دانشگاه سمنان
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درس استاتیک

STATICS

Chapter 1 - Introduction to Statics
Class Lecture

❑ CONTENTS:

1.1 MECHANICS

- Mechanics is the physical science which deals with the effects of forces on objects

- The principles of mechanics are central to research and development in many fields…

- The subject of mechanics is logically divided into two parts:
  - ***Statics***, which concerns the equilibrium of bodies under action of forces
  - ***Dynamics***, which concerns the motion of bodies

1.2 BASIC CONCEPTS

- ***Space*** is the geometric region occupied by bodies whose positions are described by linear and angular measurements relative to a coordinate system.

- ***Time*** is the measure of the succession of events and is a basic quantity in dynamics.
  - Time is not directly involved in the analysis of statics problems.

1.2 BASIC CONCEPTS

- ***Mass*** is a measure of the inertia of a body, which is its resistance to a change of velocity.

- ***Force*** is the action of one body on another.
  - A force tends to move a body in the direction of its action.
  - The action of a force is characterized by its *magnitude*, by the *direction* of its action, and by its *point of application*.
  - Thus force is a vector quantity.

1.2 BASIC CONCEPTS

- A ***particle*** is a body of negligible dimensions. In the mathematical sense, a particle is a body whose dimensions are considered to be near zero.

- ***Rigid body.*** A body is considered rigid when the change in distance between any two of its points is negligible for the purpose at hand.

1.3 SCALARS AND VECTORS

- ***Scalar quantities***: only a magnitude is associated.
  - Examples: time, volume, density, speed, energy, and mass.

- ***Vector quantities***: possess direction as well as magnitude
  - Obey the parallelogram law of addition.
  - Examples: displacement, velocity, acceleration, force, moment, momentum

  - Vectors representing physical quantities can be classified as:
    - ✓ Free
    - ✓ Sliding
    - ✓ Fixed

1.3 SCALARS AND VECTORS

- A ***free vector*** is one whose action is not confined to or associated with a unique line in space.
  - For example, displacement of any point in the body that moves without rotation.

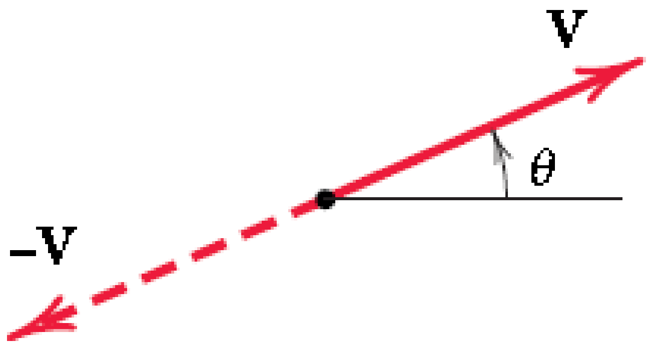
- A ***sliding vector*** has a unique line of action in space but not a unique point of application.
  - For example, external force acts on a rigid body, can be applied at any point along its line of action

- A ***fixed vector*** is one for which a unique point of application is specified.
  - The action of a force on a deformable or nonrigid body must be specified by a fixed vector at the point of application of the force.

1.3 SCALARS AND VECTORS

- Conventions for <u>Equations</u> and <u>Diagrams</u>

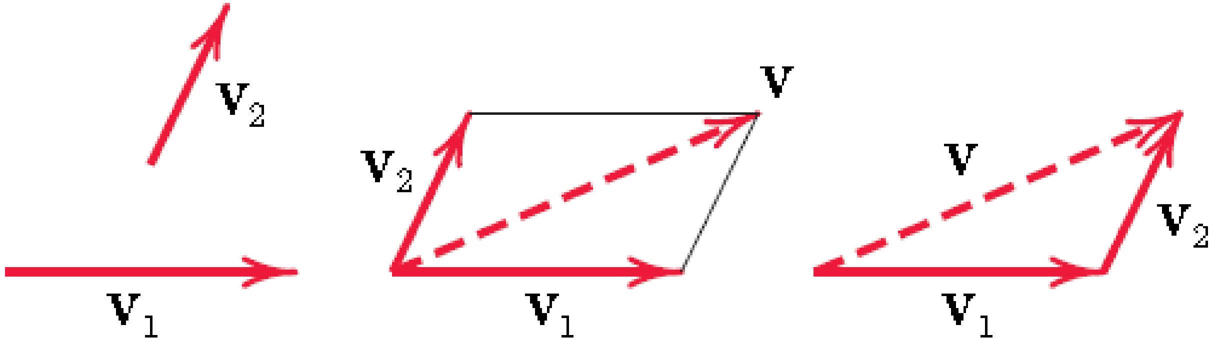
- A vector quantity **V** is represented by a line segment
  - Direction of the vector
  - Magnitude of the vector |**V**|



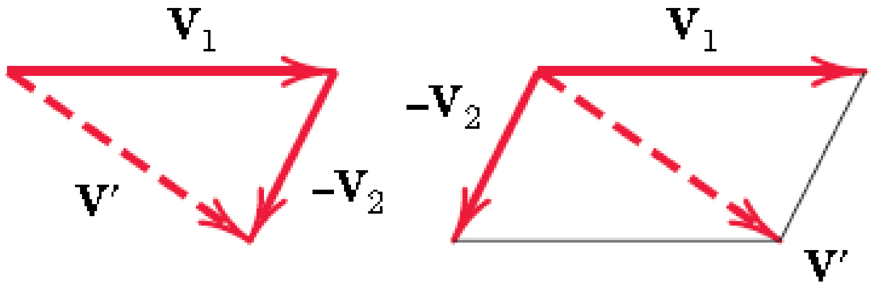
1.3 SCALARS AND VECTORS

- Vector Summation:

$$\mathbf{V} = \mathbf{V}_1 + \mathbf{V}_2$$

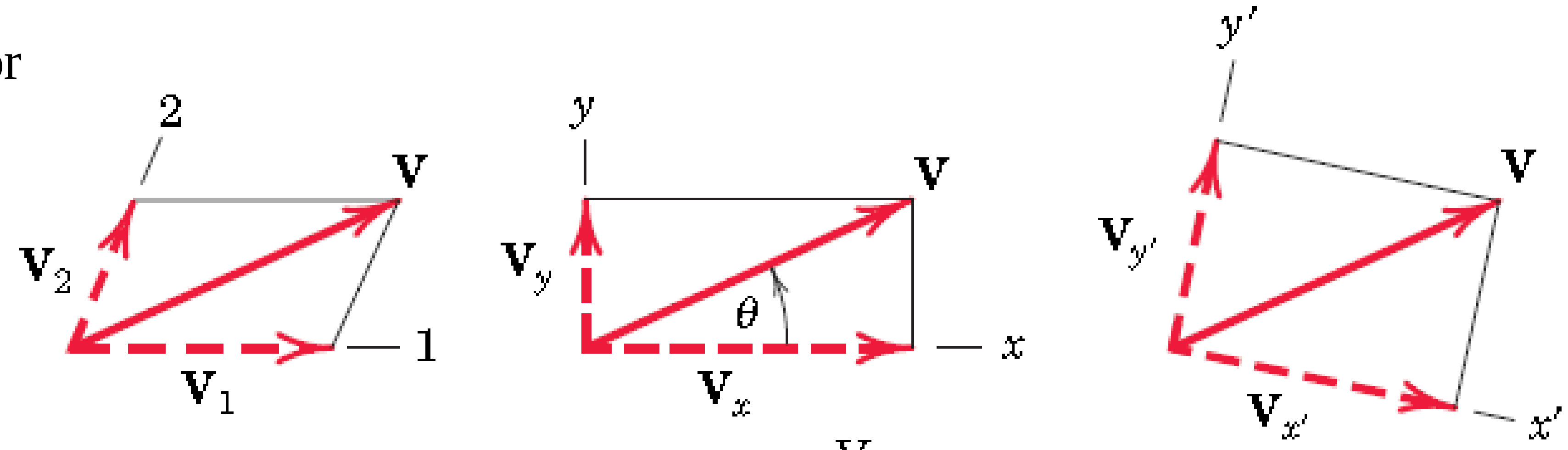


$$\mathbf{V}' = \mathbf{V}_1 - \mathbf{V}_2$$



1.3 SCALARS AND VECTORS

- *Components* of vector



$$\theta = \tan^{-1}\frac{V_y}{V_x}$$

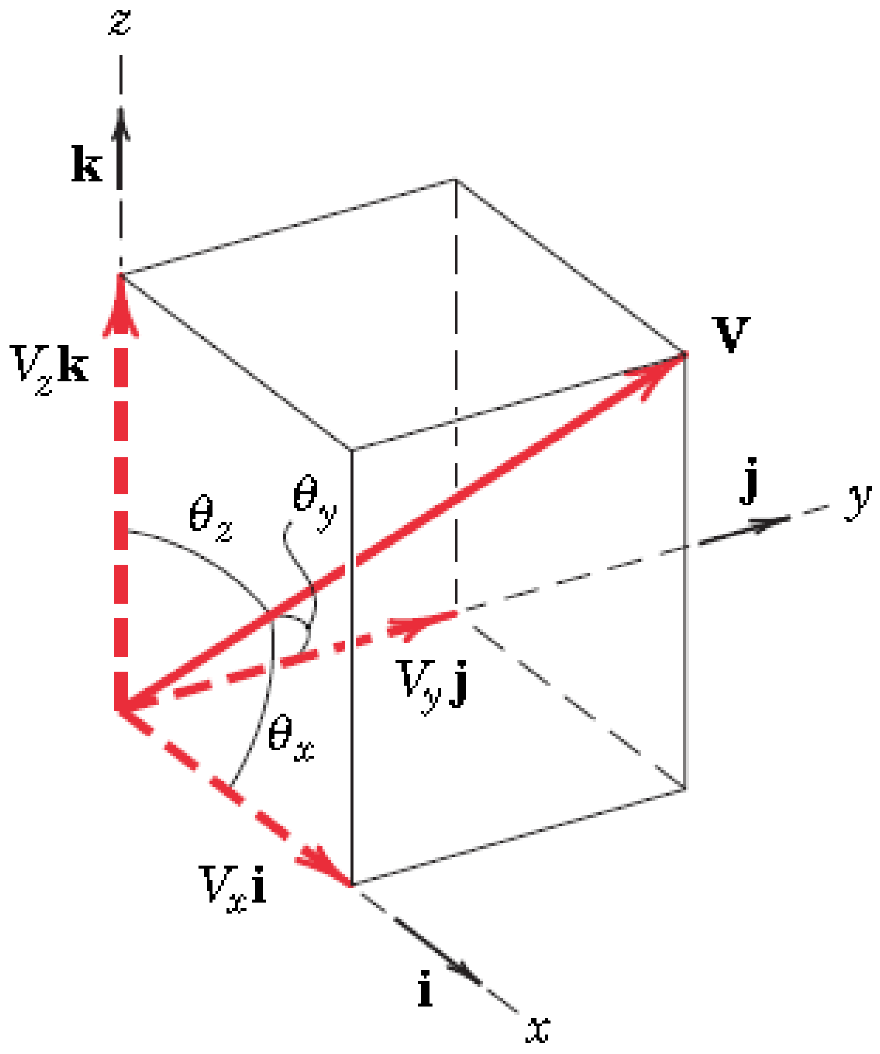
- Unit vector **n**:

$$\mathbf{V} = V\mathbf{n}$$

1.3 SCALARS AND VECTORS

- Three-dimensional vectors:
  - Unit vectors **i** , **j** , and **k** , which are vectors in the *x*-, *y*-, and *z*-directions

$$\mathbf{V} = V_x\mathbf{i} + V_y\mathbf{j} + V_z\mathbf{k}$$



1.3 SCALARS AND VECTORS

- Direction cosines l, m, and n of **V**

$$l = \cos\theta_x \qquad m = \cos\theta_y \qquad n = \cos\theta_z$$

$$V_x = lV \qquad V_y = mV \qquad V_z = nV$$

- Pythagorean theorem

$$V^2 = {V_x}^2 + {V_y}^2 + {V_z}^2 \qquad l^2 + m^2 + n^2 = 1$$

1.4 NEWTON'S LAWS

- **Law I**. A particle remains at rest or continues to move with uniform velocity (in a straight line with a constant speed) if there is no unbalanced force acting on it.

  - Newton's first law contains the principle of the equilibrium of forces, which is the main topic of concern in statics

1.4 NEWTON’S LAWS

- **Law II**. The acceleration of a particle is proportional to the vector sum of forces acting on it and is in the direction of this vector sum.

$$\mathbf{F} = m\mathbf{a}$$

- **Law III**. The forces of action and reaction between interacting bodies are equal in magnitude, opposite in direction, and collinear (they lie on the same line)

1.5 UNITS

- In mechanics we use four fundamental quantities called dimensions.
  - These are length, mass, force, and time.
  - The units used to measure these quantities cannot all be chosen independently

QUANTITY	DIMENSIONAL SYMBOL	SI UNITS			U.S. CUSTOMARY UNITS		
			UNIT	SYMBOL		UNIT	SYMBOL
Mass	M	Base units	kilogram	kg		slug	—
Length	L	Base units	meter	m	Base units	foot	ft
Time	T	Base units	second	s	Base units	second	sec
Force	F		newton	N	Base units	pound	lb

1.5 UNITS

- SI Units The International System of Units, abbreviated SI

$$\text{force (N)} = \text{mass (kg)} \times \text{acceleration (m/s}^2) \qquad \text{N} = \text{kg}\cdot\text{m/s}^2$$

- U.S. Customary Units The U.S. customary, or British system of units, also called the foot pound-second (FPS) system

$$\text{force (lb)} = \text{mass (slugs)} \times \text{acceleration (ft/sec}^2), \qquad \text{slug} = \frac{\text{lb-sec}^2}{\text{ft}}$$

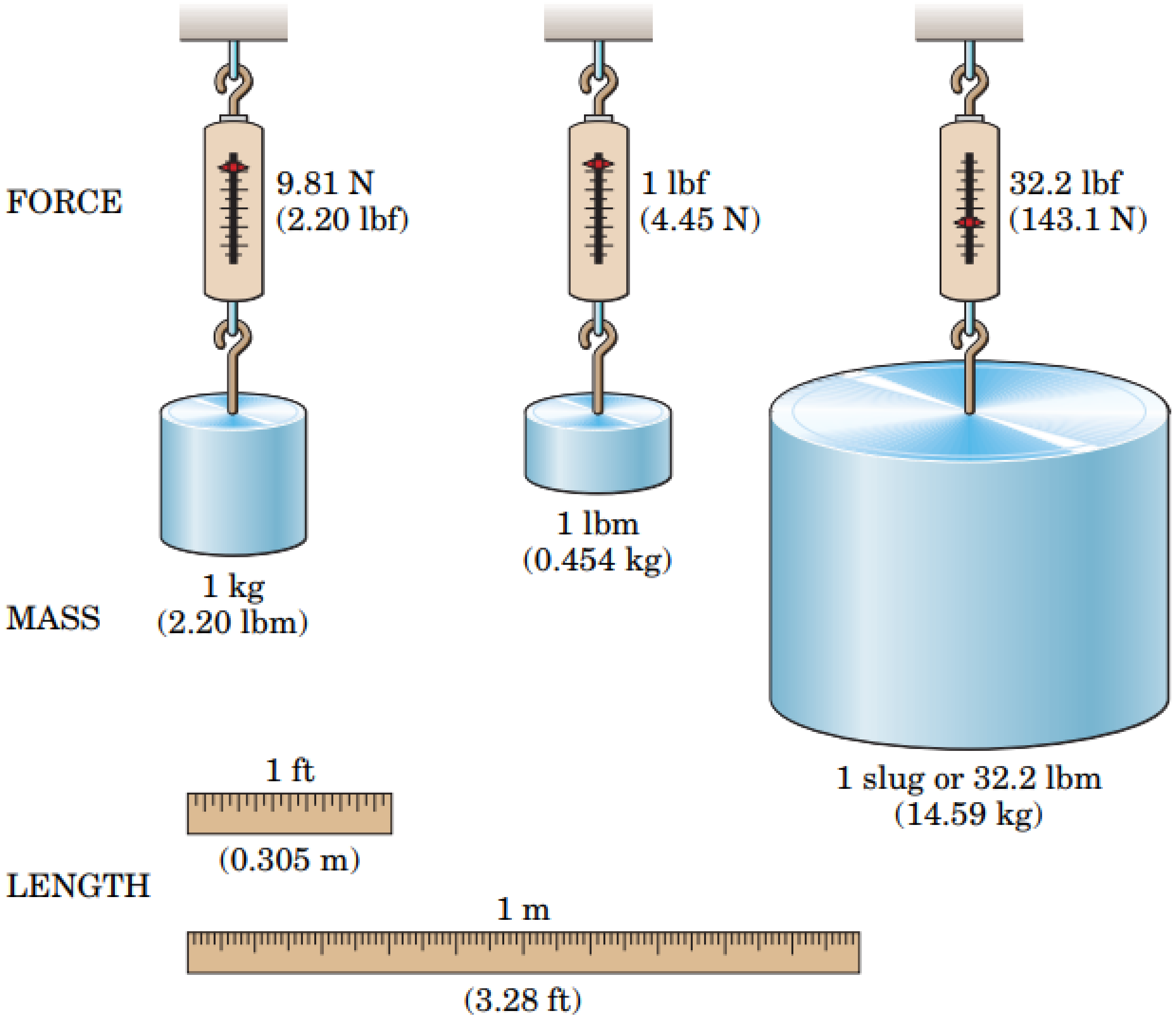
- Standard value for gravitational acceleration g:

SI units $g = 9.806\,65 \text{ m/s}^2$

U.S. units $g = 32.1740 \text{ ft/sec}^2$

1.5 UNITS

- Unit Conversions

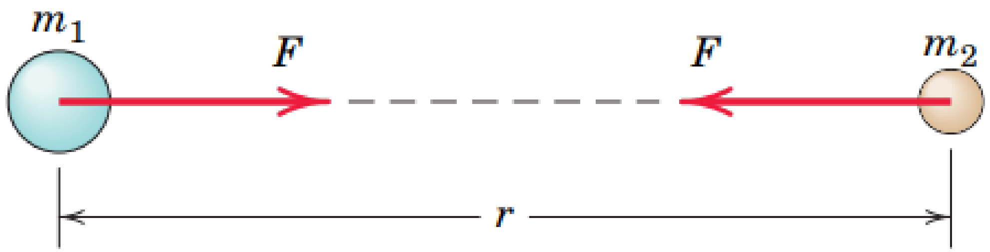


1.6 LAW OF GRAVITATION

- To compute the weight of a body: the gravitational force acting on it

- Law of gravitation:

$$F = G\frac{m_1 m_2}{r^2}$$



where F = the mutual force of attraction between two particles

G = a universal constant known as the *constant of gravitation* $G = 6.673(10^{-11})\ \mathrm{m^3/(kg\cdot s^2)}$

m_1, m_2 = the masses of the two particles

r = the distance between the centers of the particles

- Gravitational Attraction of the Earth:

$$W = mg$$

Appendix C - Trigonometry

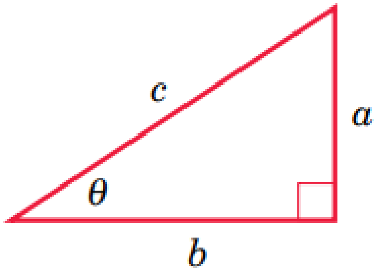
C/6 Trigonometry

1. Definitions

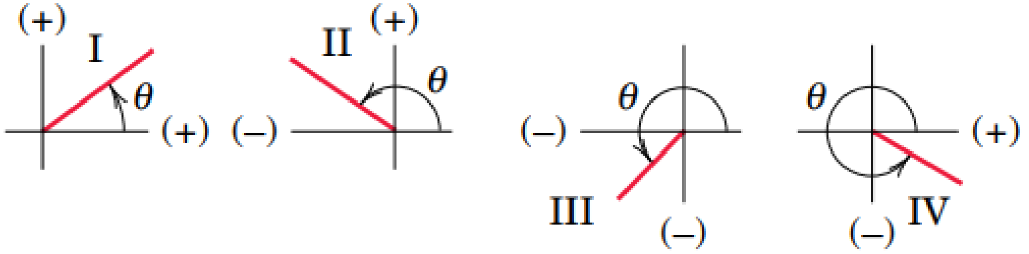
$\sin\theta = a/c \qquad \csc\theta = c/a$

$\cos\theta = b/c \qquad \sec\theta = c/b$

$\tan\theta = a/b \qquad \cot\theta = b/a$



2. Signs in the four quadrants



	I	II	III	IV
$\sin\theta$	+	+	–	–
$\cos\theta$	+	–	–	+
$\tan\theta$	+	–	+	–
$\csc\theta$	+	+	–	–
$\sec\theta$	+	–	–	+
$\cot\theta$	+	–	+	–

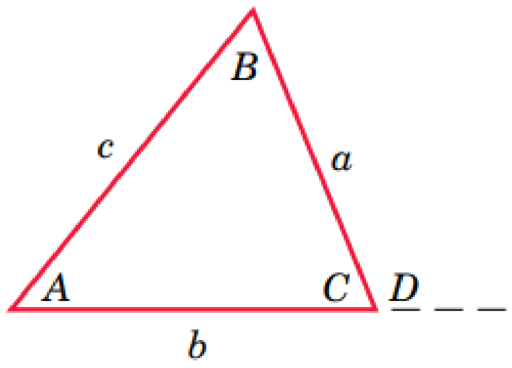
Appendix C - Trigonometry

3. Miscellaneous relations

$$\sin^2\theta + \cos^2\theta = 1$$
$$1 + \tan^2\theta = \sec^2\theta$$
$$1 + \cot^2\theta = \csc^2\theta$$
$$\sin\frac{\theta}{2} = \sqrt{\frac{1}{2}(1 - \cos\theta)}$$
$$\cos\frac{\theta}{2} = \sqrt{\frac{1}{2}(1 - \cos\theta)}$$
$$\sin 2\theta = 2\sin\theta\cos\theta$$
$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$
$$\sin(a \pm b) = \sin a\cos b \pm \cos a\sin b$$
$$\cos(a \pm b) = \cos a\cos b \mp \sin a\sin b$$

4. Law of sines

$$\frac{a}{b} = \frac{\sin A}{\sin B}$$

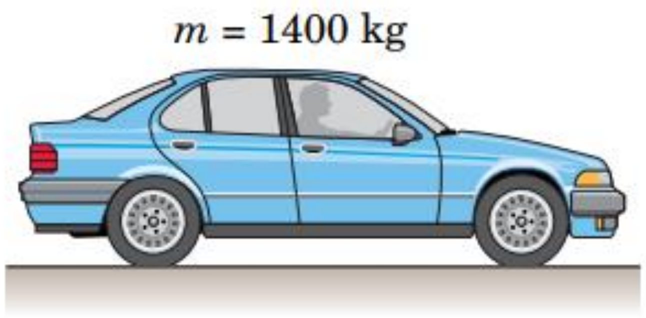


5. Law of cosines

$$c^2 = a^2 + b^2 - 2ab\cos C$$
$$c^2 = a^2 + b^2 + 2ab\cos D$$

Sample Problem 1/1

Determine the weight in newtons of a car whose mass is 1400 kg. Convert the mass of the car to slugs and then determine its weight in pounds.



$$W = mg = 1400(9.81) = 13\ 730\ \text{N}$$

$$m = 1400\ \text{kg}\left[\frac{1\ \text{slug}}{14.594\ \text{kg}}\right] = 95.9\ \text{slugs}$$

$$W = mg = (95.9)(32.2) = 3090\ \text{lb}$$

$$m = 1400\ \text{kg}\left[\frac{1\ \text{lbm}}{0.45359\ \text{kg}}\right] = 3090\ \text{lbm}$$

Sample Problem 1/3

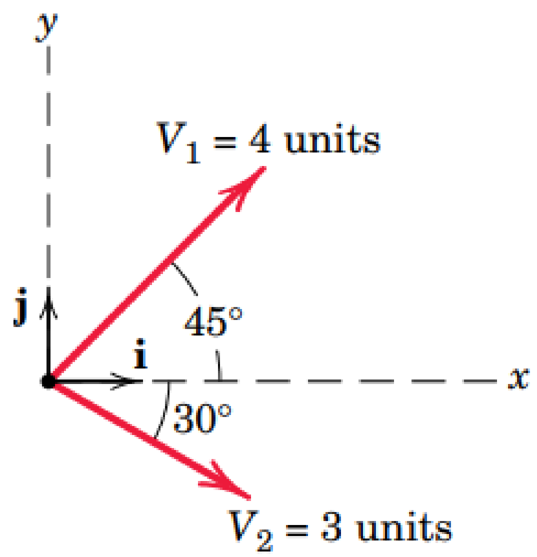
For the vectors $\mathbf{V}_1$ and $\mathbf{V}_2$ shown in the figure,

(*a*) determine the magnitude S of their vector sum $\mathbf{S} = \mathbf{V}_1 + \mathbf{V}_2$

(*b*) determine the angle α between $\mathbf{S}$ and the positive x-axis

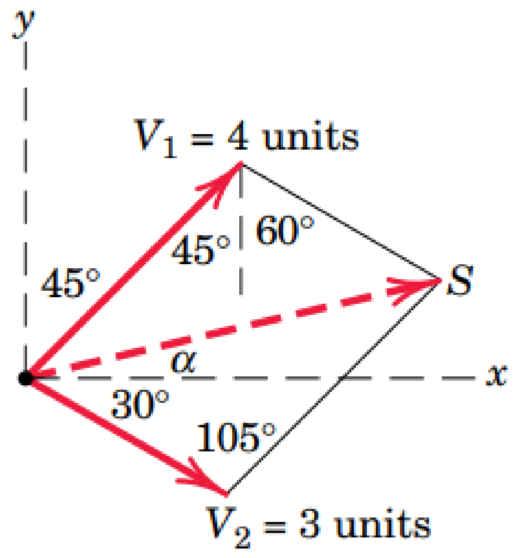
(*c*) write $\mathbf{S}$ as a vector in terms of the unit vectors $\mathbf{i}$ and $\mathbf{j}$ and then write a unit vector $\mathbf{n}$ along the vector sum $\mathbf{S}$

(*d*) determine the vector difference $\mathbf{D} = \mathbf{V}_1 - \mathbf{V}_2$



$$S^2 = 3^2 + 4^2 - 2(3)(4)\cos 105°$$

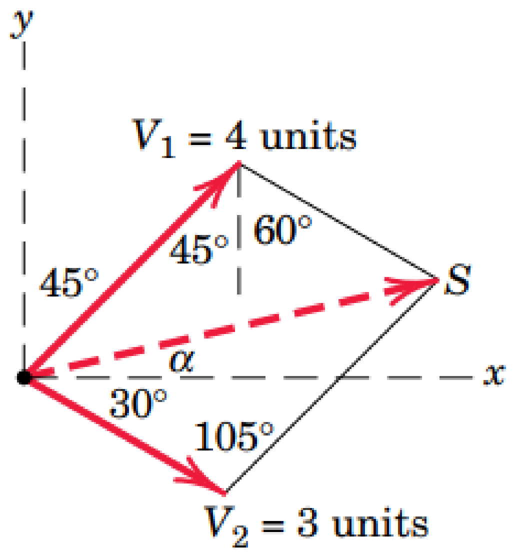
$$S = 5.59 \text{ units}$$



$$\frac{\sin 105°}{5.59} = \frac{\sin(\alpha + 30°)}{4}$$

$$\sin(\alpha + 30°) = 0.692$$

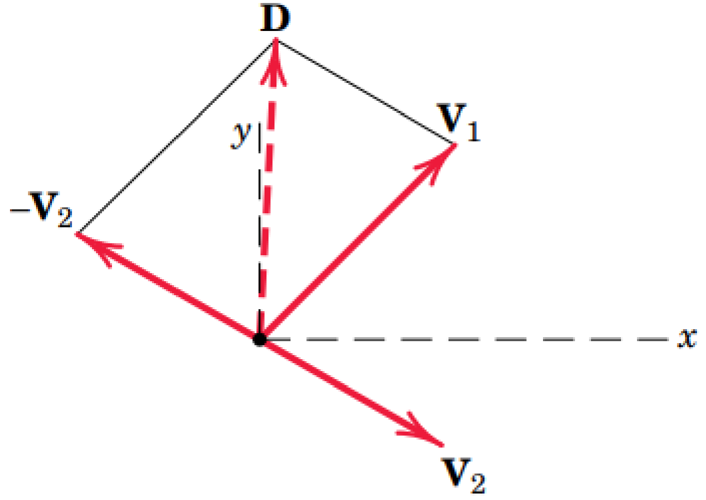
$$(\alpha + 30°) = 43.8° \qquad \alpha = 13.76°$$



$$\mathbf{S} = S[\mathbf{i}\cos\alpha + \mathbf{j}\sin\alpha]$$

$$= 5.59[\mathbf{i}\cos 13.76° + \mathbf{j}\sin 13.76°] = 5.43\mathbf{i} + 1.328\mathbf{j} \text{ units}$$

$$\mathbf{n} = \frac{\mathbf{S}}{S} = \frac{5.43\mathbf{i} + 1.328\mathbf{j}}{5.59} = 0.971\mathbf{i} + 0.238\mathbf{j}$$



$$\mathbf{D} = \mathbf{V}_1 - \mathbf{V}_2 = 4(\mathbf{i}\cos 45° + \mathbf{j}\sin 45°) - 3(\mathbf{i}\cos 30° - \mathbf{j}\sin 30°)$$

$$= 0.230\mathbf{i} + 4.33\mathbf{j} \text{ units}$$