isolation and the Free-Body Diagram

- Examples ----- Incomplete

5. Loaded bracket supported by pin connection at *A* and fixed pin in smooth slot at *B*.		

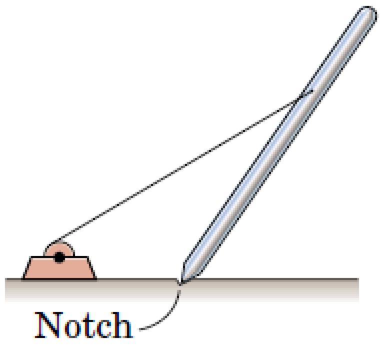
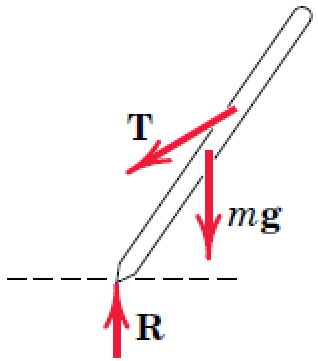
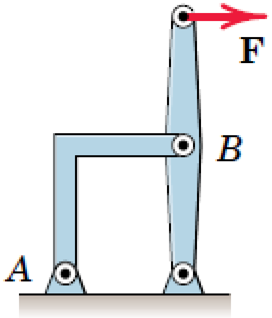
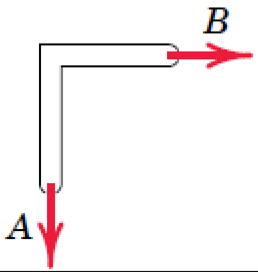
3.2 System Isolation and the Free-Body Diagram

- Examples ----- Incomplete P3.B

	Body	Wrong or Incomplete FBD
1. Lawn roller of mass m being pushed up incline θ.		
2. Prybar lifting body A having smooth horizontal surface. Bar rests on horizontal rough surface.		

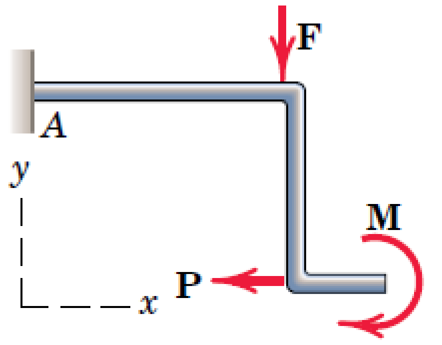
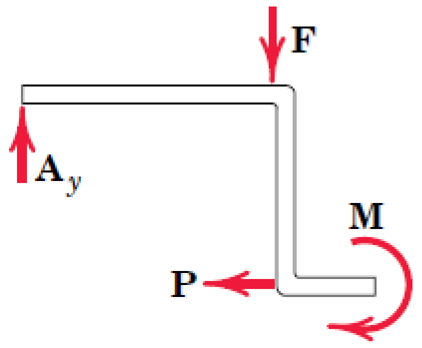
3.2 System Isolation and the Free-Body Diagram

- Examples ----- Incomplete

3. Uniform pole of mass m being hoisted into position by winch. Horizontal supporting surface notched to prevent slipping of pole.		
4. Supporting angle bracket for frame; pin joints.		

3.2 System Isolation and the Free-Body Diagram

- Examples ----- Incomplete

5. Bent rod welded to support at *A* and subjected to two forces and couple.		

3.3 Equilibrium Conditions

- Equilibrium:
  - The condition in which the resultant of all forces and moments acting on a body is zero.
  - A body is in equilibrium if all forces and moments applied to it are in balance.
  - Vector Equation:

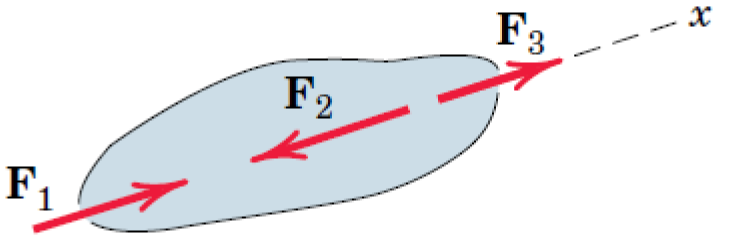
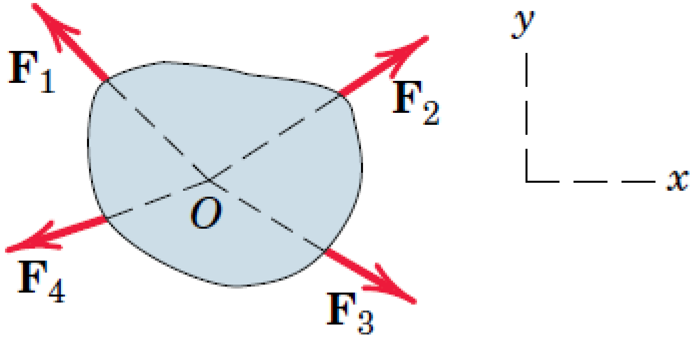
$$\mathbf{R} = \Sigma\mathbf{F} = 0 \qquad \mathbf{M} = \Sigma\mathbf{M} = 0$$

  - Components Equation:
    - ✓ about any point O on or off the body

$$\Sigma F_x = 0 \qquad \Sigma F_y = 0 \qquad \Sigma M_O = 0$$

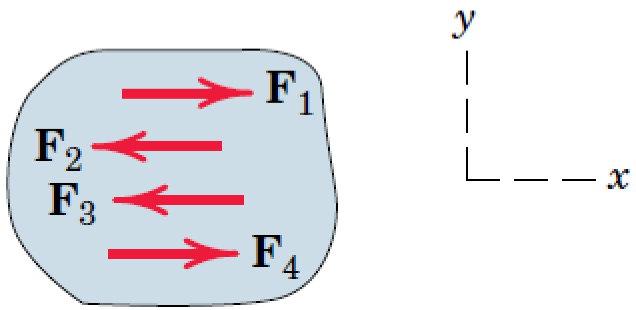
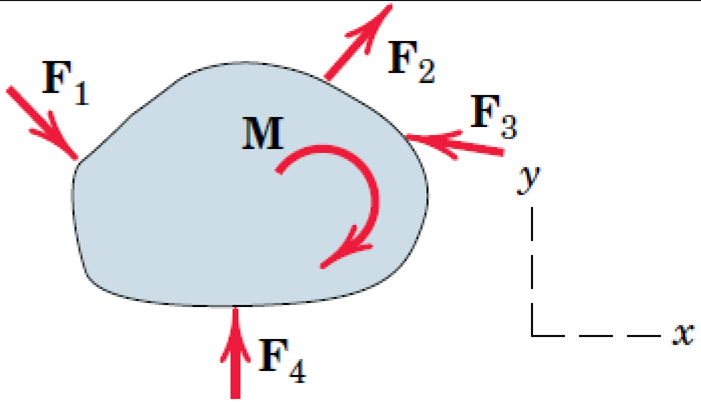
3.3 EQUILIBRIUM CONDITIONS

- Categories of Equilibrium

Force System	Free-Body Diagram	Independent Equations
1. Collinear		$\Sigma F_x = 0$
2. Concurrent at a point		$\Sigma F_x = 0$ $\Sigma F_y = 0$

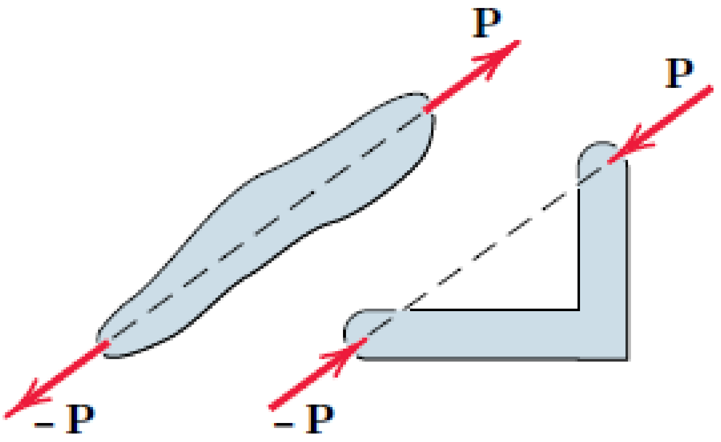
3.3 Equilibrium Conditions

- Categories of Equilibrium

Force System	Free-Body Diagram	Independent Equations
3. Parallel		$\Sigma F_x = 0 \quad \Sigma M_z = 0$
4. General		$\Sigma F_x = 0 \quad \Sigma M_z = 0$ $\Sigma F_y = 0$

3.3 EQUILIBRIUM CONDITIONS

- Two-Force Members in equilibrium:
  - The forces must be:
    - ✓ *Equal*
    - ✓ *Opposite*
    - ✓ *Collinear*



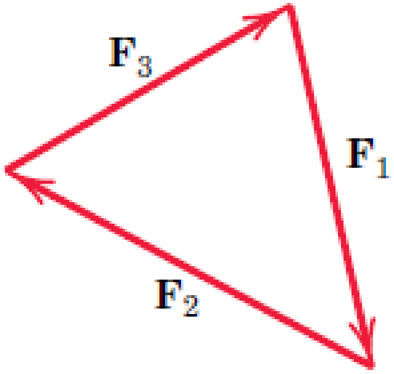
- ✓ The shape of the member does not affect, the weights of the member is negligible

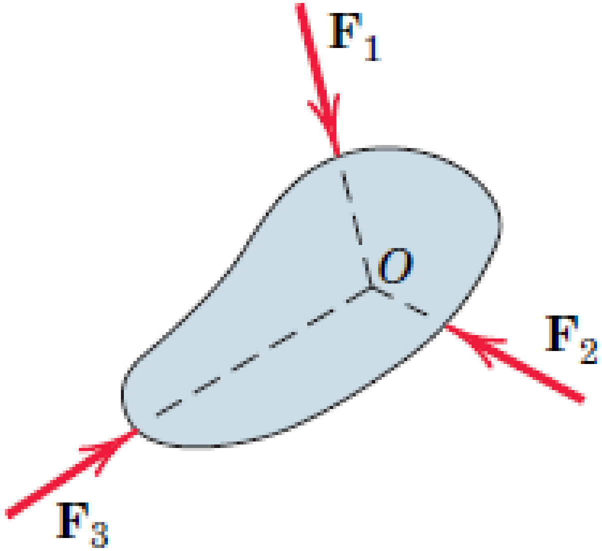
3.3 Equilibrium Conditions

- Three-Force Members in equilibrium:
  - Lines of action of the three forces to be *concurrent*

    (Except for 3 parallel forces)

    ✓ Three forces make closed polygon.





3.3 EQUILIBRIUM CONDITIONS

- Alternative Equilibrium Equations

$$\Sigma F_x = 0 \qquad \Sigma M_A = 0 \qquad \Sigma M_B = 0$$

✓ The two points *A* and *B* must not lie on a line perpendicular to the *x*-direction.

$$\Sigma M_A = 0 \qquad \Sigma M_B = 0 \qquad \Sigma M_C = 0$$

✓ *A*, *B*, and *C* are any three points not on the same straight line.

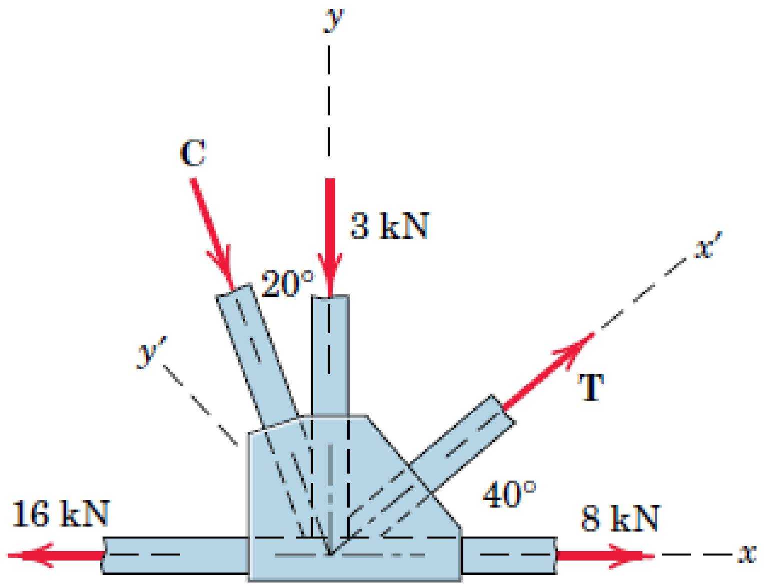
3.3 Equilibrium Conditions

- Approach to Solving Problems

1) Identify clearly the quantities which are known and unknown.
2) Choose body (or system of connected bodies) to be isolated.
3) Choose a convenient set of reference axes.
4) Identify and state the applicable force and moment principles or equations.
5) Match the number of independent equations with the number of unknowns.
6) Carry out the solution and check the results.

Sample Problem 3/1

Determine the magnitudes of the forces **C** and **T**, which, along with the other three forces shown, act on the bridge-truss joint.



Solution I (scalar algebra).

$[\Sigma F_x = 0]$ $\quad 8 + T\cos 40^\circ + C\sin 20^\circ - 16 = 0$

$$0.766T + 0.342C = 8$$

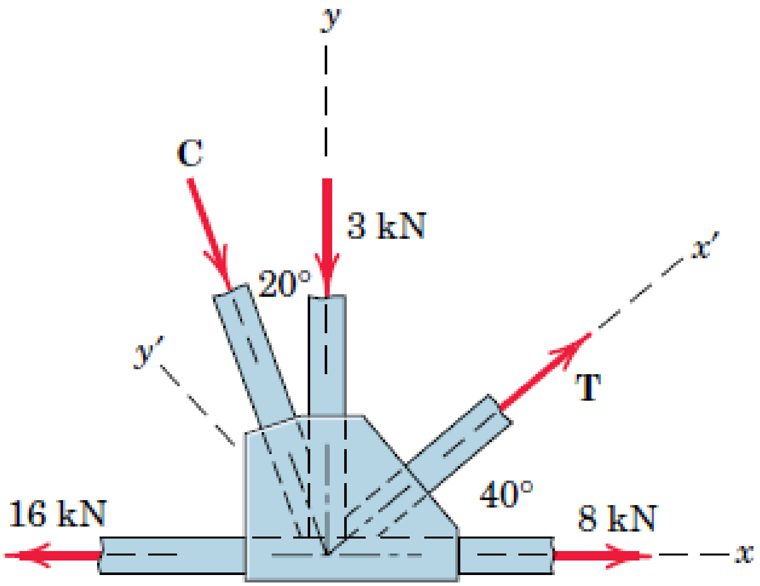
$[\Sigma F_y = 0]$ $\quad T\sin 40^\circ - C\cos 20^\circ - 3 = 0$

$$0.643T - 0.940C = 3$$

$$T = 9.09 \text{ kN} \qquad C = 3.03 \text{ kN}$$

Sample Problem 3/1

Determine the magnitudes of the forces **C** and **T**, which, along with the other three forces shown, act on the bridge-truss joint.



Solution II (scalar algebra).

$[\Sigma F_{y'} = 0]$ $\quad -C \cos 20° - 3 \cos 40° - 8 \sin 40° + 16 \sin 40° = 0$

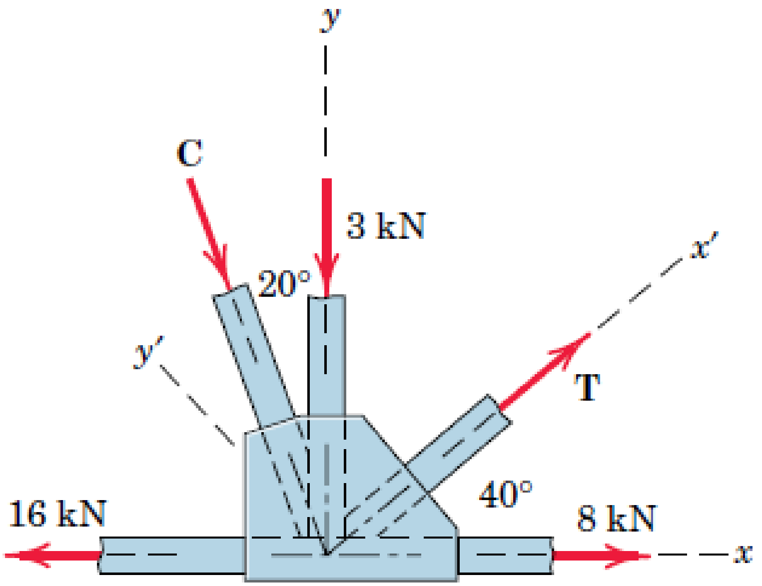
$C = 3.03 \text{ kN}$

$[\Sigma F_{x'} = 0]$ $\quad T + 8 \cos 40° - 16 \cos 40° - 3 \sin 40° - 3.03 \sin 20° = 0$

$T = 9.09 \text{ kN}$

Sample Problem 3/1

Determine the magnitudes of the forces C and T, which, along with the other three forces shown, act on the bridge-truss joint.



Solution III (vector algebra).

$$[\Sigma \mathbf{F} = \mathbf{0}] \qquad 8\mathbf{i} + (T\cos 40^\circ)\mathbf{i} + (T\sin 40^\circ)\mathbf{j} - 3\mathbf{j} + (C\sin 20^\circ)\mathbf{i} - (C\cos 20^\circ)\mathbf{j} - 16\mathbf{i} = \mathbf{0}$$

$$8 + T\cos 40^\circ + C\sin 20^\circ - 16 = 0$$

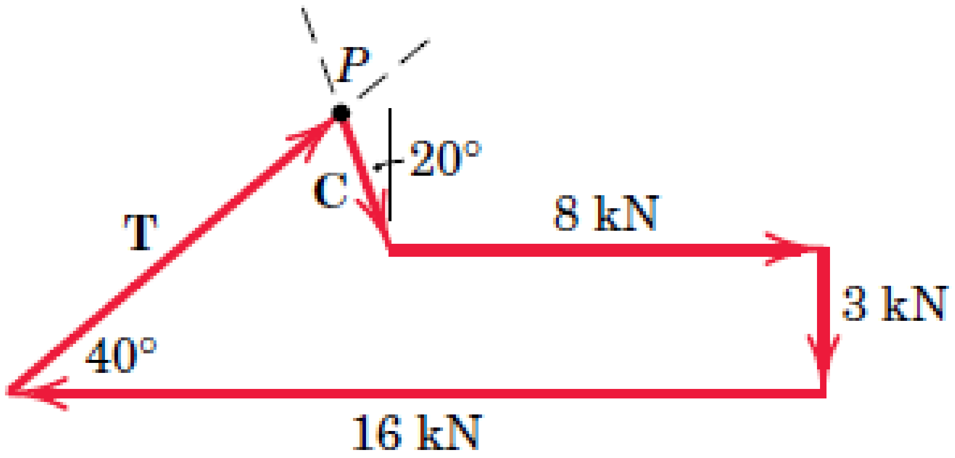
$$T\sin 40^\circ - 3 - C\cos 20^\circ = 0$$

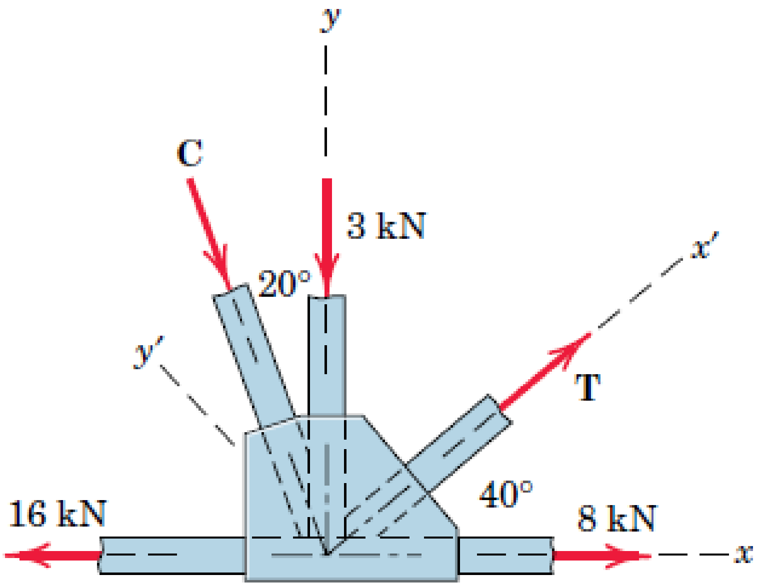
$$T = 9.09\text{ kN} \qquad C = 3.03\text{ kN}$$

Sample Problem 3/1

Determine the magnitudes of the forces **C** and **T**, which, along with the other three forces shown, act on the bridge-truss joint.

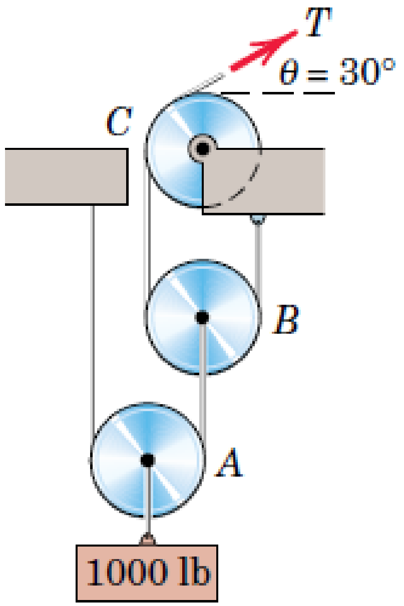
Solution IV (geometric).





Sample Problem 3/2

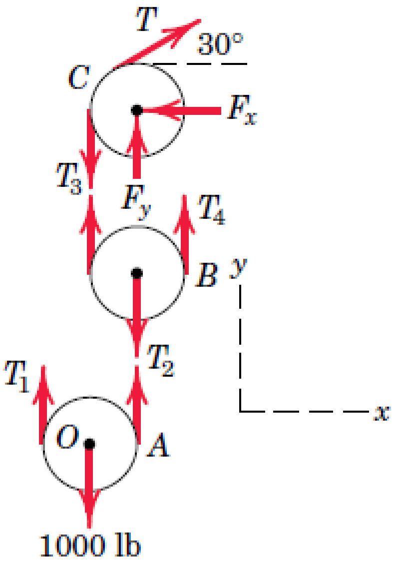
Calculate the tension T in the cable which supports the 1000-lb load with the pulley arrangement shown. Each pulley is free to rotate about its bearing, and the weights of all parts are small compared with the load. Find the magnitude of the total force on the bearing of pulley C.



$[\Sigma M_O = 0] \qquad T_1 r - T_2 r = 0 \qquad T_1 = T_2$

$[\Sigma F_y = 0] \qquad T_1 + T_2 - 1000 = 0 \qquad 2T_1 = 1000 \qquad T_1 = T_2 = 500 \text{ lb}$

$T_3 = T_4 = T_2/2 = 250 \text{ lb} \qquad T = T_3 \quad \text{or} \quad T = 250 \text{ lb}$



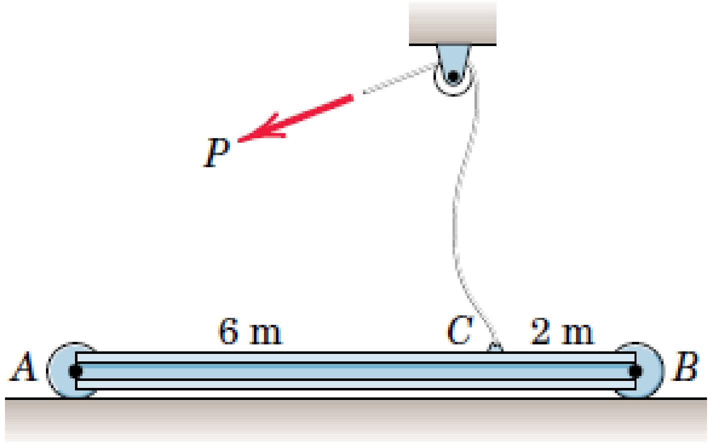
$[\Sigma F_x = 0] \qquad 250 \cos 30^\circ - F_x = 0 \qquad F_x = 217 \text{ lb}$

$[\Sigma F_y = 0] \qquad F_y + 250 \sin 30^\circ - 250 = 0 \qquad F_y = 125 \text{ lb}$

$[F = \sqrt{F_x^2 + F_y^2}] \qquad F = \sqrt{(217)^2 + (125)^2} = 250 \text{ lb}$

Sample Problem 3/3

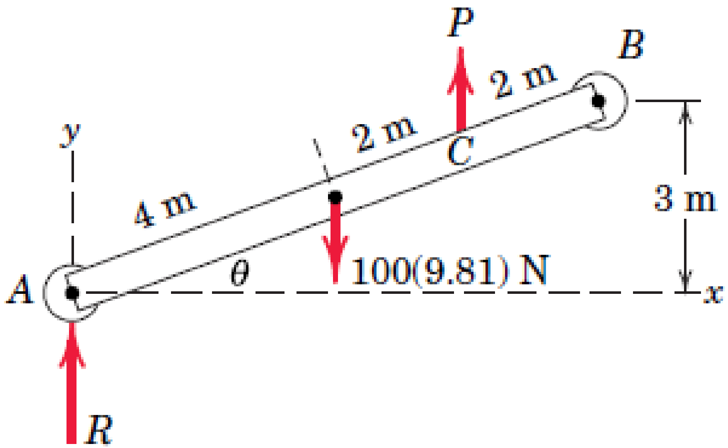
The uniform 100-kg I-beam is supported initially by its end rollers on the horizontal surface at A and B. By means of the cable at C, it is desired to elevate end B to a position 3 m above end A. Determine the required tension P, the reaction at A, and the angle θ made by the beam with the horizontal in the elevated position.



$[\Sigma M_A = 0]$ $P(6 \cos \theta) - 981(4 \cos \theta) = 0$ $P = 654 \text{ N}$

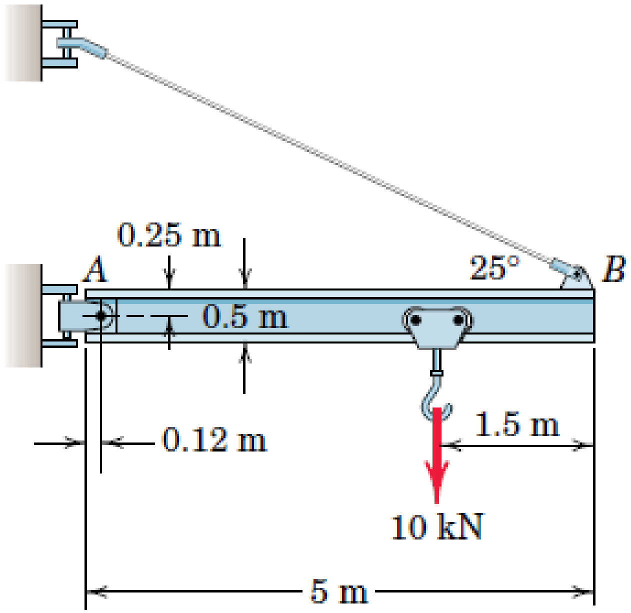
$[\Sigma F_y = 0]$ $654 + R - 981 = 0$ $R = 327 \text{ N}$

$\sin \theta = 3/8$ $\theta = 22.0^\circ$



Sample Problem 3/4

Determine the magnitude T of the tension in the supporting cable and the magnitude of the force on the pin at A for the jib crane shown. The beam AB is a standard 0.5-m I-beam with a mass of 95 kg per meter of length.



The weight of the beam is $95(10^{-3})(5)9.81 = 4.66$ kN

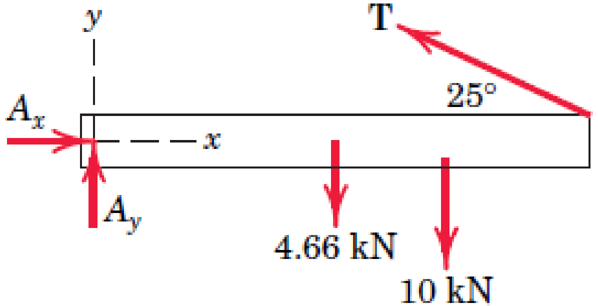
$[\Sigma M_A = 0]$ $\quad (T\cos 25°)0.25 + (T\sin 25°)(5 - 0.12) - 10(5 - 1.5 - 0.12) - 4.66(2.5 - 0.12) = 0$

$T = 19.61$ kN

$[\Sigma F_x = 0]$ $\quad A_x - 19.61\cos 25° = 0 \quad A_x = 17.77$ kN

$[\Sigma F_y = 0]$ $\quad A_y + 19.61\sin 25° - 4.66 - 10 = 0 \quad A_y = 6.37$ kN

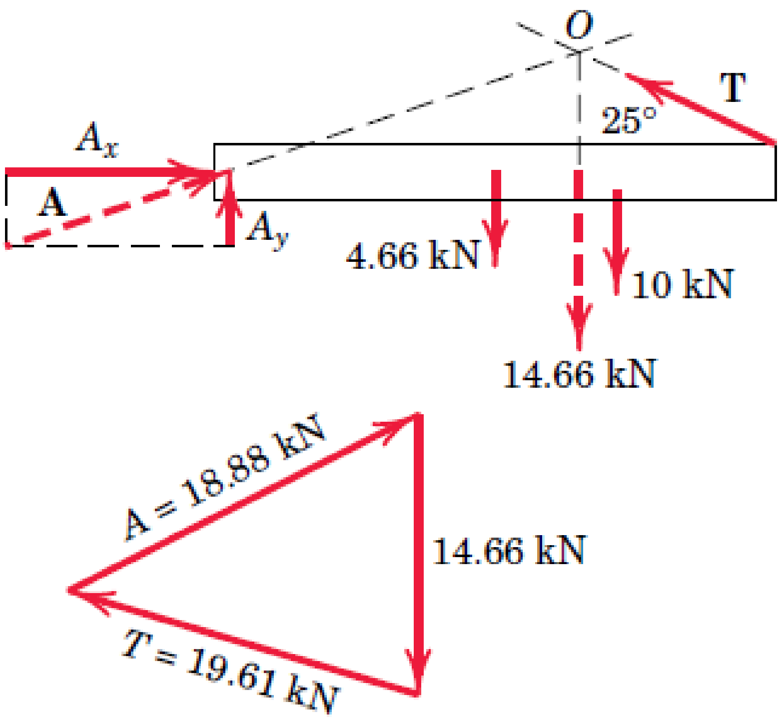
$[A = \sqrt{A_x^2 + A_y^2}] \quad A = \sqrt{(17.77)^2 + (6.37)^2} = 18.88$ kN



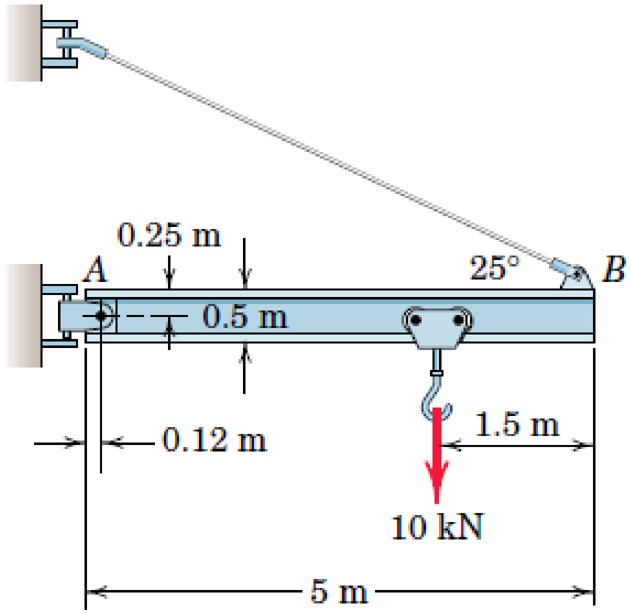
Free-body diagram

Sample Problem 3/4

Determine the magnitude T of the tension in the supporting cable and the magnitude of the force on the pin at A for the jib crane shown. The beam AB is a standard 0.5-m I-beam with a mass of 95 kg per meter of length.



Graphical solution



3.4 Equilibrium in 3D: Equilibrium Conditions

- The general conditions for the equilibrium of a body:
  - The resultant force and resultant couple on a body in equilibrium be zero

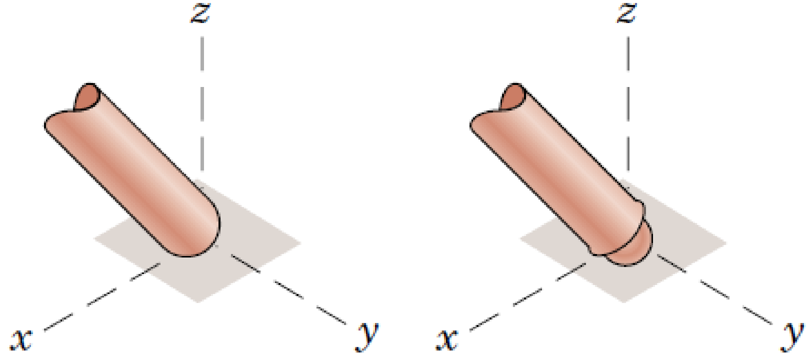
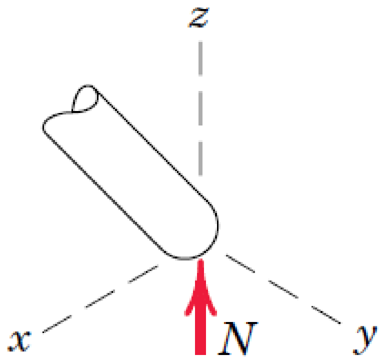
$$\Sigma \mathbf{F} = \mathbf{0} \quad \text{or} \quad \begin{cases} \Sigma F_x = 0 \\ \Sigma F_y = 0 \\ \Sigma F_z = 0 \end{cases}$$

$$\Sigma \mathbf{M} = \mathbf{0} \quad \text{or} \quad \begin{cases} \Sigma M_x = 0 \\ \Sigma M_y = 0 \\ \Sigma M_z = 0 \end{cases}$$

- Free-Body Diagrams…

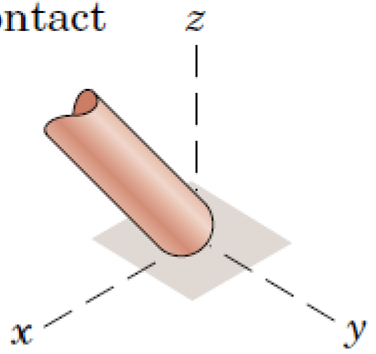
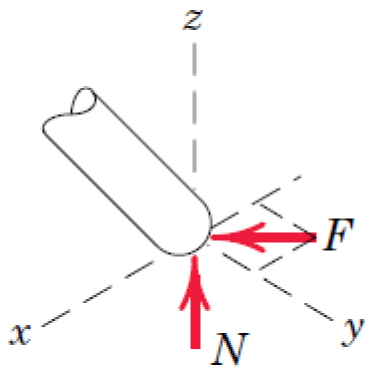
3.4 EQUILIBRIUM IN 3D: EQUILIBRIUM CONDITIONS

- Modeling the action of forces in three-dimensional analysis

Type of Contact and Force Origin	Action on Body to Be Isolated
1. Member in contact with smooth surface, or ball-supported member 	 Force must be normal to the surface and directed toward the member.

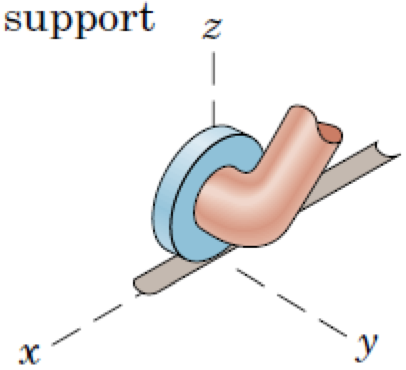
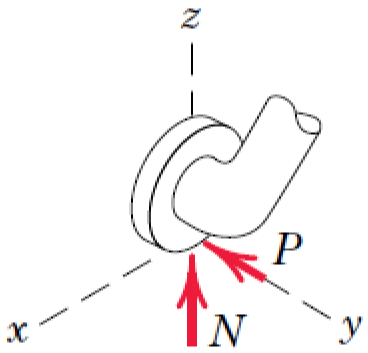
3.4 Equilibrium in 3D: Equilibrium Conditions

- Modeling the action of forces in three-dimensional analysis

Type of Contact and Force Origin	Action on Body to Be Isolated
2. Member in contact with rough surface 	 The possibility exists for a force F tangent to the surface (friction force) to act on the member, as well as a normal force N.

3.4 EQUILIBRIUM IN 3D: EQUILIBRIUM CONDITIONS

- Modeling the action of forces in three-dimensional analysis

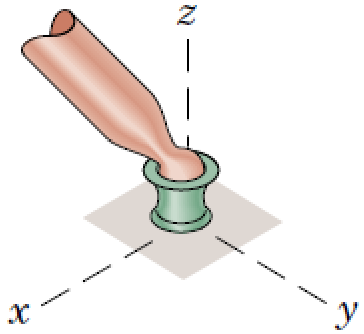
Type of Contact and Force Origin		Action on Body to Be Isolated	
3. Roller or wheel support with lateral constraint			A lateral force P exerted by the guide on the wheel can exist, in addition to the normal force N.

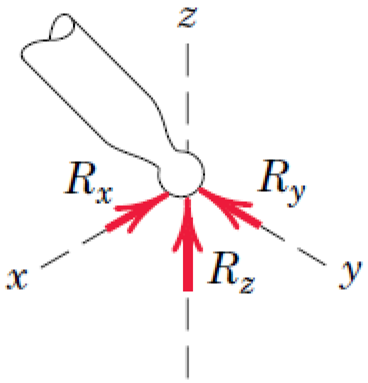
3.4 Equilibrium in 3D: Equilibrium Conditions

- Modeling the action of forces in three-dimensional analysis

Type of Contact and Force Origin | Action on Body to Be Isolated

4. Ball-and-socket joint





A ball-and-socket joint free to pivot about the center of the ball can support a force **R** with all three components.

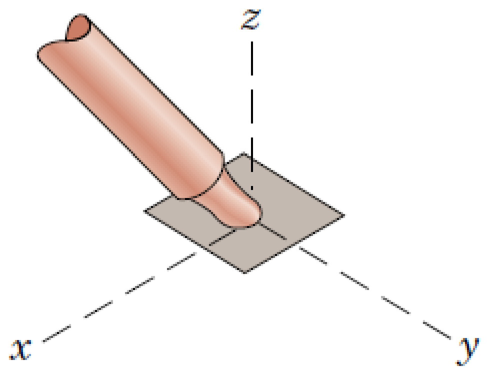
3.4 EQUILIBRIUM IN 3D: EQUILIBRIUM CONDITIONS

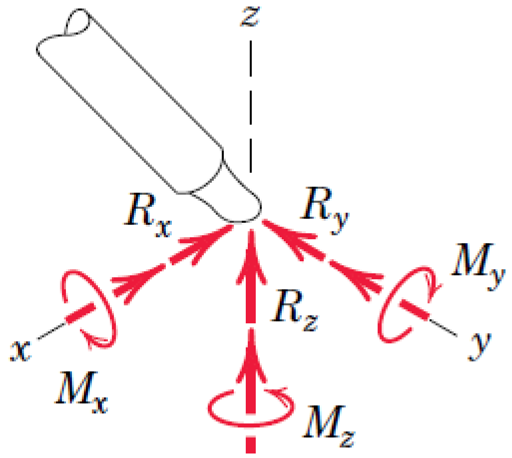
- Modeling the action of forces in three-dimensional analysis

Type of Contact and Force Origin

Action on Body to Be Isolated

5. Fixed connection (embedded or welded)





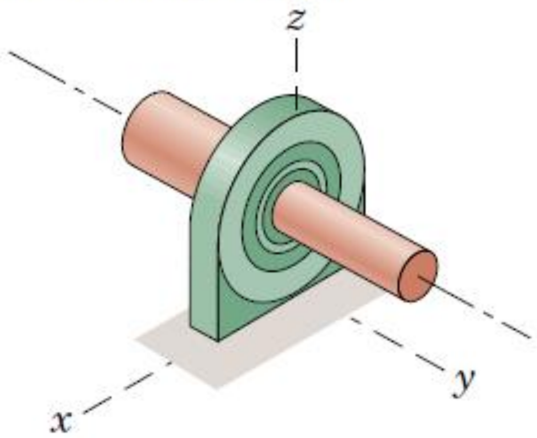
In addition to three components of force, a fixed connection can support a couple **M** represented by its three components.

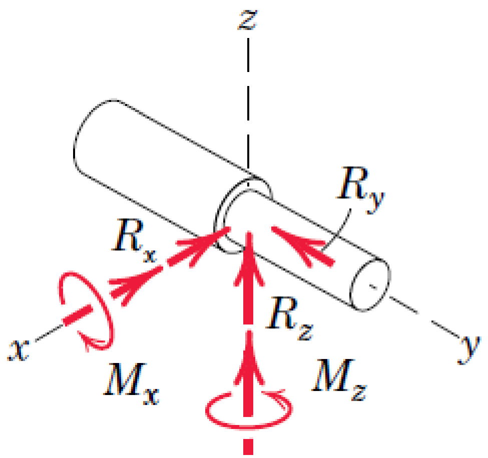
3.4 Equilibrium in 3D: Equilibrium Conditions

- Modeling the action of forces in three-dimensional analysis

Type of Contact and Force Origin | Action on Body to Be Isolated

6. Thrust-bearing support





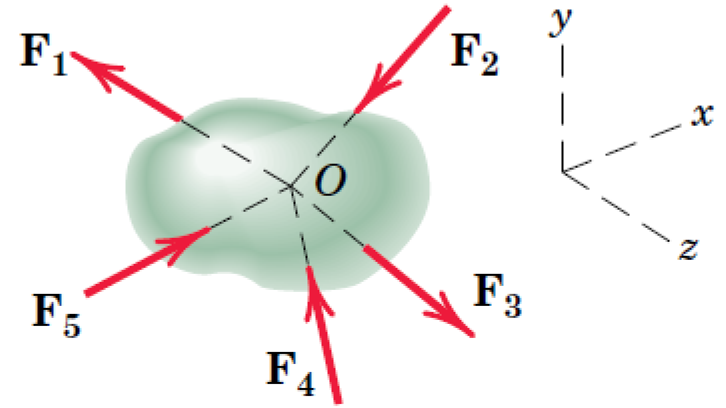
Thrust bearing is capable of supporting axial force R_y as well as radial forces R_x and R_z. Couples M_x and M_z must, in some cases, be assumed zero in order to provide statical determinacy.

3.4 Equilibrium in 3D: Equilibrium Conditions

- Categories of Equilibrium
  - ***Category 1,*** equilibrium of forces all concurrent at point *O*, requires all three force equations, but no moment equations because the moment of the forces about any axis through *O* is zero.
  - ***Category 2,*** equilibrium of forces which are concurrent with a line, requires all equations except the moment equation about that line, which is automatically satisfied.
  - ***Category 3,*** equilibrium of parallel forces, requires only one force equation, the one in the direction of the forces (*x*-direction as shown), and two moment equations about the axes (*y* and *z*) which are normal to the direction of the forces.
  - ***Category 4,*** equilibrium of a general system of forces, requires all three force equations and all three moment equations.

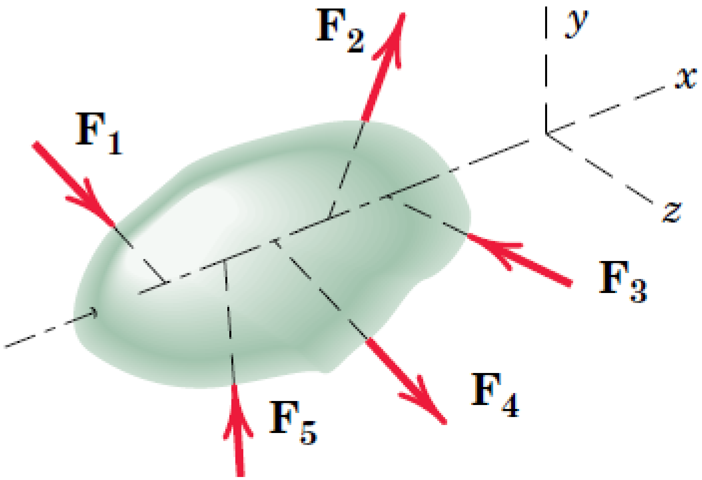
3.4 EQUILIBRIUM IN 3D: EQUILIBRIUM CONDITIONS

- Categories of equilibrium in three dimensions

Force System	Free-Body Diagram	Independent Equations
1. Concurrent at a point		$\Sigma F_x = 0$ $\Sigma F_y = 0$ $\Sigma F_z = 0$

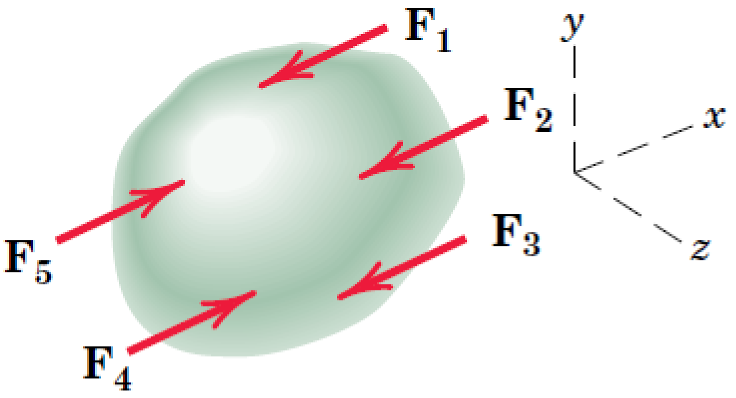
3.4 Equilibrium in 3D: Equilibrium Conditions

- Categories of equilibrium in three dimensions

2. Concurrent with a line		$\Sigma F_x = 0$ $\Sigma M_y = 0$ $\Sigma F_y = 0$ $\Sigma M_z = 0$ $\Sigma F_z = 0$

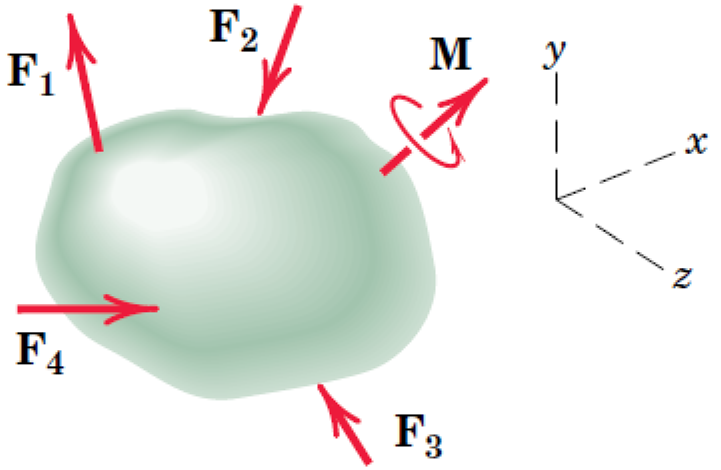
3.4 Equilibrium in 3D: Equilibrium Conditions

- Categories of equilibrium in three dimensions

3. Parallel		$\Sigma F_x = 0$ $\quad \Sigma M_y = 0$ $\Sigma M_z = 0$

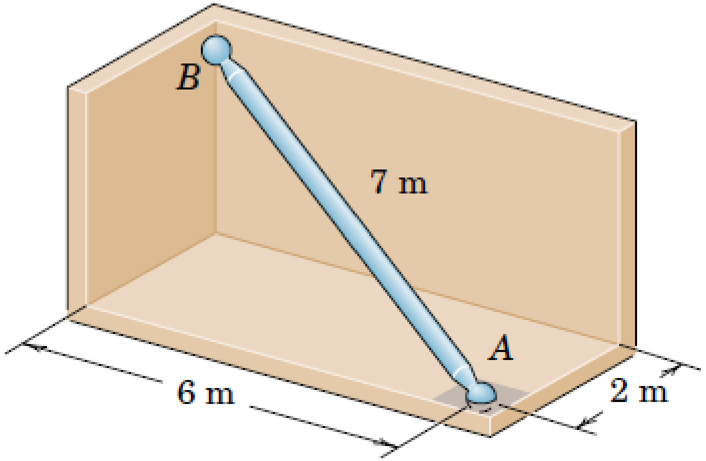
3.4 Equilibrium in 3D: Equilibrium Conditions

- Categories of equilibrium in three dimensions

4. General		$\Sigma F_x = 0$ $\Sigma M_x = 0$ $\Sigma F_y = 0$ $\Sigma M_y = 0$ $\Sigma F_z = 0$ $\Sigma M_z = 0$

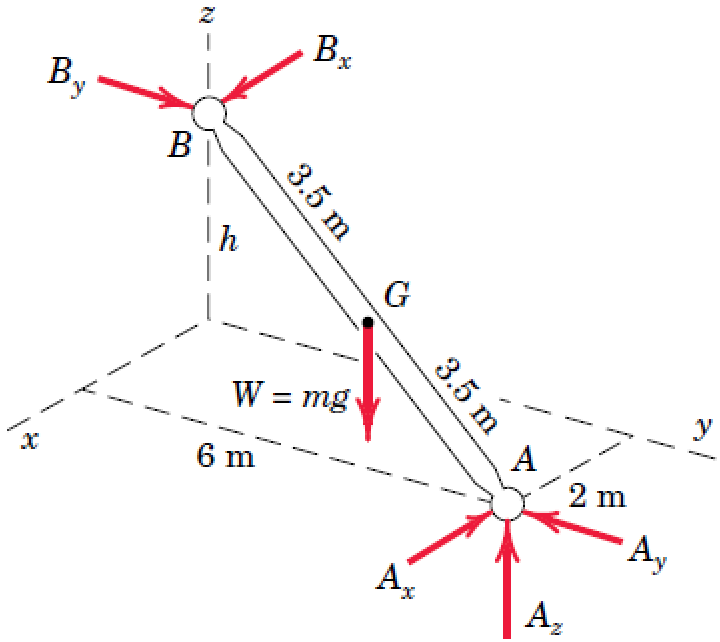
Sample Problem 3/5

The uniform 7-m steel shaft has a mass of 200 kg and is supported by a ball-and-socket joint at A in the horizontal floor. The ball end B rests against the smooth vertical walls as shown. Compute the forces exerted by the walls and the floor on the ends of the shaft.



$$W = mg = 200(9.81) = 1962 \text{ N}$$

$$7 = \sqrt{2^2 + 6^2 + h^2}, h = 3 \text{ m}$$



Vector solution.

$$\mathbf{r}_{AG} = -1\mathbf{i} - 3\mathbf{j} + 1.5\mathbf{k}\ \text{m} \qquad \text{and} \qquad \mathbf{r}_{AB} = -2\mathbf{i} - 6\mathbf{j} + 3\mathbf{k}\ \text{m}$$

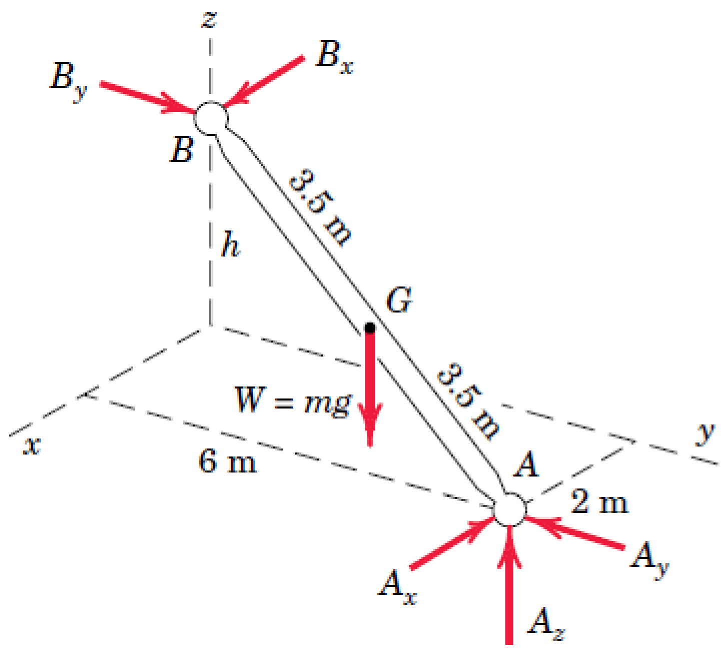
$$[\Sigma \mathbf{M}_A = \mathbf{0}] \qquad \mathbf{r}_{AB} \times (\mathbf{B}_x + \mathbf{B}_y) + \mathbf{r}_{AG} \times \mathbf{W} = \mathbf{0}$$

$$(-2\mathbf{i} - 6\mathbf{j} + 3\mathbf{k}) \times (B_x\mathbf{i} + B_y\mathbf{j}) + (-\mathbf{i} - 3\mathbf{j} + 1.5\mathbf{k}) \times (-1962\mathbf{k}) = \mathbf{0}$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -2 & -6 & 3 \\ B_x & B_y & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1 & -3 & 1.5 \\ 0 & 0 & -1962 \end{vmatrix} = 0$$

$$(-3B_y + 5890)\mathbf{i} + (3B_x - 1962)\mathbf{j} + (-2B_y + 6B_x)\mathbf{k} = \mathbf{0}$$

$$B_x = 654\ \text{N} \qquad \text{and} \qquad B_y = 1962\ \text{N}$$



$$[\Sigma \mathbf{F} = \mathbf{0}] \qquad (654 - A_x)\mathbf{i} + (1962 - A_y)\mathbf{j} + (-1962 + A_z)\mathbf{k} = \mathbf{0}$$

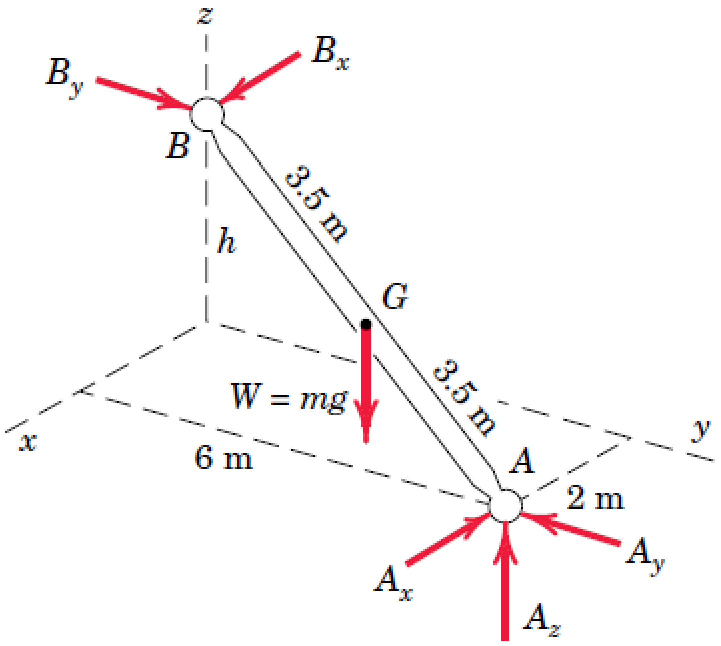
$$A_x = 654\ \text{N} \qquad A_y = 1962\ \text{N} \qquad A_z = 1962\ \text{N}$$

$$A = \sqrt{A_x^2 + A_y^2 + A_z^2}$$
$$= \sqrt{(654)^2 + (1962)^2 + (1962)^2} = 2850\ \text{N}$$

Scalar solution.

$[\Sigma M_{A_x} = 0] \qquad 1962(3) - 3B_y = 0 \qquad B_y = 1962 \text{ N}$

$[\Sigma M_{A_y} = 0] \qquad -1962(1) + 3B_x = 0 \qquad B_x = 654 \text{ N}$



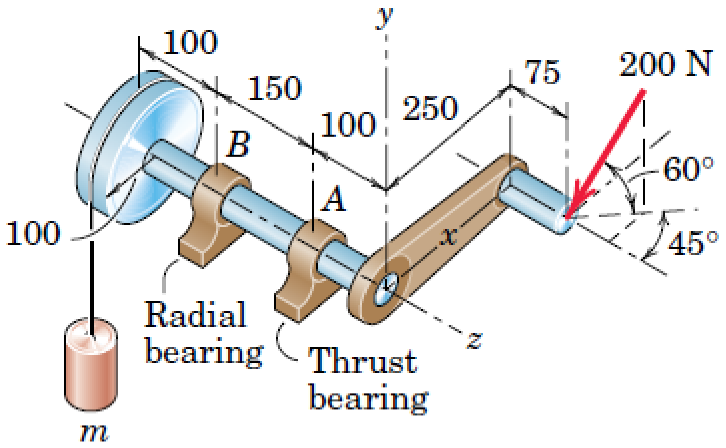
$[\Sigma F_x = 0] \qquad -A_x + 654 = 0 \qquad A_x = 654 \text{ N}$

$[\Sigma F_y = 0] \qquad -A_y + 1962 = 0 \qquad A_y = 1962 \text{ N}$

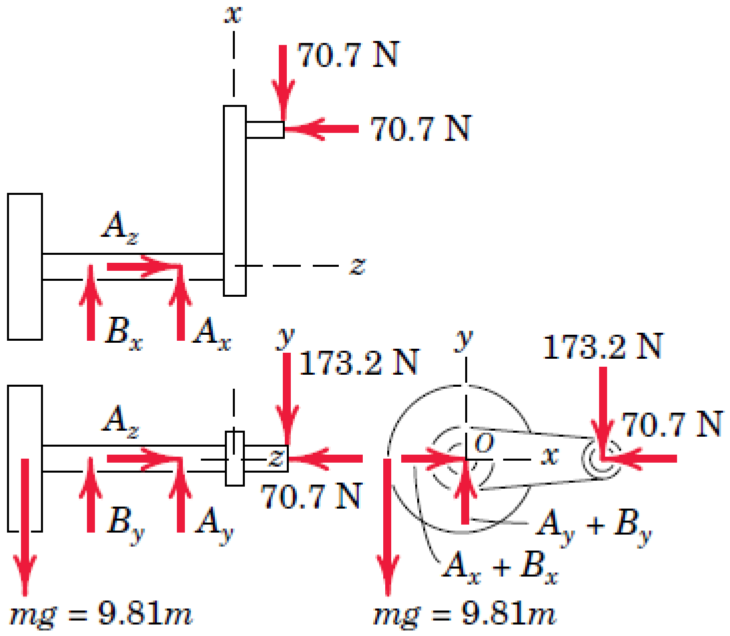
$[\Sigma F_z = 0] \qquad A_z - 1962 = 0 \qquad A_z = 1962 \text{ N}$

Sample Problem 3/6

A 200-N force is applied to the handle of the hoist in the direction shown. The bearing *A* supports the thrust (force in the direction of the shaft axis), while bearing *B* supports only radial load (load normal to the shaft axis). Determine the mass *m* which can be supported and the total radial force exerted on the shaft by each bearing. Assume neither bearing to be capable of supporting a moment about a line normal to the shaft axis.



Dimensions in millimeters



$[\Sigma M_O = 0] \qquad 100(9.81m) - 250(173.2) = 0 \qquad m = 44.1 \text{ kg}$

$[\Sigma M_A = 0] \qquad 150B_x + 175(70.7) - 250(70.7) = 0 \qquad B_x = 35.4 \text{ N}$

$[\Sigma F_x = 0] \qquad A_x + 35.4 - 70.7 = 0 \qquad A_x = 35.4 \text{ N}$

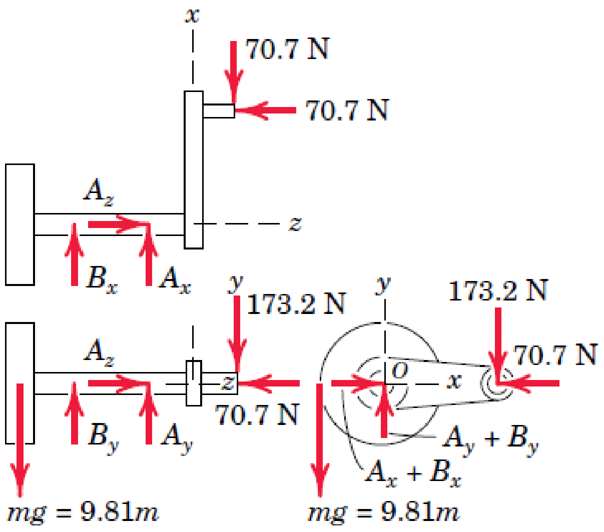
$[\Sigma M_A = 0] \qquad 150B_y + 175(173.2) - 250(44.1)(9.81) = 0 \qquad B_y = 520 \text{ N}$

$[\Sigma F_y = 0] \qquad A_y + 520 - 173.2 - (44.1)(9.81) = 0 \qquad A_y = 86.8 \text{ N}$

$[\Sigma F_z = 0] \qquad A_z = 70.7 \text{ N}$

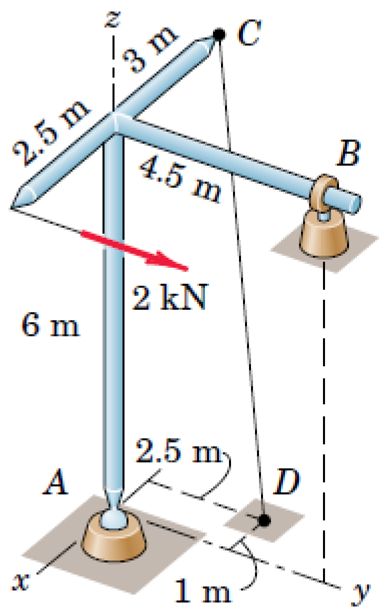
$[A_r = \sqrt{A_x{}^2 + A_y{}^2}] \qquad A_r = \sqrt{(35.4)^2 + (86.8)^2} = 93.5 \text{ N}$

$[B = \sqrt{B_x{}^2 + B_y{}^2}] \qquad B = \sqrt{(35.4)^2 + (520)^2} = 521 \text{ N}$



Sample Problem 3/7

The welded tubular frame is secured to the horizontal *x-y* plane by a ball-and-socket joint at *A* and receives support from the loose-fitting ring at *B*. Under the action of the 2-kN load, rotation about a line from *A* to *B* is prevented by the cable *CD*, and the frame is stable in the position shown. Neglect the weight of the frame compared with the applied load and determine the tension *T* in the cable, the reaction at the ring, and the reaction components at *A*.

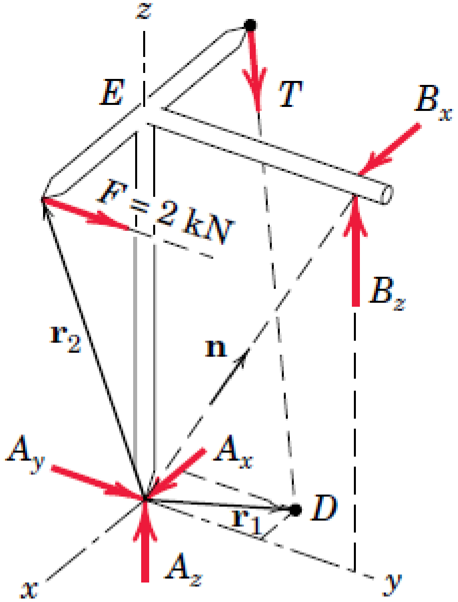


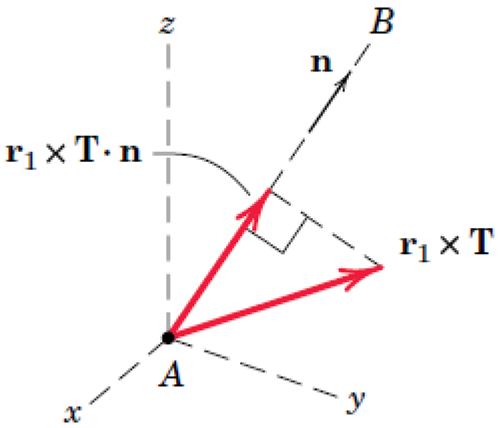
$$\mathbf{n} = \frac{1}{\sqrt{6^2 + 4.5^2}}(4.5\mathbf{j} + 6\mathbf{k}) = \frac{1}{5}(3\mathbf{j} + 4\mathbf{k})$$

$$\overline{CD} = \sqrt{46.2}\text{ m}$$

$$\mathbf{T} = \frac{T}{\sqrt{46.2}}(2\mathbf{i} + 2.5\mathbf{j} - 6\mathbf{k}) \qquad \mathbf{F} = 2\mathbf{j}\text{ kN}$$

$$\mathbf{r}_1 = -\mathbf{i} + 2.5\mathbf{j}\text{ m} \qquad \mathbf{r}_2 = 2.5\mathbf{i} + 6\mathbf{k}\text{ m}$$





$$[\Sigma M_{AB} = 0] \quad (-\mathbf{i} + 2.5\mathbf{j}) \times \frac{T}{\sqrt{46.2}}(2\mathbf{i} + 2.5\mathbf{j} - 6\mathbf{k}) \cdot \frac{1}{5}(3\mathbf{j} + 4\mathbf{k}) + (2.5\mathbf{i} + 6\mathbf{k}) \times (2\mathbf{j}) \cdot \frac{1}{5}(3\mathbf{j} + 4\mathbf{k}) = 0$$

$$-\frac{48T}{\sqrt{46.2}} + 20 = 0 \qquad T = 2.83 \text{ kN}$$

$$T_x = 0.833 \text{ kN} \qquad T_y = 1.042 \text{ kN} \qquad T_z = -2.50 \text{ kN}$$

$$[\Sigma M_z = 0] \quad 2(2.5) - 4.5B_x - 1.042(3) = 0 \qquad B_x = 0.417 \text{ kN}$$

$$[\Sigma M_x = 0] \quad 4.5B_z - 2(6) - 1.042(6) = 0 \qquad B_z = 4.06 \text{ kN}$$

$$[\Sigma F_x = 0] \quad A_x + 0.417 + 0.833 = 0 \qquad A_x = -1.250 \text{ kN}$$

$$[\Sigma F_y = 0] \quad A_y + 2 + 1.042 = 0 \qquad A_y = -3.04 \text{ kN}$$

$$[\Sigma F_z = 0] \quad A_z + 4.06 - 2.50 = 0 \qquad A_z = -1.556 \text{ kN}$$