دانشگاه سمنان

دانشکده مهندسی مکانیک

Semnan University
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درس استاتیک

STATICS

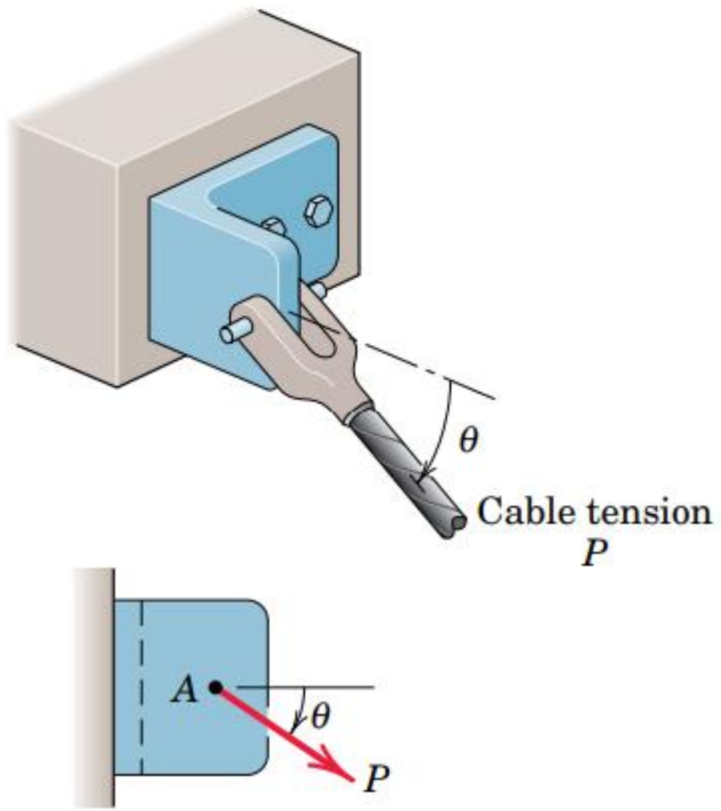
Chapter 2 - Force Systems
Class Lecture

❑ CONTENTS:

دانشگاه سمنان

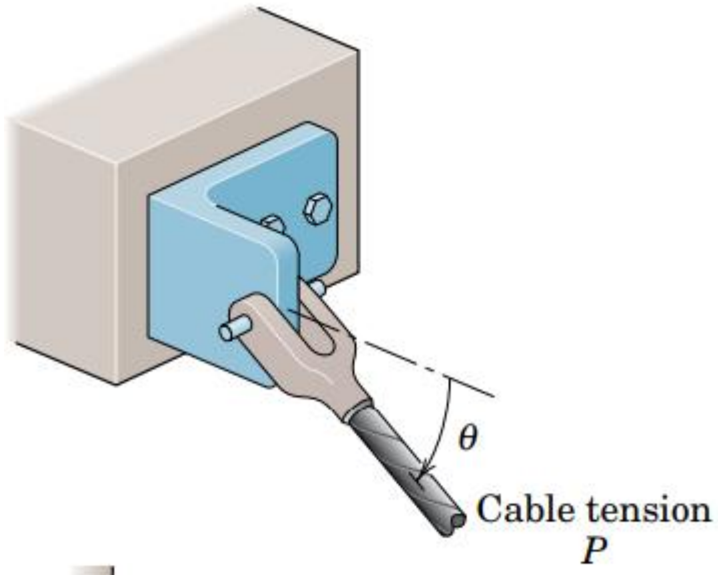
2.2 Force

- Properties of a single force:
  - Action of one body on another
  - Action which tends to cause acceleration
  - Vector quantity (Magnitude and Direction)
  - Forces may be combined by vector addition

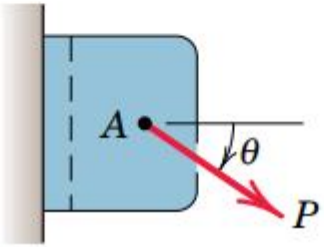


2.2 Force

- Complete specification of the action of this force must include:
  - Magnitude
  - Direction
  - Point of application
    - ✓ We must treat it as a fixed vector



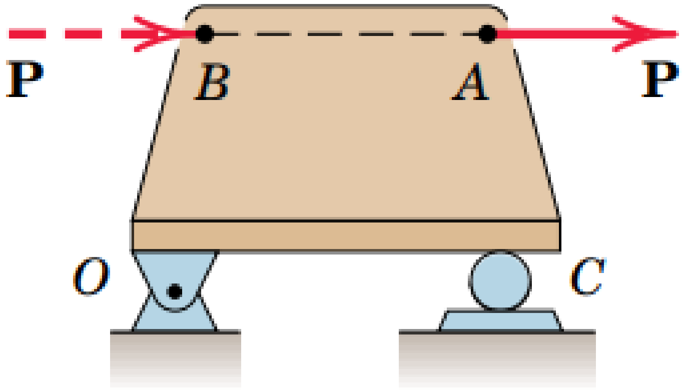
- External and Internal Effects
  - External Forces:
    - ✓ Applied forces
    - ✓ Reactive forces
  - External forces lead to creation of internal forces



2.2 FORCE

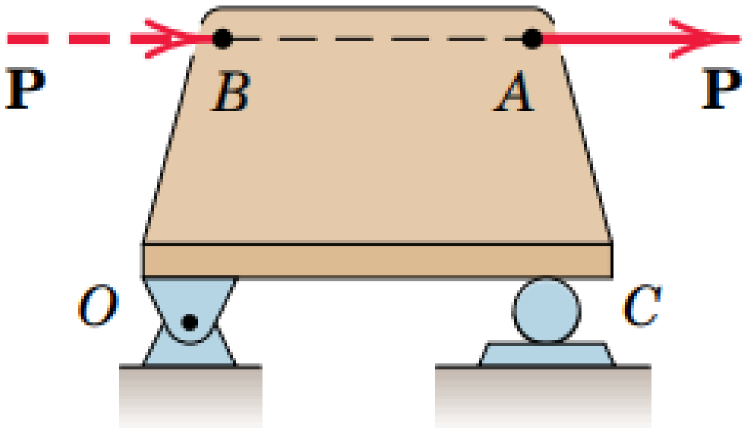
- Principle of Transmissibility
  - When dealing with the mechanics of a rigid body, we ignore deformations in the body
  - The external effects of the exerted force should be same
  - So it is not necessary to restrict the action of an applied force to a given point

  - For example:
    - ✓ Force P may be applied at A or at B or at any other point on its line of action
    - ✓ External effects: bearing support at O and roller support at C



2.2 FORCE

- Principle of Transmissibility:
  - A force may be applied at any point on its given line of action without altering the resultant effects of the force external to the rigid body on which it acts.

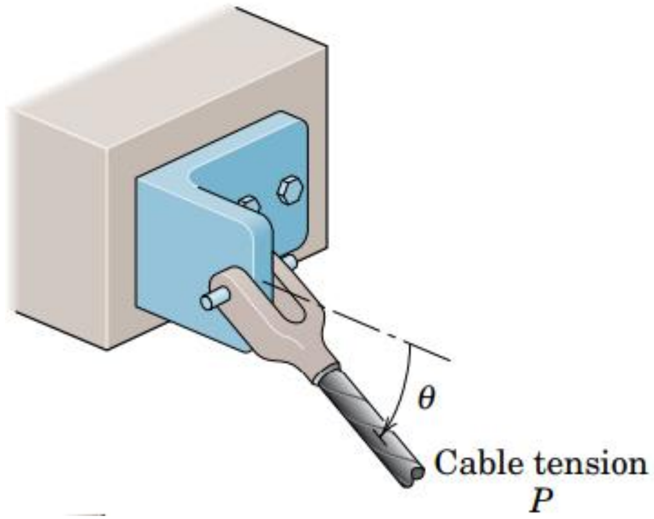


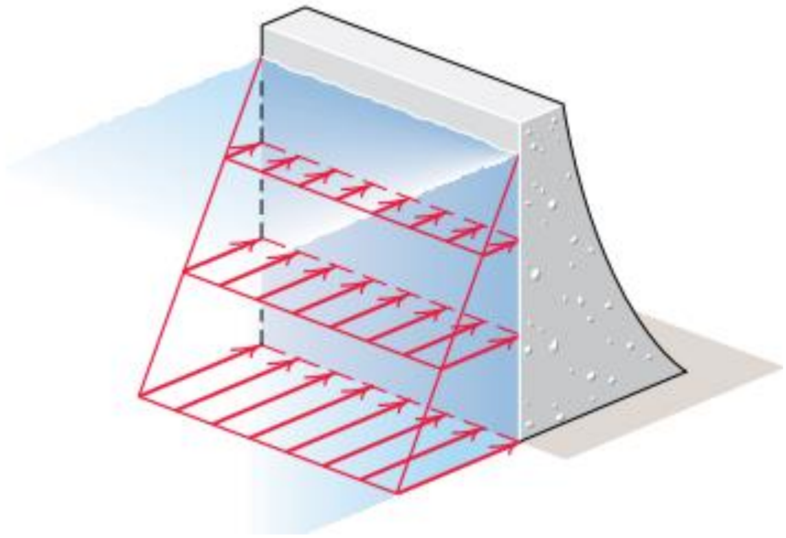
  - The force may be treated as a sliding vector:
    - ✓Magnitude
    - ✓Direction
    - ✓Line of action

2.2 Force

Force Classification

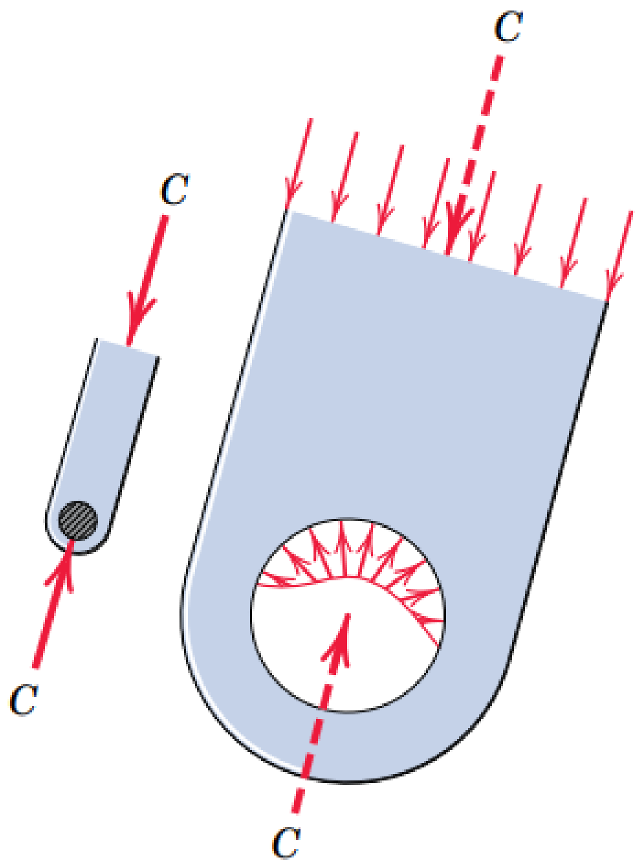
- Contact or Body forces:
  - ✓ A contact force is produced by direct physical contact
  - ✓ A body force is generated by virtue of the position of a body within a force field (such as a gravitational)





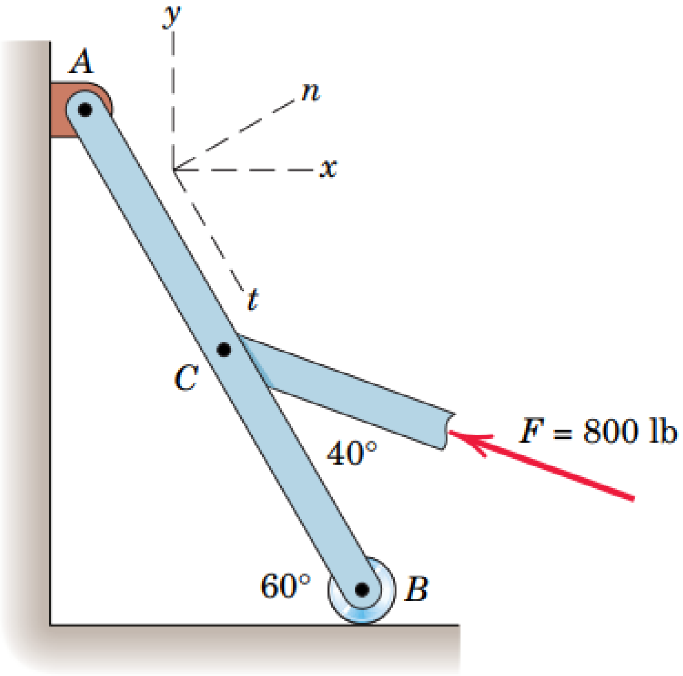
2.2 Force

- Force Classification
  - Concentrated or Distributed forces
    - ✓ Actually, almost all forces are distributed forces.
    - ✓ When the dimensions of the area are very small compared with the other dimensions of the body, we may consider the force to be concentrated



2.2 Force

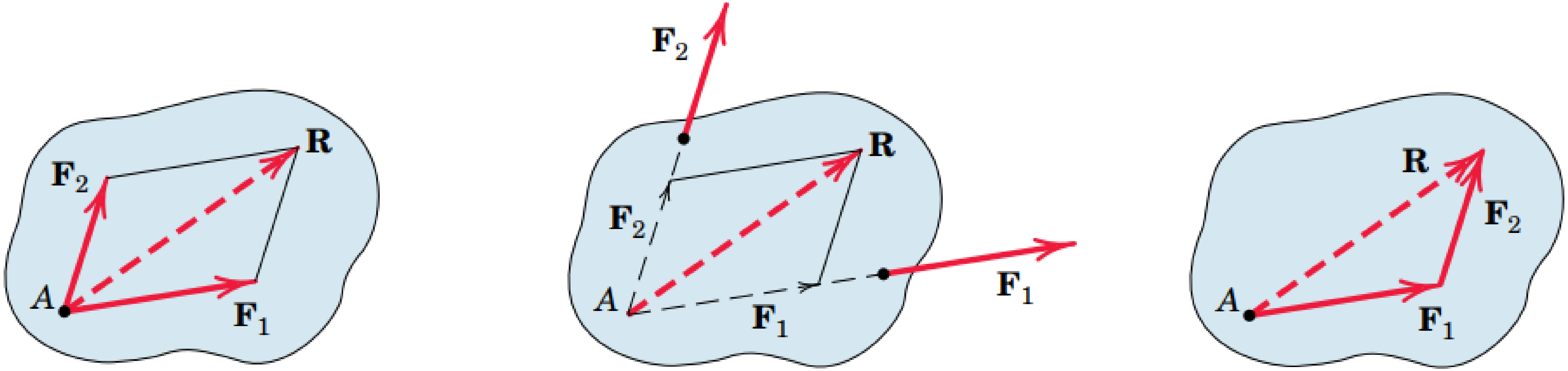
- Action and Reaction
  - According to Newton's third law, the action of a force is always accompanied by an equal and opposite reaction



2.2 Force

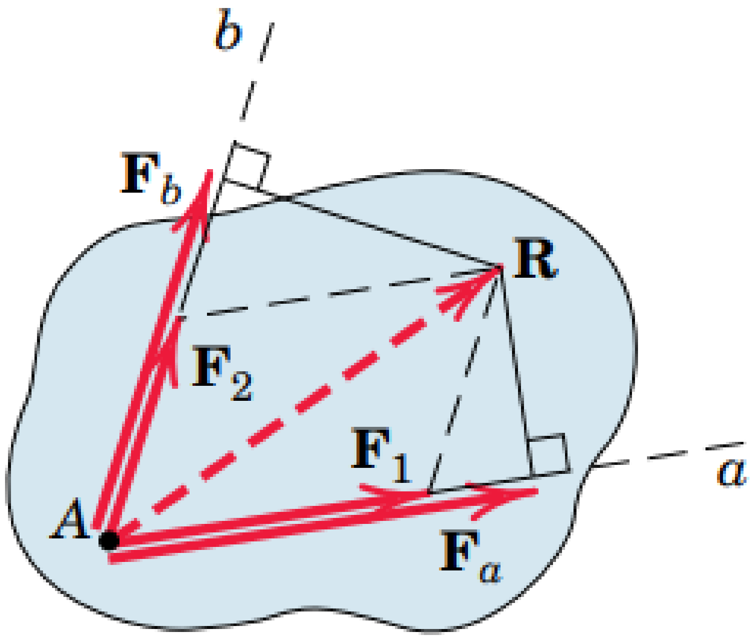
- Concurrent Forces
  - Their lines of action intersect at that point
  - They can be added using the parallelogram law in their common plane

$$\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2$$



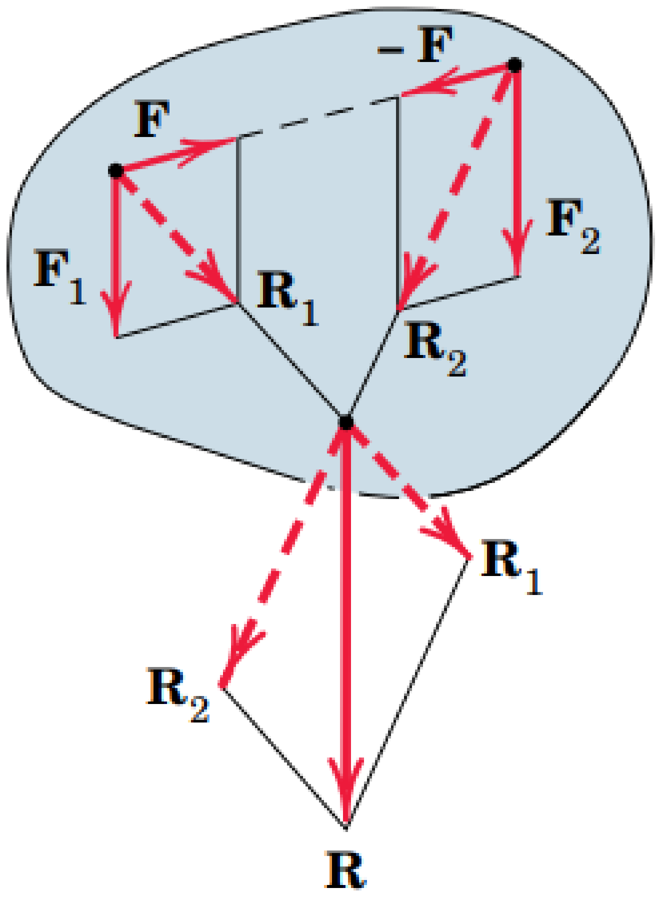
2.2 Force

- Vector Components
  - We often need to replace a force by its vector components in directions which are convenient for a given application



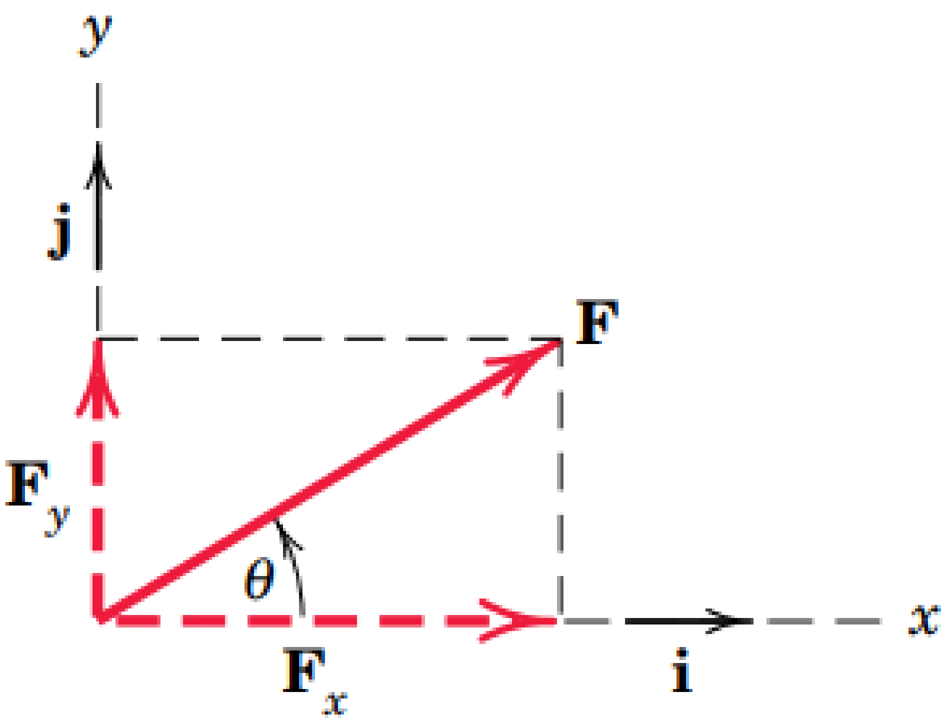
2.2 Force

- A Special Case of Vector Addition
  - Parallel Forces
    - ✓ Finding correct line of action



2.3 Rectangular Components

- The most common two-dimensional resolution of a force vector: Rectangular Components



$$\mathbf{F} = \mathbf{F}_x + \mathbf{F}_y$$

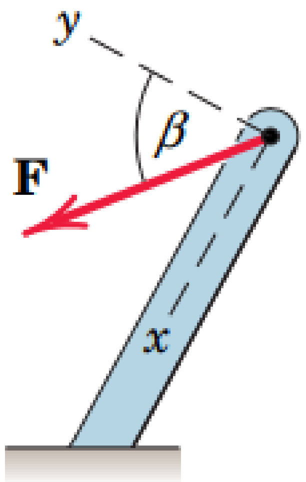
$$\mathbf{F} = F_x\mathbf{i} + F_y\mathbf{j}$$

$$F_x = F\cos\theta \qquad F = \sqrt{{F_x}^2 + {F_y}^2} = |\mathbf{F}|$$

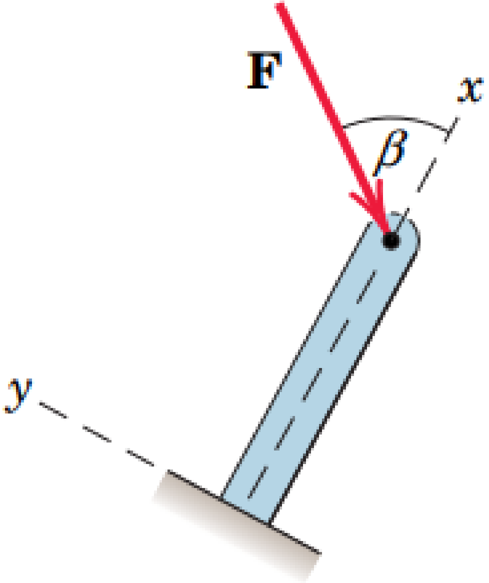
$$F_y = F\sin\theta \qquad \theta = \tan^{-1}\frac{F_y}{F_x}$$

2.3 Rectangular Components

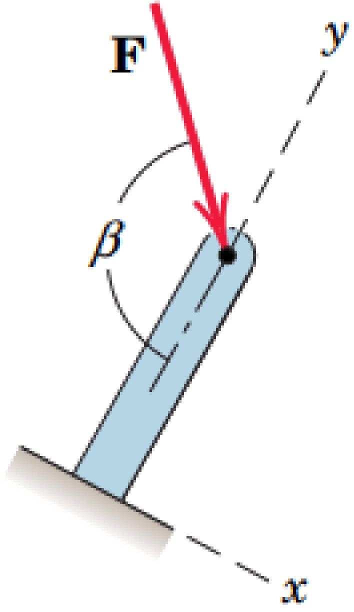
- Determining the Components of a Force



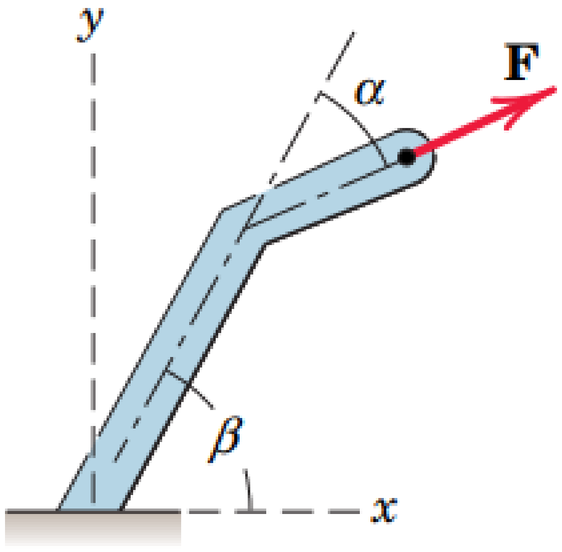
$$F_x = F \sin \beta$$
$$F_y = F \cos \beta$$



$$F_x = -F \cos \beta$$
$$F_y = -F \sin \beta$$



$$F_x = F \sin(\pi - \beta)$$
$$F_y = -F \cos(\pi - \beta)$$



$$F_x = F \cos(\beta - \alpha)$$
$$F_y = F \sin(\beta - \alpha)$$

2.3 Rectangular Components

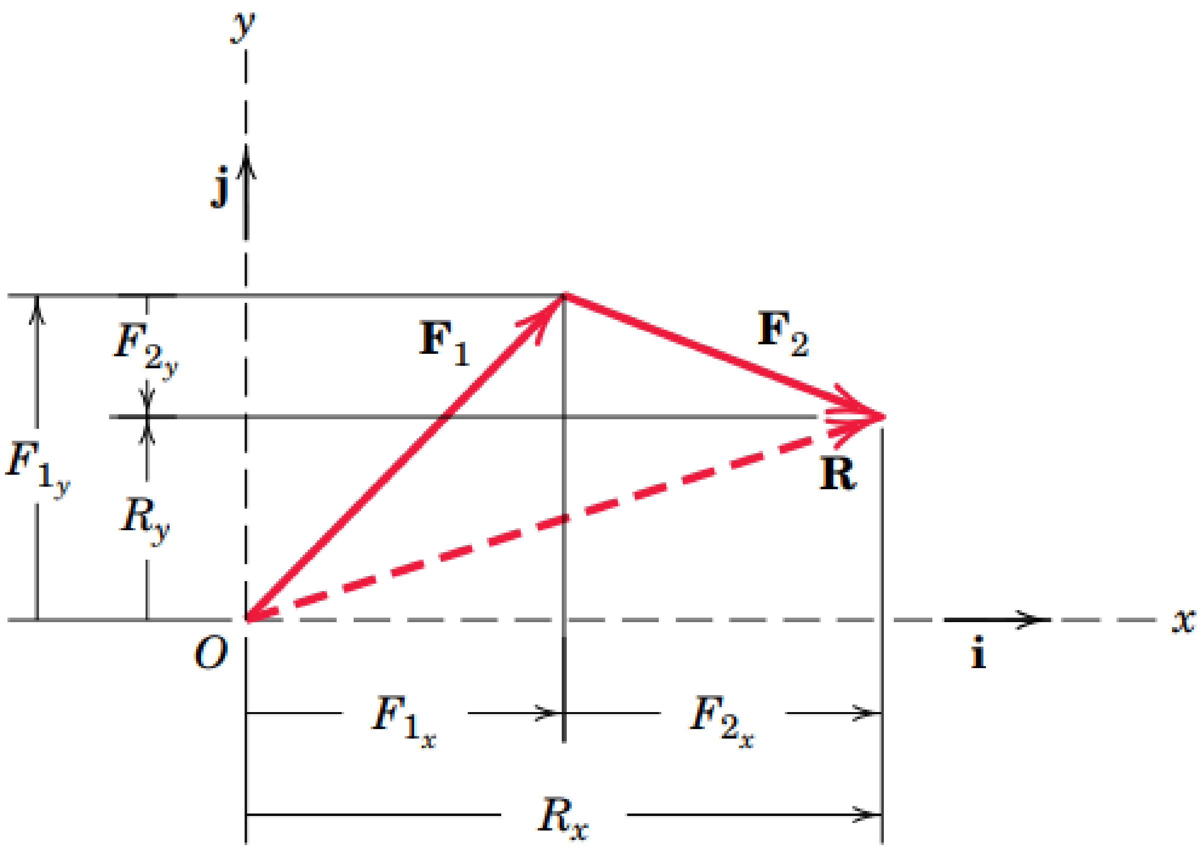
- Finding the sum or resultant R of two forces (which are concurrent)
  - Summing each component separately

$$\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2 = (F_{1_x}\mathbf{i} + F_{1_y}\mathbf{j}) + (F_{2_x}\mathbf{i} + F_{2_y}\mathbf{j})$$

$$R_x\mathbf{i} + R_y\mathbf{j} = (F_{1_x} + F_{2_x})\mathbf{i} + (F_{1_y} + F_{2_y})\,\mathbf{j}$$

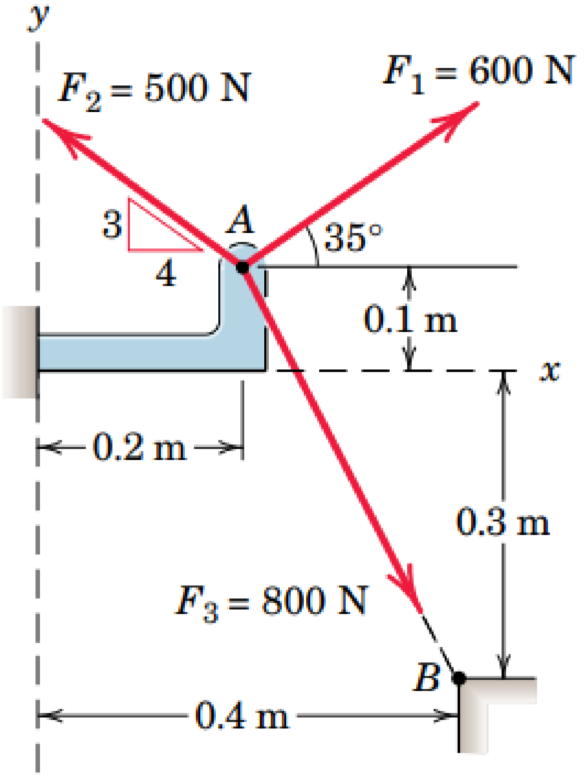
$$R_x = F_{1_x} + F_{2_x} = \Sigma F_x$$

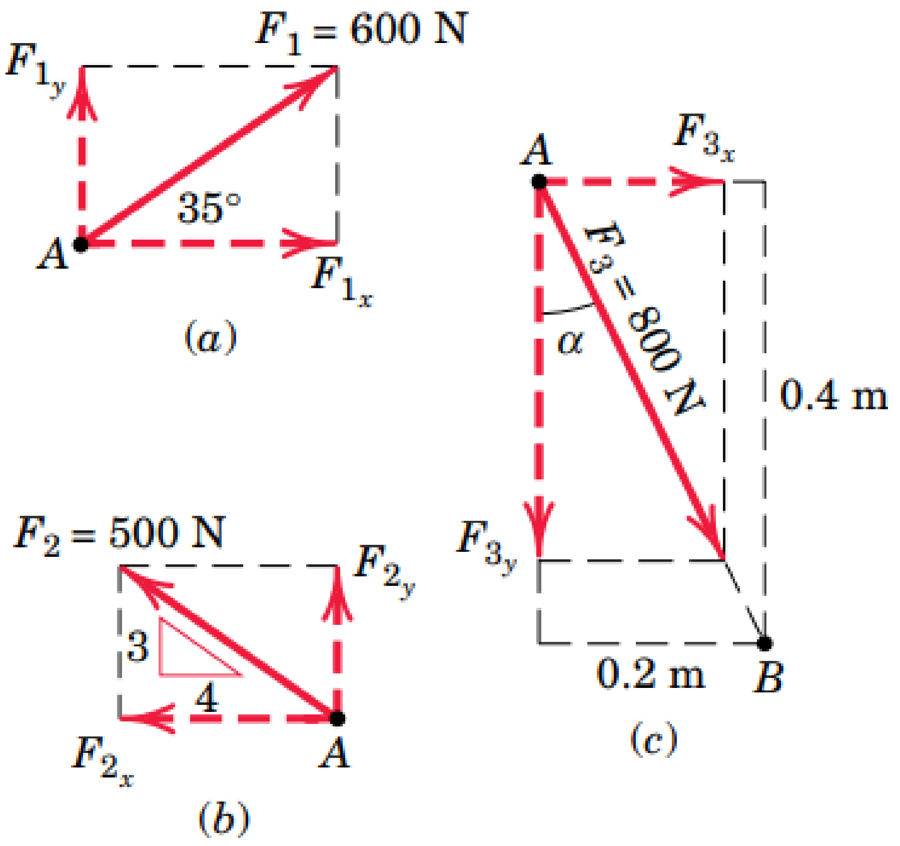
$$R_y = F_{1_y} + F_{2_y} = \Sigma F_y$$



Sample Problem 2/1

The forces $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$, all of which act on point A of the bracket, are specified in three different ways. Determine the x and y scalar components of each of the three forces.





$$F_{1_x} = 600 \cos 35° = 491 \text{ N}$$

$$F_{1_y} = 600 \sin 35° = 344 \text{ N}$$

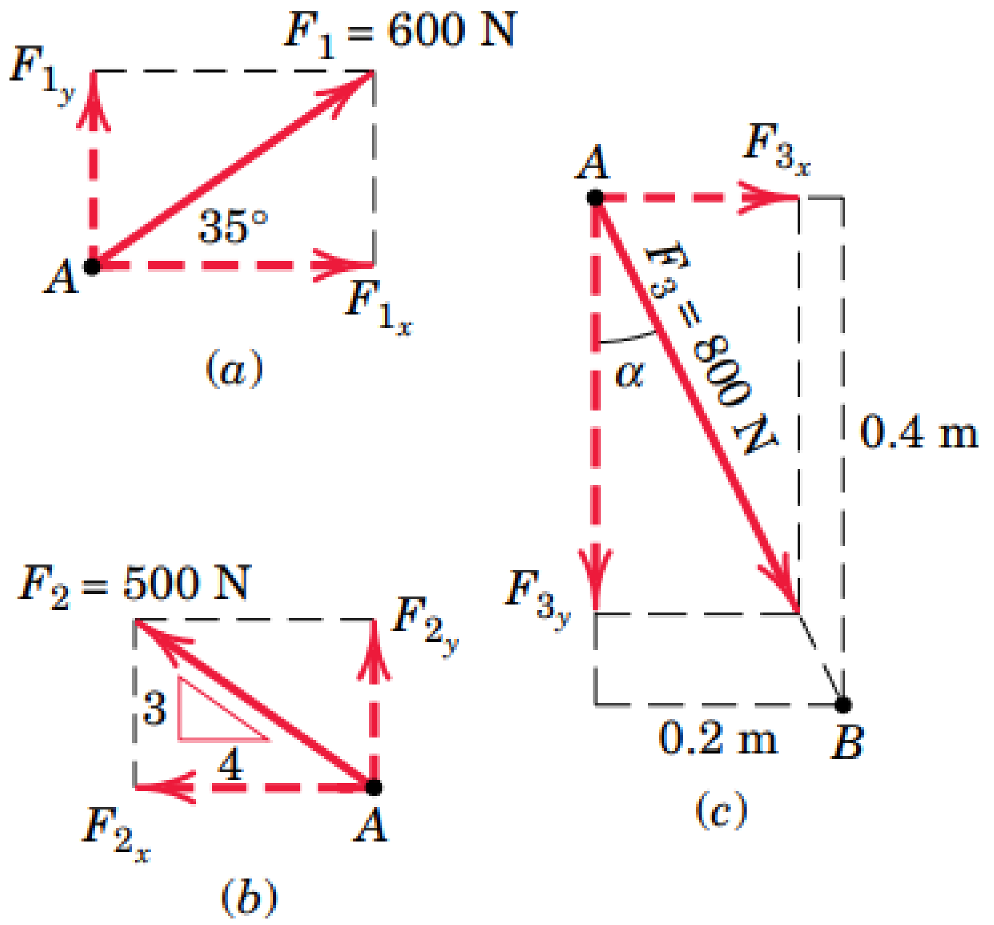
$$F_{2_x} = -500(\tfrac{4}{5}) = -400 \text{ N}$$

$$F_{2_y} = 500(\tfrac{3}{5}) = 300 \text{ N}$$

$$\alpha = \tan^{-1}\left[\frac{0.2}{0.4}\right] = 26.6°$$

$$F_{3_x} = F_3 \sin \alpha = 800 \sin 26.6° = 358 \text{ N}$$

$$F_{3_y} = -F_3 \cos \alpha = -800 \cos 26.6° = -716 \text{ N}$$



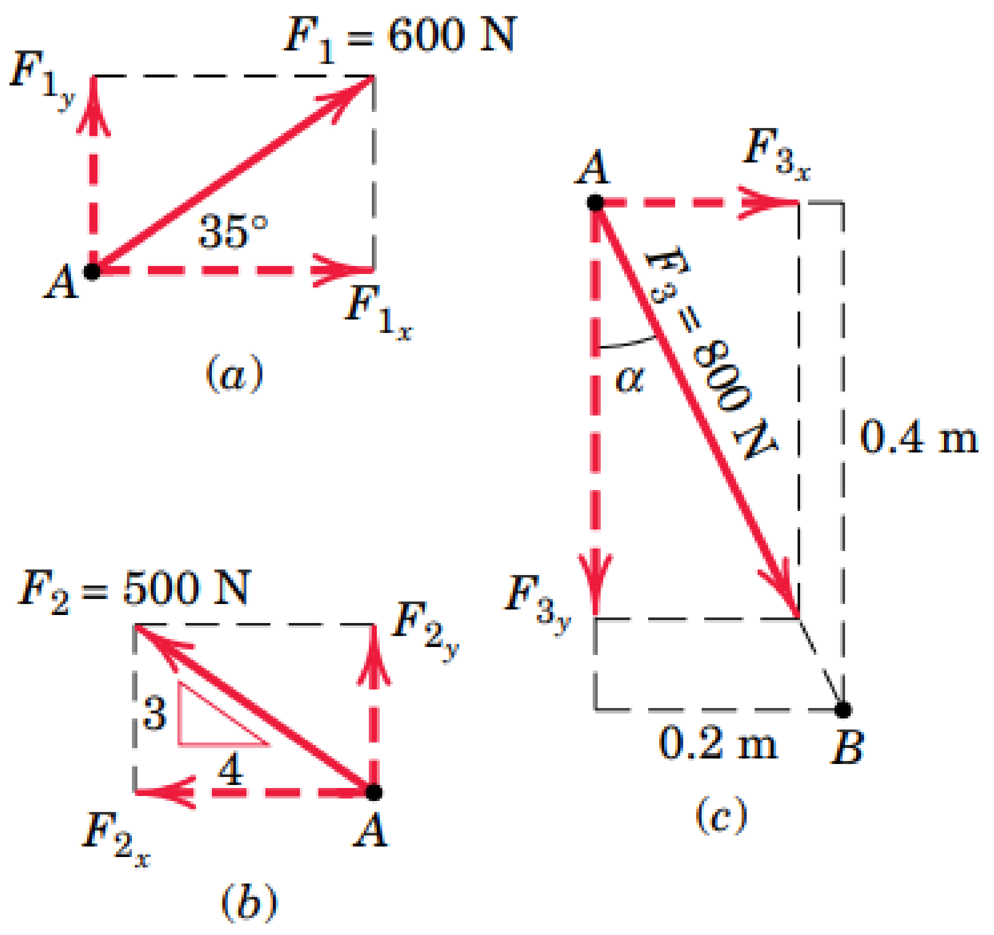
$$\mathbf{F}_3 = F_3\mathbf{n}_{AB} = F_3\frac{\overrightarrow{AB}}{\overline{AB}} = 800\left[\frac{0.2\mathbf{i} - 0.4\mathbf{j}}{\sqrt{(0.2)^2 + (-0.4)^2}}\right]$$

$$= 800\,[0.447\mathbf{i} - 0.894\mathbf{j}]$$

$$= 358\mathbf{i} - 716\mathbf{j}\ \text{N}$$

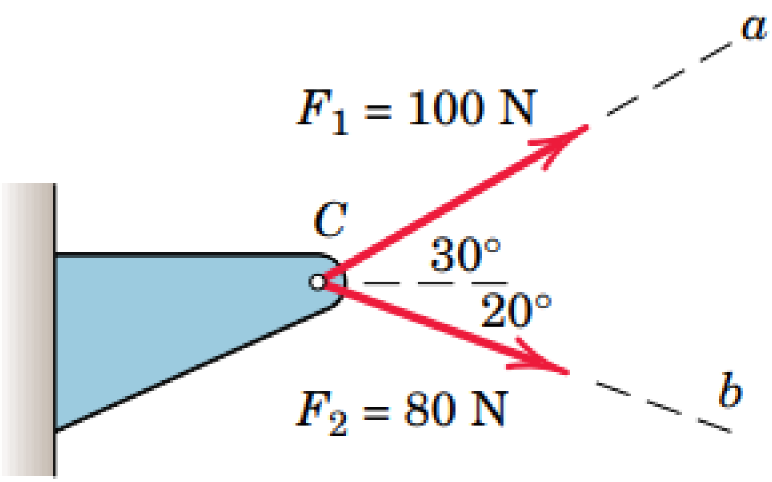
$$F_{3_x} = 358\ \text{N}$$

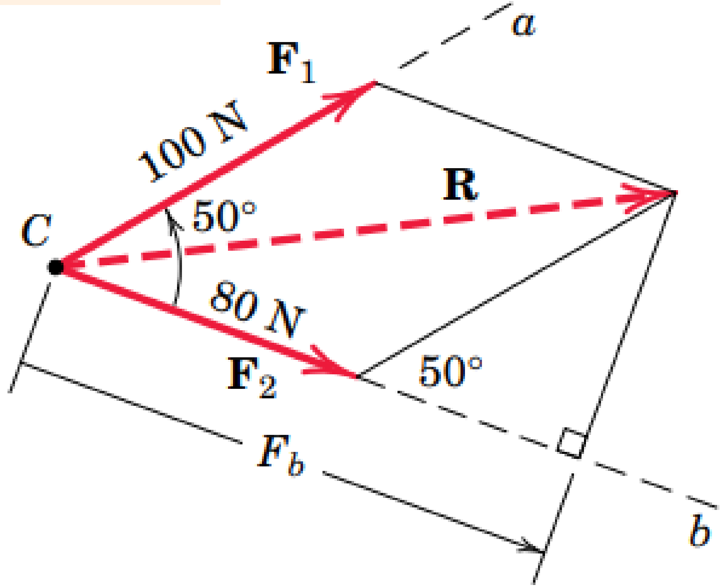
$$F_{3_y} = -716\ \text{N}$$



Sample Problem 2/4

Forces $\mathbf{F}_1$ and $\mathbf{F}_2$ act on the bracket as shown. Determine the projection F_b of their resultant $\mathbf{R}$ onto the b-axis.





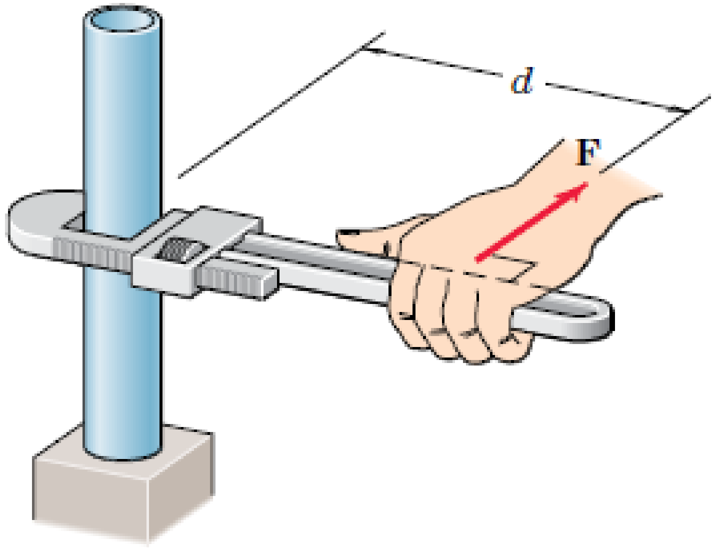
$$R^2 = (80)^2 + (100)^2 - 2(80)(100)\cos 130^\circ \qquad R = 163.4\text{ N}$$

$$F_b = 80 + 100\cos 50^\circ = 144.3\text{ N}$$

2.4 Moments

- A force can also tend to rotate a body about an axis
- Moment is also referred to as *torque*

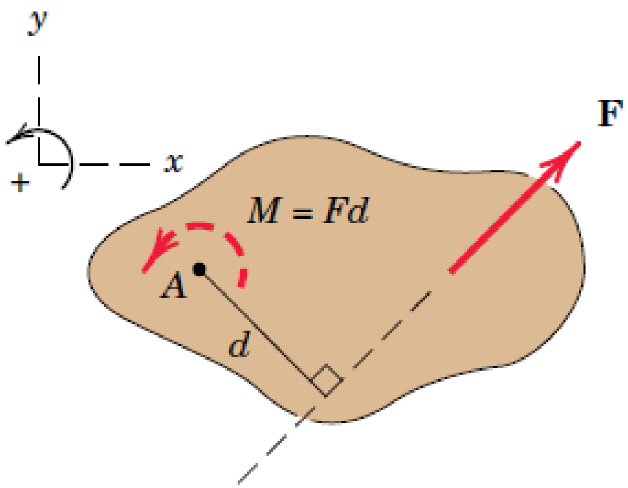
- The magnitude of this tendency depends on:
  - Magnitude *F* of the force
  - Effective length *d* of the wrench handle



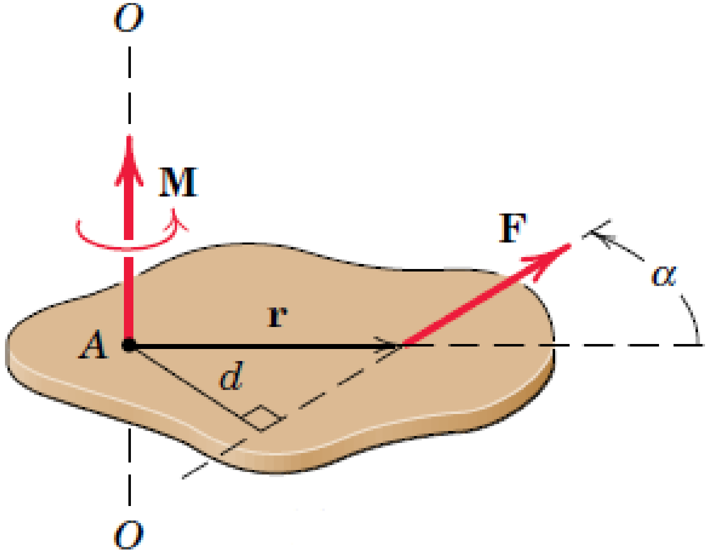
2.5 Moments

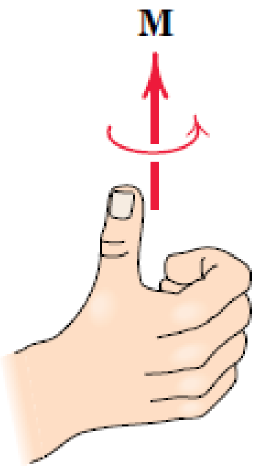
- Moment about a Point

$$M = Fd$$



- Plus sign for counterclockwise moments
- Minus sign for clockwise moments
- Sign consistency within a given problem is essential.



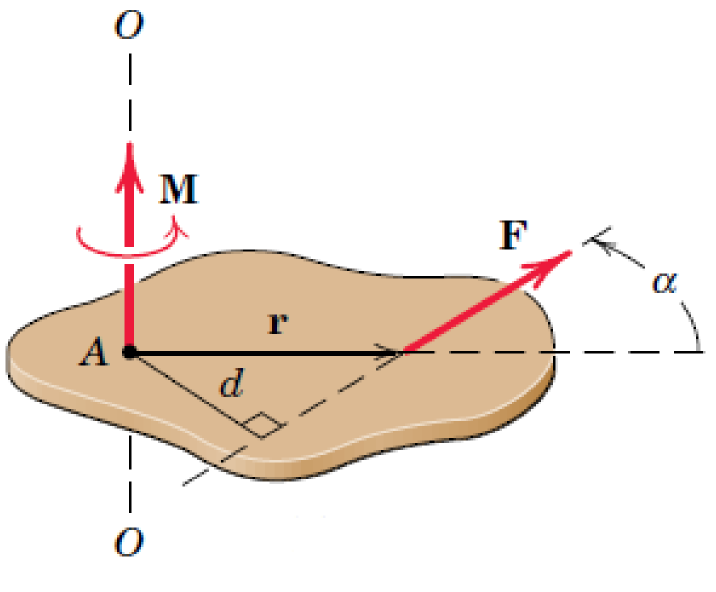


2.5 Moments

- The Cross Product

$$\mathbf{M} = \mathbf{r} \times \mathbf{F}$$

- The moment of **F** about point *A*
- **r** is a position vector which runs from the moment reference point *A* to *any* point on the line of action of **F**



$$M = Fr \sin \alpha = Fd$$

- We must maintain the sequence **r** x **F**, because the sequence **F** x **r** would produce a vector with a sense opposite to that of the correct moment.

2.5 MOMENTS

- Varignon's Theorem
  - The moment of a force about any point is equal to the sum of the moments of the components of the force about the same point.

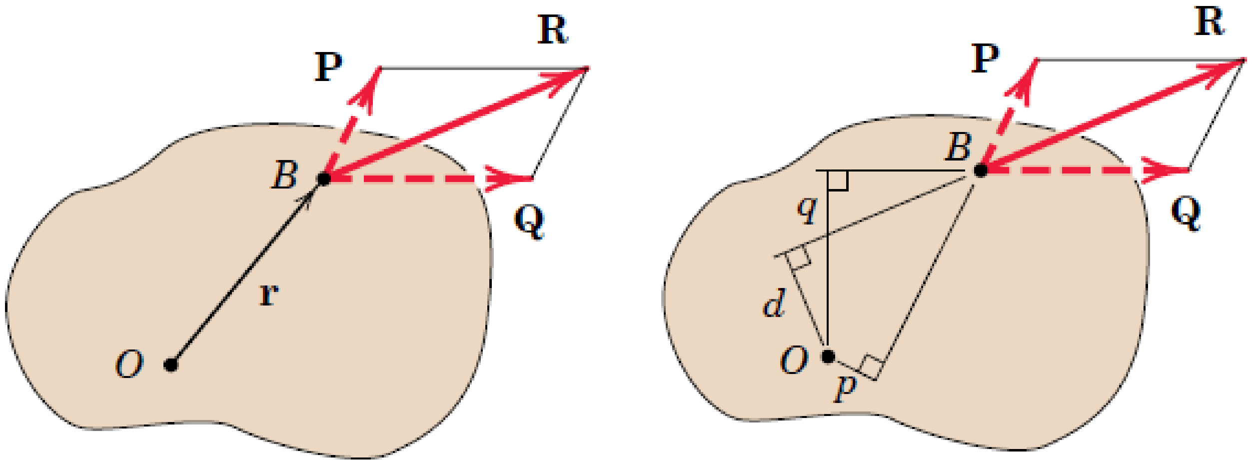
$$\mathbf{M}_O = \mathbf{r} \times \mathbf{R}$$

$$\mathbf{R} = \mathbf{P} + \mathbf{Q}$$

$$\rightarrow \quad \mathbf{r} \times \mathbf{R} = \mathbf{r} \times (\mathbf{P} + \mathbf{Q})$$

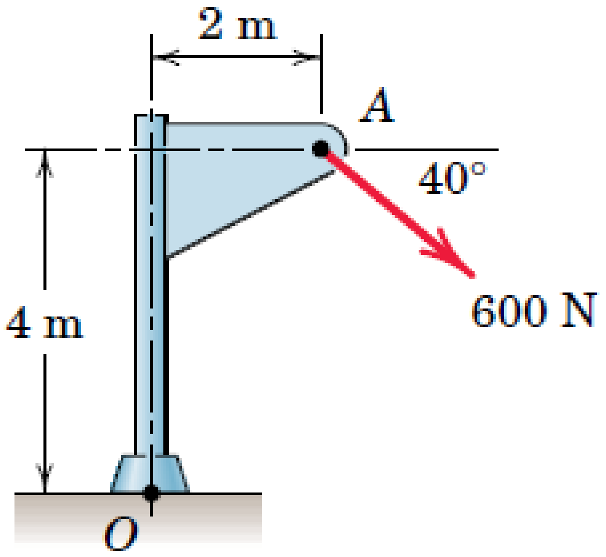
$$\rightarrow \quad \mathbf{M}_O = \mathbf{r} \times \mathbf{R} = \mathbf{r} \times \mathbf{P} + \mathbf{r} \times \mathbf{Q}$$

$$\rightarrow \quad M_O = Rd = -pP + qQ$$



Sample Problem 2/5

Calculate the magnitude of the moment about the base point O of the 600-N force in five different ways.

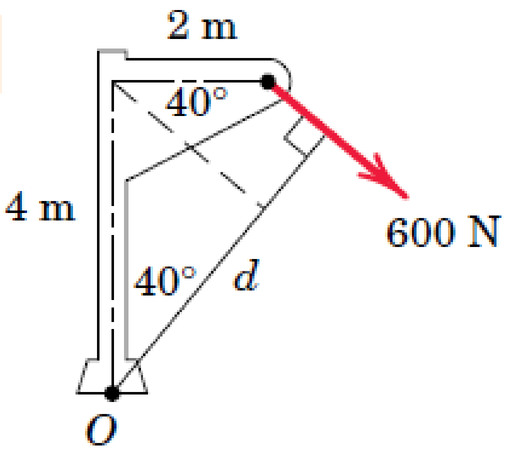


Solution. (I)

$$d = 4\cos 40^\circ + 2\sin 40^\circ = 4.35\ \text{m}$$

$$M = Fd$$

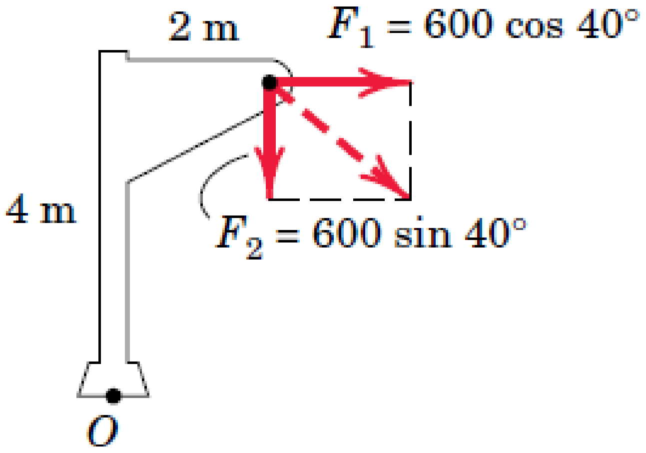
$$M_O = 600(4.35) = 2610\ \text{N}\cdot\text{m}$$



Solution. (II)

$$F_1 = 600\cos 40^\circ = 460\ \text{N}, \qquad F_2 = 600\sin 40^\circ = 386\ \text{N}$$

$$M_O = 460(4) + 386(2) = 2610\ \text{N}\cdot\text{m}$$



Solution. (III)

$$d_1 = 4 + 2 \tan 40^\circ = 5.68 \text{ m}$$

$$M_O = 460(5.68) = 2610 \text{ N}\cdot\text{m}$$

Solution. (IV)

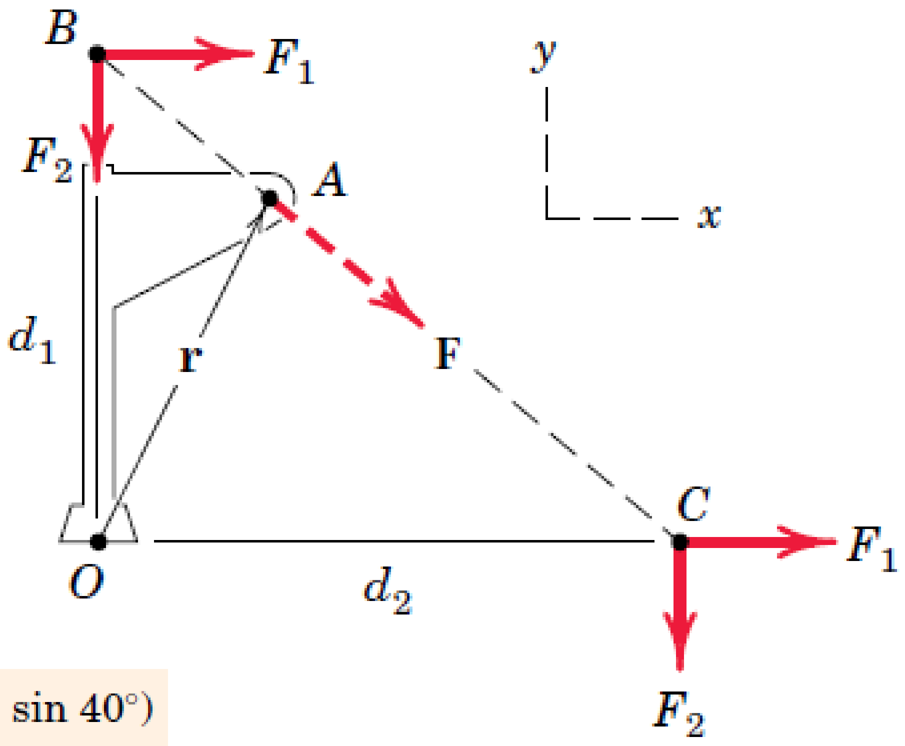
$$d_2 = 2 + 4 \cot 40^\circ = 6.77 \text{ m}$$

$$M_O = 386(6.77) = 2610 \text{ N}\cdot\text{m}$$

Solution. (V)

$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F} = (2\mathbf{i} + 4\mathbf{j}) \times 600(\mathbf{i} \cos 40^\circ - \mathbf{j} \sin 40^\circ)$$
$$= -2610\mathbf{k} \text{ N}\cdot\text{m}$$

$$M_O = 2610 \text{ N}\cdot\text{m}$$



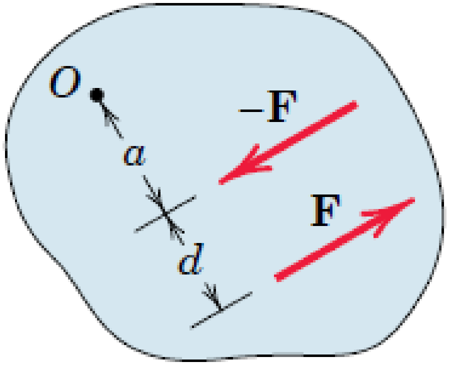
2.5 Couple

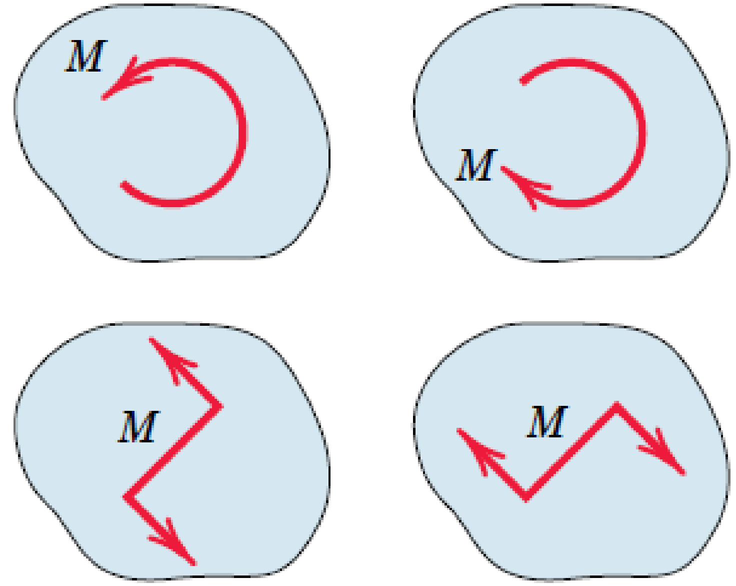
- The moment produced by two equal, opposite, and noncollinear forces is called a *couple*.

- The forces only effect is to produce a tendency of rotation

$$M = F(a + d) - Fa$$

$$\rightarrow \quad M = Fd$$





Counterclockwise couple

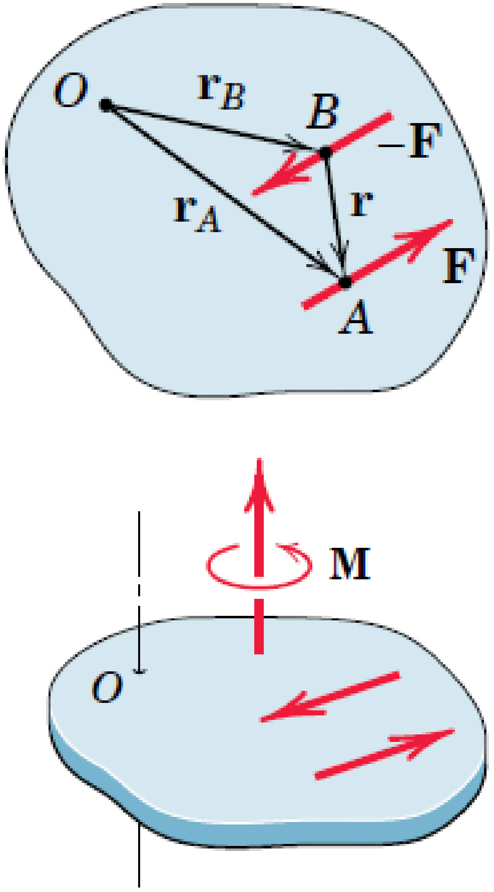
Clockwise couple

2.5 COUPLE

- Vector Algebra Method

$$\mathbf{M} = \mathbf{r}_A \times \mathbf{F} + \mathbf{r}_B \times (-\mathbf{F}) = (\mathbf{r}_A - \mathbf{r}_B) \times \mathbf{F}$$

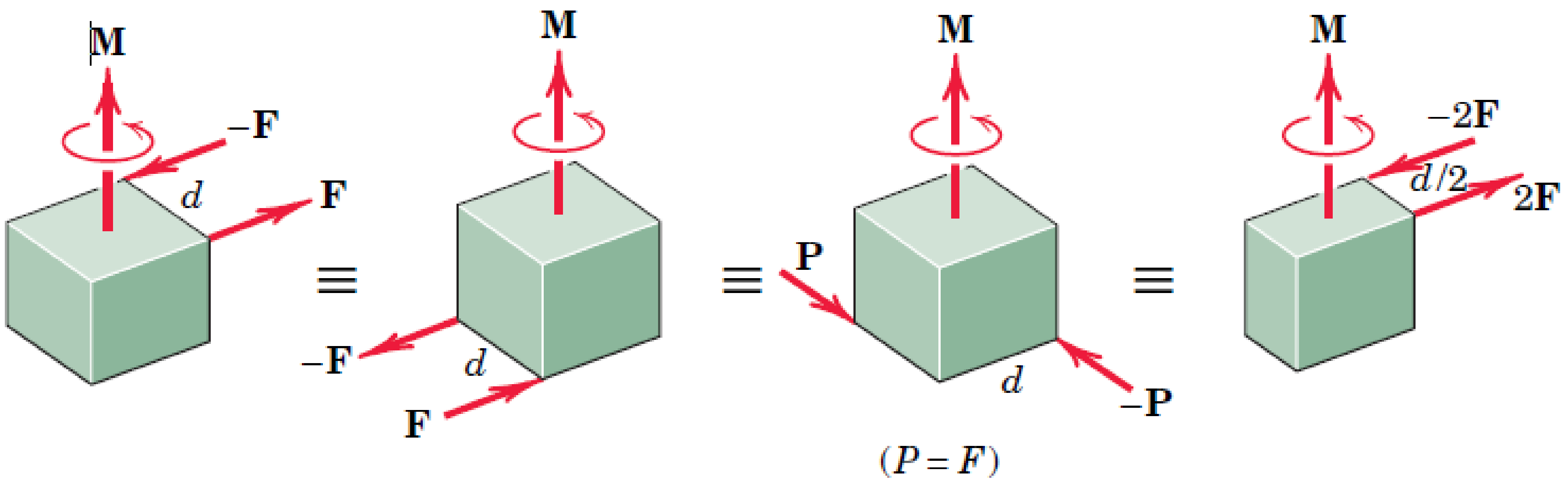
$$\rightarrow \quad \mathbf{M} = \mathbf{r} \times \mathbf{F}$$



- The moment expression contains no reference to the moment center *O* and, therefore, is the same for all moment centers. Thus, we may represent **M** by a free vector.

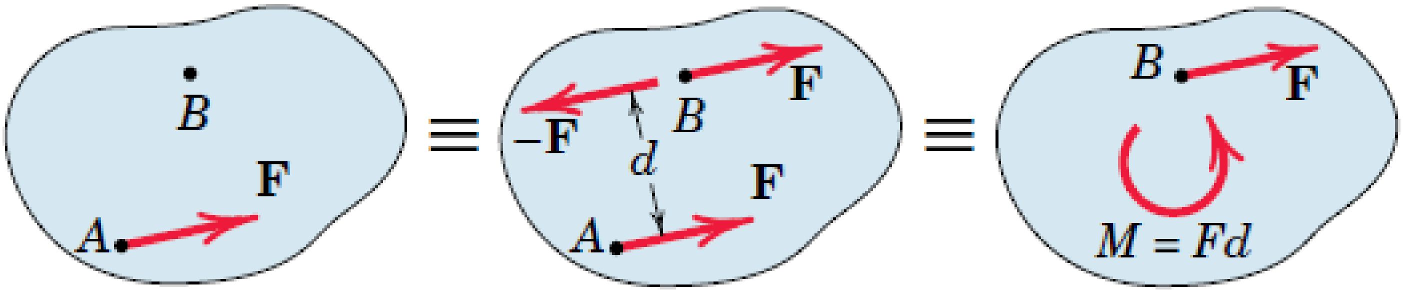
2.5 COUPLE

- Equivalent Couples
  - Changing the values of *F* and *d* does not change a given couple as long as the product *Fd* remains the same.
  - Likewise, a couple is not affected if the forces act in a different but parallel plane.



2.5 COUPLE

- Force–Couple Systems
  - The effect of a force acting on a body:
    - ✓ Push or pull the body in the direction of the force
    - ✓ Rotate the body about any fixed axis which does not intersect the line of the force



  - By reversing this process, we can combine a given couple and a force which lies in the plane of the couple (normal to the couple vector) to produce a single, equivalent force.

Sample Problem 2/7

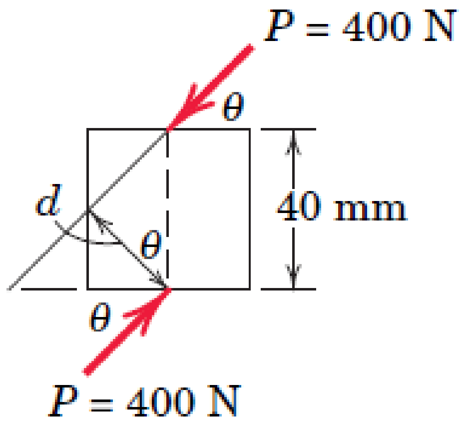
The rigid structural member is subjected to a couple consisting of the two 100-N forces. Replace this couple by an equivalent couple consisting of the two forces **P** and −**P**, each of which has a magnitude of 400 N. Determine the proper angle θ.

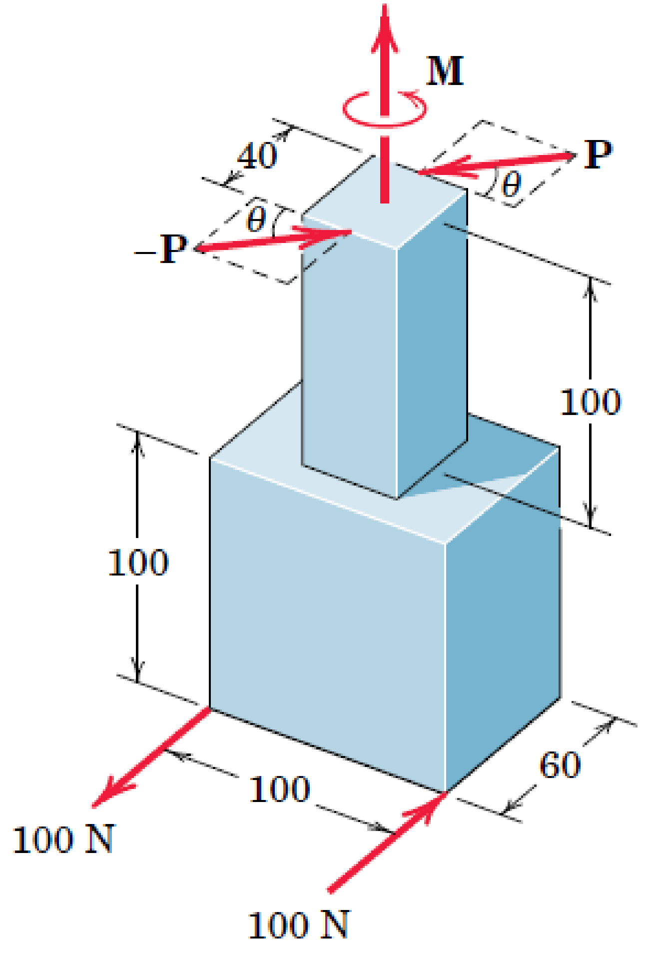
$[M = Fd] \qquad M = 100(0.1) = 10 \text{ N}\cdot\text{m}$

$$M = 400(0.040)\cos\theta$$

$$10 = (400)(0.040)\cos\theta$$

$$\theta = \cos^{-1}\frac{10}{16} = 51.3^\circ$$

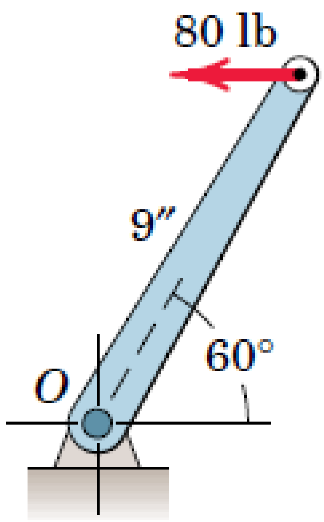




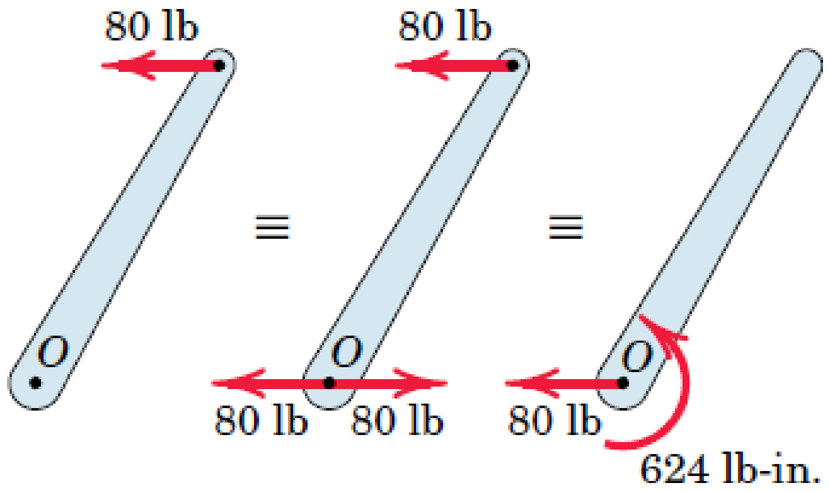
Dimensions in millimeters

Sample Problem 2/8

Replace the horizontal 80-lb force acting on the lever by an equivalent system consisting of a force at O and a couple.



$[M = Fd]$ $\qquad$ $M = 80(9 \sin 60°) = 624$ lb-in.



2.6 RESULTANT

- The *resultant* of a system of forces:
  - The simplest force combination which can replace the original forces without altering the external effect on the rigid body to which the forces are applied

- *Equilibrium* of a body:
  - The condition in which the resultant of all forces acting on the body is zero.

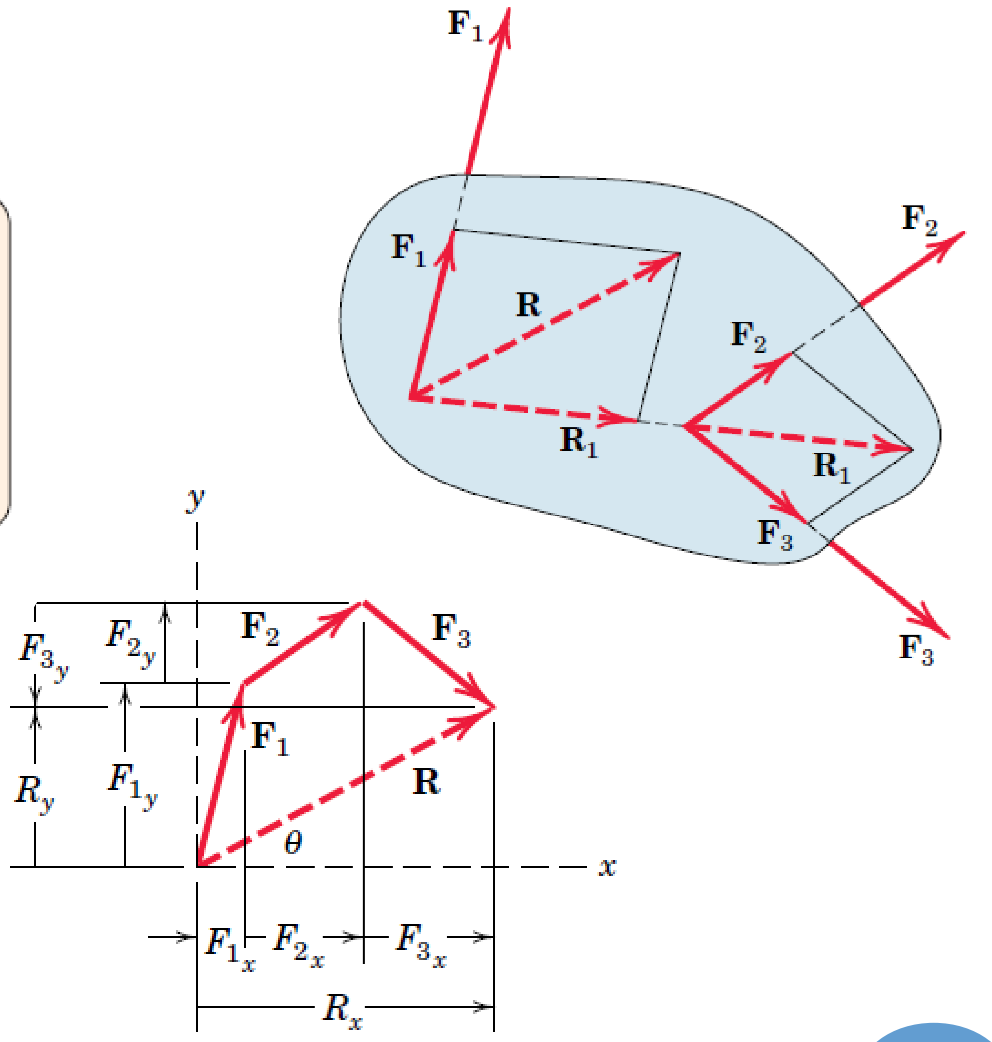
2.6 Resultant

- The *resultant* of a system of forces

$$\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \cdots = \Sigma\mathbf{F}$$

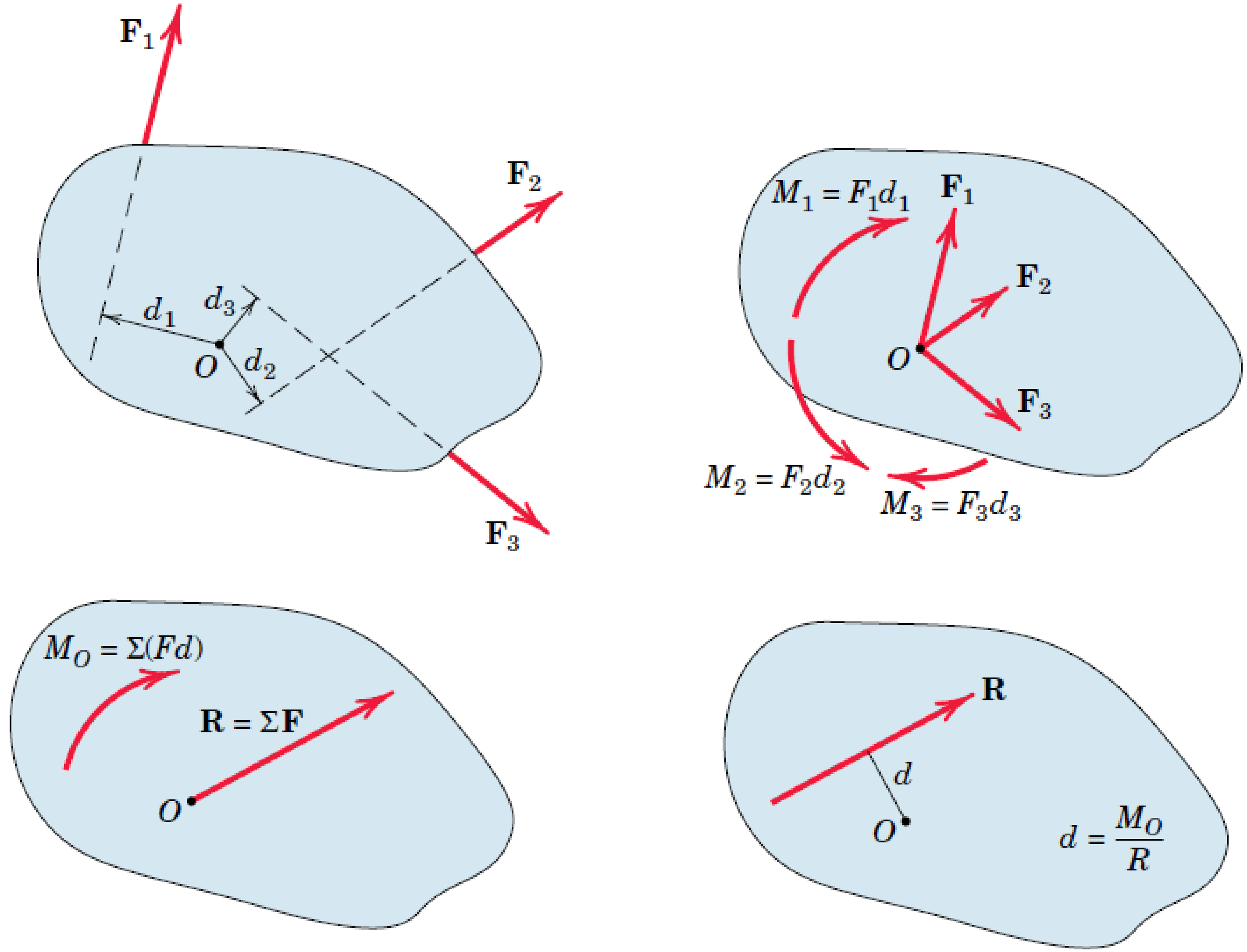
$$R_x = \Sigma F_x \qquad R_y = \Sigma F_y \qquad R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$

$$\theta = \tan^{-1}\frac{R_y}{R_x} = \tan^{-1}\frac{\Sigma F_y}{\Sigma F_x}$$



2.6 Resultant

- Algebraic Method



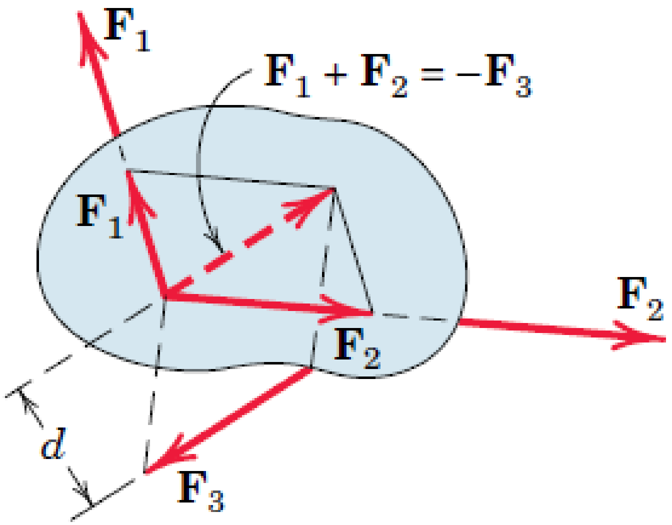
2.6 RESULTANT

- Principle of Moments

$$\mathbf{R} = \Sigma \mathbf{F}$$
$$M_O = \Sigma M = \Sigma(Fd)$$
$$Rd = M_O$$

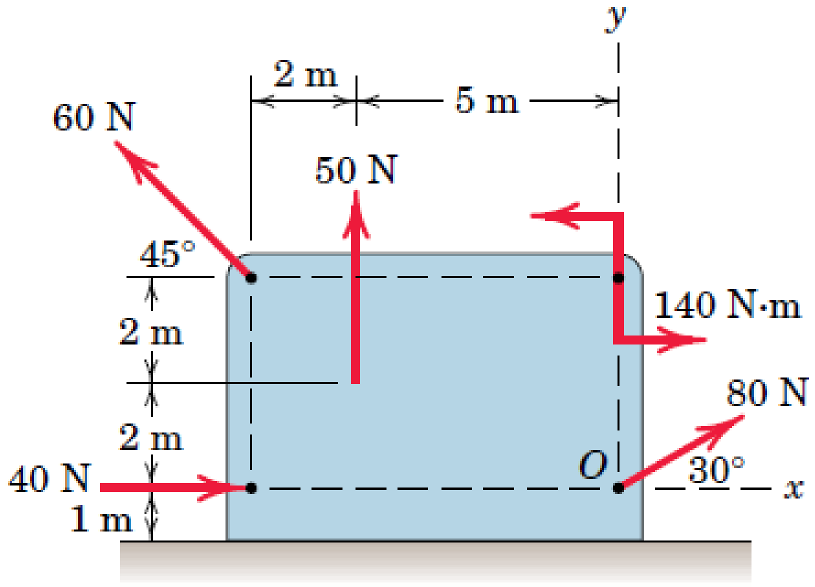
- This extends Varignon's theorem to the case of *nonconcurrent* force systems.

- The three forces have a zero resultant force but have a resultant clockwise couple ($M = F_3 d$)



Sample Problem 2/9

Determine the resultant of the four forces and one couple which act on the plate shown.



Solution. Point O is selected as a convenient reference point

$[R_x = \Sigma F_x]$ $\quad R_x = 40 + 80 \cos 30^\circ - 60 \cos 45^\circ = 66.9\text{ N}$

$[R_y = \Sigma F_y]$ $\quad R_y = 50 + 80 \sin 30^\circ + 60 \cos 45^\circ = 132.4\text{ N}$

$[R = \sqrt{R_x^{\,2} + R_y^{\,2}}]$ $\quad R = \sqrt{(66.9)^2 + (132.4)^2} = 148.3\text{ N}$

$\left[\theta = \tan^{-1} \dfrac{R_y}{R_x}\right]$ $\quad \theta = \tan^{-1} \dfrac{132.4}{66.9} = 63.2^\circ$

$[M_O = \Sigma(Fd)]$ $\quad M_O = 140 - 50(5) + 60 \cos 45^\circ(4) - 60 \sin 45^\circ(7)$

$\quad = -237\text{ N}\cdot\text{m}$

$[Rd = |M_O|]$ $\quad 148.3d = 237 \qquad d = 1.600\text{ m}$

$$R_y b = |M_O| \qquad \text{and} \qquad b = \frac{237}{132.4} = 1.792 \text{ m}$$

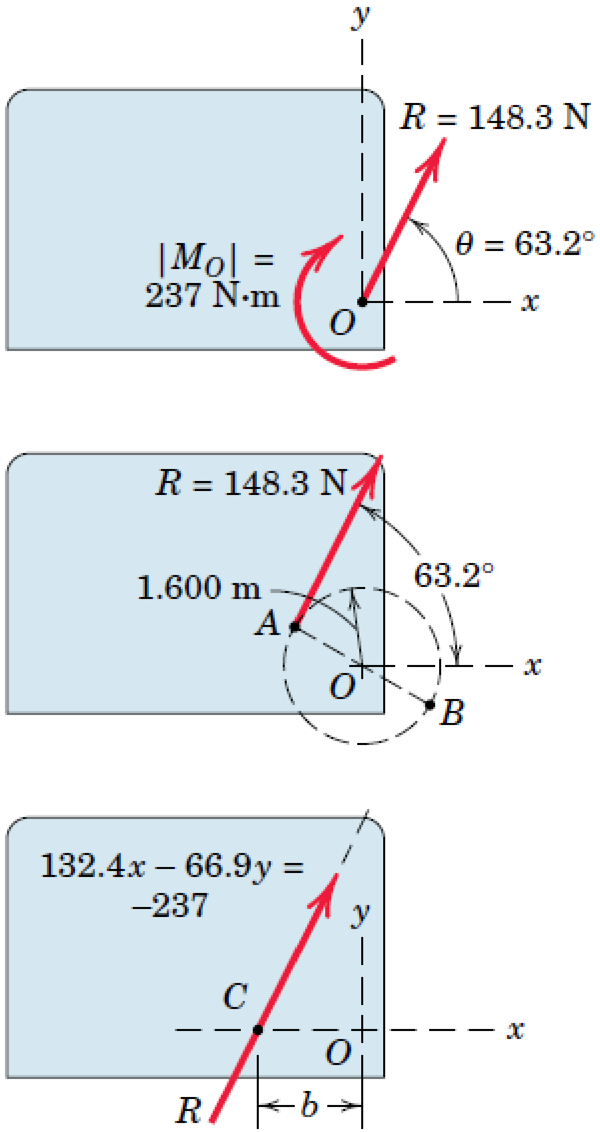
$$\mathbf{r} \times \mathbf{R} = \mathbf{M}_O$$

$$(x\mathbf{i} + y\mathbf{j}) \times (66.9\mathbf{i} + 132.4\mathbf{j}) = -237\mathbf{k}$$

$$(132.4x - 66.9y)\mathbf{k} = -237\mathbf{k}$$

$$132.4x - 66.9y = -237$$

By setting $y = 0$, we obtain $x = -1.792$ m



2.7 3D Force Systems: Rectangular Components

- It is often necessary to resolve a force into its three mutually perpendicular components

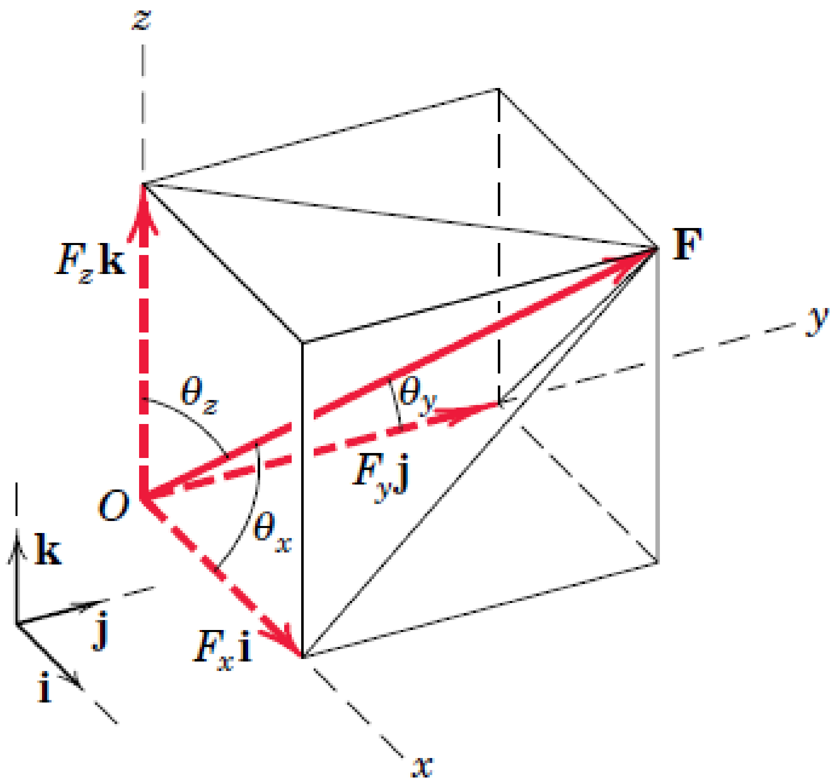
$$F_x = F\cos\theta_x \qquad F = \sqrt{F_x^2 + F_y^2 + F_z^2}$$

$$F_y = F\cos\theta_y \qquad \mathbf{F} = F_x\mathbf{i} + F_y\mathbf{j} + F_z\mathbf{k}$$

$$F_z = F\cos\theta_z \qquad \mathbf{F} = F(\mathbf{i}\cos\theta_x + \mathbf{j}\cos\theta_y + \mathbf{k}\cos\theta_z)$$

$$\mathbf{F} = F(l\mathbf{i} + m\mathbf{j} + n\mathbf{k})$$

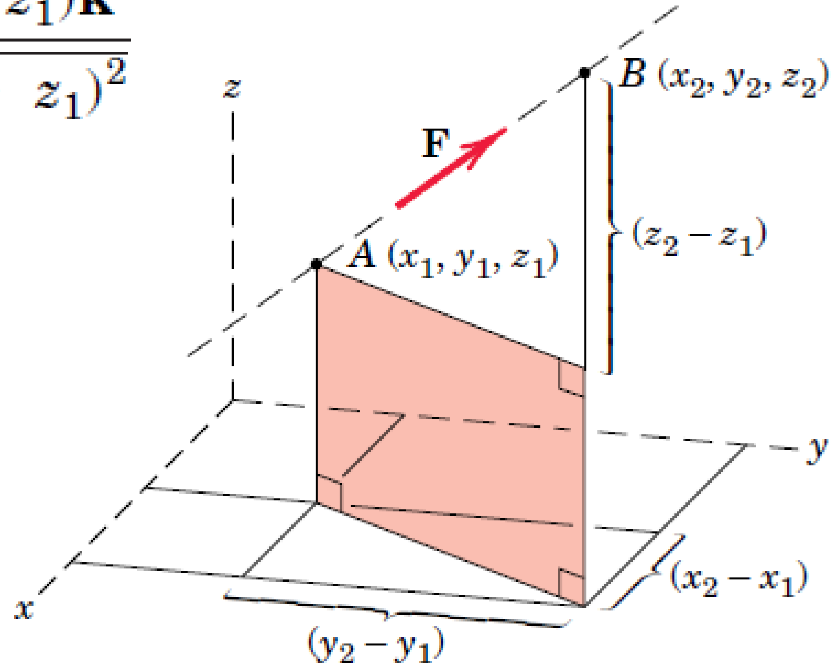
$$\mathbf{F} = F\mathbf{n}_F$$



2.7 3D Force Systems: Rectangular Components

- Direction of a force:
  - *(a) Specification by two points on the line of action of the force.*

$$\mathbf{F} = F\mathbf{n}_F = F\frac{\overrightarrow{AB}}{\overline{AB}} = F\frac{(x_2 - x_1)\mathbf{i} + (y_2 - y_1)\mathbf{j} + (z_2 - z_1)\mathbf{k}}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}}$$



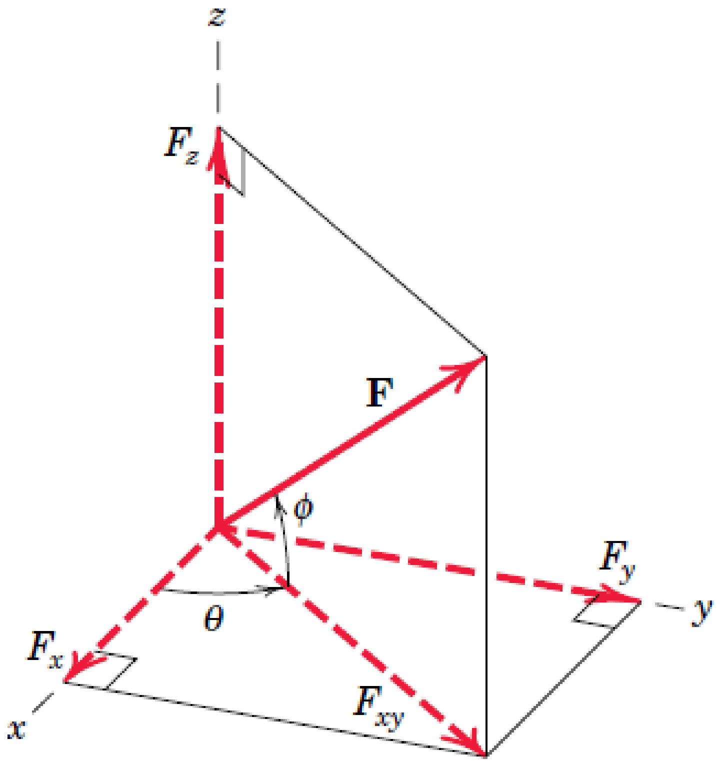
2.7 3D Force Systems: Rectangular Components

- Direction of a force:
  - *(b) Specification by two angles which orient the line of action of the force.*

$$F_{xy} = F\cos\phi$$
$$F_z = F\sin\phi$$

→

$$F_x = F_{xy}\cos\theta = F\cos\phi\cos\theta$$
$$F_y = F_{xy}\sin\theta = F\cos\phi\sin\theta$$



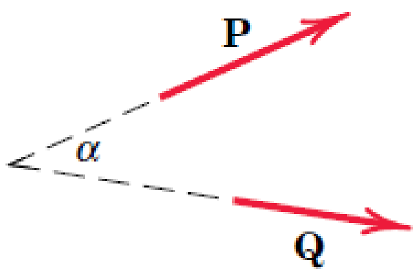
2.7 3D Force Systems: Rectangular Components

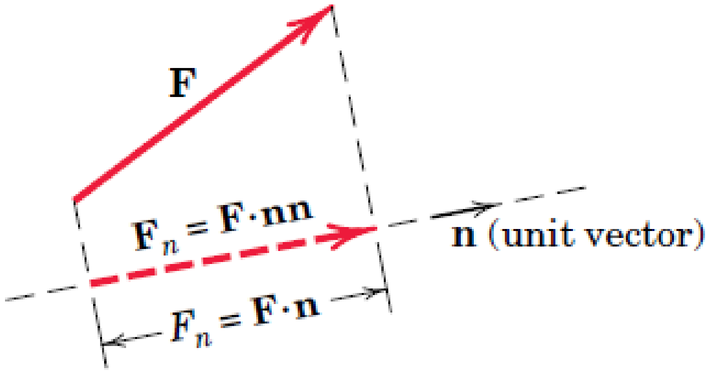
- Dot Product
  - Expressing the rectangular components of a force **F** (or any other vector) with the aid of the vector operation known as the *dot* or *scalar product.*

$$\mathbf{i}\cdot\mathbf{i} = \mathbf{j}\cdot\mathbf{j} = \mathbf{k}\cdot\mathbf{k} = 1$$

$$\mathbf{i}\cdot\mathbf{j} = \mathbf{j}\cdot\mathbf{i} = \mathbf{i}\cdot\mathbf{k} = \mathbf{k}\cdot\mathbf{i} = \mathbf{j}\cdot\mathbf{k} = \mathbf{k}\cdot\mathbf{j} = 0$$

$$\mathbf{P}\cdot\mathbf{Q} = PQ\cos\alpha$$





$$\mathbf{n} = \alpha\mathbf{i} + \beta\mathbf{j} + \gamma\mathbf{k}$$

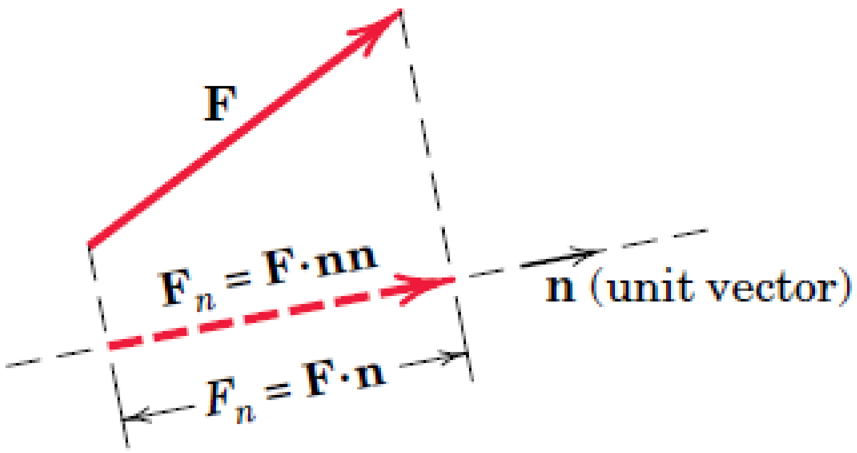
$$F_n = \mathbf{F}\cdot\mathbf{n} = F(l\mathbf{i} + m\mathbf{j} + n\mathbf{k})\cdot(\alpha\mathbf{i} + \beta\mathbf{j} + \gamma\mathbf{k})$$
$$= F(l\alpha + m\beta + n\gamma)$$

2.7 3D Force Systems: Rectangular Components

- Dot Product

$$\mathbf{n} = \alpha\mathbf{i} + \beta\mathbf{j} + \gamma\mathbf{k}$$

$$F_n = \mathbf{F}\cdot\mathbf{n} = F(l\mathbf{i} + m\mathbf{j} + n\mathbf{k})\cdot(\alpha\mathbf{i} + \beta\mathbf{j} + \gamma\mathbf{k})$$
$$= F(l\alpha + m\beta + n\gamma)$$

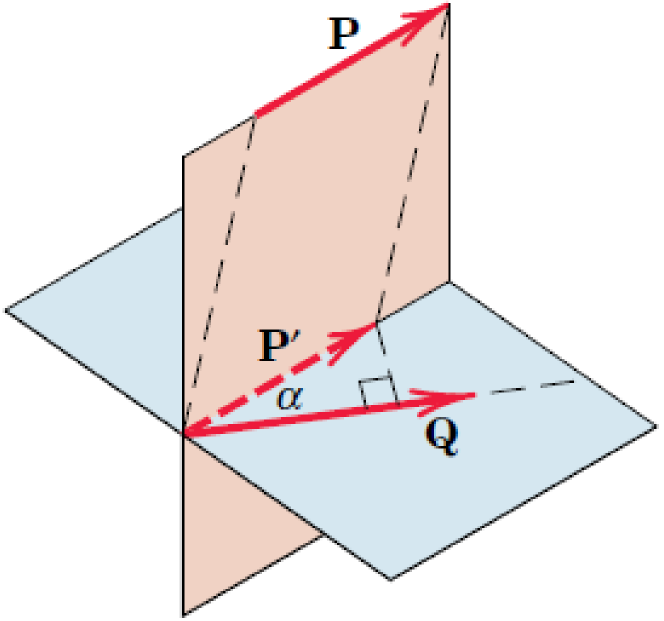


2.7 3D Force Systems: Rectangular Components

- Angle between Two Vectors

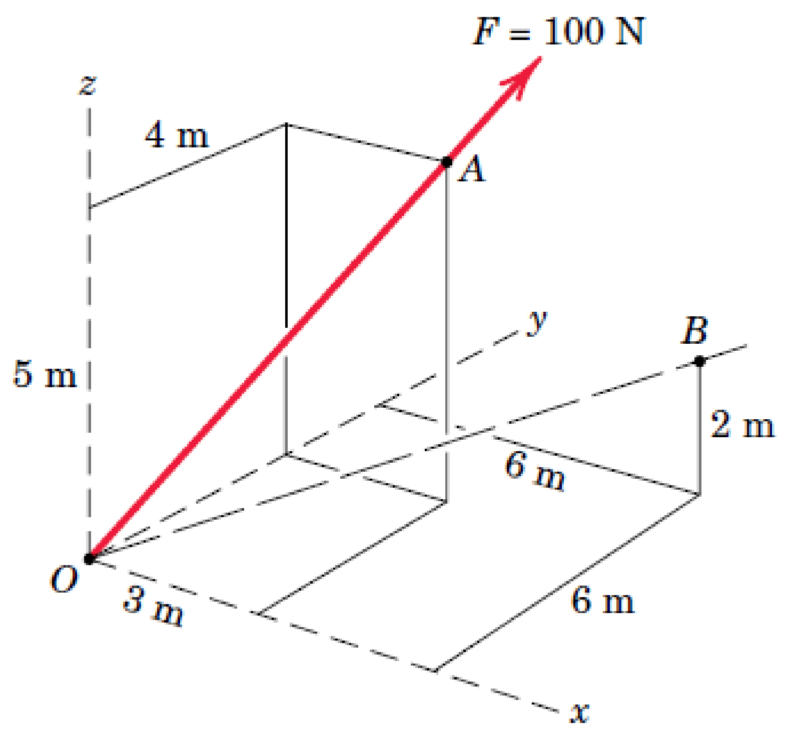
$$\theta = \cos^{-1}\frac{\mathbf{P}\cdot\mathbf{Q}}{PQ}$$

$$\theta = \cos^{-1}\frac{\mathbf{F}\cdot\mathbf{n}}{F}$$



Sample Problem 2/10

A force **F** with a magnitude of 100 N is applied at the origin *O* of the axes *x*-*y*-*z* as shown. The line of action of **F** passes through a point *A* whose coordinates are 3 m, 4 m, and 5 m. Determine (*a*) the *x*, *y*, and *z* scalar components of **F**, (*b*) the projection F_{xy} of **F** on the *x*-*y* plane, and (*c*) the projection F_{OB} of **F** along the line *OB*.



$$\mathbf{F} = F\mathbf{n}_{OA} = F\frac{\overrightarrow{OA}}{\overline{OA}} = 100\left[\frac{3\mathbf{i} + 4\mathbf{j} + 5\mathbf{k}}{\sqrt{3^2 + 4^2 + 5^2}}\right]$$

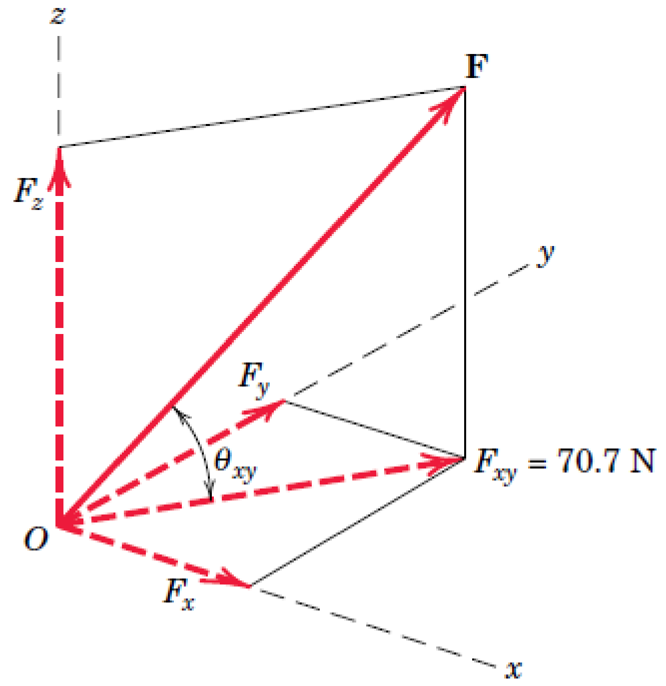
$$= 100[0.424\mathbf{i} + 0.566\mathbf{j} + 0.707\mathbf{k}]$$

$$= 42.4\mathbf{i} + 56.6\mathbf{j} + 70.7\mathbf{k}\ \text{N}$$

$F_x = 42.4$ N $\quad F_y = 56.6$ N $\quad F_z = 70.7$ N

$$\cos\theta_{xy} = \frac{\sqrt{3^2 + 4^2}}{\sqrt{3^2 + 4^2 + 5^2}} = 0.707$$

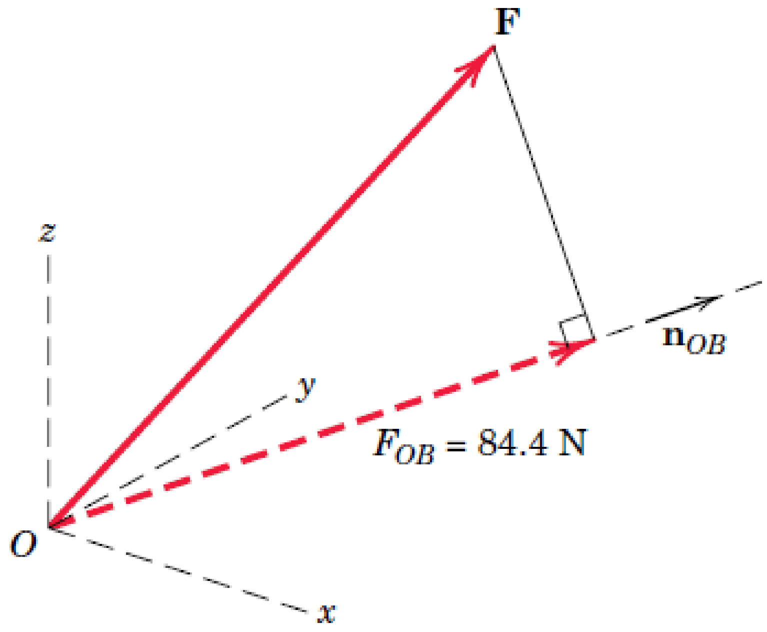
$$F_{xy} = F\cos\theta_{xy} = 100(0.707) = 70.7\ \text{N}$$



$$\mathbf{n}_{OB} = \frac{\overrightarrow{OB}}{\overline{OB}} = \frac{6\mathbf{i} + 6\mathbf{j} + 2\mathbf{k}}{\sqrt{6^2 + 6^2 + 2^2}} = 0.688\mathbf{i} + 0.688\mathbf{j} + 0.229\mathbf{k}$$

$$\begin{aligned} F_{OB} = \mathbf{F}\cdot\mathbf{n}_{OB} &= (42.4\mathbf{i} + 56.6\mathbf{j} + 70.7\mathbf{k})\cdot(0.688\mathbf{i} + 0.688\mathbf{j} + 0.229\mathbf{k}) \\ &= (42.4)(0.688) + (56.6)(0.688) + (70.7)(0.229) \\ &= 84.4 \text{ N} \end{aligned}$$

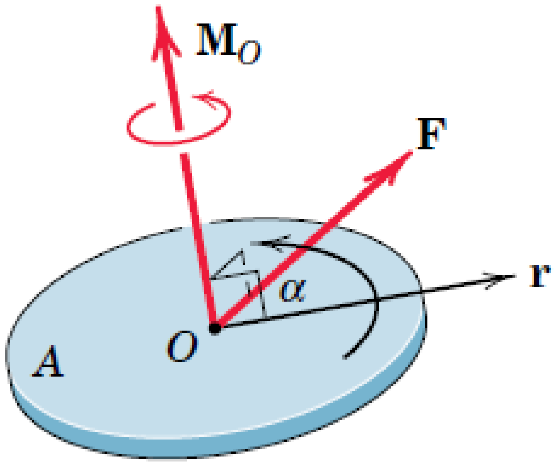
$$\begin{aligned} \mathbf{F}_{OB} &= \mathbf{F}\cdot\mathbf{n}_{OB}\mathbf{n}_{OB} \\ &= 84.4(0.688\mathbf{i} + 0.688\mathbf{j} + 0.229\mathbf{k}) \\ &= 58.1\mathbf{i} + 58.1\mathbf{j} + 19.35\mathbf{k} \text{ N} \end{aligned}$$

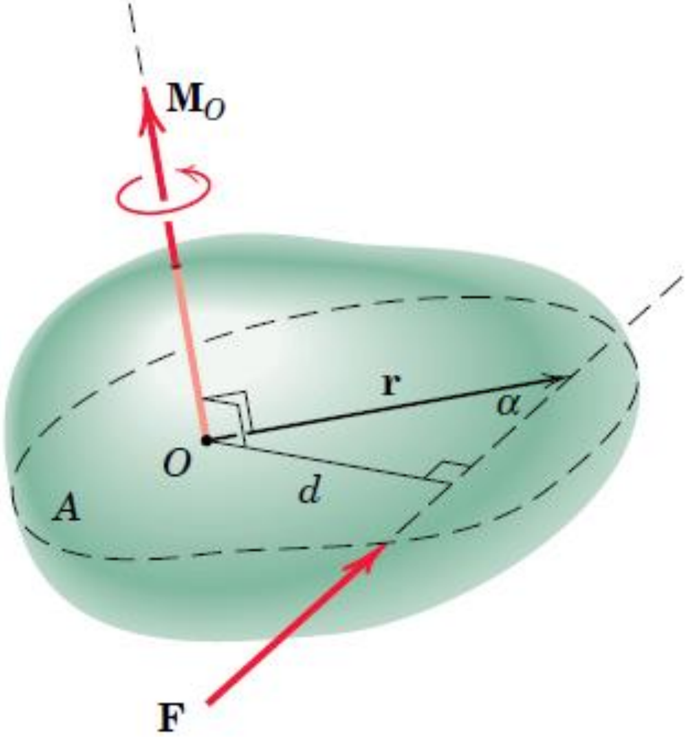


2.8 3D FORCE SYSTEMS: MOMENTS AND COUPLE

- Moments in Three Dimensions

$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F}$$

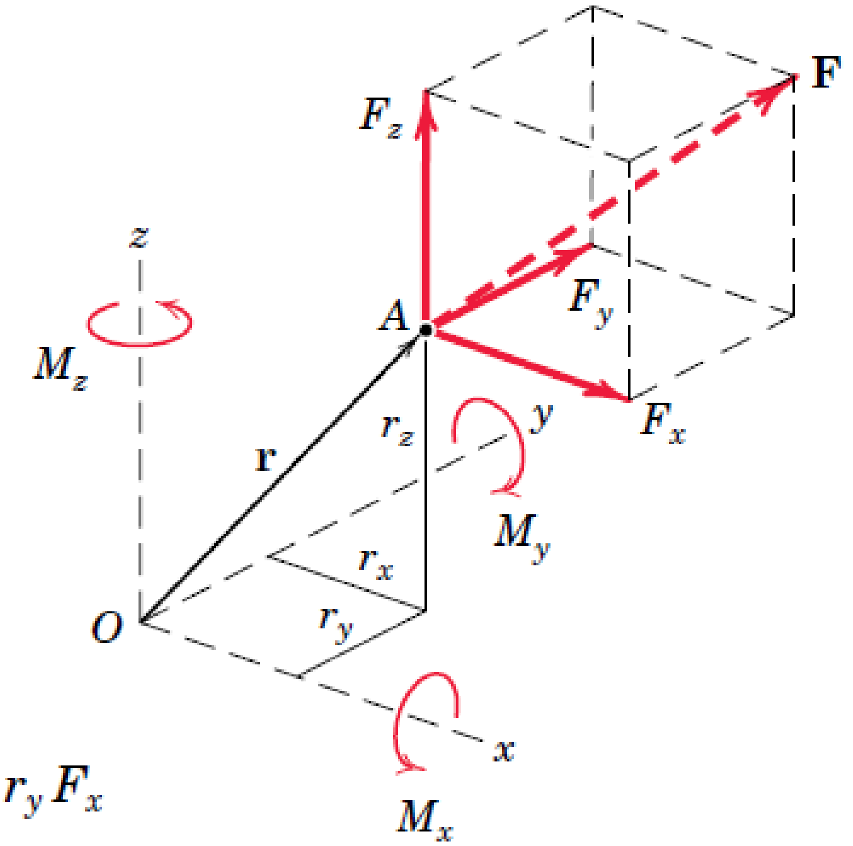




2.8 3D FORCE SYSTEMS: MOMENTS AND COUPLE

- Evaluating the Cross Product

$$\mathbf{M}_O = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$



$$\mathbf{M}_O = (r_y F_z - r_z F_y)\mathbf{i} + (r_z F_x - r_x F_z)\mathbf{j} + (r_x F_y - r_y F_x)\mathbf{k}$$

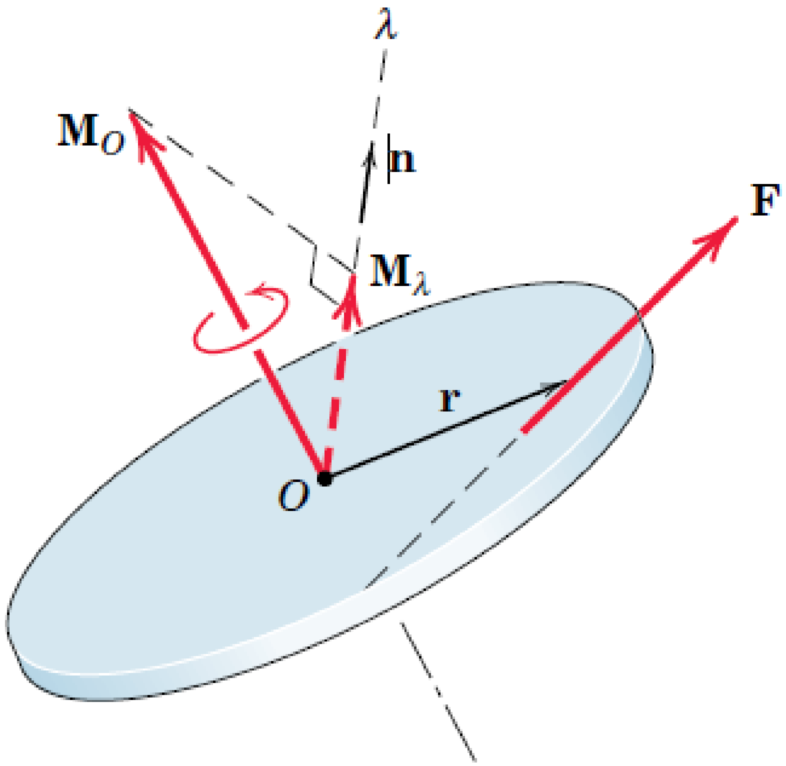
$$M_x = r_y F_z - r_z F_y \qquad M_y = r_z F_x - r_x F_z \qquad M_z = r_x F_y - r_y F_x$$

2.8 3D Force Systems: Moments and Couple

- Moment about an Arbitrary Axis

$$\mathbf{M}_\lambda = (\mathbf{r} \times \mathbf{F} \cdot \mathbf{n})\mathbf{n}$$

$$|\mathbf{M}_\lambda| = M_\lambda = \begin{vmatrix} r_x & r_y & r_z \\ F_x & F_y & F_z \\ \alpha & \beta & \gamma \end{vmatrix}$$

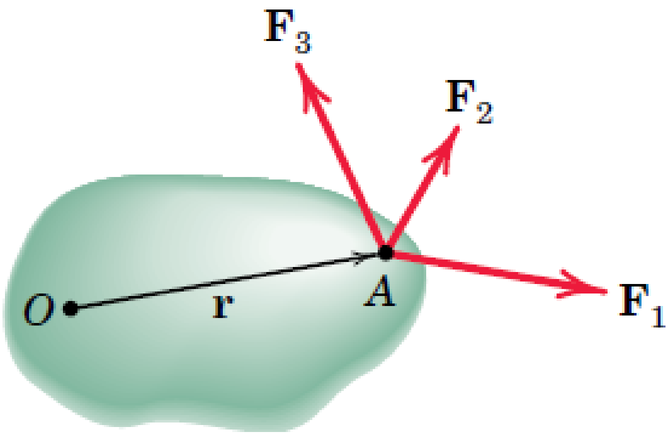


2.8 3D Force Systems: Moments and Couple

- Varignon's Theorem in Three Dimensions:
  - For a system of concurrent forces **F**1, **F**2, **F**3, ... The sum of the moments about *O*

$$\mathbf{r} \times \mathbf{F}_1 + \mathbf{r} \times \mathbf{F}_2 + \mathbf{r} \times \mathbf{F}_3 + \cdots = \mathbf{r} \times (\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \cdots)$$
$$= \mathbf{r} \times \Sigma \mathbf{F}$$

$$\mathbf{M}_O = \Sigma(\mathbf{r} \times \mathbf{F}) = \mathbf{r} \times \mathbf{R}$$

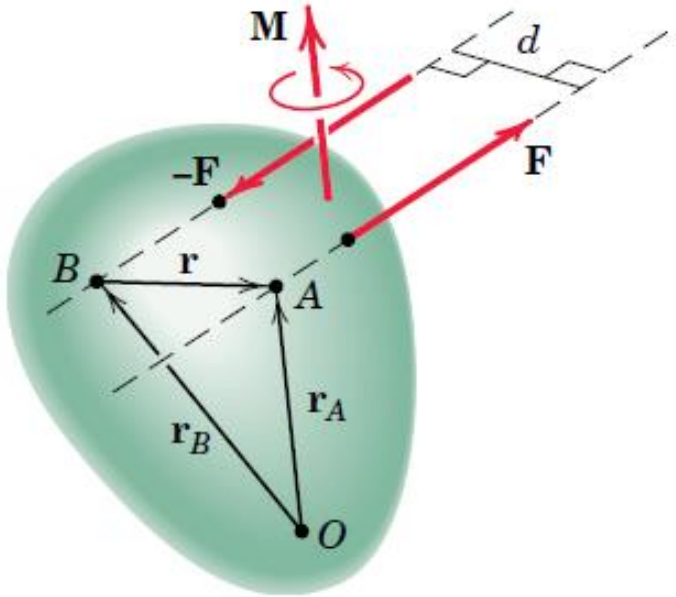


2.8 3D Force Systems: Moments and Couple

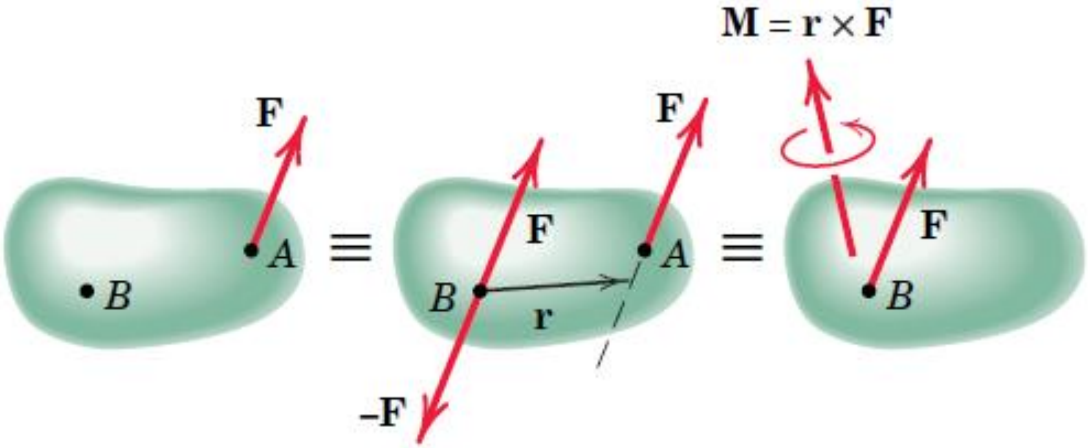
- Couples in Three Dimensions:

$$\mathbf{M} = \mathbf{r}_A \times \mathbf{F} + \mathbf{r}_B \times (-\mathbf{F}) = (\mathbf{r}_A - \mathbf{r}_B) \times \mathbf{F}$$

$$\rightarrow \quad \boxed{\mathbf{M} = \mathbf{r} \times \mathbf{F}}$$

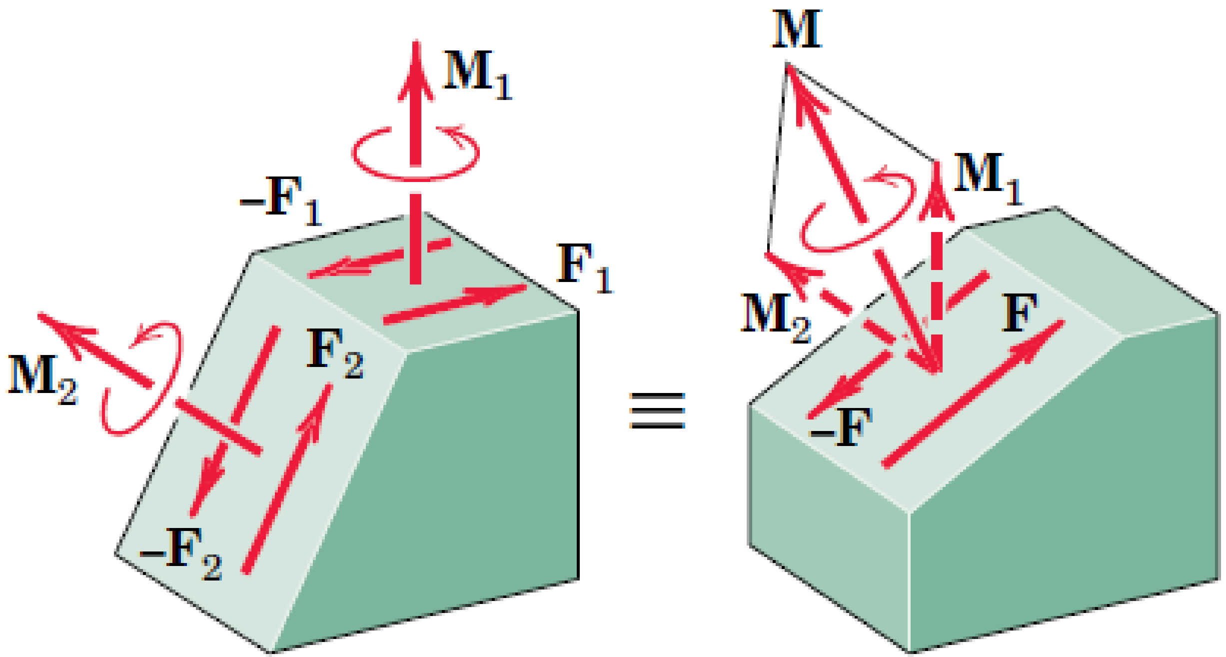


- The moment of a couple is a *free vector*.
- The moment of a force about a point is a *sliding vector*.



2.8 3D Force Systems: Moments and Couple

- Couple vectors obey all of the rules which govern vector quantities.

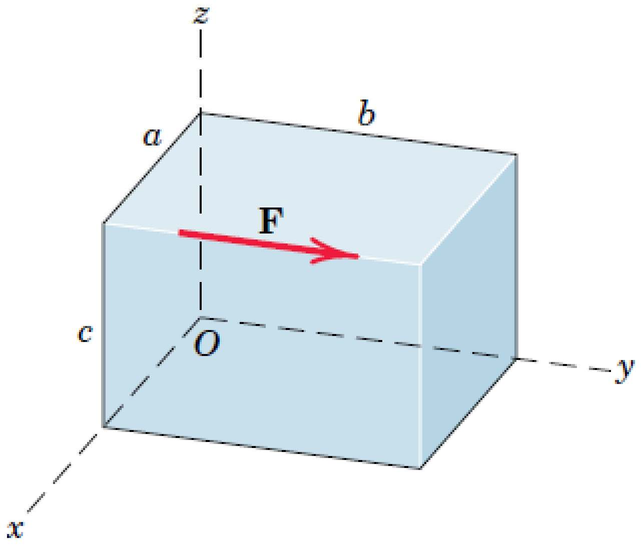


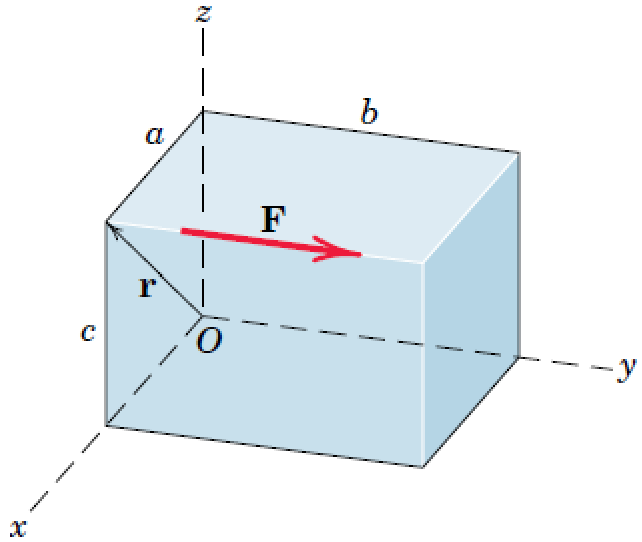
Sample Problem 2/11

Determine the moment of force **F** about point O (a) by inspection and (b) by the formal cross-product definition $\mathbf{M}_O = \mathbf{r} \times \mathbf{F}$.

$$\mathbf{M}_O = -cF\mathbf{i} + aF\mathbf{k} = F(-c\mathbf{i} + a\mathbf{k})$$

$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F} = (a\mathbf{i} + c\mathbf{k}) \times F\mathbf{j} = aF\mathbf{k} - cF\mathbf{i} = F(-c\mathbf{i} + a\mathbf{k})$$





Sample Problem 2/12

The turnbuckle is tightened until the tension in cable *AB* is 2.4 kN. Determine the moment about point *O* of the cable force acting on point *A* and the magnitude of this moment.

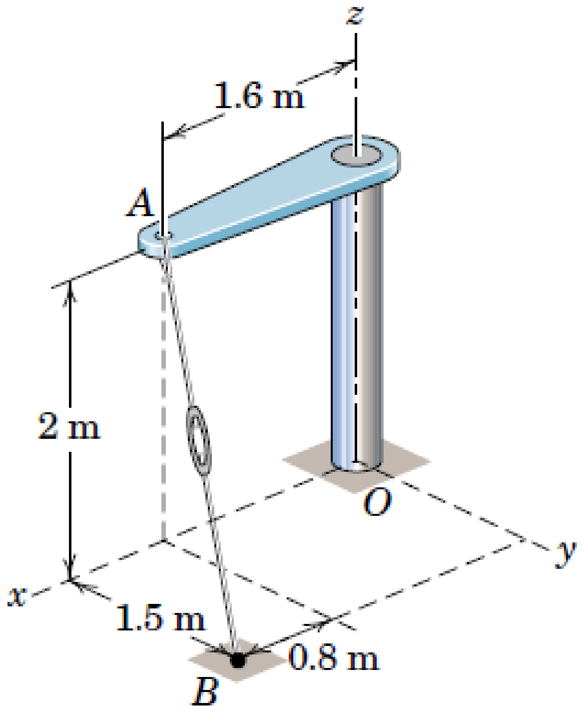
$$\mathbf{T} = T\mathbf{n}_{AB} = 2.4\left[\frac{0.8\mathbf{i} + 1.5\mathbf{j} - 2\mathbf{k}}{\sqrt{0.8^2 + 1.5^2 + 2^2}}\right]$$

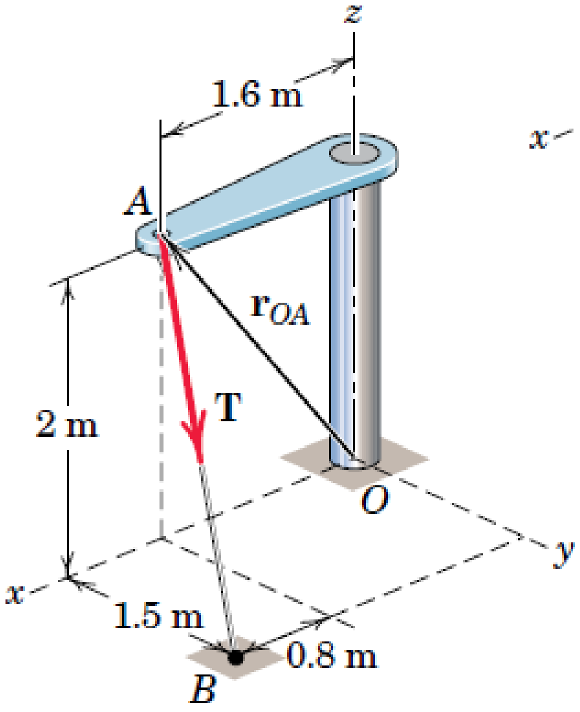
$$= 0.731\mathbf{i} + 1.371\mathbf{j} - 1.829\mathbf{k}\ \text{kN}$$

$$\mathbf{M}_O = \mathbf{r}_{OA} \times \mathbf{T} = (1.6\mathbf{i} + 2\mathbf{k}) \times (0.731\mathbf{i} + 1.371\mathbf{j} - 1.829\mathbf{k})$$

$$= -2.74\mathbf{i} + 4.39\mathbf{j} + 2.19\mathbf{k}\ \text{kN·m}$$

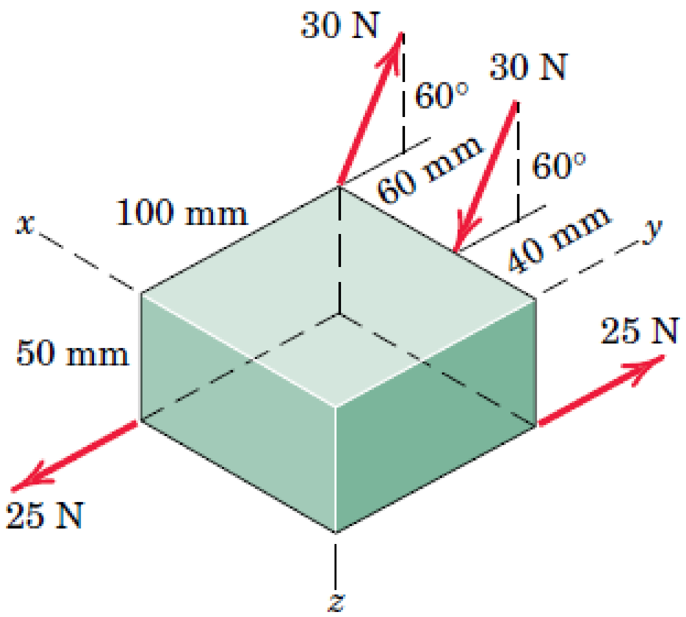
$$M_O = \sqrt{2.74^2 + 4.39^2 + 2.19^2} = 5.62\ \text{kN·m}$$





Sample Problem 2/14

Determine the magnitude and direction of the couple **M** which will replace the two given couples and still produce the same external effect on the block. Specify the two forces **F** and −**F**, applied in the two faces of the block parallel to the *y-z* plane, which may replace the four given forces. The 30-N forces act parallel to the *y-z* plane.



$$M_1 = 30(0.06) = 1.80 \text{ N·m.}$$

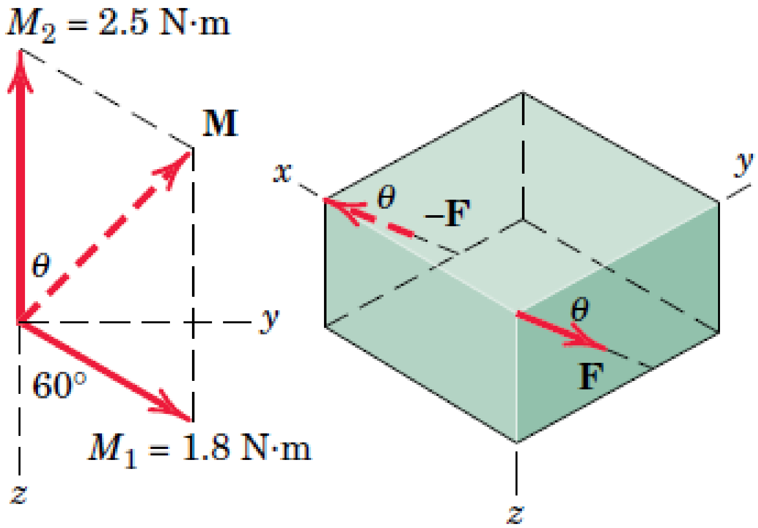
$$M_y = 1.80 \sin 60^\circ = 1.559 \text{ N·m}$$

$$M_z = -2.50 + 1.80 \cos 60^\circ = -1.600 \text{ N·m}$$

$$M = \sqrt{(1.559)^2 + (-1.600)^2} = 2.23 \text{ N·m}$$

$$\theta = \tan^{-1}\frac{1.559}{1.600} = \tan^{-1} 0.974 = 44.3^\circ$$

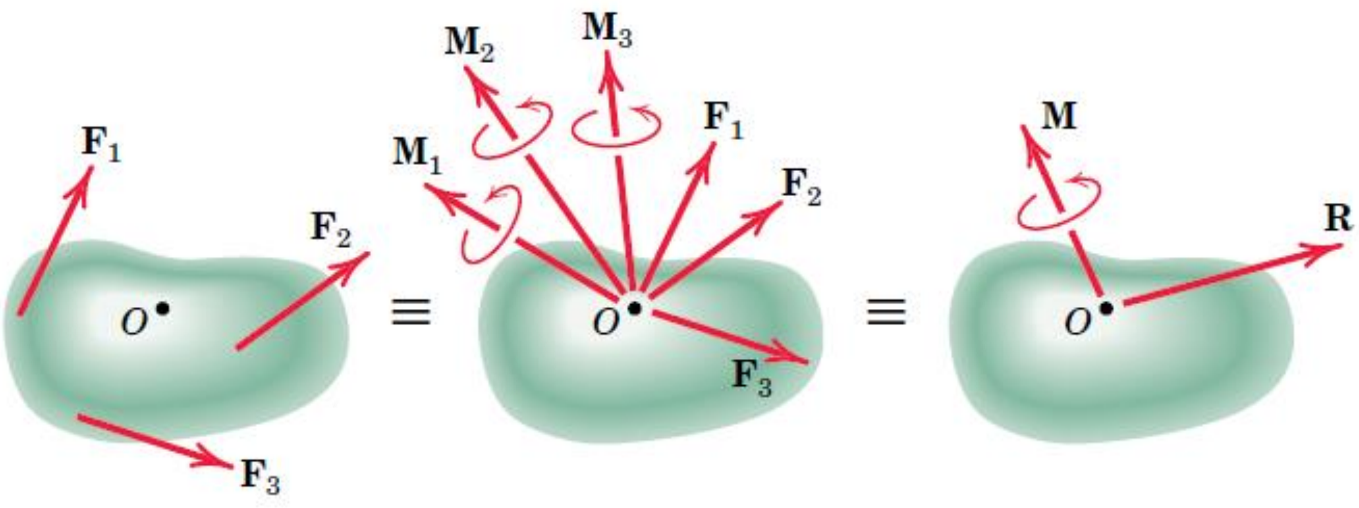
$$[M = Fd] \qquad F = \frac{2.23}{0.10} = 22.3 \text{ N}$$



2.9 3D Force Systems: Resultant

$$\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \cdots = \Sigma\mathbf{F}$$

$$\mathbf{M} = \mathbf{M}_1 + \mathbf{M}_2 + \mathbf{M}_3 + \cdots = \Sigma(\mathbf{r} \times \mathbf{F})$$



$$R_x = \Sigma F_x \qquad R_y = \Sigma F_y \qquad R_z = \Sigma F_z$$

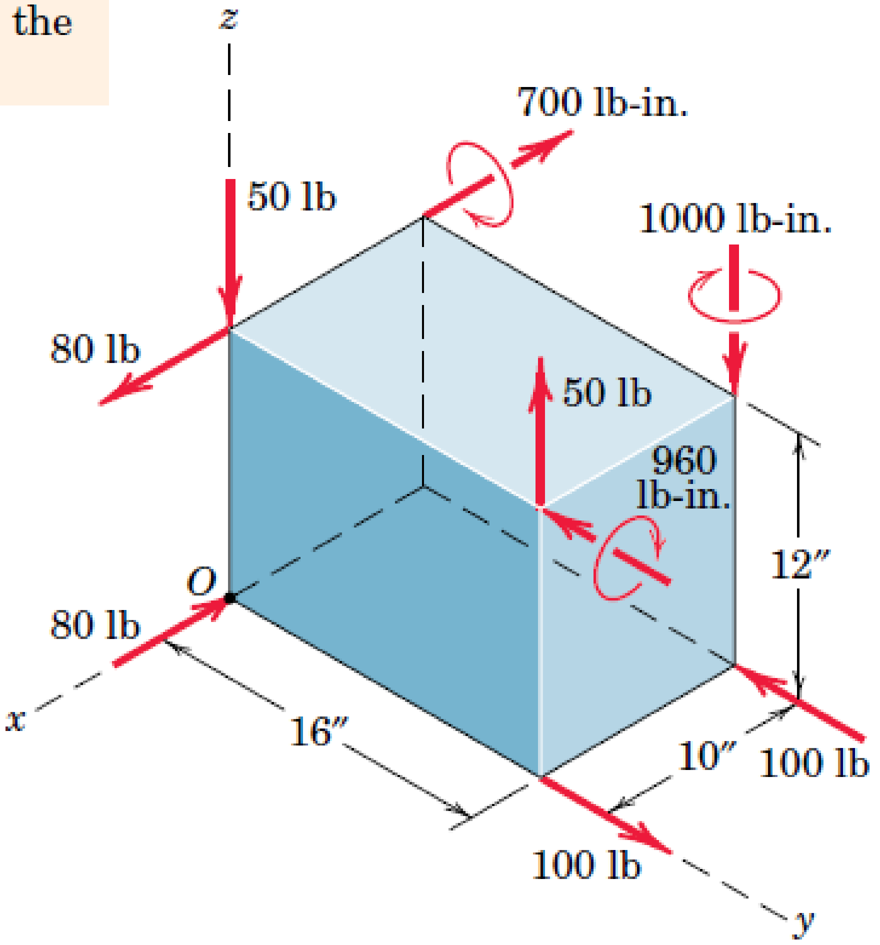
$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2 + (\Sigma F_z)^2}$$

$$\mathbf{M}_x = \Sigma(\mathbf{r} \times \mathbf{F})_x \qquad \mathbf{M}_y = \Sigma(\mathbf{r} \times \mathbf{F})_y \qquad \mathbf{M}_z = \Sigma(\mathbf{r} \times \mathbf{F})_z$$

$$M = \sqrt{M_x^2 + M_y^2 + M_z^2}$$

Sample Problem 2/16

Determine the resultant of the force and couple system which acts on the rectangular solid.

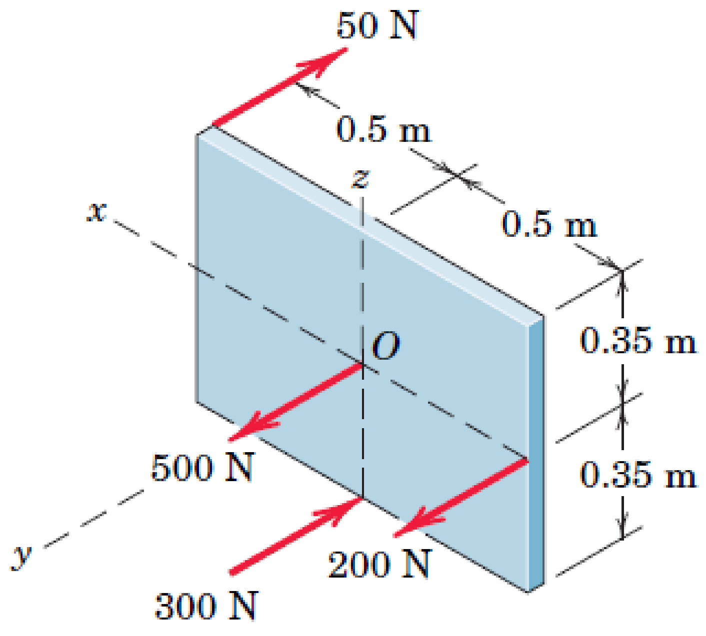


$$\mathbf{R} = \Sigma\mathbf{F} = (80 - 80)\mathbf{i} + (100 - 100)\mathbf{j} + (50 - 50)\mathbf{k} = \mathbf{0} \text{ lb}$$

$$\mathbf{M}_O = [50(16) - 700]\mathbf{i} + [80(12) - 960]\mathbf{j} + [100(10) - 1000]\mathbf{k} \text{ lb-in.}$$
$$= 100\mathbf{i} \text{ lb-in.}$$

Sample Problem 2/17

Determine the resultant of the system of parallel forces which act on the plate. Solve with a vector approach.



$$\mathbf{R} = \Sigma\mathbf{F} = (200 + 500 - 300 - 50)\mathbf{j} = 350\mathbf{j}\ \text{N}$$

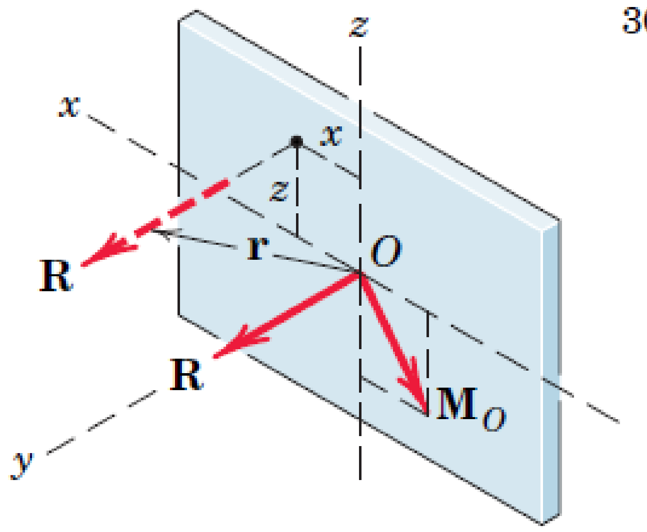
$$\mathbf{M}_O = [50(0.35) - 300(0.35)]\mathbf{i} + [-50(0.50) - 200(0.50)]\mathbf{k}$$

$$= -87.5\mathbf{i} - 125\mathbf{k}\ \text{N·m}$$

$$\mathbf{r} \times \mathbf{R} = \mathbf{M}_O$$

$$(x\mathbf{i} + y\mathbf{j} + z\mathbf{k}) \times 350\mathbf{j} = -87.5\mathbf{i} - 125\mathbf{k}$$

$$350x\mathbf{k} - 350z\mathbf{i} = -87.5\mathbf{i} - 125\mathbf{k}$$



$x = -0.357$ m and $z = 0.250$ m

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